# Ground-State Doublet of $P^{32}$†

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Angular distributions obtained for each member of the ground-state doublet of $P^{32}$ in the reaction $P^{31}(d,p)P^{32}$ indicate that for both levels the neutron is captured with $l_n=2$, and that the $s$-wave admixture for the ground state is about 5%. Estimates of the purity of the $P^{31}$ ground-state wave function are made on the basis of the available experimental data.

## I. INTRODUCTION

THE reaction $P^{31}(d,p)P^{32}$ was suggested by Bethe and Butler[1] as one which would be a good test of the validity of the shell model, since according to the shell model the target nucleus should accept a neutron with two units of orbital angular momentum only, while conservation of angular momentum allows both zero and two units. Previous measurements[2] of the unresolved ground-state doublet indicated that the neutron was captured primarily with $l_n=2$ as predicted by the shell model, the $s$-wave admixture being 5% or less. Angular distributions and relative intensities have since been obtained for each member of the doublet and it is now possible to make more definite statements about the wave functions of the nuclear states involved.

## II. EXPERIMENTAL

The measurements were made using the magnetic analysis instrumentation associated with the 7.8-Mev deuteron beam of the 42-in. Michigan cyclotron.[3] Targets were prepared by evaporating $Li_3PO_4$ on a gold-leaf backing. The protons from the $(d,p)$ reaction were detected in 1×3-in. Kodak NTB-100$\mu$ plates. Typical spectra, obtained at scattering angles of 30° and 10°, and reproduced in Fig. 1, show the proton groups corresponding to the ground state ($Q_0$) and the 77-kev[4] first-excited state ($Q_1$). The abscissa is the distance in millimeters along the image plane of the analyzer and the ordinate is the relative number of proton tracks observed in a single (0.5-mm wide) scan across the 1-in. dimension of the plate.

In obtaining the angular distributions the intensity at each scattering angle was compared with that at 30°. To minimize variations in the instrumentation and in the target, each set of data was taken in cyclic fashion, one such sequence being 30°, 10°, 15°, 30°. At least two such sets were taken at each angle. The resulting angular distributions in the center-of-mass system are shown in Figs. 2 and 3. The standard deviations associated with the experimental points were obtained from the deviations from the mean of the individual sets of data, the largest contribution to the deviation being the variation in the plate scanners.[5] The variation was minimized by limiting the number of tracks in a single scan to the order of 200 and by assigning complete sets of data to one reader.

Target contamination was a serious problem at small angles. A very weak and unidentified proton group, believed due either to $K^{40}$ or $O^{18}$ peaked near zero degrees, and a second group, due to $C^{13}$, resulting from the buildup of carbon on the target during bombardment, moved into coincidence with $Q_1$ at small angles. The contribution of these groups to the total cross section accounts for the larger uncertainty in the data in this region.

The relative intensities of $Q_0$ and $Q_1$ at the peak of the angular distributions (30°) were determined from the mean of 13 separate measurements and found to be in the ratio of $Q_1/Q_0=1.45\pm0.03$.

## III. DISCUSSION

Other things being equal, the relative intensities of the ground- and first-excited states at the peak of the distributions are expected to be in the ratio of their statistical factors $(2J_f+1)$. Since the spin and parity assignment of the ground state of $P^{32}$ is $1+$,[6] the ratio $Q_1/Q_0=1.45$ suggests that the spin and parity assignment of the first-excited state is $2+$. The measured angular distributions are consistent with these spin and parity assignments; the ground-state distribution indicates an admixture of $l=0$ and $l=2$, while the first-excited state can be interpreted as an $l=2$ capture only. Since the spin and parity of $P^{31}$ are $\frac{1}{2}+$, both $s$ and $d$ waves can contribute to the $1+$ level but conservation of angular momentum prohibits direct $s$-wave capture to the $2+$ level.

The differential cross section at the peak of the

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[1] H. A. Bethe and S. T. Butler, Phys. Rev. **85**, 1045 (1952).

[2] Parkinson, Beach, and King, Phys. Rev. **87**, 387 (1952); C. F. Black, Phys. Rev. **90**, 381(A) (1953); I. B. Teplov, Zhur. Eksptl. i Teort. Fiz. **31**, 25 (1956) [translation: Soviet Phys. JETP **4**, 31 (1957)]; Dalton, Hinds, and Parry, Proc. Phys. Soc. (London) **A70**, 586 (1957).

[3] Bach, Childs, Hockney, Hough, and Parkinson, Rev. Sci. Instr. **27**, 516 (1956).

[4] Van Patter, Endt, Sperduto, and Buechner, Phys. Rev. **86**, 502 (1952).

[5] The problem of plate reading is a serious one. The variation between experienced readers may be as large as 10%, particularly when the number of tracks per scan is large. The fractional counting loss of a single reader is roughly proportional to the number of tracks in a single scan but varies from day to day.

[6] Feher, Fuller, and Gere, Phys. Rev. **107**, 1462 (1957).

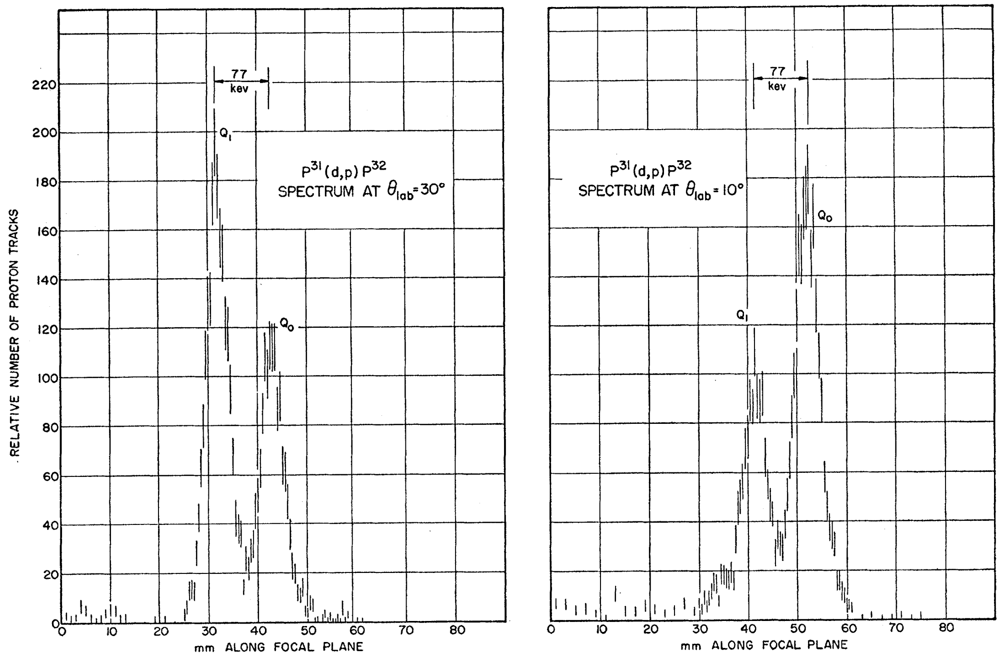


FIG. 1. The proton groups corresponding to the ground state ($Q_0$) and the first excited state ($Q_1$) in the reaction $P^{31}(d,p)P^{32}$. The two spectra have not been normalized to the same monitor count.

angular distributions for single-particle $l=0$ and $l=2$ transitions for these two levels in $P^{32}$ should, according to the Butler theory, be in the ratio of 20:1, the statistical factor $(2J_f+1)$ having been removed. It is well known, however, that the Butler theory does not give a reliable estimate of single-particle cross sections. The measured ratio[7] of the $l=0$ and $l=2$ single-particle transitions in $O^{17}$ is 29; the calculated ratio is 20. The measured ratio[8] of $l=1$ and $l=3$ single-particle transitions in $Ca^{41}$ is 13; the calculated ratio is 9. In both cases the calculated ratios are lower than the experimentally determined values. In using the calculated ratio to estimate the admixture of $l=0$ in $P^{32}$, an error as large as 40% on the high side might reasonably be expected. When one uses the calculated ratio $d\sigma_0/d\sigma_2=20$ for extreme single-particle wave functions, the percent of $l=0$ required to fit the measured angular distribution of Fig. 2 is approximately 5%. The dashed curve in Fig. 2 is computed by combining 5% of the theoretical curve for $l=0$ and 95% of the "modified" $l=2$ curve and renormalizing to unity at 30°. The modification of the $l=2$ curve takes into account the nonzero character of an actual distribution at small angles and was assumed to be that shown by the dotted curve in Fig. 2. A much better fit to the experimental points at small angles can be obtained if a radius of $7.5\times10^{-13}$ cm is used for the $l=0$ curve.

According to the shell model, the zero-order configuration in the wave function of $P^{31}$ is $(s_{\frac{1}{2}}^2)_n(s_{\frac{1}{2}}^1)_p$ outside the doubly-closed $d_{\frac{5}{2}}$ subshell, where $n$ and $p$ represent the neutron and proton configurations, respectively, and for $P^{32}$ it is $(s_{\frac{1}{2}}^2 d_{\frac{3}{2}}^1)_n(s_{\frac{1}{2}}^1)_p$ corresponding to the addition of a $d_{\frac{3}{2}}$ neutron to $P^{31}$. In attempting to determine admixtures in the wave functions, five bits of experimental evidence are now available: (1) the admixture of $l=0$ in $Q_0$, (2) the intensity ratio $Q_1/Q_0=1.45\pm0.03$, (3) the number of excited states[9] which contain $l=0$ components, (4) the anomalous magnetic moment of $P^{31}=1.132$ nm, and (5) the $\beta$ decay of $P^{32}$ which has a $\log ft=7.9$ and is $l=2$ forbidden.

More general wave functions for $P^{31}$, $P^{32}(1+)$, and $P^{32}(2+)$ are obtained by removing neutrons from the $d_{\frac{5}{2}}$ and $s_{\frac{1}{2}}$ subshells. Only those terms in the wave functions that differ from the ground state by two or less particles are considered. These may be written as

$$\begin{aligned}
\Psi(P^{31}) &= \alpha[(s_{\frac{1}{2}}^2)_n(s_{\frac{1}{2}}^1)_p]+\beta[(s_{\frac{1}{2}}^2 d_{\frac{5}{2}}^{-1} d_{\frac{3}{2}}^1)_n(s_{\frac{1}{2}}^1)_p] \\
&\quad +\gamma[(s_{\frac{1}{2}}^1 d_{\frac{3}{2}}^1)_n(s_{\frac{1}{2}}^1)_p]+\delta[(d_{\frac{3}{2}}^2)_n{}^0(s_{\frac{1}{2}}^1)_p]+\cdots, \\
\Psi[P^{32}(1+)] &= a_1[(s_{\frac{1}{2}}^2 d_{\frac{3}{2}}^1)_n(s_{\frac{1}{2}}^1)_p]+b_1[(s_{\frac{1}{2}}^1 d_{\frac{3}{2}}^2)_n{}^{\frac{1}{2},\frac{3}{2}}(s_{\frac{1}{2}}^1)_p] \\
&\quad +c_1[(d_{\frac{3}{2}}^3)_n(s_{\frac{1}{2}}^1)_p]+d_1[(s_{\frac{1}{2}}^2 d_{\frac{3}{2}}^1)_n(d_{\frac{3}{2}}^1)_p]+\cdots, \\
\Psi[P^{32}(2+)] &= a_2[(s_{\frac{1}{2}}^2 d_{\frac{3}{2}}^1)_n(s_{\frac{1}{2}}^1)_p]+b_2[(s_{\frac{1}{2}}^1 d_{\frac{3}{2}}^2)_n{}^{\frac{3}{2},\frac{5}{2}}(s_{\frac{1}{2}}^1)_p] \\
&\quad +c_2[(d_{\frac{3}{2}}^3)_n(s_{\frac{1}{2}}^1)_p]+d_2[(s_{\frac{1}{2}}^2 d_{\frac{3}{2}}^1)_n(d_{\frac{3}{2}}^1)_p]+\cdots,
\end{aligned}$$

where the superscript outside parentheses indicates the spin to which the particles are coupled. The amplitudes $\alpha$, $a_1$, and $a_2$ will be of the order of unity; the admixed amplitudes will presumably be small. The zero-order transition probabilities to the 1+ state and the 2+ state of $P^{32}$ are proportional to $|\alpha a_1|^2$ and $|\alpha a_2|^2$,

[7] W. J. Childs, thesis, University of Michigan, 1956 (unpublished).

[8] C. K. Bockelman and W. W. Buechner, Phys. Rev. **107**, 1366 (1957); see also J. B. French and B. J. Raz, Phys. Rev. **104**, 1411 (1956).

[9] E. H. Beach, Ph.D. thesis, University of Michigan, 1952 (unpublished); I. B. Teplov, reference 2; Dalton, Hinds, and Parry, reference 2.

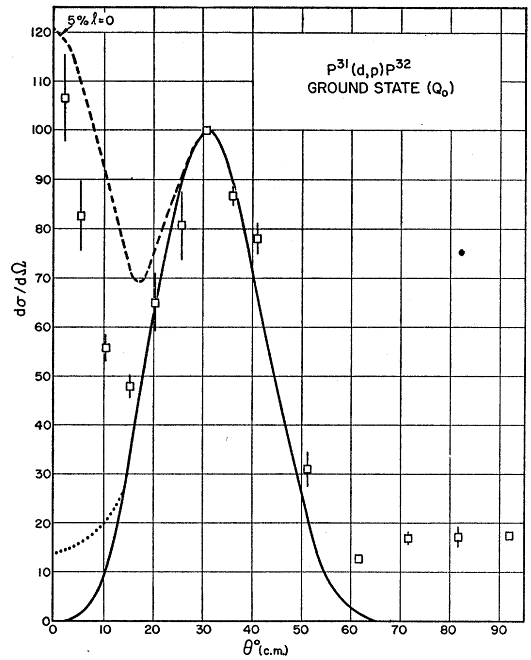


FIG. 2. The angular distribution in the center-of-mass system of the ground state proton group. The solid curve is the calculated Butler distribution for $l_n=2$, using $r_0=5.7\times10^{-13}$ cm. The dashed curve is computed by using 5% of the calculated $l_n=0$ Butler distribution and 95% of the $l_n=2$ curve, modified as small angles as shown by the dotted curve. The modified $l_n=2$ curve takes account of the nonzero character of the distribution at small angles.

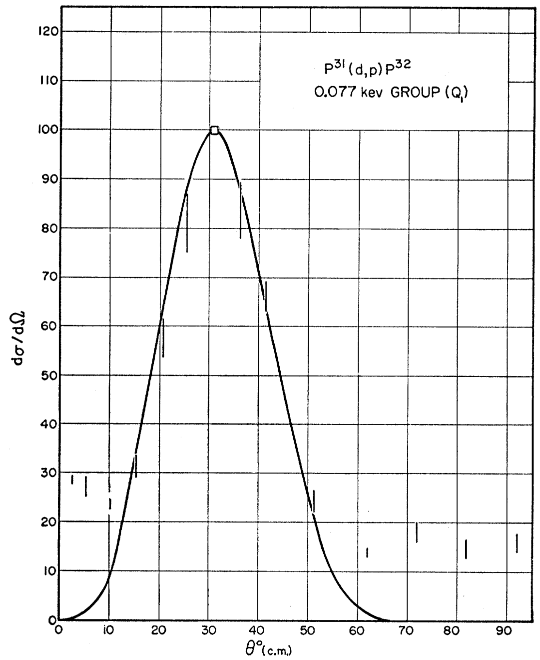


FIG. 3. The angular distribution in the center-of-mass system for the proton group corresponding to the first excited state $Q_1$. The solid curve is the Butler distribution calculated for $l_n=2$ using $r_0=5.7\times10^{-13}$ cm.

respectively. There are only two configurations which can be reached in first order, one corresponding to the capture of a $d_{\frac{5}{2}}$ particle leading to the 2+ level of $P^{32}$, with a capture probability proportional to $|\beta a_2|^2$, and one corresponding to the capture of an $s_{\frac{1}{2}}$ particle leading to the 1+ level, proportional to $|\gamma a_1|^2$. There are many second-order terms, some twelve of which can contribute to $d_{\frac{3}{2}}$ capture into both the 1+ and 2+ levels, such as $(\gamma b_1)$ and $(\gamma b_2)$, respectively, four which can contribute to $d_{\frac{5}{2}}$ capture into the 2+ level, and five which can contribute to $s_{\frac{1}{2}}$ capture into the 1+ level.

The admixture $\gamma$ allows $l=0$ capture to the 1+ state and is the only first order term to do so. Therefore, to first order, the ratio of the $l=0$ to the $l=2$ differential cross sections for $Q_0$ measured at their respective peaks gives a measure of $\gamma/\alpha$. The dimensionless reduced widths for capture of $s_{\frac{1}{2}}(l=0)$ and $d_{\frac{3}{2}}(l=2)$ nucleons in $P^{31}$ to form the 1+ level of $P^{32}$ are $\theta_s^2=\frac{2}{3}(\gamma a_1)^2$ and $\theta_d^2=(\alpha a_1)^2$, so that $(1/20)(d\sigma_0/d\sigma_2)=\theta_s^2/\theta_d^2=\frac{2}{3}(\gamma a_1)^2/(\alpha a_1)^2\simeq0.05$, or $|\gamma/\alpha|\simeq0.27$. The factor $\frac{2}{3}$ results from the proper combination of the Clebsch-Gordan coefficients. There are, however, a relatively large number of second-order terms which may have amplitudes of the order of 0.01 to 0.1 which, if the amplitudes add coherently, could contribute significantly to the transition probability.

The reduced widths for $l=2$ capture to the 1+ and 2+ states, respectively, are $\theta^2(1+)=(\alpha a_1)^2$ and $\theta^2(2+)=(\alpha a_2)^2+\frac{2}{5}(a_2\beta)^2$. The term $\frac{2}{5}(a_2\beta)^2$ corresponds to capture of a $d_{\frac{5}{2}}$ nucleon and will favor the transition to the (2+) level. The second order terms corresponding to $d_{\frac{3}{2}}$ capture such as $(\delta b_1)$ can be expected to have amplitudes of the order of 0.01 to 0.1, and again, if they add coherently could provide sizable contributions to $l=2$ capture. It is not possible to say *a priori* whether the (2+) or the (1+) level would be favored. However, the measured intensity ratio of $Q_1/Q_0=0.88$ (the statistical factor $2J+1$ having been removed) suggests that the second-order terms are relatively important, the $d_{\frac{3}{2}}$ terms favoring the 1+ level by more than 12%.

A surprisingly large number of the excited states in the reaction $P^{31}(d,p)P^{32}$ are reached[9] by admixtures of $l=0$ and $l=2$. The $l=2$ components can arise due to admixtures in the wave functions for the excited states of $P^{32}$, but the $l=0$ components can arise only from admixtures such as $\gamma$ and $\delta$ in the ground state of $P^{31}$.

According to the shell model, $P^{31}$ contains an odd proton in an $s_{\frac{1}{2}}$ shell outside the closed $d_{\frac{5}{2}}$ subshell. It might be expected, therefore, that the magnetic moment would be given by the Schmidt value 2.793 nm, whereas the observed value is 1.132 nm. Blin-Stoyle has shown[10] that for nuclei of spin $\frac{1}{2}$ the deviation from the Schmidt value can be adequately accounted for by simple configurational mixing. The most important contribution

[10] R. J. Blin-Stoyle, Revs. Modern Phys. **28**, 75 (1956); also Proc. Phys. Soc. (London) **A66**, 1158 (1953). See also M. Umezawa, Progr. Theoret. Phys. (Japan) **8**, 509 (1952).

to the deviation for $P^{31}$ arises due to mixing of the configurations $(s_{\frac{1}{2}}{}^2 d_{\frac{5}{2}}{}^{-1} d_{\frac{3}{2}}{}^1)_n (s_{\frac{1}{2}}{}^1)_p$ (of amplitude $\beta_n$) in which a neutron from the $d_{\frac{5}{2}}$ shell is elevated to the $d_{\frac{3}{2}}$ shell, and $(s_{\frac{1}{2}}{}^2)_n (s_{\frac{1}{2}}{}^1 d_{\frac{5}{2}}{}^{-1} d_{\frac{3}{2}}{}^1)_p$ (of amplitude $\beta_p$) corresponding to the excitation of a $d_{\frac{5}{2}}$ proton to the $d_{\frac{3}{2}}$ shell. While $\beta_p$ does not contribute to the stripping cross section, it does yield the largest contribution to the deviation of the magnetic moment. To first order,[11] $\mu(P^{31}) = \alpha^2 \mu(s_{\frac{1}{2}}{}^1)_p + 3.95\alpha(\beta_n + 1.20\beta_p)$, and since $\mu(P^{31}) = 1.132$ if $\alpha \simeq 1$, $\beta_n + 1.2\beta_p \simeq -0.42$. Assuming zero range forces Satchler finds $\beta_p \simeq 3\beta_n$, so that $\beta_n \simeq 0.1$. Thus only small amplitudes of the $\beta_n$ and $\beta_p$ admixtures are needed to account for the magnetic moment.

The $\beta^-$ decay of $P^{32}(1+)$ to $S^{32}(0+)$ is an $l$-forbidden $(\Delta l = 2)$ transition with $\log ft = 7.9$. The zero-order shell model configurations for $P^{32}$ and $S^{32}$ are $[(s_{\frac{1}{2}}{}^1 d_{\frac{3}{2}}{}^1)_n (s_{\frac{1}{2}}{}^1)_p]$ and $[(s_{\frac{1}{2}}{}^2)_n (s_{\frac{1}{2}}{}^2)_p]$, respectively. Since the transition probability vanishes not only for the zero-order configurations, but also for first-order, second-order configurations are required to account for the $\beta$ decay. The $\log ft$ value of 7.9 implies a transition probability $\sim 10^{-4}$ that of an allowed transition $(\Delta I = 1$, no; $\log ft \simeq 4)$. The matrix element is therefore $\sim 10^{-2}$ corresponding to an amplitude for the admixture in the $P^{32}$ ground state of $\sim 0.1$.

In summary, the information available from the stripping reaction, the magnetic moment of $P^{31}$, and the beta decay of $P^{32}$, is consistent with the wave function of $P^{31}$ containing admixtures of amplitude $\beta \simeq 0.1$ and $\gamma \simeq 0.3$ and that these are the only first-order terms. The amplitudes of the second-order configurations may be as large as 0.1.

One further point is of interest. Conservation of angular momentum prohibits the $(2+)$ level of $P^{32}$ being reached by direct $l=0$ capture. The angular distribution for this level $(Q_1)$, however, shows a slight rise at small angles that appears statistically significant and which remained after generous corrections for target impurities were applied to the original data. The Coulomb correction, calculated for a similar case,[12] removes the zero of the normal Butler curves but does not produce a rise at small angles. While the rise could be interpreted as indicating that the spin is not $2+$, the other available data give strong support to this assignment. A possible explanation is that the rise at small angles results from spin-flip stripping, which can contribute an extra unit of angular momentum to the residual nucleus. There is evidence[13] that the cross section for this process may be as large as 10% of the direct process. In the $P^{31}(d,p)P^{32}$ reaction, spin-flip can occur only through the $\gamma$ or higher configurations. Since the $\gamma$ admixture contributes $\sim 5\%$ of $l=0$ to $Q_0$, spin-flip might contribute $\sim 0.5\%$ $l=0$ to $Q_1$. This is the right order of magnitude to account for the rise near 0°.

[11] G. R. Satchler (unpublished calculation).


### IV. ACKNOWLEDGMENTS

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[12] S. T. Butler and N. Austern, Phys. Rev. **93**, 355 (1954).

[13] N. T. S. Evans and A. P. French, Phys. Rev. **109**, 1261 (1958); J. C. Hensel and W. C. Parkinson, Phys. Rev. **110**, 128 (1958).