Polarization Effects following Beta Decay

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With a view to providing alternative methods of determining the dominant invariants in the beta interaction, various polarization effects following the beta decay of unoriented nuclei are discussed. In both K capture and beta emission the recoiling nucleus is polarized along its line of flight. Measurement of this polarization could determine whether the tensor or axial vector interaction is dominant. The hole in the K shell following K capture is also polarized relative to the recoil direction. Measurement of the hole polarization can distinguish between the scalar and vector as well as between the tensor and axial vector interactions. Methods for detecting the polarizations are discussed. The longitudinal polarization of the recoiling nucleus can in principle be detected directly by atomic beam techniques or indirectly by measuring the correlation between the direction of the recoiling nucleus and the direction and circular polarization of a subsequent radiation (gamma ray, conversion electron, or beta particle). The polarization of the hole in the K shell following K capture can perhaps be determined by a recoil-x-ray circular polarization correlation experiment.

I. INTRODUCTION

A CRUCIAL problem in beta decay is the determination of the dominant invariants. So far the only direct information about these invariants has come from electron-neutrino correlation experiments, but the results of these measurements are contradictory. Of the four nuclei studied (He^6,[1] n,[2] Ne^{19},[3–5] and A^{35} [6]), He^6, together with n and Ne^{19}, imply that S and T are the dominant invariants, while A^{35}, together with n and Ne^{19}, indicate V and A.

The recent experiments on parity nonconservation in beta decay aid little in resolving these contradictions. The determination of the angular asymmetries in the decay of polarized nuclei and of the longitudinal polarization of electrons from unpolarized sources fix only the ratio of the even to the odd part in the Hamiltonian. Explicitly, these experiments imply

$$\begin{aligned} C_S' &\simeq -C_S, \quad C_V' \simeq C_V, \\ C_T' &\simeq -C_T, \quad C_A' \simeq C_A, \end{aligned} \tag{1}$$

where C and C' are the even and odd coupling constants. The ratios C_S/C_V and C_T/C_A are left undetermined, however. The fact that these "parity experiments" do not determine the invariants uniquely can be understood from the following simple example. Consider a $0^+ \rightarrow 0^+$ positron transition. The experiments show that the positron spin points in the direction of motion. Conservation of angular momentum is then satisfied by either vector interaction (where the neutrino and the positron are emitted predominantly into the same hemisphere) with a left-handed neutrino, or scalar interaction with a right-handed neutrino. In order to distinguish between these two cases, one either has to measure the dominant interaction via the positron-neutrino correlation, or one must detect the neutrino polarization.

Recently, Treiman has suggested a type of experiment which provides a new experimental approach to the determination of the dominant coupling constant.[7] He suggests the measurement of the angular distribution of the recoils resulting from the beta decay of polarized nuclei. Since the angular distribution to be expected depends on both nonconservation of parity and the electron-neutrino correlation, one would obtain information on the relative size of the invariants, at least for Gamow-Teller or mixed transitions.

We discuss here some alternative possibilities of obtaining new information about the dominant invariants. In general they yield essentially the same information as the experiment proposed by Treiman, but seem experimentally more feasible. In either K capture or beta emission from an unpolarized source the recoiling nucleus will be strongly polarized along its line of flight for Gamow-Teller or mixed transitions. A measurement of this polarization, together with the results of the parity nonconservation experiments [Eq. (1)], distinguishes between T and A. In the picture mentioned above, this experiment corresponds to an indirect determination of the neutrino polarization via the nuclear polarization. In addition, in K capture the hole left in the atomic K shell is strongly polarized, even for pure Fermi transitions. A detection of this polarization in a pure Gamow-Teller (G-T) transition would distinguish between T and A, and in a pure Fermi (F) transition between S and V. We give in the following section formulas for the polarization of the recoiling nucleus and the hole in the K shell for allowed K capture, as well as for the polarization of the re-

[1] B. M. Rustad and S. L. Ruby, Phys. Rev. **97**, 991 (1955).
[2] J. M. Robson, Phys. Rev. **100**, 933 (1955).
[3] Maxson, Allen, and Jentschke, Phys. Rev. **97**, 109 (1955).
[4] M. L. Good and E. J. Lauer, Phys. Rev. **105**, 213 (1957).
[5] W. P. Alford and D. R. Hamilton, Phys. Rev. **105**, 673 (1957).
[6] Herrmannsfeldt, Maxson, Stähelin, and Allen, Phys. Rev. **107**, 641 (1957).

[7] S. B. Treiman (private communication). Subsequent to communicating the work on the directional asymmetry of recoils from oriented nuclei, Treiman independently considered polarization effects following beta decay [Phys. Rev. **110**, 448 (1958), preceding paper].

coiling nucleus along its line of flight in allowed beta decay. In Sec. III we describe some possible methods to detect these polarizations. In the Appendix a method is given for adapting calculations already in the literature to the problems described here or to others that may come to mind.

II. POLARIZATIONS

Consider first the case of *K capture* by unpolarized nuclei. Since the results of the parity nonconservation experiments [Eq. (1)] imply that the neutrinos emerging from K capture are polarized, it is not surprising that the recoiling nuclei are also polarized. A calculation with the approximation $\alpha Z \ll 1$ shows that the distribution in M values (projection of spin J of final nucleus) measured along the line of flight of the recoil is given by

$$W(M)=\frac{1}{2J+1}\left[1-\left(\frac{B}{1+b}\right)\frac{M}{J}\right]. \tag{2}$$

Here b is the Fierz interference term and B, except for interchange of the initial and final spins, is the coefficient of $\mathbf{J}\cdot\mathbf{p}_\nu/JE_\nu$ in the distribution function for negative electron decay from a polarized nucleus with $E_e=m_e$.[8] (Throughout the paper we take $\hbar=c=1$.) Assuming that Eq. (1) holds exactly, the Fierz term vanishes ($b=0$) and B is given by

$$\begin{aligned}\xi B=2\mathrm{Re}\Bigg\{&|M_{\mathrm{GT}}|^2\lambda_{J'J}(|C_A|^2-|C_T|^2)\\ &+2\delta_{J'J}M_{\mathrm{F}}M_{\mathrm{GT}}\left(\frac{J}{J+1}\right)^{\frac{1}{2}}(C_SC_T^*-C_VC_A^*)\Bigg\},\end{aligned} \tag{3}$$

$$\xi=2|M_{\mathrm{F}}|^2(|C_S|^2+|C_V|^2)+2|M_{\mathrm{GT}}|^2(|C_T|^2+|C_A|^2),$$

$$\begin{aligned}\lambda_{J'J}&=\quad 1 &&\text{if}\quad J'\to J=J'+1\\ &=\quad 1/(J+1) &&\text{if}\quad J'\to J=J'\\ &=-J/(J+1) &&\text{if}\quad J'\to J=J'-1.\end{aligned}$$

Note that J is the spin of the final nucleus, J' the spin of the initial nucleus. The general expressions for b and B are given by Jackson, Treiman, and Wyld.[9] From Eq. (2) one finds that along its line of flight the recoiling nucleus has a polarization:

$$P=\frac{1}{J}\sum_M MW(M)=-\frac{1}{3}\left(\frac{J+1}{J}\right)\left(\frac{B}{1+b}\right). \tag{4}$$

The important point is that $|C_T|^2$ and $|C_A|^2$ enter in Eq. (3) for B with opposite signs. Hence a measurement of the polarization of the recoil in a pure G-T transition will determine whether the G-T interaction is T or A. Speaking physically, the results of the parity nonconservation experiments [Eq. (1)] indicate that if the interaction is T the neutrino emitted is right-handed whereas if the interaction is A the neutrino emitted is left-handed. Thus we expect that for K capture the nucleus will be oppositely polarized in the two cases.

In the K-capture process by unpolarized nuclei the hole left in the K shell is polarized relative to the direction of the recoil. In the same approximation as above, the probability that the captured K-shell electron has a spin projection m ($m=\pm\frac{1}{2}$) relative to the line of flight of the recoil is given by

$$W(m)=\frac{1}{2}\left[1-\frac{H}{1+b}(2m)\right]. \tag{5}$$

Here b is the same Fierz term as before and H is the coefficient of $\boldsymbol{\sigma}\cdot\mathbf{p}_\nu/E_\nu$ in the distribution function for the electron polarization from unoriented nuclei (with $E_e=m_e$).[9] With the choices of Eq. (1), H becomes

$$\xi H=2|M_{\mathrm{F}}|^2(|C_S|^2-|C_V|^2)+\tfrac{2}{3}|M_{\mathrm{GT}}|^2(|C_A|^2-|C_T|^2). \tag{6}$$

The polarization of the hole is then just equal to $(-H)$. Again it should be noted that $|C_A|^2$ and $|C_T|^2$ enter with opposite signs, as do $|C_S|^2$ and $|C_V|^2$. Consequently a measurement of the polarization of the hole in the K shell after K capture is another way of discriminating between the invariants S and V, and T and A. In fact, with the choice of C and C' in Eq. (1), H turns out to be just the negative of the electron-neutrino correlation coefficient a which multiplies $\mathbf{p}_e\cdot\mathbf{p}_\nu/E_eE_\nu$ in the distribution function.

For beta decay of unoriented nuclei, the distribution in M values of the recoiling nucleus along its line of flight can be obtained by a reinterpretation[8] of Treiman's formula for the directional asymmetry of recoils from oriented nuclei:

$$W(M)=\frac{1}{2J+1}\left[1-(A+B)x_1\frac{M}{J}-\frac{2}{3}cx_2\left(\frac{J(J+1)-3M^2}{J(2J-1)}\right)\right]. \tag{7}$$

Dropping all Coulomb corrections and assuming Eq. (1) to hold rigorously, we find

$$\begin{aligned}\xi(A+B)=4\Bigg\{&\pm|M_{\mathrm{GT}}|^2\lambda_{J'J}|C_T|^2\\ &-2\delta_{J'J}M_{\mathrm{F}}M_{\mathrm{GT}}\left(\frac{J}{J+1}\right)^{\frac{1}{2}}\mathrm{Re}(C_VC_A^*)\Bigg\},\end{aligned} \tag{8}$$

$$c\xi=2|M_{\mathrm{GT}}|^2\Lambda_{J'J}(|C_T|^2-|C_A|^2),$$

[8] The reinterpretation of existing formulas for the various coefficients is discussed in the Appendix.

[9] Jackson, Treiman, and Wyld, Phys. Rev. **106**, 517 (1957).

$$\begin{aligned} \Lambda_{J'J} &= 1 && \text{if} \quad J'\to J=J'+1 \\ &= -(2J-1)/(J+1) && \text{if} \quad J'\to J=J' \\ &= J(2J-1)/[(J+1)(2J+3)] && \text{if} \quad J'\to J=J'-1. \end{aligned}$$

x_1 and x_2 are integrals over the momentum distribution given by Treiman.[7] For a pure Gamow-Teller transition the recoil polarization in beta decay determines directly the strength of the tensor interaction.

III. DETECTION OF POLARIZATION

Measurement of the longitudinal polarization of the recoils in K capture or beta decay is evidently difficult. A direct determination by an atomic beam technique may barely be feasible in favorable cases. Indirect methods, based on the observation of a subsequent radiation, offer other possibilities. We outline here two different approaches.[10] (1) One can record coincidences between a recoiling nucleus and the direction and circular (longitudinal) polarization of a subsequent radiation (gamma ray or conversion electron). (2) If the recoiling nucleus, after traveling a certain distance, is caught on a surface, then the position of the surface determines the direction of the recoil. If, in addition, the surface is chosen so that the nucleus does not lose its spin orientation, then one can measure the circular polarization of gamma rays or the longitudinal polarization of conversion electrons from this "secondary" source. If the beta decay or K capture which causes the recoil, is followed by a second beta emission, it may be possible to observe the asymmetry of these beta rays. Such a measurement would be the most sensitive polarization analyzer.

In the following, we discuss the distribution and correlation function to be expected in the various experiments. The formulas apply to both experimental approaches, (1) and (2).

(a) Gamma Radiation

As is outlined in the Appendix, if the general population of M states of the recoil along its line of flight (or in any fixed direction) is of the form

$$W(M)=\frac{1}{2J+1}\left[1+\alpha\frac{M}{J}+\beta\left(\frac{J(J+1)-3M^2}{J(2J-1)}\right)\right], \quad (9)$$

then the angular and circular polarization distribution of a subsequent gamma ray emitted in a transition $J\to J'$ is proportional to

$$W_\gamma(\theta,\tau)=\frac{1}{8\pi}\Bigg\{\sum_\lambda|\delta_\lambda|^2-\alpha\tau\left(\frac{J+1}{3J}\right)^{\frac{1}{2}} \times\sum_{\lambda\lambda'}\delta_\lambda\delta_{\lambda'}F_1(\lambda\lambda'J'J)P_1(\cos\theta) -\beta\left[\frac{(J+1)(2J+3)}{5J(2J-1)}\right]^{\frac{1}{2}} \times\sum_{\lambda\lambda'}\delta_\lambda\delta_{\lambda'}F_2(\lambda\lambda'J'J)P_2(\cos\theta)\Bigg\}, \quad (10)$$

where the angle θ is measured from the line of flight or the fixed direction. $\tau=+1$ corresponds to right circularly polarized radiation (photon spin parallel to its line of flight), $\tau=-1$ to left circularly polarized radiation.[11] The sums over λ,λ' correspond to sums over the gamma multipole mixtures, the notation being that of Alder, Stech, and Winther.[12] The functions $F_k(\lambda\lambda'J'J)$ are defined and tabulated in their paper.

For recoiling nuclei in K capture or beta decay, the M-state populations are given by Eqs. (2) or (7), and the recoil-gamma circular polarization correlation function can be calculated readily from Eq. (10) for any special case. In beta decay it is of interest that the *angular* distribution of the gamma radiation relative to the recoil direction is not isotropic in general ($J=\frac{1}{2}$ is an exception), regardless of the state of circular polarization. Measurement of this anisotropy would determine the coefficient c and distinguish between T and A.

(b) Conversion Electrons

If the gamma radiation subsequent to the beta decay is internally converted, the detection of the longitudinal polarization of the conversion electrons affords an equivalent method of determining the initial polarization of the recoiling nucleus. A calculation of the longitudinal polarization of conversion electrons in the approximation of Dancoff and Morrison[13] (relativistic electrons, but $Ze^2/\hbar v\ll 1$) shows that for magnetic multipole conversion the conversion electrons in a given direction are longitudinally polarized to exactly the same degree as the competing photons in the same direction are circularly polarized (and with the same sense of polarization). For electric multipoles the polarization of the conversion electrons is the same as for magnetic multipoles at very high energies, but smaller at low energies. Although more detailed calculations must be performed on the conversion process, it is clear that the detection of the longitudinal polarization of the conversion electrons in correlation with the line

[10] During the preparation of the present paper, we have learned that M. Goldhaber and co-workers have devised a very ingenious method which is based on resonance scattering. They have applied it successfully to the detection of the polarization of the recoiling nucleus from Eu^{152}. [See Goldhaber, Grodzins, and Sunyar, Phys. Rev. **109**, 1015 (1958).]

[11] This convention is opposite to the ordinary optical definition of right and left circular polarization.

[12] Alder, Stech, and Winther, Phys. Rev. **107**, 728 (1957).

[13] S. M. Dancoff and P. Morrison, Phys. Rev. **55**, 122 (1939).

of flight of the recoil is entirely equivalent to the observation of the circular polarization of the gamma radiation.

(c) Secondary Beta Decay

If the initial K capture or beta emission is followed by a second beta process, the polarization of the residual nucleus along its line of flight can be detected by measurement of the *directional* asymmetry of the secondary beta particles relative to that axis. Such an experiment would probably involve the second approach described above in which the recoils are caught on a surface and allowed to undergo secondary decay. If the polarization of the recoiling nucleus after the first beta decay is P [given by Eq. (4) for K capture, and a corresponding result from Eq. (7) for beta decay], the directional asymmetry of the secondary beta particles is:

$$W_{\beta_2}(\theta)=\frac{1}{4\pi}\left(1+A_2P\frac{v_e}{c}\cos\theta\right). \tag{11}$$

In Eq. (11) the angle θ is measured relative to the recoil direction, and the asymmetry parameter A_2 is appropriate to the secondary beta transition.[9] The Fierz term has been omitted.

(d) X-Rays and Auger Electrons Following K Capture

The detection of the polarization of the hole in the K shell following K capture can perhaps be accomplished by study of the circular polarization of the x-rays emitted after K capture in coincidence with the recoil direction. If only the sum of the K x-rays is observed, there is no circular polarization because the transition is electric dipole and the spin of the electron does not enter. However, the spin-orbit coupling in the atom causes the degenerate $2p$ state to split into $2p_{\frac{1}{2}}$ and $2p_{\frac{3}{2}}$, and the x-ray transitions to the $1s_{\frac{1}{2}}$ state now depend on spin orientations. Consequently the separate x-rays are circularly polarized. With the distribution of empty K-shell m states given by Eq. (5), the x-radiation has angular and circular polarization distribution proportional to:

$$\begin{aligned} W_x(\theta,\tau)&=\frac{1}{24\pi}\left(1+\frac{H}{1+b}\tau\cos\theta\right) \quad \text{for} \quad 2p_{\frac{1}{2}}\rightarrow 1s_{\frac{1}{2}} \\ &=\frac{2}{24\pi}\left(1-\frac{H}{2(1+b)}\tau\cos\theta\right) \quad \text{for} \quad 2p_{\frac{3}{2}}\rightarrow 1s_{\frac{1}{2}}. \end{aligned} \tag{12}$$

As usual, θ is measured relative to the recoil line of flight.

Another even more difficult method for detection of the polarization of the hole is by observation of the longitudinal polarization of the Auger electrons in correlation with the recoil direction. Arguments based on the Pauli principle show that only one Auger line will be appreciably polarized, and the difficulties of resolving one line from the rest and measuring its longitudinal polarization make it a less likely possiblity than a study of the x-rays themselves.

IV. SUMMARY AND CONCLUSIONS

The question of the dominant invariants in nuclear beta decay remains unanswered at the present time. Existing "parity" experiments, while elucidating the relation between the odd and even parts of the beta decay interaction, have not given any new information on the dominant invariants. Some new possibilities, taking advantage of the different polarizations occurring *after* a beta interaction by initially unpolarized nuclei, are described in the present paper. In particular, the polarization of the recoiling nucleus along its line of flight following either K capture or beta decay is given by Eq. (2) or Eq. (7). In the K-capture process the hole in the K shell is polarized according to Eq. (5). Possible methods of detection of these polarizations are discussed.

All of these new approaches involve combinations of the beta-decay coupling constants different from those found in the distribution function for the directional asymmetry and longitudinal polarization of beta particles. With the assignment of odd coupling constants (C') relative to even ones (C) given in Eq. (1), the coefficients involved in the experiments proposed here are such that, in general, $(|C_T|^2-|C_A|^2)$ and $(|C_S|^2-|C_V|^2)$ appear. The presence of differences, rather than sums as in previous "parity" experiments, means that it will be possible to distinguish between T and A in the Gamow-Teller interaction, and between S and V in the Fermi interaction.

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APPENDIX

The formulas (2), (5), and (7) for the polarization of the recoiling nucleus along its line of flight and the polarization of the hole in the K shell after electron capture can be obtained from expressions in the papers by Jackson, Treiman, and Wyld[9] and Treiman[7] by a reinterpretation of their formulas. As an example, consider the problem of recoil polarization after K capture. The relevant term is $\mathbf{J}\cdot\mathbf{p}_\nu$, which occurs in the distribution function for recoil experiments with oriented nuclei [Eq. (2) of reference 9]. Dropping all terms involving $\mathbf{p}_e$ (these terms average to zero for the spherically symmetric state of the electron to be captured), the distribution function is

$$W(\langle\mathbf{J}\rangle|\Omega_\nu)\propto\left[1+b\frac{m_e}{E_e}+B\frac{\langle\mathbf{J}\rangle}{J}\cdot\frac{\mathbf{p}_\nu}{E_\nu}\right]. \tag{A1}$$

In the beta decay of oriented nuclei, $\langle \mathbf{J} \rangle$ is the expectation value of the spin of the original nucleus in some fixed direction. For the recoiling nucleus in K capture, $\langle \mathbf{J} \rangle \cdot \mathbf{p}_\nu / E_\nu$ is replaced by the projection M of the spin of the recoiling nucleus along the line of flight of the neutrino. Since the process of K capture is equivalent to positron emission, the signs in the coefficients b and B must be those for positron emission, and the energy of the positron must be taken as $E_e = -m_e$ in the approximation $\alpha Z \ll 1$.

The change from summing over final nuclear spin states in the beta decay of oriented nuclei to averaging over initial nuclear spin states in the K capture by non-oriented nuclei implies a change in the nuclear matrix elements. This alteration can be expressed as the interchange of the roles of J and J' (J is now the spin of the recoiling nucleus, J' the spin of the initial nucleus), and a change in the sign of $\lambda_{J'J}$, but no other modifications.

With the above substitutions and changes of sign (and the replacement of $\mathbf{p}_\nu = -\mathbf{p}_R$), the population of M states for the recoiling nucleus along its line of flight following K capture is given by Eqs. (2) and (3).

In the polarization of recoils in beta decay a similar reinterpretation of Treiman's distribution of the direction of recoils from oriented nuclei yields the M state population of Eq. (7). In general, terms like $\langle \mathbf{J} \rangle \cdot \mathbf{q}/q$ in a distribution function for initially oriented nuclei can be reinterpreted as the projection M of the spin of the recoiling nucleus along the direction defined by the vector $\mathbf{q}$, provided that the coefficient multiplying it is altered according to the rules given above.

The detection of the recoil polarization in any fixed direction can be accomplished by study of the circular polarization of gamma radiation emitted following the beta decay. Alder, Stech, and Winther[11] have described such a circular polarization correlation in which the line of flight of the beta particle serves as the reference direction, although they did not explicitly divide the beta-decay–gamma-emission process into two separate steps. For an initial gamma-emitting state of angular momentum (J,M) decaying to a final state of angular momentum J', Alder, Stech, and Winther show that angular and circular polarization distribution of the gamma radiation is of the form

$$P_M(\theta,\tau) = \frac{(2J+1)^{\frac{1}{2}}}{8\pi} \sum_{k\lambda\lambda'} (-)^{J+M} (\tau)^k \times (JJ-MM|k0)\delta_\lambda \delta_{\lambda'} F_k(\lambda\lambda' J'J) P_k(\cos\theta). \quad \text{(A2)}$$

The values $\tau = \pm 1$ correspond to right and left circular polarization, respectively.[11] The symbols δ_λ refer to the amplitudes of the 2^λ-pole radiation, and the $F_k(\lambda\lambda' J'J)$ are functions defined and tabulated by Alder, Stech, and Winther.

If the population of magnetic substates of the recoiling nucleus along some fixed direction is

$$W(M) = \frac{1}{2J+1}\left[1 + \alpha\frac{M}{J} + \beta\left(\frac{J(J+1) - 3M^2}{J(2J-1)}\right)\right], \quad \text{(A3)}$$

then the angular and circular polarization distribution of the gamma radiation is given by

$$W_\gamma(\theta,\tau) = \sum\nolimits_M W(M) P_M(\theta,\tau). \quad \text{(A4)}$$

Combining Eqs. (A2) and (A3) yields the result, Eq. (10). The factor limiting the complexity of the gamma radiation distribution is seen to be the complexity of the population of M states from allowed beta decay.

The allowed beta-gamma circular polarization correlation formula of Alder, Stech, and Winther [their Eq. (7)] can be obtained from Eq. (10) by putting $\beta = 0$ and $\alpha = A p_e / E_e$, where A is the coefficient of $\mathbf{J} \cdot \mathbf{p}_e / J E_e$ in the distribution of beta particles from oriented nuclei,[9] altered according to the rules described above.