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Recoil Effects in *K* Capture and β Decay*

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In order to provide a wider range of possible attack on the problem of experimentally establishing the β-decay coupling types, we discuss a variety of effects involving polarization and directional asymmetries of the recoils produced in *K* capture and β decay. The effects in question, though they would be difficult to detect, may well be intrinsically large. They measure, essentially, the sign of the neutrino helicity. In particular, for the *K*-capture reaction $\mu^- + C^{12} \rightarrow B^{12} + \nu$, one has the possibility to test for parity nonconservation and to determine the neutrino helicity in μ-meson capture.

I. INTRODUCTION

We shall discuss here a variety of effects involving the polarization and directional asymmetries of recoil nuclei produced in *K* capture (of μ mesons as well as electrons) and β decay. In processes of both these types, even when the initial nuclei are unpolarized the recoils can be expected to have an appreciable longitudinal polarization at production; and when the parent nuclei are polarized, the recoils can be expected to show appreciable directional asymmetries. These are the analogs of similar effects involving the electrons emitted in β decay; but the information contained in the recoil phenomena is of a complementary nature. In particular, the measurement of these recoil effects would serve to determine the helicity of the emitted neutrinos, something which at present is in doubt for β decay[1] and about which nothing whatever is known in the case of μ-meson capture (where one does not even know if parity conservation is violated).

The detection of the effects discussed here appears to be technically much more difficult than the measurement of electron-neutrino correlations, which—in conjunction with what is already known about β decay—can also establish the neutrino helicity [i.e., distinguish between the *S* and *V* Fermi (F) couplings and between the *T* and *A* Gamow-Teller (GT) couplings]. Our purpose then is merely to provide for consideration a wider range of possible attack on the problem of establishing the β-decay coupling types; also to emphasize that β decay is not only a source of polarized electrons but also of polarized recoil nuclei.

As for μ-meson capture, the particular experiments discussed here would likewise be very difficult technically; but in this case this seems to be a general property of experiments which have been proposed.[2,3]

We restrict our discussion throughout to allowed transitions and we neglect Coulomb effects. In Sec. II we consider the polarization of recoil nuclei produced in *K* capture (of electron or μ meson). In general, if the daughter nucleus has nonzero spin, then for pure G-T or mixed F and G-T transitions the recoils will have an appreciable longitudinal polarization whose sign is determined by the neutrino helicity. In the case of electron *K* capture, the problem of detecting this recoil polarization and of avoiding depolarizing effects might in most cases prove insurmountable. For μ capture, however, there is a favorable circumstance which might make detection experimentally feasible. In the reaction previously discussed in another connection,[4,5] $\mu^- + C^{12} \rightarrow B^{12} + \nu$, the B^{12} (assumed to be in its ground state of spin one) decays rapidly by β^- emission. The asymmetry of the β^- particles about the direction of recoil motion is thus a natural analyzer of the recoil polarization. The μ mesons here need not be polarized.

In Sec. III we discuss the angular distribution of recoils produced in *K* capture involving polarized parent nuclei. The recoil angular distribution is again considered in Sec. IV, this time for β decay from polarized nuclei. Here a rather remarkable effect occurs. In G-T transitions, an appreciable fore-aft asymmetry would be produced in the case of a pure tensor coupling; whereas pure axial vector coupling leads to isotropy. The detection of any asymmetry would thus imply the existence of some tensor coupling. Finally, in Sec. V we calculate the longitudinal polarization of recoils produced in β decay of unpolarized nuclei.

In what follows we use the now standard notation for the discussion of β decay. In the case of μ capture we shall assume that the Hamiltonian is written with the same ordering of spinors as in β decay, with the sole exception that electron is replaced by μ meson.

II. POLARIZATION OF RECOILS IN *K* CAPTURE

Let J' and J be, respectively, the initial and final nuclear spins in a process of electron or μ-meson *K* capture; and let

$$P = \langle J_z \rangle / J$$

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[1] We refer of course to the well-known conflicting results of electron-neutrino correlation experiments: B. H. Rustad and S. L. Ruby, Phys. Rev. **97**, 991 (1955); Maxson, Allen, and Jentschke, Phys. Rev. **105**, 213 (1957); M. L. Good and E. J. Lauer, Phys. Rev. **105**, 213 (1957); W. P. Alford and D. R. Hamilton, Phys. Rev. **105**, 213 (1957); J. M. Robson, Phys. Rev. **100**, 933 (1955); Herrmannsfeldt, Maxson, Stähelin, and Allen, Phys. Rev. **107**, 641 (1957).

[2] H. Überall, Nuovo cimento **6**, 533 (1957).

[3] Huang, Yang, and Lee, Phys. Rev. **108**, 1340 (1957).

[4] T. N. K. Godfrey, Princeton University thesis, 1954 (unpublished).

[5] Jackson, Treiman, and Wyld, Phys. Rev. **107**, 327 (1957).

be the polarization of the recoil nucleus along its line of flight. It is easy to show that

$$P=-\left(\frac{J+1}{3J}\right)\left(\frac{B_-}{1+b}\right), \tag{1}$$

where, in terms of standard notation for the coupling constants,

$$\begin{aligned}\xi B_{\mp}=2\,\mathrm{Re}\Big\{&|M_{\mathrm{GT}}|^2\lambda_{JJ'}[\pm(C_TC_T'^*+C_AC_A'^*)\\&\pm(C_TC_A'^*+C_T'C_A^*)]-\delta_{J'J}M_{\mathrm{F}}M_{\mathrm{GT}}\left(\frac{J}{J+1}\right)^{\frac{1}{2}}\\&\times[+(C_SC_A'^*+C_S'C_A^*+C_VC_T'^*+C_V'C_T^*)\\&+(C_VC_A'^*+C_V'C_A^*+C_SC_T'^*+C_S'C_T^*)]\Big\};\end{aligned} \tag{2}$$

$$\begin{aligned}\xi b=2\,\mathrm{Re}\{&|M_{\mathrm{GT}}|^2(C_TC_A^*+C_T'C_A'^*)\\&+|M_{\mathrm{F}}|^2(C_SC_V^*+C_S'C_V'^*)\};\end{aligned} \tag{3}$$

$$\begin{aligned}\xi=&|M_{\mathrm{F}}|^2(|C_S|^2+|C_V|^2+|C_S'|^2+|C_V'|^2)\\&+|M_{\mathrm{GT}}|^2(|C_T|^2+|C_A|^2+|C_T'|^2+|C_A'|^2);\end{aligned} \tag{4}$$

$$\begin{aligned}\lambda_{JJ'}&=1 &&\text{for } J'\to J=J'+1,\\&=1/(J+1) &&\text{for } J'\to J=J',\\&=-J/(J+1) &&\text{for } J'\to J=J'-1.\end{aligned} \tag{5}$$

The result contained in Eq. (1) of course refers to the polarization immediately after the K capture. Hyperfine interaction and other effects may give rise to a subsequent depolarization. This would have to be considered separately for each experimental situation and we shall attempt no general discussion here.

Let us now consider the special case of μ-meson capture in C^{12}. In general this process leads to nucleon emission; but it has been argued that in 13% of the cases B^{12} is formed directly in its ground state, and also that this transition can be treated as an allowed one in the usual sense, despite the large energy release involved.[4,5] The spin-parity assignments here are $J'=0^+$, $J=1^+$, and one sees that the polarization might be as large as two-thirds (which would be the case if the Fierz term vanishes and the two-component neutrino theory is correct). It has been estimated that the boron recoil will in fact be depolarized by a factor of about one-half as a result of hyperfine interactions.[5]

B^{12} undergoes β^- decay with half-life of 0.025 sec and end-point energy 13.4 Mev. The sign and magnitude of the boron longitudinal polarization could now in principle be determined by observing the directional asymmetries, about the recoil line of flight, of the β-decay electrons. This distribution would have the form $1-\alpha P\cos\theta$, where we in fact know $\alpha\approx 1$.[6] One sees then that the effects might be very large; but one must select β particles coming from recoils moving in a known direction, something which may not be impossible to do. It should be noted that the expression given above for the recoil polarization assumes *unpolarized* μ mesons.

We remark here that if the μ mesons have polarization $\langle\sigma_z\rangle$ along say the z axis, then the angular distribution of the recoils, irrespective of their polarization, is given by

$$\omega(\theta)d\cos\theta=\left[1-\langle\sigma_z\rangle\left(\frac{H}{1+b}\right)\cos\theta\right]d\cos\theta, \tag{6}$$

where, for G-T transitions,

$$\begin{aligned}\xi H=\tfrac{2}{3}|M_{\mathrm{GT}}|^2\,\mathrm{Re}(&C_TC_T'^*+C_AC_A'^*\\&+C_TC_A'^*+C_T'C_A^*).\end{aligned} \tag{7}$$

Thus, in measuring the recoil asymmetry one measures the same quantity as in the polarization experiment discussed above.

III. DIRECTIONAL ASYMMETRIES IN K CAPTURE ON POLARIZED NUCLEI

We give here an expression for the angular distribution of the recoils produced in K capture involving polarized parent nuclei. Let J now refer to the parent nuclear spin and suppose the polarization is along the z axis. One then finds for the recoil angular distribution

$$\omega(\theta)d\cos\theta=\left[1-\frac{\langle J_z\rangle}{J}\left(\frac{B_+}{1+b}\right)\cos\theta\right]d\cos\theta, \tag{8}$$

where the symbols have been defined above.

IV. RECOIL ASYMMETRIES IN β DECAY

We turn now from K capture to β decay. Suppose the parent nucleus has polarization $\langle J_z\rangle/J$ along the z axis. We want to find the angular distribution of the recoils, all other variables, including recoil momentum, being averaged out. To illustrate qualitatively the effects involved, let us first consider the special case of a β^- transition involving the spin change $(J\to J'=J-1)$; and let us suppose the electron mass can be neglected, as well as Coulomb effects. Now in this case we know the electron prefers to come out in the backward hemisphere relative to the nuclear polarization direction. If the coupling is axial vector, the neutrino has an equal preference for the forward hemisphere. Since with the assumption of zero electron mass the electron and neutrino are kinematically equivalent, the recoil can have no preference for either hemisphere—it is distributed with fore-aft symmetry. If the coupling is tensor the neutrino as well as the electron now prefers the backward hemisphere; and the recoil is therefore forced into a preference for the forward hemisphere.

It turns out that these results hold even when the finite electron mass is taken into account: for a pure G-T transition a forward-backward asymmetry can be

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[6] Wu, Ambler, Hayward, Hoppes, and Hudson, Phys. Rev. **105**, 1413 (1957).

produced only if there is some tensor coupling. If the parent nucleus has spin $>\frac{1}{2}$, there will in general also appear a $\cos^2\theta$ term in the recoil angular distribution. Here both T and A contribute, but with opposite sign.

Omitting the Fierz interference term (known to be small in β decay), one finds for the distribution in recoil angle—averaged over recoil momentum—

$$\omega(\theta)d\cos\theta=\{(1+\tfrac{1}{3}c'x_2)-(\langle J_z\rangle/J)(A+B)x_1\cos\theta - c'x_2\cos^2\theta\}d\cos\theta; \quad (9)$$

where

$$c'=c\left\{\frac{J(J+1)-3\langle(\mathbf{J}\cdot\mathbf{j})^2\rangle}{J(2J-1)}\right\}, \quad (10)$$

and $\mathbf{j}$ is a unit vector in the direction of polarization. The coefficients A, B, and c are as given by Jackson *et al.*[7] The coefficients x_1 and x_2 depend on E_0, the total energy released in the β decay, and are given by

$$\begin{aligned} &x_1=X_1/X;\quad x_2=X_2/X;\\ &X=4(E_0^2-1)^{\frac{1}{2}}(2E_0^4-9E_0^2-8) +60E_0\ln[E_0+(E_0^2-1)^{\frac{1}{2}}];\\ &X_1=5(E_0^2-6E_0^3+3E_0+2/E_0+12E_0\ln E_0);\\ &X_2=(E_0^2-1)^{\frac{1}{2}}(4E_0^4-28E_0^2-81) +15(6E_0+1/E_0)\ln[E_0+(E_0^2-1)^{\frac{1}{2}}]. \end{aligned} \quad (11)$$

Here E_0 is measured in units of the electron rest energy.

The crucial point is that the coefficients A and B enter only as a sum in Eq. (9). The full expressions for these coefficients are contained in reference 7. Here we shall simplify the writing by assuming, as is implied by our current knowledge of β decay, that $C_S=-C_S'$, $C_T=-C_T'$, $C_V=C_V'$, $C_A=C_A'$. Then

$$\xi(A+B)=-4\{\pm|M_{\mathrm{GT}}|^2\lambda_{J'J}|C_T|^2 +2\delta_{J'J}M_{\mathrm{F}}M_{\mathrm{GT}}[J/(J+1)]^{\frac{1}{2}}\,\mathrm{Re}\,C_VC_A^*\}, \quad (12)$$

$$\xi c=2|M_{\mathrm{GT}}|^2\Lambda_{J'J}(|C_T|^2-|C_A|^2), \quad (13)$$

where the $\pm$ signs refer to $\beta_{\mp}$ decay and the remaining symbols are defined in reference 7. Note that for an initial nucleus of spin one-half, the coefficient c' in Eq. (10) vanishes. We mention here that for neutron decay the coefficient x_1 has the value: $x_1=0.56$.

V. POLARIZATION OF RECOILS IN β DECAY

Our final topic concerns the longitudinal polarization of the daughter nuclei produced in allowed β decay. We again revert to the notation where J refers to the daughter spin. Let $P=\langle J_z\rangle/J$ denote the polarization along the line of flight. One finds

$$P=-[(J+1)/3J](A'+B')x_1. \quad (14)$$

Here A' and B' are the same, respectively, as the coefficients A and B in reference 7, with the following changes: the symbol $\lambda_{J'J}$ in the formulas of reference is to be replaced by the symbol $(-\lambda_{JJ'})$ defined here in Eq. (5). These changes come about of course because of the reversal in roles of initial and final state polarization.

Just as in the previous section, one sees that for pure G-T transitions there will be appreciable polarization if the coupling is pure tensor, none at all if pure axial vector.

We finally remark that in addition to a possible longitudinal polarization, the recoils will in general have an alignment along the axis of flight, proportional to the coefficient c of Eqs. (10) and (13). Thus, if the recoil undergoes γ decay, there will be a correlation between γ ray and final recoil. Inasmuch as the effect depends on the coefficient c it could serve to distinguish between the T and A couplings. The calculations for such effects are lengthy and will not be presented here. Possible experiments, however, are under study.

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It is a pleasure to thank Professor R. Sherr for many stimulating discussions. Calculations similar to those contained in the present note, but going beyond it in a number of respects, have been made independently by Frauenfelder, Jackson, and Wyld,[8] whom I wish to thank for interesting communications.

Notes added in proof.—Goldhaber, Grodzins, and Sunyar [Phys. Rev. **109**, 1015 (1958)], have recently in effect measured the recoil polarization in electron K capture on Eu^{152m}. They conclude that the Gamow-Teller coupling is mainly axial vector.

Our discussion of μ capture fails to take into account the hyperfine interaction of μ meson and nucleus, the effects of which have recently been pointed out by Bernstein, Lee, Yang, and Primakoff (to be published). However, since such effects can arise only when the nucleus has nonzero spin, our discussion of μ capture on C^{12} requires no change.

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[7] Jackson, Treiman, and Wyld, Nuclear Phys. 4, 206 (1957).

[8] Frauenfelder, Jackson, and Wyld, Phys. Rev. **110**, 451 (1958), following paper.