carbon results, and is not contradicted by those from silicon. This might be explained by the fact that for smaller binding energies the neutron wave function extends to larger radii, thus increasing the importance of its overlap with the deuteron and proton waves far from the nucleus where these waves are less distorted by the nucleus. Assuming that either or both the deuteron and proton interactions with the nucleus are the basic cause of the polarization, the polarization would then be expected to be less for reactions involving smaller neutron binding energies.

It would be useful in nuclear spectroscopy if there were a consistent correlation between the sign of the polarization and the relative orientation of the spin of the captured neutron and its orbital angular momentum. The orientations are known for the reactions investigated in the present experiment since the initial nuclei have spin zero and the spins of the final nuclei are known.[24,25] At small angles for the $l_n=1$ proton groups, the polarization is positive if the neutron spin and the orbital angular momentum are parallel, and negative if they are antiparallel. A change in sign was observed at large angles for some proton groups. However, this simple rule could still hold because of the statistical uncertainty of these results. Theoretical calculations do not exclude different signs at larger angles.[9]

Recent experiments of Hensel and Parkinson[26] support the existence of such a unique correlation. Further experimental and theoretical work is necessary to establish more firmly a simple rule of this kind.

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[24] The spins of the levels in Si^{29} are not definitely known, but the assignments of Holt and Marsham (see reference 22) are probably correct.

[25] F. Ajzenberg and T. Lauritsen, Revs. Modern Phys. **27**, 77 (1955).

[26] J. C. Hensel and W. C. Parkinson, Bull. Am. Phys. Soc. Ser. II, **2**, 228 (1957).

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Time-Reversal Invariance and Beta-Gamma Angular Correlation. II*

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The angular-correlation function between beta and gamma rays from oriented nuclei (with or without observation of circular polarization) is given for use in testing the invariance property of the beta interactions under time reversal. The formulas are derived in the cases where the beta decay is an allowed or first forbidden transition and the gamma ray has a mixture of arbitrary multipolarities. A general form for the $\beta-\gamma_1-\gamma_2$ angular correlation in triple cascade transitions of unoriented nuclei is also shown. It does not give clear-cut information on the invariance property of beta interactions under time reversal. The correlation functions where gamma rays are replaced by alpha particles are discussed in the above two cases. The beta-alpha directional correlation from oriented nuclei may be useful in testing the invariance property of beta interactions under time reversal.

1. INTRODUCTION

RECENT experiments on the beta-gamma polarization correlations[1,2] in Sc^{46} and Au^{198} have indicated that the coupling constants in the beta interactions are probably real. Theoretical analyses of the beta spectrum of RaE by several authors[3-5] have also given the same result. As is well known, the realness of the coupling constants implies the invariance of the beta interactions under time reversal. However, the results of the beta angular distribution from polarized neutrons[6] are difficult to explain with real coupling constants.[7] All of these experiments are based on

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[1] F. Boehm and A. H. Wapstra, Phys. Rev. **106**, 1364 (1957); **107**, 1202 and 1462 (1957); and **109**, 456 (1958).

[2] R. M. Steffen and P. Alexander (to be published).

[3] Fujita, Matumoto, Yamada, and Nakamura, Phys. Rev. **108**, 1104 (1957).

[4] Matunobu, Nakamura, and Takebe, Progr. Theoret. Phys. (Kyoto) (to be published).

[5] R. R. Lewis, Phys. Rev. **108**, 904 (1957).

[6] Burgy, Epstein, Krohn, Novey, Raboy, Ringo, and Telegdi, Phys. Rev. **107**, 1731 (1957).

[7] The case of Co^{56} is somewhat complicated. From the data on the beta angular distribution from polarized Co^{56} [Ambler,

TABLE I. Pseudoscalars $(\mathbf{J}\cdot[\mathbf{p}\times\mathbf{k}])^a(\mathbf{J}\cdot\mathbf{k})^b(\mathbf{J}\cdot\mathbf{p})^c(\mathbf{p}\cdot\mathbf{k})^d$ under T and P. In the 1st to 4th columns, "odd" ("even") means odd (even) numbers. In the 5th and 6th columns, $-$ $(+)$ means pseudoscalar (scalar) under the operation of P or T. If the sign is minus, we can test the invariance of beta interactions under the relevant operation. In the 7th column, "both" means that the term appears for both aligned and polarized nuclei, while "polarized" means that the term appears only for polarized nuclei. In the 8th column, "required" means that the measurement of the circular polarization of the gamma ray is necessary, while "no" means that the term appears whether or not one observes the circular polarization of the gamma ray. For the remaining four sets of a, b, c, and d, which are not listed here,

$$(\mathbf{J}\cdot[\mathbf{p}\times\mathbf{k}])^a(\mathbf{J}\cdot\mathbf{k})^b(\mathbf{J}\cdot\mathbf{p})^c(\mathbf{p}\cdot\mathbf{k})^d$$

is scalar under both P and T.

a	b	c	d	T	P	Nuclei	Circular polarization of gamma rays
odd	even	even	even	$-$	$-$	polarized	required
odd	even	even	odd	$-$	$+$	polarized	no
odd	even	odd	even	$-$	$+$	both	required
odd	even	odd	odd	$-$	$-$	both	no
odd	odd	even	even	$-$	$-$	both	no
odd	odd	even	odd	$-$	$+$	both	required
odd	odd	odd	even	$-$	$+$	polarized	no
odd	odd	odd	odd	$-$	$-$	polarized	required
even	even	even	odd	$+$	$-$	both	required
even	even	odd	even	$+$	$-$	polarized	no
even	odd	even	odd	$+$	$-$	polarized	no
even	odd	odd	even	$+$	$-$	both	required

measurements of the real part of the products of the coupling constants. It is advisable to measure directly their imaginary part. One such experiment is the beta-gamma angular correlation from oriented nuclei, for which the theoretical calculations were made by Morita and Morita[8] and by Curtis and Lewis.[9] This experiment has been performed by Ambler *et al.*[10] for the case of Co^{58}. The observed asymmetry effect in terms of $(\mathbf{J}\cdot[\mathbf{p}\times\mathbf{k}])(\mathbf{J}\cdot\mathbf{k})$ and $(\mathbf{J}\cdot[\mathbf{p}\times\mathbf{k}])(\mathbf{J}\cdot\mathbf{k})^3$, which come from the violation of invariance under time reversal, appears to be very small. Here, $\mathbf{J}$ is a unit vector along the axis of nuclear orientation; $\mathbf{p}$ and $\mathbf{k}$ are unit vectors in the directions of the momenta of the beta ray and the gamma ray, respectively. There is, however, no definite conclusion regarding the realness of the coupling constants from the data on Co^{58}, because the ratio $M_{\mathrm{F}}/M_{\mathrm{GT}}$ of Co^{58} may be very small. Since nuclei with an incomplete shell of atomic $4f$ or $5f$ electrons are most easily oriented at low temperatures, and many of the beta-active nuclei in this region of atomic number decay by first forbidden transitions, we have extended our previous work[8] to cover these cases.

In Sec. 2, the pseudoscalars under the operation of space reflection (P) and time reversal (T) are shown. In Sec. 3, a general form of beta-gamma angular correlation (both for direction and for polarization) is given for oriented (both aligned and polarized) nuclei. It is related to the other angular-correlation functions in Sec. 4. In Sec. 5, some suggestions for application of this formula are discussed. In the Appendix, the angular correlation in triple cascade transitions is given for unoriented nuclei.

2. PSEUDOSCALARS UNDER T AND P

In the successive beta and gamma decays of the oriented nuclei, we can express the angular correlation between beta and gamma rays in terms of

$$(\mathbf{J}\cdot[\mathbf{p}\times\mathbf{k}])^a(\mathbf{J}\cdot\mathbf{k})^b(\mathbf{J}\cdot\mathbf{p})^c(\mathbf{p}\cdot\mathbf{k})^d.$$

The powers a, b, c, and d for each term in parentheses are related to ν, n, and n_1 [see Eq. (1)] by the following relations:

$$a+b+c=\nu,\quad a+c+d=n,\quad a+b+d=n_1.$$

The properties of $(\mathbf{J}\cdot[\mathbf{p}\times\mathbf{k}])^a(\mathbf{J}\cdot\mathbf{k})^b(\mathbf{J}\cdot\mathbf{p})^c(\mathbf{p}\cdot\mathbf{k})^d$ under the operator of T and P are summarized in Table I. Its behavior under charge conjugation is not shown because we have no unique information on it except in the kth forbidden transitions with $\Delta J=\pm(k+1)$. In the other cases, it is necessary to assume the values of beta matrix elements. In Table I, the terms which have a minus sign in column 5 or 6 appear only if the beta interactions are noninvariant under the relevant operation. The permissible values for a, b, c, and d will be given in the next section.

One of the simplest applications of Table I is to the case of Co^{58}. In this case, the beta decay is allowed, the gamma decay is a quadrupole transition, and the decay scheme is $2^+(\beta)2^+(\gamma)0^+$. The four terms, $(\mathbf{J}\cdot[\mathbf{p}\times\mathbf{k}])\cdot(\mathbf{J}\cdot\mathbf{k})^b$ with $b=0$, 1, 2, 3, appear in the beta-gamma angular correlations if there is violation of invariance under T. (Our discussion always assumes the strong interactions to be invariant under T; otherwise a minor

Hayward, Hoppes, and Hudson, Phys. Rev. **108**, 503 (1957)], we can say that $M_{\mathrm{F}}/M_{\mathrm{GT}}\approx 0$ and/or $\mathrm{Re}(C_S{}^*C_T'+C_S'{}^*C_T-C_V{}^*C_A'-C_V'{}^*C_A)\approx 0$. On the other hand, the angular distributions of six gamma rays following the beta decay of aligned Co^{56} are consistent with the assumption that

$$(|C_S|^2+|C_S'|^2+|C_V|^2+|C_V'|^2)M_{\mathrm{F}}^2\approx(|C_T|^2+|C_T'|^2+|C_A|^2+|C_A'|^2)M_{\mathrm{GT}}^2$$

within experimental errors [Poppema, Siekman, Wageningen, and Tolhoek, Physica **21**, 223 (1955), where they assumed all the gamma rays to be pure]. This assumption is also consistent with all of the data on gamma-ray angular distributions from aligned Co^{56} and on gamma-gamma directional correlations from unoriented Co^{56} [M. Sakai, J. Phys. Soc. Japan **10**, 729 (1955)], if the 1.75-Mev gamma ray is a mixture of $M1$ and $E2$. [The theoretical analysis was done by Morita, Ogata, and Sakai, Physica **22**, 915 (1956); a more detailed discussion was given in Bull. Kobayasi Inst. Phys. Research **6**, 69 (1956).] Therefore, it is important to reanalyze whether or not the assumption, $M_{\mathrm{F}}/M_{\mathrm{GT}}\approx 0$, is consistent with all data on both oriented and unoriented Co^{56} nuclei.

[8] M. Morita and R. S. Morita, Phys. Rev. **107**, 1316 (1957). Both beta-gamma directional and polarization correlations from oriented nuclei have been considered for allowed beta decay. See Appendix II of the present paper.

[9] R. B. Curtis and R. R. Lewis, Phys. Rev. **107**, 1381 (1957). Beta-gamma directional correlation has been considered for allowed beta decay. We wish to thank Dr. Curtis and Dr. Lewis for a valuable discussion.

[10] Ambler, Hayward, Hoppes, and Hudson, Phys. Rev. **108**, 503 (1957).

modification of the results in this paper is necessary.) If we do not measure the circular polarization of the gamma ray, the terms with $b=0$ and 2 vanish.

The term $\alpha Z(\mathbf{J}\cdot[\mathbf{p}\times\mathbf{k}])^a(\mathbf{J}\cdot\mathbf{k})^b(\mathbf{J}\cdot\mathbf{p})^c(\mathbf{p}\cdot\mathbf{k})^d$ is a scalar (pseudoscalar) if the term $(\mathbf{J}\cdot[\mathbf{p}\times\mathbf{k}])^a(\mathbf{J}\cdot\mathbf{k})^b\cdot(\mathbf{J}\cdot\mathbf{p})^c(\mathbf{p}\cdot\mathbf{k})^d$ is a pseudoscalar (scalar) under T. Here, αZ is the Coulomb phase shift in the wave function of the beta particle.

3. BETA-GAMMA ANGULAR CORRELATION FROM ORIENTED NUCLEI

A general form for the beta-gamma angular correlation from oriented nuclei is derived from a previous similar calculation.[11] We assume the decay scheme to be $j(\beta)j_1(\gamma)j_2$ and the gamma ray to be a mixture of 2^{L_1}, $2^{L_1'}$, $\cdots$. The unit vector $\mathbf{J}$ along the orientation axis of the nucleus is chosen as the z axis. The beta and gamma rays are assumed to be emitted in the directions of $\mathbf{p}$ and $\mathbf{k}$ with polar angles $(\theta_1, \varphi_1\equiv 0)$ and (θ_2,φ), respectively (see Fig. 1). The result is

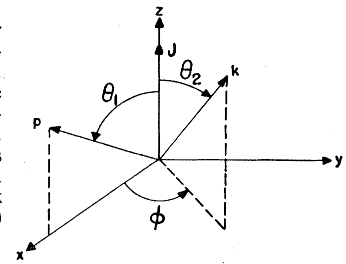

FIG. 1. Geometry for beta-gamma angular correlation from oriented nuclei. The unit vector $\mathbf{J}$ along the orientation axis of the nucleus is chosen as the z axis. The beta and gamma rays are assumed to be emitted in the directions of $\mathbf{p}$ and $\mathbf{k}$ with polar angles $(\theta_1, \varphi_1\equiv 0)$ and (θ_2,φ), respectively.

$$W(\theta_1,\theta_2,\varphi,p_1)=\sum_{L\leq L'}\sum_{L_1}\sum_{L_1'}\sum_{\nu}\sum_{n}\sum_{n_1}(-)^{L'+n}\bar{f}_\nu(j)\{(2j_1+1)(2\nu+1)(2n+1)\}^{\frac{1}{2}}X\begin{bmatrix} j & j_1 & L \\ j & j_1 & L' \\ \nu & n_1 & n\end{bmatrix}$$
$$\times[B_{LL'}{}^{(n)}\textstyle\sum_\mu(-)^\mu(\nu n0\mu|n_1\mu)D_{-\mu,0}{}^{(n)}(0,\theta_1,0)D_{\mu,0}{}^{(n_1)}(\varphi,\theta_2,0)+\text{c.c.}](-)^{L_1+L_1'}p_1{}^{n_1}$$
$$\times(j_1\|L_1\|j_2)(j_1\|L_1'\|j_2)F_{n_1}(L_1L_1'j_2j_1),\quad (1)$$

with the condition $L_1+L_1'+\delta_1+\delta_1'=$even, where

$$\bar{f}_\nu(j)=\textstyle\sum_m(-)^{j-m}(jjm-m|\nu 0)a_m,$$

$a_m=$relative population of the initial magnetic substate,

$$X\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k\end{bmatrix}=\textstyle\sum_\lambda(2\lambda+1)W(abkf;c\lambda)\times W(dfhb;e\lambda)W(adkh;g\lambda),\text{[12]}$$

$$F_{n_1}(L_1L_1'j_2j_1)=F_{n_1}(L_1'L_1j_2j_1)=(-)^{j_2-j_1-1}\{(2j_1+1)(2L_1+1)(2L_1'+1)\}^{\frac{1}{2}}\times(L_1L_1'1-1|n_10)W(j_1j_1L_1L_1';n_1j_2),$$

$$D_{m,0}{}^{(l)}(\varphi,\theta,0)=e^{-im\varphi}D_{m,0}{}^{(l)}(0,\theta,0)=[4\pi/(2l+1)]^{\frac{1}{2}}Y_{l,m}{}^*(\theta,\varphi),$$

$p_1=1(-1)$ for a left (right) circularly polarized gamma ray, and $\delta_1=0(+1)$ for magnetic (electric) radiation. The L and L' indicate the ranks of the nuclear matrix elements in the beta decay. (For example, the ranks of $\int 1$, $\int \boldsymbol{\sigma}$, $\int \boldsymbol{\sigma}\times\mathbf{r}$, and B_{ij} are 0, 1, 1, and 2, respectively.) The $B_{LL'}{}^{(n)}$'s are related to the $b_{LL'}{}^{(n)}$'s in the following way:

$$2B_{LL'}{}^{(n)}=b_{LL'}{}^{(n)}\text{ with }\mathrm{Re}(C_i{}^*C_j{}^{(\prime)})\text{ and }\mathrm{Im}(C_i{}^*C_j{}^{(\prime)})\text{ replaced by }C_i{}^*C_j{}^{(\prime)}\text{ and }-iC_i{}^*C_j{}^{(\prime)},\text{ respectively.}\quad (2)$$

Here $i=j$ is included. The $b_{LL'}{}^{(n)}$'s for the allowed transitions are given in Appendix II of reference 8 for the most general interaction. Those for the first forbidden transition are given by Morita and Morita[13] with an assumption of no interferences between STP and VA.[14] This assumption is compatible with all of the present data on beta decay. Therefore, we can immediately obtain the explicit forms of $B_{LL'}{}^{(n)}$ from $b_{LL'}{}^{(n)}$ up to the first forbidden transition. In the case of more highly forbidden transitions the $B_{LL'}{}^{(n)}$'s may be deduced by a similar but tedious calculation.[15]

The restrictions on the values of the integral numbers ν, n, and n_1 are evident from the algebraic coefficients in Eq. (1). They depend on the ranks of the beta-decay matrix elements, the multipolarity of the gamma ray, and the spins of the nuclear levels. Furthermore, ν has only even values if nuclei are aligned. n_1 is also restricted to be even, if the circular polarization of gamma ray is not observed. The terms, $(\mathbf{J}\cdot[\mathbf{p}\times\mathbf{k}])^a(\mathbf{J}\cdot\mathbf{k})^b(\mathbf{J}\cdot\mathbf{p})^c\cdot(\mathbf{p}\cdot\mathbf{k})^d$, which are pseudoscalar with respect to T, appear only when $\nu+n+n_1$ is odd (namely, when a is odd). And if $\nu+n+n_1$ is odd, the $9j$ coefficient X vanishes, unless $L\neq L'$. Therefore, the term with which we can test the invariance property of beta interactions

[11] M. Morita, Progr. Theoret. Phys. (Kyoto) **15**, 445 (1956).

[12] For example, see Sharp, Kennedy, Sears, and Hoyle, Chalk River Technical Report CRT-556, 1956 (unpublished).

[13] M. Morita and R. S. Morita, Phys. Rev. **109**, 2048 (1958). The reason for no interferences between STP and VA is discussed in its reference 16.

[14] Alder, Stech, and Winther, Phys. Rev. **107**, 728 (1957). Formulas for $s_k(LL')$, whose definition is somewhat different from that of our $b_{LL'}{}^{(n)}$, have been given by them, in the special case of STP with an assumption of no interferences between $B_{ij}{}^\beta$ and the other matrix elements.

[15] M. Yamada and M. Morita, Progr. Theoret. Phys. (Kyoto) **8**, 443 (1952), and their later papers. (A list is given in reference 13.)

under time reversal comes from interferences among beta matrix elements with different ranks. Here, we do not discuss the terms $\alpha Z(\mathbf{J}\cdot[\mathbf{p}\times\mathbf{k}])^a(\mathbf{J}\cdot\mathbf{k})^b(\mathbf{J}\cdot\mathbf{p})^c(\mathbf{p}\cdot\mathbf{k})^d$, because their effect on the angular correlations is usually small and within the experimental errors at the present time.

4. RELATION TO THE OTHER ANGULAR CORRELATIONS

Upon integrating $W(\theta_1,\theta_2,\varphi,p_1)$ over the direction of emission of the gamma ray, one obtains the angular-distribution function for beta rays from oriented nuclei[13,14,16]:

$$\int W(\theta_1,\theta_2,\varphi,p_1)d\Omega_\gamma=W(\theta_1:\beta)$$

$$=\sum_n\sum_{L\leq L'} f_n(j)(-)^{j_1-j+L+L'+n}\times W(jjLL';nj_1)b_{LL'}^{(n)}P_n(\cos\theta). \quad (3)$$

In the second line of Eq. (3), the irrelevant common factor has been omitted.

By integrating $W(\theta_1,\theta_2,\varphi,p_1)$ over the direction of emission of the beta ray and the azimuthal angle φ of the gamma ray and also over the electron energy, one obtains the angular-distribution function for gamma rays from oriented nuclei[17]:

$$W(\theta_2,p_1:\gamma_1)=\sum_\nu(-)^{j+j_1+\nu}\bar{f}_\nu(j)$$

$$\times\Bigg[\sum_L(-)^L W(jjj_1j_1;\nu L)(2j_1+1)^{\frac{1}{2}}$$

$$\times\int_1^{W_0} a_L pWK^2F(Z,W)dW\Bigg]\Bigg[\sum_{L_1,L_1'}(-)^{L_1+L_1'}p_1^\nu$$

$$\times(j_1\|L_1\|j_2)(j_1\|L_1'\|j_2)F_\nu(L_1L_1'j_2j_1)\Bigg]P_\nu(\cos\theta), \quad (4)$$

with the condition $L_1+L_1'+\delta_1+\delta_1'=\text{even}$, where $a_L=(-)^L(2L+1)^{-\frac{1}{2}}b_{LL}^{(0)}$ is the correction factor for the beta spectrum of the matrix elements with rank L.

5. CONCLUSION

If the beta interactions are noninvariant under time reversal, the beta-gamma correlations (both in direction and polarization) from oriented (both aligned and polarized) nuclei have an up-down asymmetry of beta intensity with respect to the plane containing the direction of nuclear orientation and the gamma-ray momentum. The angle θ_2, which gives the maximum asymmetry, depends on the decay scheme, the forbiddenness of the beta decay, the multipolarity of the gamma ray, and the degree of nuclear orientation.

Since this asymmetry appears in the interferences among beta-decay matrix elements of different ranks, we must choose the nuclei which decay in kth forbidden transitions with a spin change $\Delta J=\pm k$. In the case of the first forbidden transitions, we may also use the decay with $\Delta J=0$. There are some suitable nuclei with incomplete shells of atomic $4f$ or $5f$ electrons. For example, Ce^{141} and Nb^{147} have been aligned by Ambler *et al.*[18]

When alpha decay follows the beta decay, the beta-alpha directional correlation[19] from oriented nuclei is derived from Eq. (1), by replacing $p_1^{n_1}F_{n_1}(L_1L_1'j_2j_1)$ by

$$(-)^{j_2-j_1}(L_1L_100|n_10)\{(2j_1+1)(2L_1+1)(2L_1'+1)\}^{\frac{1}{2}}\times W(j_1j_1L_1L_1';n_1j_2).$$

Here, L_1 (L_1') represents the L_1th (L_1'th) partial wave of the alpha particle and n_1 is even. This correlation function may be useful to test the invariance of beta interactions under T.

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APPENDIX I. ANGULAR CORRELATION IN TRIPLE CASCADE TRANSITIONS FROM UNORIENTED NUCLEI

The angular correlation function for a beta ray followed by two gamma rays in triple cascade transitions of unoriented nuclei is calculated very similarly to that between beta and gamma rays from oriented nuclei. The decay scheme is $j(\beta)j_1(\gamma_1)j_2(\gamma_2)j_3$. The direction of the momentum of the beta ray is chosen as the z axis. The γ_1 and γ_2 rays are assumed to be emitted in the directions represented by unit vector $\mathbf{k}_1$ and $\mathbf{k}_2$, with polar angles (θ_1, $\varphi_1\equiv 0$) and (θ_2,φ), respectively (see Fig. 2).

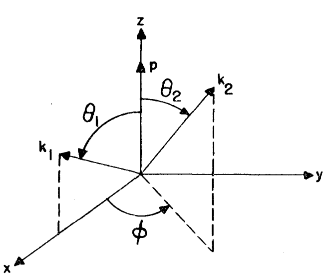

FIG. 2. Geometry for beta-gamma-gamma angular correlation from unoriented nuclei. The direction of the momentum $\mathbf{p}$ of beta ray is chosen as the z axis. Two gamma rays are assumed to be emitted in the directions of $\mathbf{k}_1$ and $\mathbf{k}_2$, with polar angles (θ_1, $\varphi_1\equiv 0$) and (θ_2,φ), respectively.

16 M. Morita, Phys. Rev. **107**, 1729 (1957).

17 See, for example, S. R. de Groot and H. A. Tolhoek, in *Beta- and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (North-Holland Publishing Company, Amsterdam, 1955), p. 613; Morita, Ogata, and Sakai (see reference 7).

18 Ambler, Hudson, and Temmer, Phys. Rev. **97**, 1212 (1955).

19 On the beta-alpha directional correlation from unoriented nuclei, theoretical work has been done by M. Morita and M. Yamada, Progr. Theoret. Phys. (Kyoto) **13**, 114 (1955), and M. Morita, Eqs. (42) and (43) of reference 11.

The result is[20]

$$W(\theta_1,\theta_2,\varphi,p_1,p_2:\beta-\gamma_1-\gamma_2)=\sum_{L\leq L'}\sum_{L_1}\sum_{L_1'}\sum_{L_2}\sum_{L_2'}\sum_{n}\sum_{n_1}\sum_{n_2}(-)^{1-j+j_1+L_1'}(2j_1+1)\{(2n+1)(2n_1+1)(2L_1+1)$$
$$\times(2L_1'+1)(2j_2+1)\}^{\frac{1}{2}}W(j_1j_1LL';nj)b_{LL'}{}^{(n)}(j_1\|L_1\|j_2)(j_1\|L_1'\|j_2)$$
$$\times p_1{}^{n_1}(L_1L_1'1-1|n_10)X\begin{bmatrix} j_1 & j_2 & L_1 \\ j_1 & j_2 & L_1' \\ n & n_2 & n_1 \end{bmatrix}[\textstyle\sum_\mu(-)^\mu(nn_10\mu|n_2\mu)D_{-\mu,0}{}^{(n_1)}(0,\theta_1,0)$$
$$\times D_{\mu,0}{}^{(n_2)}(\varphi,\theta_2,0)](-)^{L_2+L_2'}p_2{}^{n_2}(j_2\|L_2\|j_3)(j_2\|L_2'\|j_3)F_{n_2}(L_2L_2'j_3j_2),\quad\text{(A1)}$$

where $n+n_1+n_2$ is even, and $L_i+L_i'+\delta_i+\delta_i'$ is even, $i=1$, 2. The symbols in Eq. (A1) are similar to those in Eq. (1).

Upon integrating $W(\theta_1,\theta_2,\varphi,p_1,p_2:\beta-\gamma_1-\gamma_2)$ over the direction of emission of the γ_2 ray, one obtains the $\beta-\gamma$ angular correlation given in Eq. (3) of reference 13, except for a common factor. By integrating $W(\theta_1,\theta_2,\varphi,p_1,p_2:\beta-\gamma_1\text{-}\gamma_2)$ over the direction of emission of the γ_1 ray and over the azimuthal angle φ of the γ_2 ray, one obtains the function $W(\theta_2,p_2:\beta-\gamma_2)$ given in Eq. (5) of reference 13.

When the γ_i ray ($i=1$ or 2) is replaced by an alpha particle, the angular-correlation function of the three emitted particles is obtained from Eq. (A1) by replacing $-p_i{}^{n_i}(L_iL_i1-1|n0)$ by $(L_iL_i00|n_i0)$ with even n_i. Here, the L_i (L_i') represents the L_ith (L_i'th) partial wave of the alpha particle.

The angular correlation function in triple cascade transitions of unoriented nuclei has terms which are pseudoscalar under T, $\alpha Z(\mathbf{p}\cdot[\mathbf{k}_1\times\mathbf{k}_2])^a(\mathbf{p}\cdot\mathbf{k}_1)^b(\mathbf{p}\cdot\mathbf{k}_2)^c\cdot(\mathbf{k}_1\cdot\mathbf{k}_2)^d$ with a even and an appropriate choice for b, c, d. (All the terms $(\mathbf{p}\cdot[\mathbf{k}_1\times\mathbf{k}_2])^a(\mathbf{p}\cdot\mathbf{k}_1)^b(\mathbf{p}\cdot\mathbf{k}_2)^c\cdot(\mathbf{k}_1\cdot\mathbf{k}_2)^d$ with a odd, cancel out by assuming the invariance of strong interactions under T.[21]) However, an experiment of this type does not give us clear-cut information about the invariance of beta interactions under T.[22]

[20] A similar calculation has been independently done by T. Kotani. We wish to thank Dr. Kotani for his valuable communication.

[21] These terms may appear in the successive $\gamma_1-\beta-\gamma_2$ transitions. There is, however, no nucleus with so short a beta-decay lifetime that the angular correlation theory is valid.

[22] T. D. Lee and C. N. Yang, Brookhaven National Laboratory Report BNL-443 (T-91), 1957 (unpublished). We are indebted to Dr. Lee and Dr. Yang for valuable discussions.

APPENDIX II. ERRATA IN TIME-REVERSAL INVARIANCE AND BETA-GAMMA ANGULAR CORRELATION. I, M. MORITA AND R. S. MORITA, PHYS. REV. 107, 1316 (1957)

In Eqs. (6) and (A1) of I,

$$-2\,\mathrm{Im}(C_T{}^*C_S'+\cdots)\pm2\,\mathrm{Re}(C_A{}^*C_S'+\cdots)(\alpha Z/p)$$

should be read as

$$\mp2\,\mathrm{Im}(C_T{}^*C_S'+\cdots)+2\,\mathrm{Re}(C_A{}^*C_S'+\cdots)(\alpha Z/p).$$

(The same error occurs in the paper of Curtis and Lewis.[9] In the right-hand side of their expression for E, the $\pm$ signs should be added.) Consequently, all of the expressions for the "Anisotropy" should be understood to hold for electron decay. For positron decay, the signs of the anisotropy and of α' in Table I should be reversed. The right-hand side of Eq. (3′) should be multiplied by the relative sign of M_{F} and M_{GT} and replaced by C_S instead of C_S'. The footnote b to Table I should be replaced by the following: "In experiments, it is much easier to normalize α' by

$$\{W(\pi/2,\pi/6,\pi/2)-W(\pi/2,\pi/6,-\pi/2)\}/\{W(\pi/2,\pi/6,\pi/2)+W(\pi/2,\pi/6,-\pi/2)\}$$
$$=\alpha''(p/W)\,\mathrm{Im}(C_T{}^*C_S)\times(\text{relative sign of } M_{\mathrm{F}} \text{ and } M_{\mathrm{GT}}),$$

instead of Eq. (3′). For this case, the third line of Table I should be read as $\alpha''=0.002$, 0.008, 0.019, 0.058, 0.092, 0.163, and 0.275."