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Quadrupole Moments of As^{75}, La^{139}, and Hg^{201}

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It is shown that the level $5d^34s\ {}^5F_1$ of Ta II is approximately free from perturbation, making the calculation of the quadrupole moment simple. Then it is shown that the quadrupole coupling constant (measured by Brown and Tomboulian) in the hyperfine structure (hfs) of this level can be interpreted if one assumes a rather large antishielding effect $\Delta(5d^34s)=-0.77$. In the course of the calculation the magnetic moment $\mu(Ta^{181})=2.4\pm0.2$ nm was obtained. Upon assuming the shielding corrections $\Delta(5d6s)=-0.3$ and $\Delta(6s6p)=-0.1$ for any heavy element around Lu, the quadrupole moments $Q(La^{139})=0.2_1\pm0.04$ barn and $Q(Hg^{201})=0.4_2\pm0.04$ barn were obtained. In the case of La II $5d4f\ {}^3D_1$, a negative (instead of positive) quadrupole moment was obtained by neglecting the shielding correction. This can be interpreted by assuming a large antishielding effect in the $4f$ electron. In connection with this a tentative value $Q(Er^{167})=4$ barns was deduced. Upon assuming $\Delta(4p5s)=-0.16$ for the hfs of the spectrum of As II, the value $Q(As^{75})=0.27\pm0.04$ barn was obtained. From the hfs of a level of the configuration $4p5p$ of As II, the value 0.12 ± 0.07 barn was obtained, the shielding correction being neglected; and this is interpreted to be due to a possible large antishielding effect in the $5p$ electron. The same tendency seems to be detectable in the published determination of $Q(K^{39})$ from the $5p$ state of K I.

I. INTRODUCTION

THE calculations of Sternheimer[1] show that the atomic core shields or antishields the nuclear quadrupole coupling. In a previous paper[2] based on this idea, quadrupole moments of some nuclei were determined by hyperfine structure (hfs) measurements. The present work will treat the problem more rigorously, and we shall see that in certain cases of heavy elements the antishielding effect is so large that the neglect of the effect in the calculation of a quadrupole moment leads not only to an erroneous magnitude but also to the wrong sign.

The quadrupole moment deduced from the hfs neglecting the shielding or antishielding effect will be denoted by Q', while the true quadrupole moment will be denoted by Q. Putting $Q=(1+\Delta)Q'$, we shall call Δ the shielding correction, regardless of whether Δ is positive or negative. [$1+\Delta$ is equal to Sternheimer's $1/(1-R)$.] The shielding or antishielding effect predominates according as Δ is positive or negative, respectively. If there are two kinds of non-s electrons, we need two Δ's.[3]

II. MAGNETIC MOMENT OF Ta^{181}

In order to test the purity of a level of Ta II by means of a magnetic moment determination, the hfs of some levels of Ta I was first examined. On the plates which Kamei had taken,[4] the hfs of the lines Ta I λ3996 [$5d^36s^2\ {}^4P_{\frac{1}{2}}-5d^36s({}^5F)6p\ {}^6P_{\frac{3}{2}}$], λ4692 [$5d^46s\ {}^6D_{\frac{1}{2}}-5d^36s({}^5F)6p\ {}^6P_{\frac{3}{2}}$], and λ4740 [$5d^46s\ {}^6D_{\frac{3}{2}}-5d^36s({}^5F)6p\ {}^6P_{\frac{3}{2}}$][5] were measured; the result is summarized in Table I. The interval factor A and the quadrupole coupling constant B were calculated according to the usual formula

$$E=E_0+\tfrac{1}{2}AC+BC(C+1),$$
$$C=F(F+1)-I(I+1)-J(J+1).$$

Kamei[4] solved the energy matrix for the configuration $5d^36s^2$ given by Marvin,[6] and obtained $\zeta=1650$, $F_2=578$ and $F_4=60$. If we assume that these parameter values together with $G_2=2200$ hold for the configuration $5d^46s$ and that there is no mixing of $5d^36s^2$ and $5d^46s$, then we can determine the composition of the wave function of each level of $5d^46s$, using the energy matrix given by Bozman and Trees.[7] It can be easily shown that the purity of $5d^46s\ {}^6D_{\frac{3}{2}}$ and ${}^6D_{\frac{1}{2}}$ is about 95%. Then we can calculate the approximate magnetic moment (μ) by the LS-coupling formulas:

$$A(d^4s\ {}^6D_{\frac{3}{2}})=(13/75)a(s)+(473a_d'-163a_d''+664a_d''')/375,$$
$$A(d^4s\ {}^6D_{\frac{1}{2}})=(7/15)a(s)+(98a_d'-58a_d''+544a_d''')/75,$$
$$a_d'=a(d_{\frac{5}{2}}),\quad a_d''=a(d_{\frac{3}{2}}),\quad a_d'''=a(d_{\frac{5}{2}},d_{\frac{3}{2}}). \tag{1}$$

From Kamei's work on the configuration $5d^36s^2$, we get

TABLE I. Hfs of the spectrum of Ta I.

Level	Intervals (cm^{-1})	A ($10^{-3}\ cm^{-1}$)	B ($10^{-3}\ cm^{-1}$)
$5d^36s^2\ {}^4P_{\frac{1}{2}}$	-0.116_3	$-29._1$	
$5d^36s({}^5F)6p\ {}^6P_{\frac{3}{2}}$	0.359_4, 0.297_6, 0.229_1	73.5_6	-0.07
$5d^46s\ {}^6D_{\frac{1}{2}}$	0.414_9	103.7	
$5d^46s\ {}^6D_{\frac{3}{2}}$	0.202_1, 0.161_6, 0.120_0	40.0_6	~0.02

[1] R. M. Sternheimer, Phys. Rev. **95**, 736 (1954); **105**, 158 (1957). The kindness of Dr. Sternheimer in showing me the manuscript of the latter article prior to publication is greatly appreciated.
[2] K. Murakawa and T. Kamei, Phys. Rev. **105**, 671 (1957).
[3] K. Murakawa, Phys. Rev. **98**, 1285 (1955).
[4] T. Kamei, Phys. Rev. **99**, 789 (1955).
[5] The classification and the term notation of the spectrum of Ta I were taken from Klinkenberg, van den Berg, and van den Bosch, Physica **16**, 861 (1950); **18**, 221 (1952).
[6] H. H. Marvin, Phys. Rev. **47**, 521 (1935).
[7] W. R. Bozman and R. E. Trees, J. Research Natl. Bur. Standards **58**, 95 (1957).

$a_d'=0.0186$ cm^{-1}.[8] Putting this and the observed value of $A(5d^46s\ ^6D_{\frac{3}{2}})$ (see Table I) into the formula (1), we get $a(s)=0.235$ cm^{-1}. Putting in the Goudsmit-Fermi-Segrè formula

$$\mu = a(s)\frac{117.8I}{ZZ_0^2F(1-\delta)(dn^*/dn)/n^{*3}}, \qquad (2)$$

this value of $a(s)$, $Z=73$, $Z_0=1$, $1-\delta$ (finite nuclear volume correction factor) $=0.913$, F (relativity correction factor) $=1.895$, $(dn^*/dn)/n^{*3}=0.360$, and $I=\frac{7}{2}$, we get $\mu=2.1$ nm. Similarly we get from $A(5d^46s\ ^6D_{\frac{1}{2}})$ the value $\mu=2.2$ nm.

In reality the levels $5d^46s\ ^6D_{\frac{3}{2}}$ and $^6D_{\frac{1}{2}}$ are somewhat perturbed by the configuration $5d^36s^2$ (see reference 5), so we conclude that the true value of μ is somewhat greater than 2.2 nm.

We now turn our attention to the level $5d^36s\ ^5F_1$ of Ta II. Brown and Tomboulian[9] measured the hfs of this level, and obtained $A=(-0.079\pm0.001)$ cm^{-1}, $B=(-0.77\pm0.04)\times10^{-3}$ cm^{-1}. Under the assumption of an LS coupling they got $a(s)=0.405\pm0.005$ cm^{-1}. From the value of B, Trees[10] obtained $Q'=11.8$ barns. Later Brown and Tomboulian[11] expressed the supposition that this level might be perturbed and that the correct value of Q' would be around 6 barns. Recent work of Trees *et al.*[12] on the analysis of the spectrum of Ta II shows that this level is not appreciably perturbed. Their table lists $g_{\mathrm{obs}}=0.000$, $g_{\mathrm{calc}}=0.058$, $g_{LS}=0.000$ for this level.

Putting the above-mentioned value of $a(s)$ into the formula (1) [$Z_0=2$, $F=1.89$, $1-\delta=0.913$, $(dn^*/dn)/n^{*3}=0.141$], one gets

$$\mu(\mathrm{Ta}^{181})=2.4\pm0.2 \text{ nm}.$$

This value is just what one would expect from the hfs of Ta I, so this might be considered to be another proof of the conclusion that the LS-coupling calculation (without perturbation) for $5d^36s\ ^5F_1$ is still a good approximation for our purpose. The value of μ given here is somewhat larger than the one published in the previous literature which neglected the perturbation in the spectrum of Ta I.

III. QUADRUPOLE MOMENTS OF La^{139} AND Er^{167}

The foregoing discussion justifies again the process by which Trees[10] obtained $Q'=11.8$ barns from Ta II $5d^36s\ ^5F_1$. Since in the previous work[2] $Q(\mathrm{Ta}^{181})=2.7\pm0.3$ barns was obtained, we conclude that $\Delta=-0.77$ for the configuration $5d^36s$ of Ta II. This is quite different from the value $\Delta=-0.3$ that was assumed for the configuration $5d^n$. This means that we cannot assume that in general $\Delta(5d^n6s)=\Delta(5d^n)$, although the s electron contributes nothing to the quadrupole coupling.

From the measurement of the hfs of the levels $5d6s\ ^3D_{3,2,1}$ of Lu II by Gollnow,[13] it can be concluded that $Q'(\mathrm{Lu}^{175})=5.4$ barns, whereas it was shown previously[2] that $Q(\mathrm{Lu}^{175})=3.9$ barns. This means that $\Delta=-0.3$ for the configuration $5d6s$ of Lu II. On the other hand Gollnow's measurement gave $Q'=4.2$ barns for the configuration $6s6p$ of Lu II. He concluded that this configuration is perturbed and that therefore the value of Q' ($=4.2$ barns) is to be discarded. A closer examination of the level system of Lu II fails, however, to reveal any level that can perturb this configuration, so his conclusion is invalid. We get, therefore, $\Delta=-0.1$ for the configuration $6s6p$ of Lu II.

We shall assume that $\Delta(5d6s)=-0.3$, and $\Delta(6s6p)=-0.1$ for the spectrum of any heavy element around Lu.

With respect to La^{139}, it was shown previously[2] that $Q'=+0.5$ barn for the configuration $5d^26s$ of La I, and $Q'=+0.3$ barn for the configuration $5d6s$ of La II. From the latter and $\Delta(5d6s)=-0.3$, we get $Q(\mathrm{La}^{139})=(0.2_1\pm0.04)$ barn [instead of the previous 0.3_5 barn which was deduced under the assumption $\Delta(5d^26s)=\Delta(5d^2)$ that is now known to have a poor validity], and from the former we get $\Delta=-0.6$ for the configuration $5d^26s$ of La I. The probable error comes from the experimental uncertainty but does not include the uncertainty of the value of Δ.

Using a water-cooled hollow-cathode discharge tube and a Fabry-Pérot etalon, the hfs of the line La II λ6174 ($5d^2\ ^3P_0-5d4f\ ^3D_1$) was measured. This gave directly the hfs of the upper level: the hfs intervals 0.0626 and 0.0481 cm^{-1} were obtained, giving $A=0.0138$ and $B=0.005\times10^{-3}$ cm^{-1}. The configuration $5d4f$ of La II is approximately of LS-coupling,[14] so we can use the LS-coupling formula:

$$\begin{aligned}(df\ ^3D_1{}^1|\omega/r^3|df\ ^3D_1{}^1)=&(1/35)[(246/175)R_2' \\ &+(58/25)R_2''+(48/175)S_2]\langle r_d^{-3}\rangle \\ &+(1/35)[(205/49)R_3'-(816/245)R_3'' \\ &-(552/245)S_3]\langle r_f^{-3}\rangle,\end{aligned}$$

in which we have put $\omega=3\cos^2\theta-1$ for brevity. R', R'', and S are relativity correction factors for $\langle r^{-3}\rangle$ according to Casimir.[15] Neglecting the shielding correction and putting $R_2'=1.027$, $R_2''=1.095$, $S_2=1.027$ and $R_3'=1.006$, $R_3''=1.012$, $S_3=1.006$ in the above formula and then putting $\zeta(5d)=542$, $\zeta(4f)=443$, $Z^*(5d)=39.7$, $Z^*(4f)=24$ in a formula very similar to the one (7) of

[8] It is assumed that the deviation from the theoretical hfs formulas for d electrons that was observed in medium-heavy elements [K. Murakawa, J. Phys. Soc. (Japan) **11**, 422 (1956); **11**, 774 (1956)] does not occur in heavy elements.

[9] B. M. Brown and D. H. Tomboulian, Phys. Rev. **88**, 1158 (1952).

[10] R. E. Trees, Phys. Rev. **92**, 308 (1953).

[11] B. M. Brown and D. H. Tomboulian, Phys. Rev. **91**, 1580 (1953).

[12] Trees, Cahill, and Rabinowitz, J. Research Natl. Bur. Standards **55**, 335 (1955).

[13] H. Gollnow, Z. Physik **103**, 443 (1936).

[14] E. U. Condon and G. H. Shortley, *The Theory of Atomic Spectra* (Cambridge University Press, New York, 1935), p. 206.

[15] H. Casimir, Verhandel. Teylers Tweede Genootschap, Haarlem **11**, (1936).

reference 3, we get $Q'=-0.2_5$ barn, whereas the correct value is $Q=(0.2_1\pm0.04)$ barn. This discrepancy arises apparently from the neglect of the shielding correction. Let us assume that $\Delta=-0.3$ for the $5d$ electron in the configuration $5d4f$. Then in order to get Q with the correct sign and with approximately the correct magnitude, it can easily be shown that we have to choose $\Delta(4f)=-0.7$. Since we are not treating the configuration $5d^n$, the assumption $\Delta(5d)=-0.3$ would not be accurate, and therefore the result $\Delta(4f)=-0.7$ would be only roughly correct. On the other hand, we have assumed that $\Delta(s)=0$, $\Delta(6p^n)\approx-0.1$, and $\Delta(5d^n)=-0.3$. Therefore we expect $\Delta(4f)=-0.6$ by extrapolation, so that the large antishielding of the $4f$ electron is not surprising. We shall adopt $\Delta(4f)=-0.6$ for the configuration $4f^n$.

Fitting their paramagnetic resonance measurements to Elliott and Stevens' calculation[16] for the ion Er^{+++} $4f^{11}\,{}^4I$, Bogle, Duffus, and Scovil[17] obtained $Q(Er^{167})=10.2$ barns. In the work of Elliott and Stevens, the shielding correction was neglected. If we adopt $\Delta(4f)=-0.6$, we get $Q(Er^{167})=4$ barns. This would be nearer the true value than the one given by Bogle *et al.*

From Lew's preliminary report[18] on the hfs of the level $4f^3\,{}^4I_{9/2}$ of Pr I, Suwa and the author[19] deduced $Q'=-0.05$ barn. Lew[20] published the details of his investigation and obtained $Q'=-0.054$ barn. If we adopt $\Delta(4f)=-0.6$, we get $Q(Pr^{141})\approx-0.02$ barn.

IV. QUADRUPOLE MOMENT OF Hg^{201}

In a previous paper,[3] necessary data and formulas for deriving $Q(Hg^{201})$ from the hfs of the levels $6s6p\,{}^3P_2$ and ${}^1P_1$ were presented. We now see that $\Delta(6p)$ and $\Delta(5d)$ are -0.1 and -0.3 instead of the previous 0.019 and 0.085, respectively. Making these changes, we get $Q=0.4_2$ and 0.3_8 barn from the levels ${}^3P_2$ and ${}^1P_1$, respectively. The former can be given a somewhat larger weight than the latter. We might thus consider

$$Q(Hg^{201})=0.4_2\pm0.04 \text{ barn}$$

the best value available at present. Pound and Wertheim[21] determined $Q(Hg^{201})$ by quite a different kind of experiment to be $0.46_{-0.11}^{+0.28}$ barn. This is in agreement with the result obtained here.[22]

V. QUADRUPOLE MOMENT OF As^{75}

From the hfs of the level $4s4p^3\,{}^3D_1$ of As II, Suwa and the author[23] deduced $Q'=0.32\pm0.05$ barn. From the measurement of Tolansky[24] on the hfs of the line $\lambda5657$ $(4p5s\,{}^3P_2-4p5p\,{}^3P_1)$ of As II, Schüler and Marketu[25] had previously deduced $Q'=0.29$ barn. A closer examination of the level system of As II reveals the fact that the configuration $4p5s$ is perturbed by the configuration $4s4p^3$, and a more accurate calculation using the data of Tolansky for the line $\lambda5657$ shows that $Q'=0.32$ barn. Concerning the shielding correction, Sternheimer's calculation[1] is available for the $4p$ state of K I. Since no calculation was carried out for $\Delta(4p5s)$ of As II, we shall tentatively assume that it is the same as in the case of $4p$ of K I, namely -0.16. Then we get

$$Q(As^{75})=0.27\pm0.04 \text{ barn}.$$

By using a water-cooled hollow cathode discharge tube, the hfs of the line As II $\lambda4986$ $(5s\,{}^3P_1-5p\,{}^3P_1)$[26] was investigated, and this was found to consist of three components: 0 (3), 0.1954 (2), 0.4191 (1) cm^{-1}. The relative intensity is in parentheses. The final level has the hfs intervals 0.1982 and 0.1273 cm^{-1}. From these it is found that $A=0.0025$ and $B=(0.15\pm0.09)\times10^{-3}$ cm^{-1} for the initial level.

Green and Barrows[26] solved the energy matrix for $4p5p$ (As II) given by Johnson[27] and obtained $\zeta(4p)=1840$ and $\zeta(5p)=380$, etc. Inserting the parameter values given by Green and Barrows in the energy matrix and using the usual procedure, we get the actual wave function of $4p5p\,{}^3P_1$ (As II) decomposed into LS-coupling wave functions:

$$4p5p\,{}^3P_1'=K_1\,{}^3D_1+K_2\,{}^3P_1+K_3\,{}^1P_1+K_4\,{}^3S_1, \quad (3)$$

$K_1=0.275$, $K_2=0.834$, $K_3=-0.369$, $K_4=-0.303$. Then the matrix element of ω/r^3 is given by

$$({}^3P_1'|\omega/r^3|{}^3P_1')=K_1^2({}^3D_1|\omega/r^3|{}^3D_1)+K_2^2({}^3P_1|\omega/r^3|{}^3P_1)+\cdots+2K_1K_2({}^3D_1|\omega/r^3|{}^3P_1)+\cdots. \quad (4)$$

On the other hand, we get for the configuration $npn'p$ the following LS-coupling formulas:

$$({}^3D_1|\omega/r^3|{}^3D_1)=\sum(-1/150)(R'+20S)\langle r^{-3}\rangle,$$
$$({}^3P_1|\omega/r^3|{}^3P_1)=\sum(-1/10)R'\langle r^{-3}\rangle,$$
$$({}^1P_1|\omega/r^3|{}^1P_1)=\sum(1/15)(R'+2S)\langle r^{-3}\rangle,$$
$$({}^3S_1|\omega/r^3|{}^3S_1)=0,$$

[16] R. J. Elliott and K. W. H. Stevens, Proc. Phys. Soc. (London) **A64**, 205 (1951).
[17] Bogle, Duffus, and Scovil, Proc. Phys. Soc. (London) **A65**, 760 (1952).
[18] H. Lew, Phys. Rev. **89**, 530 (1953).
[19] K. Murakawa and S. Suwa, J. Phys. Soc. (Japan) **9**, 93 (1954).
[20] H. Lew, Phys. Rev. **91**, 619 (1953).
[21] R. V. Pound and G. K. Wertheim, Phys. Rev. **102**, 396 (1956).
[22] In a recent paper J. Blaise and H. Chantrel [J. phys. **18**, 193 (1957)] have calculated $Q(Hg^{201})$ from the hfs of the levels $6s6p\,{}^3P_2$, ${}^3P_1$ and ${}^1P_1$, without taking the perturbation of the configuration $5d^96s^26p$ into account. Referring to the formulas of G. Breit and L. A. Wills [Phys. Rev. **44**, 470 (1933)], they conclude that the hfs theory is inadequate to account for the observed A's of the three levels mentioned above and therefore for the observed $B(6s6p\,{}^1P_1)$. This uncertainty disappears as soon as we introduce the perturbation, as was done in reference 3. If their calculations are modified according to this idea, their final result would be somewhat modified and would leave no essential disagreement. The kindness of Dr. Blaise in showing me their manuscript prior to publication is greatly appreciated.
[23] K. Murakawa and S. Suwa, Rept. Inst. Sci. and Technol. Univ. Tokyo **6**, 209 (1952).
[24] S. Tolansky, Proc. Roy. Soc. (London) **137**, 541 (1932).
[25] H. Schüler and M. Marketu, Z. Physik **102**, 703 (1936).
[26] J. B. Green and W. M. Barrows, Phys. Rev. **47**, 131 (1935).
[27] M. H. Johnson, Phys. Rev. **38**, 1628 (1931).

$$({}^3D_1|\omega/r^3|{}^3P_1)=\sum{}'[-1/\{30(15)^{\frac{1}{2}}\}]\times(5R'+22S)\langle r^{-3}\rangle, \quad (5)$$

$$({}^3D_1|\omega/r^3|{}^1P_1)=\sum[-1/\{15(30)^{\frac{1}{2}}\}](R'-S)\langle r^{-3}\rangle,$$

$$({}^3D_1|\omega/r^3|{}^3S_1)=\sum[-4/\{15(20)^{\frac{1}{2}}\}](R'+2S)\langle r^{-3}\rangle,$$

$$({}^3P_1|\omega/r^3|{}^1P_1)=\sum{}'(2^{\frac{1}{2}}/30)(R'-S)\langle r^{-3}\rangle,$$

$$({}^3P_1|\omega/r^3|{}^3S_1)=\sum{}'[-2/\{15(3)^{\frac{1}{2}}\}](R'-S)\langle r^{-3}\rangle,$$

$$({}^1P_1|\omega/r^3|{}^3S_1)=\sum[2(2)^{\frac{1}{2}}/\{15(3)^{\frac{1}{2}}\}](R'-S)\langle r^{-3}\rangle.$$

$\sum$ means that similar expressions for np and $n'p$ should be added. $\sum'$ means that the expression for np and similar one for $n'p$ with reversed sign should be added. In these formulas it is always meant that $M=J$. The sign of the off-diagonal element is compatible with the sign of the spin-orbit interaction matrix element given by Johnson,[27] when his indices 1 and 2 refer to the np and $n'p$ electron, respectively.

Inserting $R'=1.034$, $S=1.058$ [$Z^*(4p)=29$, $Z^*(5p)=28$] in (5) and then in (4), it is seen that the contributions to $\langle\omega/r^3\rangle$ from the $4p$ and $5p$ electrons are $-0.136\langle r_{4p}^{-3}\rangle$ and $+0.089\langle r_{5p}^{-3}\rangle$, respectively. Inserting these values and the value of $B(4p5p\,{}^3P_1)$ and $I=\frac{3}{2}$ in the usual formula for obtaining Q, we get $Q'=(0.12\pm0.07)$ barn. This Q' is definitely smaller than $Q=0.27$ barn. If we assume that $\Delta(4p)=-0.16$ in the configuration $4p5p$, then we conclude that $\Delta(5p)\approx-0.8$, namely the antishielding effect in $5p$ is larger than in $4p$. If $\Delta(4p)>0$ contrary to the above-mentioned assumption, then the conclusion would have to be modified. Anyhow, this shows that in deducing the value of Q in a medium-heavy element we must be careful, if a highly excited p electron is involved.

In this connection two recent papers concerning $Q(K^{39})$ may be quoted. The values $Q'=0.07\pm0.02$ barn and $Q'=0.11\pm0.02$ barn were obtained from the $4p$ state[28] and the $5p$ state[29] of K I, respectively. Since the shielding correction is neglected in these two values, and since $\Delta(4p)$ and $\Delta(5p)$ are in general not necessarily equal, it is not necessary that the two values of Q' are equal. Actually the above-mentioned values can be considered to be in agreement only in the extreme case of the combined errors, but this is not likely. It might be assumed in the case of K I that most probably the antishielding effect is larger in the $5p$ state than in the $4p$ state.

In summary, it may be concluded that although the present work is still confined to a one-digit discussion of the shielding effect, it serves to demonstrate that, without considering the shielding effect, different values of Q would be obtained from different terms of an isotope.*

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[28] P. Buck and I. I. Rabi, Phys. Rev. **107**, 1291 (1957).

[29] G. J. Ritter and G. W. Series, Proc. Roy. Soc. (London) **A238**, 473 (1957).

* *Note added in proof.*—After sending the manuscript to the editor, we received the article of Y. Ting [Phys. Rev. **108**, 295 (1957)] who measured the hfs of La^{139} in the level $5d6s^2\,{}^2D$ and obtained $Q=(0.268\pm0.010)$ barn. In addition to the procedures described by him in deducing Q, two points seem to require further consideration. First, from the theory of G. Racah [Phys. Rev. **63**, 367 (1943)] it follows that $(d^2s\,{}^2D|e^2/r|ds^2\,{}^2D)=-(70)^{\frac{1}{2}}H_2$, [$H_2=R^2(dd,ds)/35$], and therefore $5d6s^2\,{}^2D$ is perturbed by $5d^26s\,{}^2D$. Second, Sternheimer's correction consists of shielding part (considered by Ting) and antishielding part (note considered by Ting). The latter is also important in this case.

In the calculation of Q it is sufficient in this case to put $5d6s^2\,{}^2D'=K_1\,{}^2D(5d6s^2)+K_2\,{}^2D(5d^2\,6s)$, $K_1^2=0.825$, $K_2^2=0.175$. Putting this relation and $\Delta(5d6s^2)=-0.3$ and $\Delta(5d^2\,6s)=-0.6$ (see Sec. III of the present work), and the value of B measured by Ting in the usual formula for obtaining Q, we get $Q(La^{139})=0.2_1$ barn from both $J=\frac{3}{2}$ and $\frac{5}{2}$. This is in complete agreement with the result of the present work.