the ionization limit gave a best fit to the dispersion curve. The reason for the discrepancy between their results and ours for the resonance oscillator strengths is not immediately clear. However, the Wolf-Herzfeld curve fitting covers only a relatively small region of the spectrum, and some clarification might result from an extension of their analysis toward the resonance energies.[23]

One final independent check on the wave functions used in this calculation should be mentioned. The ground state functions were found by the Hartrees[7] to give as accurate a prediction of the diamagnetic susceptibility as could be checked by existing experiments, i.e., to within 5 or 10%.

In view of the foregoing facts, the predicted absorption oscillator strengths as derived from a computed dipole matrix element and spectral term values, i.e., $^1f(1049\text{ A})=0.20$ and $^3f(1067\text{ A})=0.05$, may be considered reliable probably to within 10 or 20%.

[23] The region covered is such that the denominator of the polarizability expression, $\nu_0^2-\nu^2$, varies by only 10% from one end of the region to the other, where ν_0 is the resonance frequency used in the analysis

ACKNOWLEDGMENTS

The present calculation of Hartree-Fock functions was made possible by the generous help of Dr. W. W. Piper of the General Electric Research Laboratories, Schenectady, New York, who has made his digital computer program available to us and who actively participated in performing the first set of iterations of the triplet 4s argon function. It is a pleasure to acknowledge several fruitful discussions with Dr. Piper. The author is also indebted to Professor T. A. Keenan of the University of Rochester Computing Center for his aid in adapting the wave function program to the Center's input-output scheme, to Professor D. L. Dexter of this laboratory for many helpful suggestions, and to Mr. M. P. Rimmer for his assistance in programming the numerical integrations used in computing atomic interaction energies.

PHYSICAL REVIEW VOLUME 110, NUMBER 2 APRIL 15, 1958

K Capture—Positron Ratios for First-Forbidden Transitions: Sb^{122}, Rb^{84}, I^{126}, As^{74}†

M. L. PERLMAN, JOAN P. WELKER, AND M. WOLFSBERG
Chemistry Department, Brookhaven National Laboratory, Upton, New York

(Received December 26, 1957)

The theory for the relative probabilities of *K*-electron capture and of positron emission is reinvestigated in the cases of allowed and first-forbidden transitions. Effects of screening and of finite nuclear size are discussed and calculated. Careful comparison is made between theory and available experimental results for allowed and "unique" first-forbidden transitions; details and results of a new measurement for the transition $Sb^{122}\rightarrow Sn^{122}$ ($\Delta J=2$, yes) are presented. Other types of first-forbidden transitions are analyzed in terms of the relative contributions of the various matrix elements which appear in the interaction combinations *S*, *T*, *P* and *V*, *A*; and the experimental results for the $\Delta J=0$, yes transitions in As^{74}, Rb^{84}, and I^{126} are discussed. Simple formulas are presented for the evaluation of the coefficients of the several matrix elements.

INTRODUCTION

THE study of the shapes of beta spectra has been used to gain insight into the nature of beta decay. Another valuable tool of similar nature is the study of the relative probabilities of *K* capture and positron emission in effecting transitions between characterized nuclear states. In this paper special emphasis will be placed on the comparison between theory and experiment for first-forbidden transitions. New experimental data will be presented for a first-forbidden transition with spin change two which occurs in the decay of Sb^{122}.

The compilations of theoretical beta-decay probabilities for the well-known five types of interactions, pure and mixed,[1] provide a simple basis from which one may easily calculate the theoretical *K*-capture probabilities. Using Pursey's notation for cross terms, the probability of *n*th-forbidden positron decay[2] is given by

$$P_+=\frac{1}{2\pi^3}\sum_{X,Y}G_XG_Y\tfrac{1}{2}(1+\delta_{XY})\times\int_1^{W_0}pW(W_0-W)^2F_0(W,Z)C_n(X,Y)dW$$

$$=\frac{1}{2\pi^3}\sum_{X,Y}G_XG_Y\tfrac{1}{2}(1+\delta_{XY})\mathcal{C}_n(X,Y)\times\int_1^{W_0}pW(W_0-W)^2F_0(W,Z)L_0dW,\quad(1)$$

† Research performed under the auspices of the U. S. Atomic Energy Commission.

[1] E. J. Konopinski and G. E. Uhlenbeck, Phys. Rev. **60**, 308 (1941); E. Greuling, *ibid.* **61**, 568 (1942); A. M. Smith, *ibid.* **82**, 955 (1951); D. Pursey, Phil. Mag. **42**, 1193 (1951); M. E. Rose and R. K. Osborne, Phys. Rev. **93**, 1315 (1954).

[2] S. R. De Groot and H. A. Tolhoek, Physica **16**, 456 (1950).

where the nomenclature is the usual one, X and Y referring to the five types of interaction, $C_n(X,Y)$ being $C_n(X)$ for $X=Y$, and $\mathcal{C}_n(X,Y)$ being newly defined by the equation. The G's are the respective coupling constants. Expression (1) holds only if both time-reversal invariance and parity conservation are valid with respect to the decay process. In the absence of their validity, G_XG_Y should be replaced by $G_XG_Y^* + G_X'G_Y'^*$, where the primes refer to the coupling constants for the parity nonconserving interaction.[3] Complex interaction constants would result from time-reversal noninvariance. For the present discussion, we retain expression (1) [and the corresponding expression (2) below] since this simplification does not affect any of the conclusions except in one instance. The subsequent evaluation of the ratio of the contributions of the tensor and axial vector interactions does depend upon the use of the simple equations (1) and (2), but it appears proper to proceed in this manner so that the present work may be compared with previous evaluations of the Fierz interference term in allowed transitions. In the same notation K-capture probability becomes

$$P_K=\frac{1}{4\pi^2}\sum_{X,Y}G_XG_Y\tfrac{1}{2}(1+\delta_{XY})(W_0+\epsilon_K)^2C_{nK}(X,Y)$$
$$=\frac{1}{4\pi^2}\sum_{X,Y}G_XG_Y\tfrac{1}{2}(1+\delta_{XY})\times(W_0+\epsilon_K)^2g_K^2\mathcal{C}_{nK}(X,Y), \quad (2)$$

where ϵ_K is the total relativistic energy of the K electron. The factors $C_{nK}(X,Y)$ are easily obtainable from the known functions $C_n(X,Y)$ by the following procedure. Let f_K and g_K be the small and large components, respectively, of the relativistic K-electron wave function.[4] The positron functions may be formed from positive-energy electron wave functions by the transformation $\psi_{\text{positron}}=C\psi^*_{\text{electron}}$.[5] One may then identify the positron wave function in terms of the quantum numbers of the sphere-normalized electron wave function from which it is obtained. Then the K-electron function for $m=\frac{1}{2}$ is i times the positron function for $j=\frac{1}{2}$, $l=1$, $m=-\frac{1}{2}$, if one identifies f_K with g_{-2} and g_K with f_{-2}; likewise the K-electron function for $m=-\frac{1}{2}$ is minus i times the positron function for $j=\frac{1}{2}$, $l=1$, $m=\frac{1}{2}$.[6] f_{-2} and g_{-2} appear only in the above two positron functions and in none of the other wave functions involved in allowed and first-forbidden β^+ transitions. Thus the recipe, for these transitions, for conversion from $C_n(X,Y)$ to $C_{nK}(X,Y)$ becomes: (1) multiply $C_n(X,Y)$ by $2p^2F_0$ [to take care of the facts that $C_n(X,Y)$ is only a correction factor and that one has to take into account the change from sphere normalization to energy normalization in the positron wave function], and (2) replace f_{-2} by g_K, g_{-2} by f_K, and all other component positron wave functions by zero. The formulas then obtained agree in general but not in detail[7] with those previously given by Good, Peaslee, and Deutsch,[8] by Zweifel,[9] and by Nataf and Bouchez.[10] A similar method may be employed for finding the proper formulas for other electron-capture processes.

Simple formulas for the $\mathcal{C}$ factors for both positron and K-capture decay in first-forbidden transitions are given in Table I. The factors for each matrix element have been expanded in powers of $\kappa=\alpha Z/2R$, according to the expressions of Konopinski and Uhlenbeck and of Smith[1] for $\alpha Z\ll 1$. Also, for the case of the positron decay factors, an averaging process was used in order to avoid tedious numerical integration associated with more exact calculation. For the averaging procedure, the method of Davidson[11] was employed, except for the term $\sum|B_{ij}|^2$. This method utilizes an averaging with the function $(W-1)(W_0-W)$ taken as an approximation to the shape of the spectrum. For the case of the $\sum|B_{ij}|^2$ term, a method proposed by Brysk and Rose[12] was used, since over the Z and energy ranges tested[13] it gave better results than those obtained by use of the Davidson procedure. The formulas of Table I for the interaction combination S, T, P were tested[14] for Z values as large as 53 and for the W_0 range from 1.9 to 5.0. In this range, individual coefficients of matrix elements were found to be given with better than 5% accuracy by the approximate formulas, except for the $\sum|B_{ij}|^2$ coefficients which, at low W_0 values, were in error by as much as 10%. The ratios of the individual coefficients for K-capture and positron emission were found all to be given with better than 5% accuracy by the approximate formulas.

Two forms of pseudoscalar correction factors, I and II, are given in Table I. Formulation I is that of Rose

[3] T. D. Lee and C. N. Yang, Phys. Rev. **104**, 254 (1956).

[4] H. A. Bethe and E. E. Salpeter, *Handbuch der Physik* (Springer-Verlag, Berlin, 1957), Vol. 35, p. 155.

[5]
$$C=\begin{bmatrix}0&0&0&1\\0&0&-1&0\\0&-1&0&0\\1&0&0&0\end{bmatrix}.$$

[6] The f, g nomenclature of the electron wave functions is the usual one.

[7] It would appear that previous workers have followed the same procedure as above, except for the fact that they all replaced g_0 by g_K and f_0 by f_K. This procedure probably derives from a comparison of the K-electron wave functions with the negative-electron wave functions and is, therefore, not valid since the neutrino emitted in K capture is an antiparticle to that emitted in negatron decay. Thus if g_0 is replaced by g_K and f_0 by f_K, q must also be replaced by $-q$. Then the two methodologies become equivalent. [Care must still be used, however, in evaluating $C_1(P)$ if the correction factor of Rose and Osborne[1] is used.]

[8] Good, Peaslee, and Deutsch, Phys. Rev. **69**, 313 (1946).

[9] P. F. Zweifel, Phys. Rev. **96**, 1572 (1954).

[10] R. Nataf and R. Bouchez, J. phys. radium **13**, 190 (1952).

[11] J. Davidson, Phys. Rev. **82**, 48 (1951).

[12] H. Brysk and M. E. Rose, Oak Ridge National Laboratory Report, ORNL-1830, 1955 (unpublished).

[13] One can see from Davidson[11] that in other Z and energy ranges the Brysk and Rose approximation may not represent an improvement.

[14] Formulas involving $\mathcal{C}_1(V)$ or $\mathcal{C}_1(A)$ were not tested because the original interest was in the interaction combination S, T, P.

TABLE I. Simple approximate formulas for positron emission and K-capture probabilities for first-forbidden transitions—Interactions S, T, P and V, A.[a,b]

$$P_+(G_S,G_T,G_P)=P_+(\text{permitted})\{G_S^2|\int\beta\mathbf{r}|^2\mathcal{C}_1^A(S)+G_T^2[|\int\beta\boldsymbol{\sigma}\cdot\mathbf{r}|^2\mathcal{C}_1^A(T)+|\int\beta\boldsymbol{\alpha}|^2\mathcal{C}_1^B(T)+|\int\beta\boldsymbol{\sigma}\times\mathbf{r}|^2\mathcal{C}_1^C(T)$$
$$-((\int\beta\boldsymbol{\sigma}\times\mathbf{r})\cdot(\int\beta\boldsymbol{\alpha})^*+\text{c.c.})\mathcal{C}_1^D(T)+\Sigma|B_{ij}|^2\mathcal{C}_1^E(T)]+G_SG_T[-i((\int\beta\boldsymbol{\sigma}\times\mathbf{r})\cdot(\int\beta\mathbf{r})^*-\text{c.c.})\mathcal{C}_1^A(S,T)$$
$$+i((\int\beta\boldsymbol{\alpha})\cdot(\int\beta\mathbf{r})^*-\text{c.c.})\mathcal{C}_1^B(S,T)]+\begin{cases}G_P^2|\int\boldsymbol{\sigma}\cdot\mathbf{r}|^2\mathcal{C}_1^A(P)+G_TG_P|\int\boldsymbol{\sigma}\cdot\mathbf{r}|^2\mathcal{C}_1^A(P,T)\} & \text{(formulation I)}\\ G_P^2|\int\beta\gamma_5|^2\mathcal{C}_1^B(T)+G_TG_Pi((\int\beta\gamma_5)(\int\beta\boldsymbol{\sigma}\cdot\mathbf{r})^*-\text{c.c.})\mathcal{C}_1^B(S,T)\} & \text{(formulation II)}\end{cases}$$

$$P_K(G_S,G_T,G_P)=P_K(\text{permitted})\{G_S^2|\int\beta\mathbf{r}|^2\mathcal{C}_{1K}^A(S)+\cdots\}$$

$$P_+(G_V,G_A)=P_+(\text{permitted})\{G_V^2[|\int\mathbf{r}|^2\mathcal{C}_1^A(V)+|\int\boldsymbol{\alpha}|^2\mathcal{C}_1^B(T)+i((\int\boldsymbol{\alpha})\cdot(\int\mathbf{r})^*-\text{c.c.})\mathcal{C}_1^D(T)]$$
$$+G_A^2[|\int\boldsymbol{\sigma}\cdot\mathbf{r}|^2\mathcal{C}_1^A(A)+|\int\gamma_5|^2\mathcal{C}_1^B(T)-i((\int\boldsymbol{\sigma}\cdot\mathbf{r})(\int\gamma_5)^*-\text{c.c.})\mathcal{C}_1^D(T)+|\int\boldsymbol{\sigma}\times\mathbf{r}|^2\mathcal{C}_1^D(A)+\Sigma|B_{ij}|^2\mathcal{C}_1^E(T)]$$
$$+G_VG_A[-i((\int\boldsymbol{\sigma}\times\mathbf{r})\cdot(\int\mathbf{r})^*-\text{c.c.})\mathcal{C}_1^A(S,T)-((\int\boldsymbol{\sigma}\times\mathbf{r})\cdot(\int\boldsymbol{\alpha})^*+\text{c.c.})\mathcal{C}_1^B(S,T)]\}$$

$$P_K(G_V,G_A)=P_K(\text{permitted})\{G_V^2[|\int\mathbf{r}|^2\mathcal{C}_{1K}^A(V)+\cdots\}$$

$$P_+(\text{permitted})=\frac{1}{2\pi^3}\int_1^{W_0}pW(W_0-W)^2F_0(W,Z)L_0dW;\quad P_K(\text{permitted})=\frac{1}{4\pi^2}g_K^2(W_0+\epsilon_K)^2$$

$\kappa=\alpha Z/2R$; $R=$nuclear radius$\simeq\frac{1}{2}\alpha A^{\frac{1}{3}}$

$Z=Z(\text{daughter})>0$ for β^+ emission; $Z=Z(\text{parent})-0.3>0$ for K-capture

$W_0=$end-point energy of β^+ spectrum in units of the electron rest energy; $M=$proton mass/electron mass

$$\delta=\frac{W_0+1}{(W_0-1)^2}-\frac{2W_0\ln W_0}{(W_0-1)^3};\quad \eta=\frac{W_0(W_0+1)}{(W_0-1)^2}-\frac{2W_0^2\ln W_0}{(W_0-1)^3}=W_0\delta;\quad \xi_K=\frac{4}{\alpha^2Z^2}\left(\frac{1-\gamma_0}{1+\gamma_0}\right)$$

$\gamma_0=(1-\alpha^2Z^2)^{\frac{1}{2}}$; $\xi=2/(1+\gamma_0)$

Positron emission	K capture
$\mathcal{C}_1^A(S)=\xi\{\kappa^2+\kappa(-\frac{2}{3}+2\delta)+[(7/45)W_0^2-(4/45)W_0-(13/45)+\frac{2}{3}\eta]\}$	$\mathcal{C}_{1K}^A(S)=\xi_K[\kappa^2+\kappa(\frac{2}{3}W_0+\frac{2}{3})+(\frac{1}{3}W_0^2+\frac{2}{3}W_0+\frac{1}{3})]$
$\mathcal{C}_1^A(T)=\xi\{\kappa^2+\kappa(-\frac{2}{3}+2\delta)+[(1/45)W_0^2-(2/45)W_0-\frac{1}{5}+\frac{2}{3}\eta]\}$	$\mathcal{C}_{1K}^A(T)=\xi_K\{\kappa^2+\kappa(\frac{2}{3}W_0+\frac{2}{3})+[\frac{1}{9}W_0^2+(2/9)W_0+\frac{1}{9}]\}$
$\mathcal{C}_1^B(T)=1$	$\mathcal{C}_{1K}^B(T)=1$
$\mathcal{C}_1^C(T)=\xi\{\kappa^2+\kappa(-\frac{2}{3}W_0+2\delta)+[(13/90)W_0^2-(1/90)W_0+(4/45)-\frac{2}{3}\eta]\}$	$\mathcal{C}_{1K}^C(T)=\xi_K[\kappa^2+\kappa(-\frac{2}{3}W_0-\frac{2}{3})+(\frac{1}{6}W_0^2+\frac{1}{3}W_0+\frac{1}{6})]$
$\mathcal{C}_1^D(T)=\xi[-\kappa+(\frac{1}{3}W_0-\delta)]$	$\mathcal{C}_{1K}^D(T)=\xi_K^{\frac{1}{2}}[-\kappa+(\frac{1}{3}W_0+\frac{1}{3})]$
$\mathcal{C}_1^E(T)=(1/24)(W_0^2-1)$	$\mathcal{C}_{1K}^E(T)=\frac{1}{12}(W_0+1)^2$
$\mathcal{C}_1^A(ST)=\xi[-\kappa^2+\kappa(\frac{1}{3}W_0+\frac{1}{3}-2\delta)]$	$\mathcal{C}_{1K}^A(ST)=-\xi_K\kappa^2$
$\mathcal{C}_1^B(ST)=\xi[\kappa+(-\frac{1}{3}+\delta)]$	$\mathcal{C}_{1K}^B(ST)=\xi_K^{\frac{1}{2}}[\kappa+(\frac{1}{3}W_0+\frac{1}{3})]$
$\mathcal{C}_1^A(V)=\xi\{\kappa^2+\kappa(-\frac{2}{3}W_0+2\delta)+[(11/45)W_0^2-(2/45)W_0+(1/45)-\frac{2}{3}\eta]\}$	$\mathcal{C}_{1K}^A(V)=\xi_K[\kappa^2+\kappa(-\frac{2}{3}W_0-\frac{2}{3})+(\frac{1}{3}W_0^2+\frac{2}{3}W_0+\frac{1}{3})]$
$\mathcal{C}_1^A(A)=\xi[\kappa^2+\kappa(-\frac{2}{3}W_0+2\delta)+(\frac{1}{9}W_0^2+\frac{1}{9}-\frac{2}{3}\eta)]$	$\mathcal{C}_{1K}^A(A)=\xi_K\{\kappa^2+\kappa(-\frac{2}{3}W_0-\frac{2}{3})+[\frac{1}{9}W_0^2+(2/9)W_0+\frac{1}{9}]\}$
$\mathcal{C}_1^D(A)=\xi\{\kappa^2+\kappa(-\frac{2}{3}+2\delta)+[(1/18)W_0^2-(1/18)W_0-(2/9)+\frac{2}{3}\eta]\}$	$\mathcal{C}_{1K}^D(A)=\xi_K[\kappa^2+\kappa(+\frac{2}{3}W_0+\frac{2}{3})+(\frac{1}{6}W_0^2+\frac{1}{3}W_0+\frac{1}{6})]$
Pseudoscalar interaction formulation I[c]	
$\mathcal{C}_1^A(P)=(\xi/4M^2)\{4\kappa^4+\kappa^3[-(8/3)W_0-4-4\delta]+\kappa^2[(57/45)W_0^2+(26/45)W_0+(67/45)+2\eta]\}$	$\mathcal{C}_{1K}^A(P)=(\xi_K/4M^2)\{4\kappa^4+\kappa^3[(28/3)+\frac{4}{3}W_0]+\kappa^2[(13/9)W_0^2+(38/9)W_0+(61/9)]\}$
$\mathcal{C}_1^A(PT)=(\xi/M)\{-2\kappa^3+\kappa^2[\frac{2}{3}W_0+(5/3)-\delta]+\kappa[-(2/9)W_0^2+(4/9)-\frac{2}{3}\eta]\}$	$\mathcal{C}_{1K}^A(PT)=(\xi_K/M)\{-2\kappa^3+\kappa^2(-W_0-3)+\kappa[-(4/9)W_0^2-(14/9)W_0-(10/9)]\}$

[a] Note that $\mathcal{C}_1(S)=|\int\beta\mathbf{r}|^2\mathcal{C}_1^A(S)$, $\mathcal{C}_1(T)=|\int\beta\boldsymbol{\sigma}\cdot\mathbf{r}|^2\mathcal{C}_1^A(T)+|\int\beta\boldsymbol{\alpha}|^2\mathcal{C}_1^B(T)+\cdots$, etc.
[b] With the assumption $\alpha Z\ll 1$, one would expect to set $\xi=1$; however, in the formulas of this table ξ's are included in such a way as to make the leading term in each $\mathcal{C}$ correct to higher powers of αZ. Trials have shown that the use of the ξ's does improve the $\mathcal{C}$ factors slightly; however, the K/β^+ ratios from individual $\mathcal{C}$ factors are slightly improved by setting both ξ and ξ_K equal to unity. Further, it is found empirically that the best over-all results are obtained by use of $\xi_K=\xi$.
[c] Only the three leading powers in κ are given.

and Osborn.[1] This formulation is based on the result that, with standard types of nuclear forces, the contribution from the pseudoscalar interaction vanishes unless terms containing the gradient of the lepton wave functions are not disregarded. This formulation needs very large pseudoscalar coupling constants, as will be shown later, in order that there may be a pseudoscalar contribution to first-forbidden transitions. Formulation II is that of Ruderman[15] and of Peaslee.[16] This formulation is based on the fact that, if pseudoscalar-coupled forces are present in the nucleus, the pseudoscalar interaction may be treated like the other interactions. Formulation II does not require abnormally large coupling constants in order that the pseudoscalar inter-

[15] M. Ruderman, Phys. Rev. **89**, 1227 (1953).
[16] D. C. Peaslee, Phys. Rev. **91**, 1447 (1953).

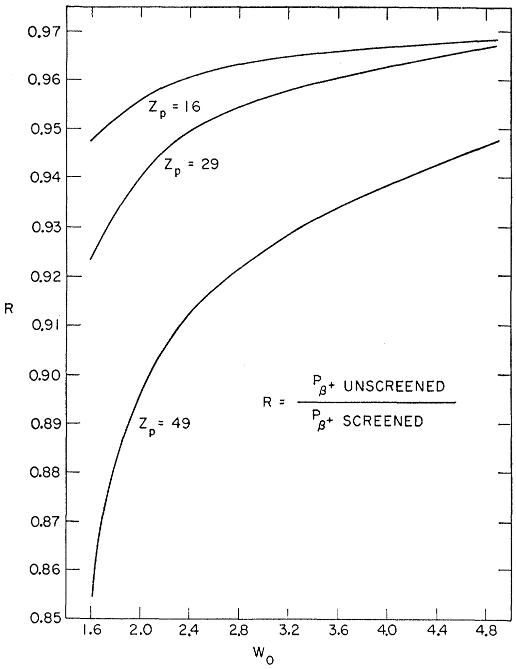


FIG. 1. Ratio R for allowed transitions of positron decay probability calculated with unscreened Coulomb functions to the decay probability calculated with screened functions, as a function of energy W_0 (in units of electron rest energy).

action may make a contribution to first-forbidden transitions.

In the above theoretical calculations the values of the lepton wave functions at the nuclear radius are substituted wherever these wave functions appear in nuclear integrals. This approximation may be looked upon as an idealized shell-model approximation. The effects resulting from the use of alternative approximations are not investigated here. Moreover, the wave functions which are employed for the charged leptons are hydrogen-like Coulomb wave functions for a point nucleus. In order to correct for the fact that one is actually dealing with multi-electron atoms and finite nuclei, screening corrections and finite size corrections must be considered.

The effect of screening on positron wave functions has been considered by Reitz[17] for several values of Z and a range of energies. By use of his results, it is possible to calculate for allowed transitions the ratio of the positron emission probability obtained with unscreened positron wave functions to the probability obtained with screened positron wave functions. Figure 1 shows this ratio as a function of the decay energy for three values of Z. It must be noted here that, on account of errors involved in the measurement of areas under decay-probability curves, the ratios shown in Fig. 1 may be in error by a much as 0.01.

Zweifel[9] has compiled a table of allowed K to positron branching ratios, calculated with screened functions. The present calculations show that his values for $Z=16$ are much too high, while for $Z=49$ they are too low.[18] Zweifel concluded, on the basis of unpublished work by J. R. Reitz, that the effect of screening in forbidden transitions may be approximated, at any given positron energy, by the screening effect in allowed transitions. This approximation has been extended somewhat further in the present calculations with the assumption that the integrated effect of screening on a positron spectrum with end point W_0 is the same for allowed and for first-forbidden transitions. Screening corrections for the K wave functions are taken into account by setting Z_K equal to $Z(\text{parent})-0.3$.

The finite-size correction has been discussed for positron emission by Rose and Holmes[19] and for K-capture by Brysk and Rose.[12] For positron emission no numerical correction factors are available for Z less than 60, and the experimental determinations to be discussed here involve values of Z less than 60. Nevertheless, in order to estimate the importance of this correction, calculations of the effect were made for a Z value equal to 60. The calculations were carried out for the interaction combination[20] S, T, P at two transition energies, $W_0=2$ and $W_0=6$ (in units of m_0c^2). For the case of capture the procedure is straightforward; for positron emission the correction was taken to be that evaluated at a mean positron energy, $(W_0+1)/2$. This approximation is justifiable because of a combination of two facts: the positron spectra are roughly symmetrical about the mean energy; and the correction factors here employed, when they have appreciable effect and also an energy dependence, are approximately linear functions of energy in the range considered. The results of these calculations are presented in Table II.

TABLE II. Effect of the finite nuclear size on the probabilities of K-electron capture and positron emission. Ratios of finite-size to point-charge probabilities for various $\mathcal{C}$ factors.

Factor	$(P_K)_{fs}/(P_K)_{pc}$ $W_0=2$	$(P_K)_{fs}/(P_K)_{pc}$ $W_0=6$	$(P_{\beta^+})_{fs}/(P_{\beta^+})_{pc}$ $W_0=2$	$(P_{\beta^+})_{fs}/(P_{\beta^+})_{pc}$ $W_0=6$	$\frac{(P_K/P_{\beta^+})_{fs}}{(P_K/P_{\beta^+})_{pc}}$ $W_0=2$	$\frac{(P_K/P_{\beta^+})_{fs}}{(P_K/P_{\beta^+})_{pc}}$ $W_0=6$
$\mathcal{C}_1{}^A(S)$	0.61	0.66	0.58	0.57	1.06	1.16
$\mathcal{C}_1{}^A(T)$	0.61	0.64	0.58	0.57	1.06	1.12
$\mathcal{C}_1{}^B(T)$	0.97	0.97	0.98	0.98	0.99	1.00
$\mathcal{C}_1{}^C(T)$	0.55	0.52	0.56	0.50	0.97	1.04
$\mathcal{C}_1{}^D(T)$	0.73	0.70	0.74	0.71	0.99	0.99
$\mathcal{C}_1{}^E(T)$	0.97	0.97	0.98	0.99	0.99	0.98
$\mathcal{C}_1{}^A(S,T)$	0.58	0.58	0.57	0.53	1.02	1.10
$\mathcal{C}_1{}^B(S,T)$	0.77	0.79	0.75	0.75	1.02	1.05
$\mathcal{C}_1{}^A(V)$	0.55	0.55	0.56	0.51	0.98	1.09
$\mathcal{C}_1{}^A(A)$	0.54	0.49	0.56	0.50	0.97	0.99
$\mathcal{C}_1{}^D(A)$	0.61	0.64	0.58	0.57	1.06	1.13
Pseudoscalar formulation I						
$\mathcal{C}_1{}^A(P)$	0.61	0.46	0.55	0.56	1.11	0.82
$\mathcal{C}_1{}^A(PT)$	0.61	0.65	0.56	0.56	1.09	1.16

[17] J. R. Reitz, Phys. Rev. 77, 10 (1950).

[18] Recently published corrected ratios K/β^+ are in agreement with our conclusion [P. F. Zweifel, Phys. Rev. **107**, 329 (1957)].

[19] M. E. Rose and D. K. Holmes, Phys. Rev. **83**, 190 (1951); Oak Ridge National Laboratory Report ORNL-1022, 1951 (unpublished).

[20] The original interest was in this combination.

For permitted transitions and for first-forbidden transitions with spin change two, the coefficients of which correspond to those of the matrix elements $|\int\beta\boldsymbol{\alpha}|^2$ and $\sum|B_{ij}|^2$, respectively, the results show that the effect of finite-size correction is small on both the positron emission probabilities and the K-capture probabilities; and, on the basis of the magnitude of the effects at $Z=60$, for the transitions of these two types which are discussed in this paper, the correction cannot affect the K/β^+ ratios calculated by more than 2%. However, for first-forbidden $\Delta J=0$, ± 1 transitions, both the positron emission probabilities and the K-capture probabilities are greatly affected. The case of the $\Delta J=0$ transition in I^{126}, which will be discussed in a subsequent section, may be taken as an example. Using the correction factors for $Z=60$ and $W_0=2$, one finds, for the tensor matrix element $|\int\boldsymbol{\sigma}\cdot\mathbf{r}|^2$, that the K-capture probability is decreased to about 61% of the point-charge value; and similarly large corrections are found for some of the other matrix elements. The positron emission probabilities associated with the several matrix elements change also, and the net result is that the K/β^+ ratios calculated for the individual matrix elements are affected comparatively little by the finite-size correction. The largest ratio changes occur in matrix elements associated with the pseudoscalar interaction, and at $W_0=2$ the changes amount approximately to 10%. It may be noted that, as transition energy increases, the changes in individual K/β^+ ratios generally increase. It appears plausible that the finite-nuclear-size effect on K/β^+ ratios should decrease as Z decreases; however, the results of extrapolation from higher Z values to those of the cases later to be considered must be viewed with caution. In the calculated K/β^+ ratios presented later, finite-size corrections have been omitted. This procedure is justifiable in the cases of allowed and "unique" first-forbidden ($\Delta J=2$) transitions because of the demonstrable smallness of the effect. It is justifiable for the $\Delta J=0$, ± 1 first-forbidden cases because the energies and Z values of the particular transitions considered are such as to make the expected size of the effect not more than 10% for individual matrix elements; for combinations of matrix elements and interactions, the major effect is indistinguishable from a change in the relative values of these matrix elements or in the relative magnitudes of the interaction coupling constants, which quantities are not well known.

"UNIQUE" TRANSITIONS

For a permitted transition all pure interactions give rise to the same branching ratio K/β^+. Deviation from the ratio so calculated can occur only if there is a mixture of tensor (T) and axial vector (A) interactions or of scalar (S) and vector (V) interactions. Only the interactions T and A can give rise to first-forbidden transitions with spin change two. Again each of the two pure interactions give rise to the same branching ratio K/β^+. Deviation from this ratio may occur, however, for a mixture of the two interactions. These transitions, for which the branching ratios are uniquely given except in the case of the above mixtures of interactions, are called "unique."

In Table III the available experimental data on allowed branching ratios are listed and are compared with theoretical values calculated with screening taken into account. Although theory and experiment are in moderately good agreement generally, there is a clear discrepancy in the case of Sn^{111} and a disagreement among experiments in the cases of Cu^{61} and Cu^{64}.

The only first-forbidden "unique" transitions of known energy for which capture-positron ratios have been measured occur in the decay of I^{126} and of Rb^{84}. Evidence supporting the identification of these transitions is given in the articles on their decay schemes[21–24];

TABLE III. Comparison of observed K-capture-positron ratios with values derived from theory.[a] Allowed transitions.

Parent nuclide	W_0 (units of m_0c^2)	$P_{K\text{ capture}}/P_{\beta^+}$ observed	calculated
$_{9}F^{18}$	2.27	0.030 ± 0.002[b]	0.029
$_{11}Na^{22}$	2.061	0.103 ± 0.006[c]	0.104[d]
$_{21}Sc^{44}$	3.879	0.067 ± 0.016[e]	0.045
$_{23}V^{48}$	2.35	0.66[f]	0.73
$_{25}Mn^{52}$	2.138	1.78[g, h]	1.9
$_{26}Fe^{52}$	2.56	0.60[i]	0.71
		0.70 ± 0.16[j]	
$_{27}Co^{58}$	1.924	5.4[g]	5.2
		5.4[k]	
$_{29}Cu^{61}$	3.37	$0.22\ \pm0.03$[l]	0.29
		$0.25\ \pm0.03$	
		$0.18\ \pm0.03$	
$_{29}Cu^{64}$	2.284	$2.65\ \pm0.3$[f]	2.3
		$1.75\ \pm0.2$	
		$1.90\ \pm0.2$	
		$3.5\ \pm1.0$	
$_{40}Zr^{89}$	2.76	$\sim$2.7[m]	2.8
		$\sim$3.7[n]	
$_{48}Cd^{107}$	1.63	290[o]	310
$_{50}Sn^{111}$	3.96	$2.50\ \pm0.25$[p]	1.5

[a] Except as noted, the theoretical ratios were obtained by interpolation from the curves given by E. Feenberg and G. Trigg, Revs. Modern Phys. **22**, 399 (1950) and subsequent correction for screening. Detailed checks have shown that this procedure should give values accurate to within 5%. These ratios may be obtained more conveniently and probably with greater accuracy from Brookhaven National Laboratory Report BNL-485 (T-110) (unpublished).

[b] Drever, Moljk, and Scobie, Phil. Mag. **1**, 942 (1956).

[c] R. Sherr and R. H. Miller, Phys. Rev. **93**, 1076 (1954). This value and, where necessary, others in the table have been corrected for L capture with the factors given by M. E. Rose and J. L. Jackson, Phys. Rev. **76**, 1540 (1949).

[d] This value was calculated by P. F. Zweifel, Phys. Rev. **96**, 1572 (1954).

[e] J. W. Blue and E. Bleuler, Phys. Rev. **100**, 1324 (1955).

[f] Way, King, McGinnis, and Van Lieshout, *Nuclear Level Schemes, A=40—A=92*, Atomic Energy Commission Report TID-5300 (U. S. Government Printing Office, Washington, D. C., 1955).

[g] Good, Peaslee, and Deutsch, Phys. Rev. **69**, 313 (1946).

[h] R. Sehr, Z. Physik **137**, 523 (1954).

[i] Average of values from reference f. The datum $\epsilon/\beta^+=1.6$, which may be in error on account of uncertainty about converted transitions in Mn^{52} [G. Friedlander (private communication)], has not been included.

[j] E. Arbman and N. Svartholm, Arkiv Fysik **10**, 1 (1956).

[k] C. S. Cook and F. M. Tomnovec, Bull. Am. Phys. Soc. Ser. II, **1**, 253 (1956).

[l] R. Bouchez, Physica **18**, 1171 (1952).

[m] M. Goldhaber *et al.*, Phys. Rev. **83**, 661 (1951).

[n] Shore, Bendel, Brown, and Becker, Phys. Rev. **91**, 1203 (1953).

[o] H. Bradt *et al.*, Phys. Rev. **68**, 57 (1945).

[p] C. L. McGinnis, Phys. Rev. **81**, 734 (1951).

[21] Marty, Langevin, and Hubert, J. phys. radium **14**, 663 (1953).

[22] M. L. Perlman and J. P. Welker, Phys. Rev. **95**, 133 (1954).

[23] Koerts, Macklin, Farrelly, Van Lieshout, and Wu, Phys. Rev. **98**, 1230 (1955).

[24] J. P. Welker and M. L. Perlman, Phys. Rev. **100**, 74 (1955).

TABLE IV. Observed and calculated K-capture-positron ratios for first-forbidden "unique" transitions ($\Delta J=2$, yes).

Parent nuclide	W_0/mc^2	$P_{K\text{-capture}}/P_{\beta^+}$ Observed	Calculated $\Delta J=2$, yes: Exact calculation unscreened	Calculated $\Delta J=2$, yes: Approximation unscreened	Calculated $\Delta J=2$, yes: Approximation screened	Calculated Allowed: Exact calculation unscreened	Calculated Allowed: Exact calculation screened
$_{53}I^{126}$	3.37 ± 0.1	12_{-3}^{+7} [a]	16.4	16.2	14.7	4.4	4.0
	3.33 ± 0.1	21 ± 8 [b]	17.2[c]		15.6		4.2
	3.17 ± 0.04	20 ± 2 [c]	21.4[c]		19.6		5.1
	3.26 ± 0.04 [d]	18 ± 3 [d]			17.3 ± 1		4.6
$_{37}Rb^{84}$	4.32 ± 0.13	2.06 ± 0.36 [e]	0.85 ± 0.12 [f]	0.87	0.83	0.27	0.26
	4.24[g]						

[a] Reference 21.
[b] Reference 22.
[c] Reference 23.
[d] Weighted average.
[e] Reference 24.
[f] The uncertainty, ±0.12, corresponds to the energy uncertainty, ±0.13.
[g] Measurement of N. Benczer and C. S. Wu (privately communicated).

for Rb^{84} the ground state spin assignment has recently been confirmed by direct measurement.[25] In Table IV the observed ratios are listed together with theoretical values for "unique" first-forbidden transitions ($\Delta J=2$, yes) and for allowed transitions. The calculations have been made under the assumption that T/A is very large or very small. Columns entitled "exact calculation" are computed directly from the usual theory. Columns labeled "approximation" are derived from the allowed transition ratios by use of the factors of Table II. Subheadings "unscreened" refer to the use of unscreened Coulomb wave functions for positrons and of $Z_K=Z-0.3$ for K electrons. Subheadings "screened" refer to calculations corrected for the screening effect on the positron wave functions. It may be noted that the ratio values obtained by use of "approximation unscreened" are in excellent agreement with "exact unscreened" values. With this indication of the accuracy of the approximation, the ratio values designated "approximation screened" are taken to be the "best" now readily obtainable. For I^{126} theory and experiment are in agreement; for Rb^{84}, however, one sees that the measured K/β^+ ratio is enhanced, compared with the ratio for an allowed transition, by a factor approximately 2.5 times larger than expected. The measurements on the positron radiations of Sb^{122}, which are described below, were undertaken to provide additional information on unique first-forbidden transitions.

INVESTIGATION OF ANTIMONY-122

The decay scheme of antimony-122, shown in Fig. 2, represents the concordant results of two recent investigations carried out by Glaubman[26] and by Farrelly *et al.*[27] Values given for the fractional decay by electron-capture to the ground state of Sn^{122} are $2.5\pm0.8\%$[28] and $2.0\pm0.3\%$[27]; the value $2.1\pm0.3\%$, which is indicated in the decay scheme, is a weighted average. The Sb^{122} spin-parity assignment is deduced chiefly from the shape of the 1.99-Mev beta spectrum. Data on decay energies compiled by Way and Wood[29] show that a positron branch with end-point energy approximately 500 kev may reasonably be expected to occur in the ground-state transition to Sn^{122}. Glaubman[26] has reported the measurement $K/\beta^+=300\pm130$ for this transition; the positron energy was not determined, however. Both the energy and the abundance of the positron transition have now been measured by use of a triple-coincidence pulse analyzer, and these results are combined with the known K-capture probability to give the K/β^+ ratio.

Source Preparation

Metallic antimony, 99.4% Sb^{121}, was irradiated in the Brookhaven reactor for three days to produce the Sb^{122} used in these experiments.[30] Because of the possi-

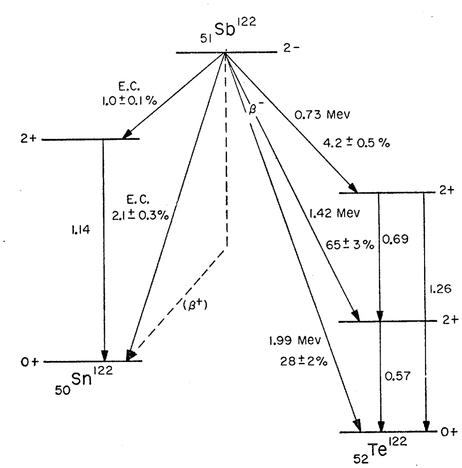


FIG. 2. Decay scheme of Sb^{122}.

[25] Sunderland, Hubbs, Nierenberg, and Silsbee, Bull. Am. Phys. Soc. Ser. II, **1**, 252 (1956).
[26] M. J. Glaubman, Phys. Rev. **98**, 645 (1955).
[27] Farrelly, Koerts, Benczer, Van Lieshout, and Wu, Phys. Rev. **99**, 1440 (1955).
[28] This value is derived from the data of M. J. Glaubman, to which small corrections for L capture and for normalization have been applied. See M. E. Rose and J. L. Jackson, Phys. Rev. **76**, 1540 (1949).
[29] K. Way and M. Wood, Phys. Rev. **94**, 119 (1954).
[30] This material was supplied by the Y-12 plant, Carbide and Carbon Corporation, through the Isotopes Division, U. S. Atomic Energy Commission, Oak Ridge, Tennessee.

bility that a very small amount of impurity could produce a positron activity sufficient to mask that of the antimony, the sample was treated chemically after irradiation. The procedure used[31] was one originally designed to separate antimony from a large number of elements produced in fission, such as As, Ge, Se, Sn, and Te. The purified source consisted of about 3 mg of metal[32] mounted as a disk, 9.3 mm in diameter, on filter paper. A small amount of Zapon lacquer, containing roughly 0.1 mg of resin, served to bind the metal particles together. The source was covered with a Mylar film having a surface density 0.8 mg/cm^2.

Decay of the beta activity of a small fraction of the purified source material was observed with an end-window proportional counter. The Sb^{122} half-life was found to be 2.73±0.03 days, and the activity of 60-day Sb^{124} was equivalent to less than 0.5% of the activity of the Sb^{122} at the time of the positron measurements.

Positron Intensity and Energy Measurements

A gray-wedge coincidence spectrometer was employed to observe the pulse distribution produced in an anthracene detector by positrons from the Sb^{122} source. Figure 3 is a diagram of the experimental arrangement. The thickness of the anthracene crystal, 5 mm, was sufficient to stop 1.3-Mev electrons. Pulses arising from the relatively overwhelming abundance of negative beta radiations were not observed, because the spectrometer was gated only by a signal representing a pulse from the anthracene detector in threefold coincidence with pulses from each of the two NaI annihilation radiation detectors, one on either side of the anthracene and all in a linear array. Single-channel analyzers were used in the coincidence circuit with the NaI scintillators; window widths were set at 3 volts with the 511-kev photopeaks occurring at 30 volts. Lead shields were placed between the source and the NaI detectors to screen these detectors from nuclear gamma rays, direct and scattered. In addition, aluminum absorbers on the ends of these detectors prevented them from responding to scattered beta rays. Effective discrimination was thus obtained against the registration of negative beta radiation in coincidence with nuclear gamma rays.

That this discrimination was adequate for the purposes of the measurements was proved by comparison of the triple-coincidence rate observed with an Sb^{122} source in the geometry described above with the rate observed when one of the NaI detectors was swung about as indicated in Fig. 3 so that the angle included by the three detectors was 90 degrees. In these two arrangements the efficiencies for nuclear coincidence events were very much the same; and indeed the individual rates in the detectors were essentially independent of the arrangement. Nevertheless, the triple-coincidence rate in the 90 degree geometry was measured to be 6%, at most, of the rate in the 180 degree geometry. The total triples count, therefore, was taken as a measure of the number of positrons incident on the anthracene. On the basis of preliminary results, which showed that the Sb^{122} positron end-point energy is not greatly different from that of Na^{22}, the gain of the anthracene detector system was adjusted so it operated in a region where the "singles" count-rate was gain-independent for Na^{22}. All beta particles from Sb^{122} incident on the anthracene were therefore counted.

With one Sb^{122} source the count-rates of anthracene "singles" and of triple coincidences were measured over a period of 8 days. This time was divided into four intervals over each of which the rates were integrated. In short periods between these intervals similar measurements were made with a Na^{22} source replacing the Sb^{122} in the apparatus. For the Na^{22} singles and triples

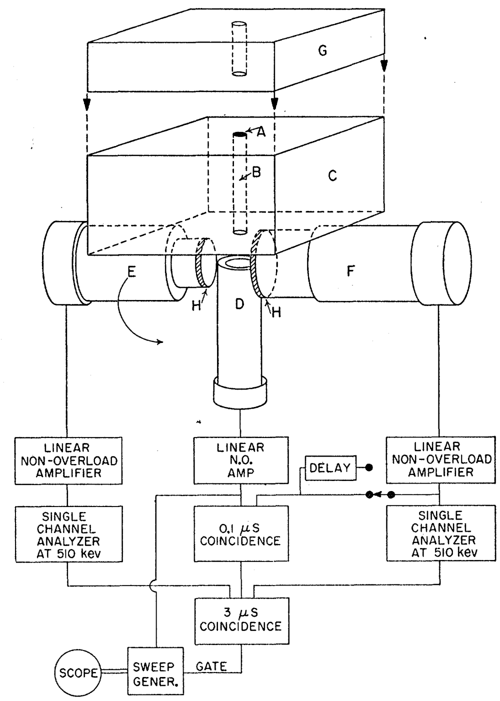


FIG. 3. Coincidence apparatus for measurements on Sb^{122} positrons. Radiations from source at *A*, which pass through collimating hole *B*, are incident on anthracene detector *D*. Area of anthracene is larger than necessary to intercept all illumination from collimator, whose diameter is 10 mm. To reduce scattering, hole in collimator is lined with 0.8 mm thickness of polystyrene. Lead of block *C*, 8 cm thick, and of block *G*, 4 cm thick, shield annihilation detectors *E* and *F* from nuclear gamma radiations, either direct or scattered. *E* and *F* are cylindrical NaI(Tl) scintillators, 1 in.×1½ in. and 2 in.×2 in. Aluminum absorbers *H*, 4.8 mm thick, serve to shield NaI detectors from beta radiations scattered from the collimator and from the anthracene.

[31] G. R. Leader and W. H. Sullivan in *Radiochemical Studies: The Fission Products*, edited by C. D. Coryell and N. Sugarman (McGraw-Hill Book Company, Inc., New York, 1951), Paper No. 133, National Nuclear Energy Series, Plutonium Project Record, Book 2, Vol. 9, p. 934.

[32] Trivalent antimony was reduced to the metal by use of $CrCl_2$ solution as described by J. W. Barnes in Los Alamos Scientific Laboratory Report LA-1721 (unpublished).

TABLE V. Ratio β^+/β^- in the decay of Sb^{122}.

Time interval (hr)	$\epsilon_T/\epsilon_{\beta^+Na}$	$X/0.97=\beta^+/\beta^-$
0 to 22.5	$(2.4\pm0.4)\times10^{-3}$	$(6.21\pm1.1)\times10^{-5}$
24.42 to 43.25	$(2.78\pm0.16)\times10^{-3}$	$(6.03\pm0.6)\times10^{-5}$
96.17 to 137.33	$(2.75\pm0.10)\times10^{-3}$	$(7.12\pm0.6)\times10^{-5}$
143.75 to 186.33	$(2.73\pm0.15)\times10^{-3}$	$(6.48\pm0.7)\times10^{-5}$
		Av $(6.46\pm0.40)\times10^{-5}$

counts, S_{Na} and T_{Na}, the relations

$$S_{Na}=\epsilon_{\beta^+Na}(0.90)D_{Na},\quad T_{Na}=\epsilon_T(0.90)D_{Na},\qquad(3)$$

may be written, where the ϵ's are efficiencies, D_{Na} is the number of Na^{22} disintegrations, and 0.90 is the fraction of Na^{22} which decays by positron emission.[33] The ratio of the measured quantities S_{Na}/T_{Na} is equal to the efficiency ratio, $\epsilon_{\beta^+Na}/\epsilon_T$. For the Sb^{122} measurements, the singles and triples rates are

$$S_{Sb}=[\epsilon_{\beta^-Sb}(0.97)+\epsilon_{\beta^+Sb}X]D_{Sb},\quad T_{Sb}=\epsilon_T X D_{Sb},\qquad(4)$$

where 0.97 is the fraction of Sb^{122} which decays by negative beta-particle emission and X is the fraction of decay by positron emission. Here $\epsilon_{\beta^+Sb}X$ is negligible compared with $\epsilon_{\beta^-Sb}(0.97)$. The triple-coincidence efficiency ϵ_T is taken to be the same for the Sb^{122} positrons and for the Na^{22} positrons, because the end-point energies of the two spectra differ very little, as is shown by measurements described below. The ratio of positrons to negatrons in the radiations of Sb^{122} is then

$$\frac{X}{0.97}=\frac{T_{Sb}\epsilon_{\beta^-Sb}}{S_{Sb}\epsilon_T}=\frac{T_{Sb}}{S_{Sb}}\left(\frac{\epsilon_{\beta^-Sb}}{\epsilon_{\beta^+Na}}\right)\left(\frac{\epsilon_{\beta^+Na}}{\epsilon_T}\right),\qquad(5)$$

where $\epsilon_{\beta^+Na}/\epsilon_T$ has been determined from the Na^{22} measurements.

Although absorption of beta rays in the sources and in the covering of the anthracene detector was negligibly small, the quantity $\epsilon_{\beta^-Sb}/\epsilon_{\beta^+Na}$ is not unity because the scattering by air in the source-detector path for the Sb^{122} beta radiations is different from the scattering for the Na^{22} positrons. Further, the properties of the collimator in the absence of gas scattering are slightly different for the two radiation sources. The magnitude of the first effect was evaluated by means of measurements of Na^{22} and Sb^{122} rates by the anthracene detector with nitrogen and with helium in the source-detector space. Rates in vacuum were determined by extrapolation. The second effect was evaluated from rate measurements of the two sources made in helium gas with and without the collimator; appropriate small corrections were made for the helium scattering. The value for $\epsilon_{\beta^-Sb}/\epsilon_{\beta^+Na}$ was found to be 1.50 ± 0.05.

Results of these abundance measurements are shown in Table V. The constancy of the β^+/β^- ratio over the duration of measurements, a time of approximately eight days, shows that the positron activity decayed with the same 2.7 day half-life that is characteristic of the negatron decay of Sb^{122}. This evidence and the fact that the source material was subjected to rigorous purification after irradiation make it quite unlikely that the observed triple coincidences could have been produced by radiations of a nuclide other than Sb^{122}. Furthermore, not more than a fraction of 1% of the coincidences can have been associated with the creation

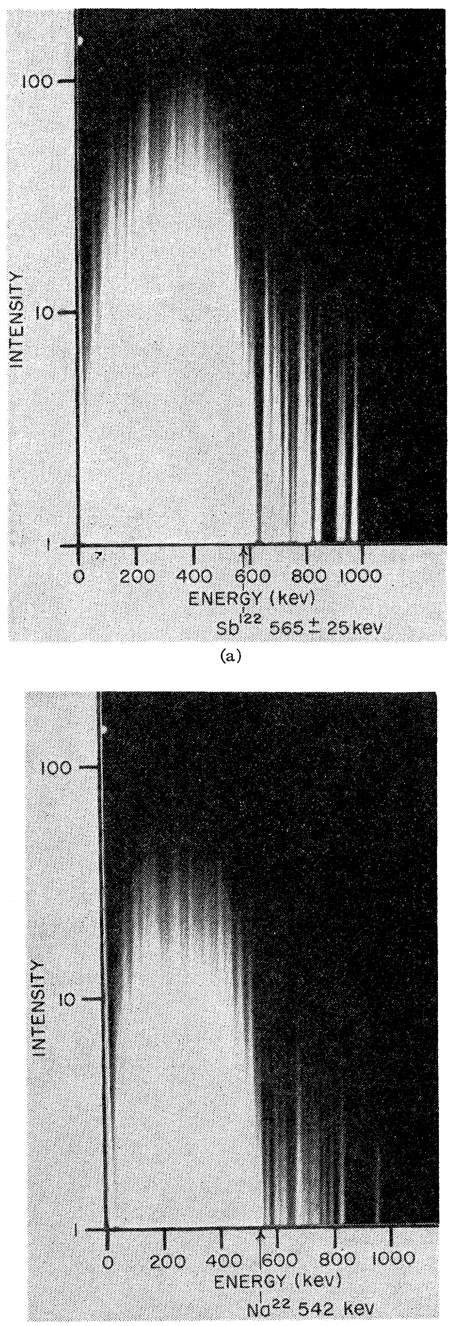


FIG. 4. (a) Gray-wedge photograph of Sb^{122} positron spectrum. (b) Gray-wedge photograph of Na^{22} positron spectrum.

[33] R. Sherr and R. H. Miller, Phys. Rev. 93, 1076 (1954).

TABLE VI. Observed and calculated K capture—positron ratios for first-forbidden "unique" transition ($\Delta J=2$, yes) in Sb^{122}.

W_0	$P_{K\text{-capture}}/P_{\beta^+}$ Observed	Calculated $\Delta J=2$, yes: Exact calc. unscreened	Approximation unscreened	Approximation screened	Allowed Exact calc. unscreened	Allowed Exact calc. screened
2.11 ± 0.05	300 ± 50	316	306	275_{-55}^{+70}	54	49

of positron-electron pairs by the 1.14-Mev and 1.26-Mev gamma radiations of Sb^{122}.[34]

Figure 4(a) shows one of several gray-wedge photographs obtained of the Sb^{122} positron spectrum. The energy scale on the abscissa of this figure was calibrated by means of a photograph such as that shown in Fig. 4(b), which was taken with the same Sb^{122} source after a relatively small Na^{22} source had been added. The end-point energy of the Na^{22} positron spectrum was taken to be 542 kev.[35] In the mixed-source photographs at least 99% of the triple coincidences are from positrons of Na^{22}; the singles rate in the anthracene detector, however, was practically unchanged by the addition of the Na^{22} to the Sb^{122} sources. Calibration error caused by change of photomultiplier gain with count rate was thus avoided. The sum pulses seen in the photographs beyond the end-points of the positron spectra are associated with the high beta rates, up to 35 000 per second, which were required in the anthracene detector because of the low abundance of the Sb^{122} positrons. Addition of random negative beta pulses to the positron pulses displayed in triple coincidence produced sum pulses of amplitude larger than that corresponding to the positron end-point.

From three sets of photographs such as those of Fig. 4 the Sb^{122} positron end-point energy was found to be 565 ± 25 kev. Positron emission to the first excited state of Sn^{122} cannot occur.

From the value for β^+/β^- of Table V and from the abundance of the ground state K-capture transition, the experimentally determined ratio K/β^+ for the ground state transition is calculated to be 300 ± 50. A comparison of the experimental result with theoretical values is given in Table VI, which may be considered a continuation of Table IV.

DISCUSSION

K capture—positron ratios have been measured in three first-forbidden transitions with spin change two. In two of these cases, which occur in the decay of I^{126} and of Sb^{122}, experiment and theory agree within the uncertainties. In the third case, that of Rb^{84}, however, the measured ratio disagrees with that given by theory; and a second investigation of the decay scheme of Rb^8 is now being carried out in this laboratory.

Since only interactions T and A can contribute to first-forbidden transitions with spin change two, it is worthwhile to consider the K/β^+ ratio for a mixed interaction. One finds then

$$\left(\frac{P_K}{P_{\beta^+}}\right)=\frac{\frac{1}{12}\left(1+\frac{G_A}{G_T}\right)^2\left(\frac{\pi}{2}\right)(W_0+\epsilon_K)^4 g_K{}^2}{\int_1^{W_0}F_0Wpq^2\left[\left(1+\frac{G_A{}^2}{G_T{}^2}\right)(\frac{1}{12}q^2L_0+\frac{3}{4}L_1)-2\left(\frac{G_A}{G_T}\right)(\frac{1}{12}q^2L_0^-+\frac{3}{4}L_1^-)\right]dW},\quad (6)$$

where the nomenclature L_0, L_1, L_0^-, L_1^- is that of Konopinski and Uhlenbeck and of Smith.[1] This expression is derived with the use of nonrelativistic nuclear wave functions. Under the assumptions $G_{A,T}/G_{T,A}\ll1$ and $\alpha Z\ll1$, the following approximate result is obtained:

$$\left(\frac{P_K}{P_{\beta^+}}\right)_{T,A}=\left(\frac{P_K}{P_{\beta^+}}\right)_{T,A=0}\times\frac{1+2G_{A,T}/G_{T,A}}{1-(2G_{A,T}/G_{T,A})\langle1/W\rangle_{\text{Av}}},\quad (7)$$

where $\langle1/W\rangle_{\text{Av}}$ is the average W^{-1} over the positron spectrum and $(P_K/P_{\beta^+})_{T,A=0}$ refers, of course, to the unique ratios which have been computed in previous sections. The experimental capture to positron ratios then lead to $G_{A,T}/G_{T,A}=0.01\pm0.06$ from I^{126} (weighted average value) and $G_{A,T}/G_{T,A}=0.03\pm0.12$ from Sb^{122}. The conclusion that $G_{A,T}/G_{T,A}$ is essentially zero has also been reached on the basis of data from the study of the shapes of allowed beta spectra[36] and from the capture to positron ratio[33] in Na^{22}.

"NONUNIQUE" TRANSITIONS

In first-forbidden transitions with spin change one and spin change zero, more than one type of nuclear matrix element generally appears in the beta-decay probabilities. As may be seen in Table I, the ratio, for a given matrix element, of the coefficients for K capture and β^+ emission, $\mathcal{C}_{1K}/\mathcal{C}_1$, is generally different from the coefficient ratios for other matrix elements. Thus the

[34] C. M. Davisson, in *Beta- and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (North-Holland Publishing Company, Amsterdam, 1955), p. 41.

[35] Macklin, Lidofsky, and Wu, Phys. Rev. **78**, 318 (1950); B. T. Wright, Phys. Rev. **90**, 159 (1953).

[36] C. S. Wu, in *Beta- and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (North-Holland Publishing Company, Amsterdam, 1955), p. 319.

TABLE VII. $\mathcal{C}_1^Z(X,Y)$ factors for positron emission and K capture.[a]

Factor[b]	As^{74}, $W_0=2.8004$ β^+	K	$\mathcal{C}_{1K}/\mathcal{C}_1$	Rb^{84}, $W_0=2.526$ β^+	K	$\mathcal{C}_{1K}/\mathcal{C}_1$	I^{126}, $W_0=1.900$ β^+	K	$\mathcal{C}_{1K}/\mathcal{C}_1$	I^{126} $W_0=5.000$ $\mathcal{C}_{1K}/\mathcal{C}_1$	As^{74} $W_0=1.900$ $\mathcal{C}_{1K}/\mathcal{C}_1$[c]	As^{74} $W_0=5.000$ $\mathcal{C}_{1K}/\mathcal{C}_1$[c]
$\mathcal{C}_1^A(S)$	6.074^{1}	8.910^{1}	1.47	7.118^{1}	9.879^{1}	1.39	1.152^{2}	1.451^{2}	1.26	1.50	1.39	1.79
$\mathcal{C}_1^A(T)$	5.990^{1}	8.594^{1}	1.43	7.062^{1}	9.609^{1}	1.36	1.149^{2}	1.433^{2}	1.25	1.47	1.37	1.75
$\mathcal{C}_1^B(T)$	1.000^{0}	1.000^{0}	1.00	1.000^{0}	1.000^{0}	1.00	1.000^{0}	1.000^{0}	1.00	1.00	1.00	1.00
$\mathcal{C}_1^C(T)$	5.028^{1}	4.643^{1}	0.923	6.134^{1}	5.657^{1}	0.922	1.079^{2}	1.023^{2}	0.948	0.986	0.914	0.954
$\mathcal{C}_1^D(T)$	-7.101^{0}	-6.756^{0}	0.951	-7.821^{0}	-7.476^{0}	0.956	-1.034^{1}	-1.009^{1}	0.976	0.979	0.946	0.953
$\mathcal{C}_1^E(T)$	2.766^{-1}	1.185^{0}	4.29	2.150^{-1}	1.015^{0}	4.72	9.870^{-2}	6.641^{-1}	6.73	2.96	6.44	3.00
$\mathcal{C}_1^A(ST)$	-5.496^{1}	-6.421^{1}	1.17	-6.513^{1}	-7.464^{1}	1.15	-1.105^{2}	-1.217^{2}	1.10	1.23	1.14	1.37
$\mathcal{C}_1^B(ST)$	7.779^{0}	9.270^{0}	1.19	8.364^{0}	9.802^{0}	1.17	1.071^{1}	1.197^{1}	1.12	1.22	1.16	1.32
$\mathcal{C}_1^A(P)$	2.490^{3}	6.150^{3}	2.47	3.512^{3}	7.854^{3}	2.24	9.872^{3}	1.784^{4}	1.81	2.45	2.27	3.37
$\mathcal{C}_1^A(PT)$	-7.750^{2}	-1.454^{3}	1.88	-9.937^{2}	-1.737^{3}	1.75	-2.133^{3}	-3.199^{3}	1.50	1.89	1.77	2.47
$\mathcal{C}_1^A(V)$[c]	5.031^{1}	4.856^{1}	0.965	6.070^{1}	5.826^{1}	0.960	1.054^{2}	1.023^{2}	0.970	1.033	0.937	1.05
$\mathcal{C}_1^A(A)$[c]	4.946^{1}	4.526^{1}	0.915	6.003^{1}	5.539^{1}	0.923	1.051^{2}	1.003^{2}	0.954	0.967	0.907	0.920
$\mathcal{C}_1^D(A)$[c]	5.863^{1}	8.728^{1}	1.49	6.848^{1}	9.749^{1}	1.43	1.116^{2}	1.451^{2}	1.30	1.57	1.38	1.76

[a] Numbers in the body of the table are to be multiplied by ten raised to the power indicated in the superscripts.
[b] The designations in this column refer both to positron emission and K capture.
[c] Evaluations made with formulas of Table I in columns and rows thus designated.

analysis of branching ratios in these "nonunique" transitions is more complex than that required for the "unique" transitions.

All five forms of interaction, including P, may lead to this type of transition; however, for simplicity, only the two more probable combinations S, T, P and V, A are treated. The procedure employed to study "nonunique" transitions here is very similar to that of Peaslee[16] and of King and Peaslee,[37] although some disagreement with their results is noted. One must make use of relationships among matrix elements in order to simplify the expressions for the decay probabilities. Thus one may set for β^+ decay or K capture

$$\begin{aligned}
&\int \boldsymbol{\alpha} = -i\Lambda(\alpha Z/2R)\int \mathbf{r},\\
&\int \beta\boldsymbol{\alpha} = -i\Lambda'(\alpha Z/2R)t\int \mathbf{r},\\
&\int \beta\boldsymbol{\sigma}\cdot\mathbf{r} = -\int \boldsymbol{\sigma}\cdot\mathbf{r},\\
&\int \boldsymbol{\sigma}\times\mathbf{r} = it\int \mathbf{r},\\
&\int \beta\boldsymbol{\sigma}\times\mathbf{r} = -it\int \mathbf{r},\\
&\int \gamma_5 = -i\Lambda''(\alpha Z/2R)\int \boldsymbol{\sigma}\cdot\mathbf{r},
\end{aligned} \tag{8}$$

where Λ, Λ', Λ'', and t are real numbers, and Z is positive. Λ, Λ', and Λ'' have been estimated from various models to be of the order of 1 to 2. The quantity t equals $(1/\hbar)[(\boldsymbol{\sigma}\cdot\mathbf{L})_f-(\boldsymbol{\sigma}\cdot\mathbf{L})_i]$, which is the case of single-particle transitions equals $\pm(2j+1)$ for $\Delta j=0$ transitions, and equals ±1 for $\Delta j=1$ transitions. If only ordinary forces exist in the nucleus, the pseudoscalar interaction is treated in the manner of Rose and Osborn (Method I). If pseudoscalar-coupled forces exist in the nucleus, then one has

$$\int \beta\gamma_5 = -ig(\alpha Z/2R)\int \boldsymbol{\sigma}\cdot\mathbf{r}, \tag{8'}$$

where g is a parameter which depends on the nuclear forces (Method II).

With the assumption that the interaction is $G_SS+G_TT+G_PP$, the positron emission probability for first-forbidden transitions becomes

$$\begin{aligned}
P_+ = P_+(\text{permitted})\Big\{ & G_T^2\sum|B_{ij}|^2\mathcal{C}_1^E(T)\\
&+\left|\int \mathbf{r}\right|^2\{G_S^2\mathcal{C}_1^A(S)+G_T^2t^2[\kappa^2\Lambda'^2\mathcal{C}_1^B(T)\\
&+2\kappa\Lambda'\mathcal{C}_1^D(T)+\mathcal{C}_1^C(T)]\\
&+2G_SG_Tt[\mathcal{C}_1^A(ST)+\Lambda'\kappa\mathcal{C}_1^B(ST)]\}\\
&+\left|\int \boldsymbol{\sigma}\cdot\mathbf{r}\right|^2\{G_T^2\mathcal{C}_1^A(T)+\tfrac{1}{4}(G_P/M)^2\mathcal{C}_1^A(P)\\
&+(G_P/M)G_T\mathcal{C}_1^A(PT)\}\Big\},
\end{aligned} \tag{9}$$

where

$$P_+(\text{permitted}) = \frac{1}{2\pi^3}\int_1^{W_0} L_0pW(W_0-W)^2dW,$$

and M is the proton mass in electron mass units. This is the result obtained with Method I. With Method II, the last term must be replaced by

$$\left|\int \boldsymbol{\sigma}\cdot\mathbf{r}\right|^2\{G_T^2\mathcal{C}_1^A(T)+G_P^2\kappa^2g^2\mathcal{C}_1^B(T)-2G_PG_T\kappa g\mathcal{C}_1^B(ST)\}. \tag{9'}$$

[37] R. W. King and D. C. Peaslee, Phys. Rev. 94, 1284 (1954).

The terms here have all been previously defined in the introductory section. P_K is found by substitution of values of $\mathcal{C}$ and P(permitted) appropriate for K-capture.

Using only the leading term in each $\mathcal{C}$, these expressions become

$$P_+=P_+(\text{permitted})\kappa^2\bigg[\sum|B_{ij}|^2G_T{}^2\kappa^{-2}(1/24)(W_0{}^2-1)+\left|\int\mathbf{r}\right|^2\{G_S+G_Tt(\Lambda'-1)\}^2+\begin{cases}\left|\int\boldsymbol{\sigma}\cdot\mathbf{r}\right|^2(G_T-G_P{}'\kappa)^2\bigg] & \text{Method I.}\\ \left|\int\boldsymbol{\sigma}\cdot\mathbf{r}\right|^2(G_T-G_Pg)^2\bigg] & \text{Method II.}\end{cases}\tag{10}$$

The corresponding equations for K capture are exactly the same, except for the coefficient of $\sum|B_{ij}|^2$, since the leading terms in all other $\mathcal{C}$ expressions are identical for both β^+ decay and for K-capture. For $\sum|B_{ij}|^2$ one has the coefficient $(1/12)(W_0+1)^2$.

Similarly, one obtains for the interaction G_VV+G_AA

$$\begin{aligned}P_+=P_+(\text{permitted})\bigg\{&G_A{}^2\sum|B_{ij}|^2\mathcal{C}_1{}^E(T)\\&+\left|\int\mathbf{r}\right|^2\{G_A{}^2t^2\mathcal{C}_1{}^D(A)\\&+G_V{}^2[\mathcal{C}_1{}^A(V)+\Lambda^2\kappa^2\mathcal{C}_1{}^B(T)+2\Lambda\kappa\mathcal{C}_1{}^D(T)]\\&+2G_AG_Vt[\mathcal{C}_1{}^A(ST)+\Lambda\kappa\mathcal{C}_1{}^B(ST)]\}\\&+\left|\int\boldsymbol{\sigma}\cdot\mathbf{r}\right|^2\{G_A{}^2[\mathcal{C}_1{}^A(A)+\Lambda''^2\kappa^2\mathcal{C}_1{}^B(T)\\&+2\Lambda''\kappa\mathcal{C}_1{}^D(T)]\}\bigg\}\\ \simeq P_+(\text{permitted})\kappa^2\bigg[&\sum|B_{ij}|^2G_A{}^2\kappa^{-2}\frac{1}{24}(W_0{}^2-1)\\&+\left|\int\mathbf{r}\right|^2\{G_At+G_V(\Lambda-1)\}^2\\&+\left|\int\boldsymbol{\sigma}\cdot\mathbf{r}\right|^2(G_A\Lambda''-G_A)^2\bigg].\end{aligned}\tag{11}$$

The same considerations apply to obtaining the K-capture expressions for V, A as for S, T, P.

It is not intended to review in detail King and Peaslee's analysis of $\log ft$ values, which is based on the interaction combination S, T, P. Their calculations are based on a modified single-particle analysis in which the assumption is made that the leading contribution to the nuclear matrix element is made by the term indicated by the $j-j$ shell model. A nuclear matrix element is then expressed M.E. $=a$(m.e.), where (m.e.) is the one-particle integral. Deviations from the shell-model picture as well as equivalent particles in unfilled shells in initial or final states within the shell model are expected to lead to $|a|^2\ll 1$. This condition leads to the slowness of the so-called "unfavored" first-forbidden transitions with which we are dealing here. The assumption is made that the interaction combination is $G_S=-G_T=1$ for β^- decay and $G_S=G_T=1$ for β^+ decay. One finds then that the $\log ft$ values indicate $\Lambda'=1$. This choice arises in part because no difference is observed between $t=+1$ and $t=-1$ transitions. The number of cases considered, however, is not very large. Roughly, thus, most of the contribution to the coefficient of $|\int\mathbf{r}|^2$ arises from the scalar interaction. One also finds, for formulation I, $|G_P|\simeq 150$ to 400, while for formulation II, $|G_Pg|\simeq 0.75$ to 2.75. A similar analysis may be made on the basis of the interaction combination A, V. Taking the interaction coefficients equal and of magnitude unity, one finds that the form of the probability function is almost the same as for S, T, P. One finds now that $\Lambda=1$, and consequently most of the contribution to the coefficient of $|\int\mathbf{r}|^2$ arises from the axial vector interaction. One also finds $|\Lambda''|\simeq 0.75$ to 2.75.

One need not discuss the unique transitions in this section, since for these only the $\sum|B_{ij}|^2$ matrix element appears.

For $\Delta J=1$, $\Delta j=1$ transitions both the $|\int\mathbf{r}|^2$ matrix element and the $\sum|B_{ij}|^2$ matrix element may make contributions. The relative values of these two matrix elements for such transitions have not been computed. One may assume for the present that the $\sum|B_{ij}|^2$ contribution to such transitions is negligible, even for the case of K capture where the coefficient of $\sum|B_{ij}|^2$ may be relatively large. Upon using the above values for the coupling constants and $\Lambda'=1$, for the cases in Table VII, where the values of the various $\mathcal{C}$ factors are given, the ratio K/β^+ is represented for S, T, P roughly by the permitted ratio multiplied by the ratio of the $\mathcal{C}_1{}^A(S)$'s. This conclusion is not changed if Λ' is varied in order better to cancel the term in t. Thus the more exact calculations bear out the naive conclusion that the ratio should be obtainable just from a knowledge of the scalar contributions. This would result in appreciable deviation from permitted ratios as is seen from Table VII. For the case of the interaction combination V, A, the ratio K/β^+ is expected to be similar since the ratio of the $\mathcal{C}_1{}^A(S)$'s is almost the same as the ratio of the $\mathcal{C}_1{}^D(A)$'s. If the $\sum|B_{ij}|^2$ term also makes an appreciable contribution to the transition probability, the deviations from the allowed ratio should be larger.

TABLE VIII. Observed and calculated K capture—positron ratios for first-forbidden transitions with $\Delta J=0$.

Parent nuclide	$_{33}As^{74}$	$_{37}Rb^{84}$	$_{53}I^{126}$
W_0	2.80	2.53[a]	1.92[b]
P_K/P_{β^+}			
Observed	1.5	5.1 ±0.4	148[b]
Calculated	1.47	4.2 ±0.4[c]	143
Allowed	1.17	3.4 ±0.3[c]	122

[a] C. S. Wu and N. Benczer (private communication).
[b] These are preliminary unpublished values reported by D. S. Harmer and M. L. Perlman, and they have been chosen in preference to the earlier values given in reference 23 because the newer values are the result of a direct measurement.
[c] The uncertainty estimates are based on the authors' estimate of the uncertainty in the W_0 value.

For $\Delta J=0$, $\Delta j=0$ ($0\not\to 0$) transitions all three matrix elements may be operative. The King and Peaslee analysis, however, indicates that most of the contribution arises from the $|\int \boldsymbol{\sigma}\cdot\mathbf{r}|^2$ term and that, moreover, there must be a sizable pseudoscalar contribution, as has already been pointed out. The actual K/β^+ ratios computed depend, of course, on the values chosen for G_P and G_T. In formulation I (S,T,P) one finds, with reasonable parameters, values for K/β^+ which differ from the permitted ratio approximately by the ratio of the $\mathcal{C}_1{}^A(PT)$'s. On the other hand, formulation II (S,T,P) and also the interaction V, A lead to ratios which are much closer to allowed ratios.

Unfortunately, no measurements have as yet been reported for first-forbidden transitions with $\Delta J=1$, $\Delta j=1$ or $\Delta J=0$, $\Delta j=0$. The predictions made above for these transitions rest on very naive and qualitative arguments, and they could be affected by detailed consideration of finite-size effects. The individual contributions of the various possible matrix elements show differing trends in their K/β^+ ratios with respect to allowed K/β^+ ratios: e.g., the ratio corresponding to $\sum|B_{ij}|^2$ relative to the ratio for allowed transitions is large and decreases with increasing energy while all other such ratios increase with increasing energy; and further the pseudoscalar contributions with formulation I lead to large ratios while with formulation II they lead to small ratios. Thus, a study of these transitions should prove very useful in determining the important contributions to beta decay.

The only nonunique first-forbidden transitions for which K/β^+ ratios have been measured[23,24,38] are $\Delta J=0$ transitions for which shell-structure arguments indicate $\Delta j=2$; the parent nuclides are $_{33}As^{74}$, $_{37}Rb^{84}$, and $_{53}I^{126}$. The logft values of such transitions show that as a class[37] these transitions are an order of magnitude less probable than $\Delta J=0$, $\Delta j=0$ transitions and an order of magnitude more probable than $\Delta J=2$, $\Delta j=2$ transitions (for $f_0=f_1$). If shell-structure arguments were completely rigorous, only the $\sum|B_{ij}|^2$ matrix element would contribute to these transitions, and the K/β^+ ratios for these transitions would be the same as for the unique first-forbidden transitions. However, if this were the situation, these transitions would be expected to have the same speed as the unique transitions. King and Peaslee reached the conclusion that the shell-structure arguments must be modified to permit contributions from the $|\int \boldsymbol{\sigma}\cdot\mathbf{r}|^2$ and $|\int \mathbf{r}|^2$ matrix elements. They estimate that the value of $|a|^2$ for these matrix elements in these transitions is approximately an order of magnitude lower than the usual value of $|a|^2$ for unfavored transitions. Even such small deviation from the shell-structure description would lead to a case where a large fraction of the transition rate is accounted for by these matrix elements with only a small fraction of the rate due to the $\sum|B_{ij}|^2$ matrix element. Proceeding on the assumption that the contributions of the $|\int \boldsymbol{\sigma}\cdot\mathbf{r}|^2$ and $|\int \mathbf{r}|^2$ matrix elements alone would have an allowed ratio, one may write the relation

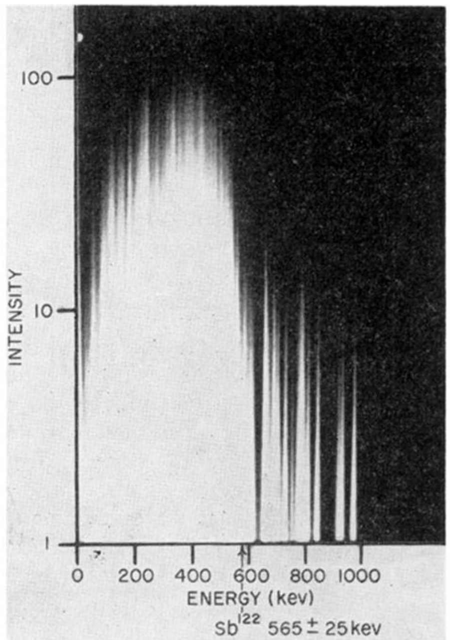
$$(P_K/P_{\beta^+})_{\Delta J=0,\,\Delta j=2} = (P_K/P_{\beta^+})_{\text{allowed}}[1+(1/12)(W_0+1)^2y]/[1+(1/24)(W_0{}^2-1)y],$$

where y would be the ratio of the rate due to the $\sum|B_{ij}|^2$ matrix element relative to that due to the $|\int \boldsymbol{\sigma}\cdot\mathbf{r}|^2$ and $|\int \mathbf{r}|^2$ matrix elements if the energy dependent factors were unity (i.e., $f_1\simeq f_0$). One then calculates with $y=0.3$ for the transitions in As^{74}, Rb^{84}, and I^{126} the values given in Table VIII. It may be noted that the value of 0.3 for y, which is necessary to achieve the agreement demonstrated in Table VIII, is somewhat higher than the value expected from a comparison of the logft values of $\Delta J=0$, $\Delta j=2$ transitions with the logf_1t values of $\Delta J=2$, $\Delta j=2$ transitions. Thus, if the preceding evidence is taken to be sufficient, it would appear very attractive, with either of the interaction combinations, to describe these transitions as being a mixture of the $\Delta j=2$ character and of the $\Delta j=0$ character, with formulation II (i.e., pseudoscalar-coupled nuclear forces) being the correct one for the pseudoscalar interaction, if the combination is S, T, P.

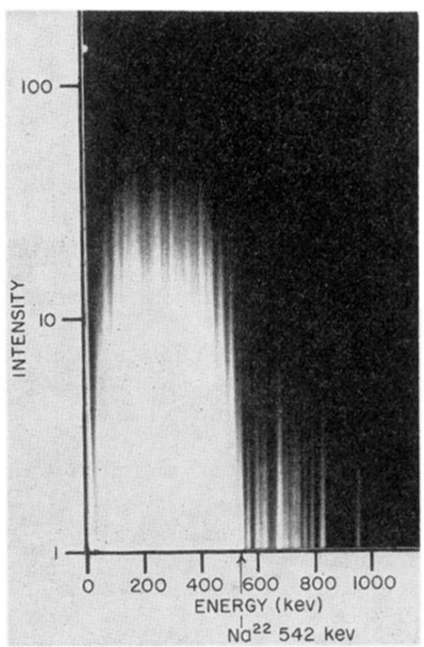
[38] Johansson, Cauchois, and Siegbahn, Phys. Rev. **82**, 275 (1951).

ACKNOWLEDGMENTS

The authors thank Dr. M. J. Glaubman, Dr. C. S. Wu, and Dr. N. Benczer for information about their work on Sb^{122} and Rb^{84}. They are grateful to Dr. M. Goldhaber for comments and to R. Chase and E. Nielsen for assistance with instrumental problems.



(a)



(b)

FIG. 4. (a) Gray-wedge photograph of Sb^{122} positron spectrum.
(b) Gray-wedge photograph of Na^{22} positron spectrum.