

With an MgO sample containing $\sim 10^{17}$ spins, the inverted state persisted for about 2.5 milliseconds at 4.2°K. Amplification was observed with a gain of 20 db at 125 microseconds after inversion, falling to 3 db at 720 microseconds.

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Energy Levels of an Asymmetric Rotor

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MICROWAVE spectroscopy is at present seriously handicapped by the lack of sufficiently extensive tables of the rotational energy of a rigid rotor. This is commonly described either by the E_{τ}, κ rotation of King, Hainer, and Cross¹ or by a series expansion² of a quantity w in powers of the Wang asymmetry parameter, b . The available tables² give E_{τ} at 0.01 intervals of κ up to $J=12$. More recent compilations^{3,4} give the coefficients in the series expansion up to $J=40$, but this converges prohibitively slowly when $|\kappa| < 0.9$ and the K value is small.

Work in progress here on bent triatomic molecules requires accurate energies for $J > 12$ and large asymmetries. A satisfactory technique has been developed by use of the method of Golden.⁵ This originally required an exceptionally good tabulation of characteristic values $M(s)$ of the Mathieu equation of parameter s . The new method instead uses the energy tables² for low J to obtain the energy levels of higher J by a "stepup" process, to be described.

Golden gives the result:

$$w = M(4\theta) - 2\theta + \alpha\theta' + \beta\theta'^2 + \dots,$$

where

$$\theta = \frac{1}{2} |b| \left[J(J+1) - 1 - \frac{1}{2J(J+1)} \right],$$

$$\theta' = \frac{1}{2} |b| \left[1 + \frac{1}{2J(J+1)} \right],$$

the terms in α and β being small corrections.

The stepup process consists of increasing J while simultaneously decreasing the asymmetry, b , so that θ remains constant. The quantity w then also remains constant, apart from small variations in θ' . The appropriate value of w for the lower J and larger asymmetry is found from the tables,² and corrected for higher J by using the values of α and β listed by Golden.⁵ In practice it was necessary to develop more complicated relations in the E_{τ}, κ rotation, but this does not affect the rapidity and convenience of the process.

Table I compares tabulated values of E_{τ} for the level $12_{12,0}$ with values calculated by the above method and by the series expansion up to b^6 .

There is good agreement for asymmetries so large that the series oscillates violently, though agreement

TABLE I. Comparison of values of E_{τ} for the level $12_{12,0}$.

	$\kappa = 0.90$	$\kappa = 0.80$
Tabulated value	151.111552	148.695733
Stepup from $11_{11,0}$	151.111558	148.695724
Stepup from 6_{60}	151.111397	148.694102
Stepup from 3_{30}	151.109561	148.572802
Stepup from 2_{20}	151.068250	
Series expansion	151.923200	230.240702

becomes less good as the asymmetry increases, making the method valueless for $|\kappa| < 0.5$. It is also inaccurate when K is comparable with J , but the series expansion is then satisfactory. The value of the technique is that it gives satisfactory energies where other methods fail, and could yield even better results by use of these as starting points for an iteration solution of the secular determinant. Much tedious computation can also be saved by the recently suggested use of energy moments.⁶

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Specific Heat of He³ below 1°K

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THE first measurements of the specific heat of liquid He³ under its saturated pressure by deVries and Daunt¹ covered the temperature range 0.5°K to 2.3°K. Subsequent measurements by Roberts and

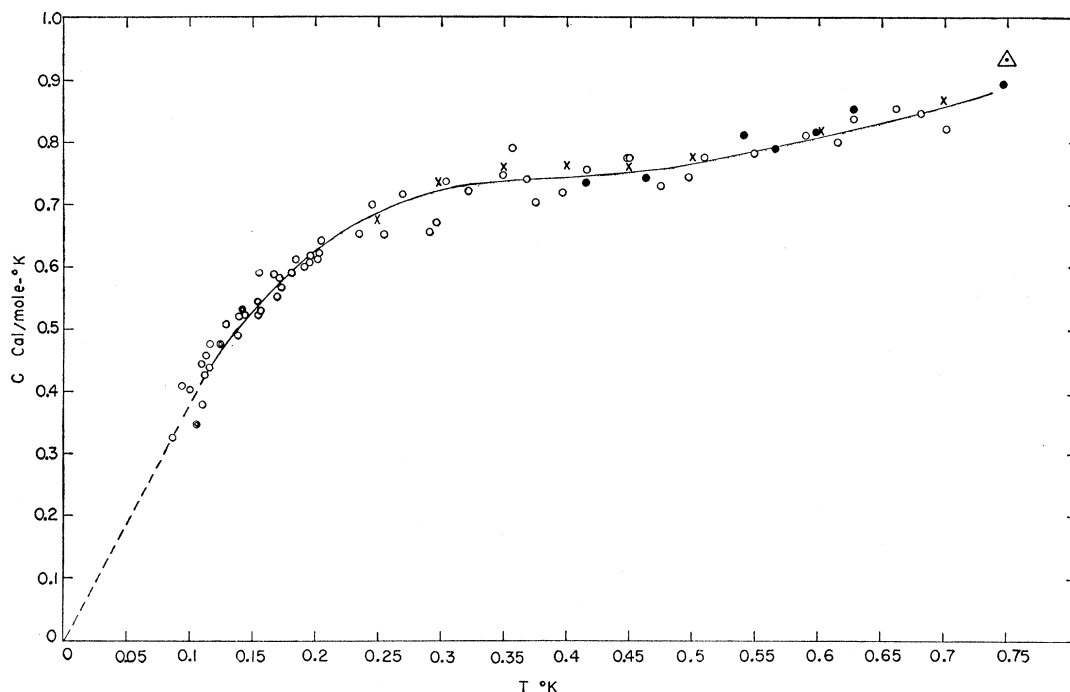


FIG. 1. Molar specific heat of liquid He³ at a pressure of 12–15 cm Hg. ○ present results. Previous measurements at the saturated vapor pressure are shown as: △ deVries and Daunt¹; ● Roberts and Sydoriak²; × Abraham, Osborne, and Weinstock.³

Sydoriak² extended the range of observation down to 0.37°K and those by Osborne, Abraham, and Weinstock³ down to 0.23°K. The later measurements, carried out with greater volumes of He³, were of higher accuracy. All the data were consistent and showed that no marked anomaly, such as a λ -point, occurred in the range of temperature studied. They also showed the curious result that between about 0.3°K and 1.0°K the specific heat varied little with temperature, and no unambiguous extrapolation of the curve could be made to $T=0^\circ\text{K}$. It was considered important therefore to extend the observations to yet lower temperatures, and measurements down to 0.085°K are reported herewith.

An adiabatic calorimeter of volume 0.5 cm³ was used which was cooled below 1°K by a paramagnetic salt, the thermal connection being made through a superconducting Pb thermal valve which could be opened and closed magnetically, as described elsewhere.⁴ A 100-ohm manganin heater was used and temperatures were determined by susceptibility measurements of a cerium magnesium nitrate specimen thermally attached to the calorimeter. This salt was chosen since it accurately follows Curie's law in the temperature range covered,⁵ and has a negligible heat capacity. Details of the apparatus and method of measurement are deferred to a later publication.

The results of our measurements of the specific heat of pure⁶ He³ under a pressure of 12 to 15 cm Hg are

shown in Fig. 1, which also gives the data from previous observations made at the saturated vapor pressure.^{1–3} It is estimated that the smooth curve drawn through our data is accurate within $\pm 2\%$. No corrections have been made for the differences between C_p and C_v , which are negligible in this temperature range.

It will be seen from Fig. 1 that the smooth curve falls close to the previous results down to 0.25°K and that below this temperature it continues to diminish smoothly with decreasing temperature down to 0.085°K. The curve appears to allow a linear extrapolation to 0°K to be made with a slope of 3.75 cal/mole-deg². The justification for this linear extrapolation is reinforced by appeal to the entropy diagram of Fig. 2, which has been calculated by using the linear extrapolation and the measured specific heat. At 0.5°K we find $S_{\text{liq}} = 1.39 \pm 0.03$ cal/mole-deg. Since the difference between the specific heat under 12 to 15 cm Hg and under the saturated vapor pressure is small in the range of our measurements, it is permissible to compare this figure with the value of 1.44 cal/mole-deg calculated for S_{liq} at saturated vapor pressure from the most recent vapor pressure data.⁷

We feel that the experimental results reported here, together with the magnetic susceptibility data of Fairbank *et al.*⁸ and the recent theoretical work of Brueckner and Gammel,⁹ now provide sufficient evidence to give a high degree of certainty to the linear

extrapolation that we have made, particularly in view of the close agreement (within about 2%) between the experimental and theoretical values of the linear term. Rigorously there always remains the experimental possibility of a transition, possibly accompanied by the appearance of superfluidity, at temperatures lower than those of our measurements; but we feel that the data presented here make this possibility now unlikely.

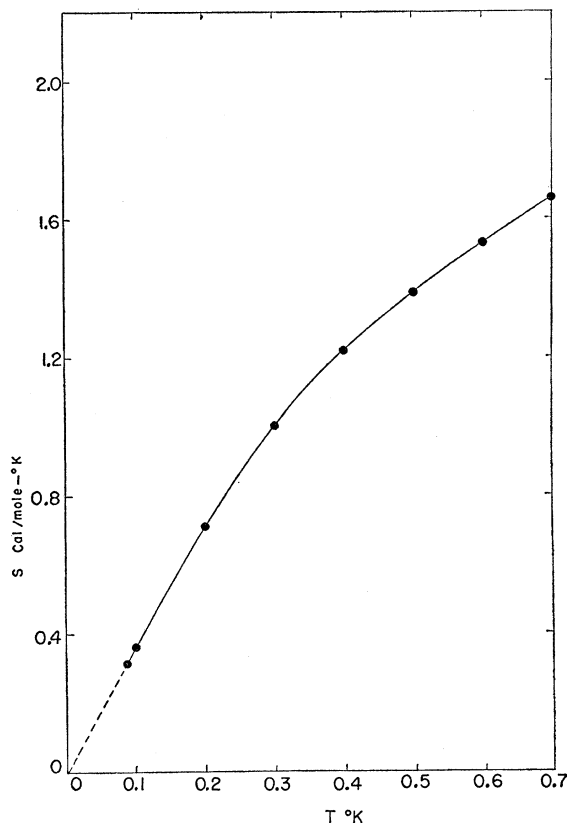


FIG. 2. Molar entropy of liquid He³ at a pressure of 12–15 cm Hg, calculated from the present specific heat measurements by using a linear extrapolation from 0.085°K to 0°K.

It is generally agreed¹⁰ that the condition leading to superfluidity in He⁴ is the absence of low-lying energy states apart from the collective phonon excitations. It follows from the linear Fermi-type specific heat that in He³ an abundance of low single-particle excitations exists down to 0°K, thus preventing the occurrence of superfluidity, a conclusion inherent in the original statistical approach of London.¹¹

The marked deviation from an ideal Fermi function above about 0.2°K indicates a more complex character for the excitations at these higher temperatures. It is anticipated that further insight into the nature of the excitations will result from measurements on the liquid under pressure, which are now in progress. Further discussion is therefore postponed.

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Electron Nuclear Double Resonance of Neodymium*

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THE method of electron nuclear double resonance has recently been employed by Feher¹ for investigation of some solid state problems. In this letter we discuss the application of the method to an investigation of Nd³⁺ in LaCl₃ crystals.

Both Nd¹⁴³ and Nd¹⁴⁵ have nuclear spin 7/2. The lowest crystal field state of Nd³⁺ (*f*³, *I*_{9/2}) in LaCl₃ is a Kramers doublet described approximately by²

$$0.941|\pm 7/2(-0.339)|\mp 5/2\rangle.$$

The energies of the hyperfine levels are therefore given by³

$$W(M, m) = -\frac{1}{2}hcA + hcP\{M^2 + m^2 + 2Mm - 5\} \\ \pm \frac{1}{2}\{[(M+m)hc(A-2P) + g_1\beta H]^2 \\ + (hcB)^2[16 - (M+m)^2]\}^{1/2} - g_N'\beta Hm, \quad (1)$$

for the case in which **H** is parallel to the *c* axis of the