element will depend primarily on possible errors in the monitoring reactions. These relative cross sections are estimated to be good to about $\pm 15\%$.

The cross section for H^3 production in high-energy reactions is roughly proportional to the geometric cross section for targets of $Z=6$ to 82,[5–7] and increases with increasing bombarding energy. At 2.2 Bev, the tritium production cross section is about 10% of geometric.[7] In all targets from Al to Au, the excitation functions for Be^7 production also rise with increasing energy.[1–4] The Be^7 cross sections are from 1–10 mb at 2.2 Bev for all of these elements.[3] The results in Table II show that the He^6 cross sections are comparable to or larger than the corresponding Be^7 cross sections for Al and heavier targets. The ratio H^3/He^6 appears to be approximately constant (at $\sim$15) in all cases.

The major production process for all three isotopes from medium to heavy target nuclides is assumed to be evaporation from the excited nucleus left following the initial high-energy events. Monte Carlo calculations, based on the evaporation model and using excitation energies from Monte Carlo calculations of the high-energy processes,[17] account for the observed yields within the rather wide limits of error of both experiment and calculation.[18]

The cross section for Be^7 production from carbon is

[17] Friedlander, Miller, Metropolis, Bivins, Storm, and Turkevich, Bull. Am. Phys. Soc. Ser. II, 2, 63 (1957).

[18] J. Hudis (private communication).

TABLE II. Cross sections (in millibarns) for He^6 production in high-energy proton bombardments.

Target \ Energy (Bev)	1.0	1.9	2.85
C	0.5	...	0.6
Al	1.1	1.3	...
Cu	2	4	4
Ag	4	7	12
Pb	10	21	29

10–12 mb at all energies in the range 0.34–3.0 Bev, and it has been suggested that the Be^7 is produced as a residue of the high-energy process, rather than by ejection as an entity.[1,3] The cross sections for production of He^6 from carbon are lower than those for Be^7 production by a factor of 20, and also do not change radically with bombarding energy. This lower cross section is plausible since production of He^6 as a residue of reactions such as $C^{12}(p,p2He^3)He^6$ or $C^{12}(p,3p\alpha)He^6$ is much less likely than the production of Be^7 in a reaction such as $C^{12}(p,p\alpha n)Be^7$.

ACKNOWLEDGMENTS

The authors wish to thank the operating crew of the Cosmotron, Mrs. J. Eliot, Mrs. G. Thieben, J. Eigner, and M. Aronchick for their assistance in performing the bombardments. The help and advice of Dr. G. Friedlander is gratefully acknowledged.

PHYSICAL REVIEW VOLUME 110, NUMBER 1 APRIL 1, 1958

Interaction of 1.5-Bev Negative π Mesons with Emulsion Nuclei*†

J. E. CREW‡ AND R. D. HILL
Physics Department, University of Illinois, Urbana, Illinois

(Received July 31, 1957)

The energy spectrum of charged pion secondaries, produced from emulsion nuclei by bombardment with 1.5-Bev negative pions, has been measured. The spectrum was observed to be sharply peaked for pions of laboratory energies from 100 to 150 Mev. The mean pion energy was 148 Mev, and 75% of the pions were in the energy range from 50 to 150 Mev. The observed number of charged pions per star was 0.66. Half of the emulsion stars had no meson track, 36% had a single meson, 12% two mesons, and 2% more than two mesons. The experimental results have been compared with Monte Carlo nuclear cascade calculations.

INTRODUCTION

FOR the most part, investigations of fundamental pion-nucleon interactions at high energies have now been performed. One might then ask whether this information can be used to describe the interaction of high-energy pions with nucleon complexes, such as nuclei. Can one assume, as has been done successfully in analyzing interactions of high-energy nucleons with nuclei, that individual pion-nucleon processes can be followed within nuclei as nuclear cascades?

Investigations of moderately high-energy pions (0.5–0.75 Bev) with nuclei, however, have shown that there are features of the inelastic processes which are difficult to understand on the basis of individual pion-nucleon

* Assisted by the joint program of the U. S. Atomic Energy Commission and the Office of Naval Research.

† Based on a thesis submitted to the Graduate College of the University of Illinois in partial fulfillment of requirements for the degree of Doctor of Philosophy.

‡ Present address: National Bureau of Standards, Washington, D. C.

interactions. For instance, Blau and co-workers[1] have found that pion inelastic interactions with emulsion nuclei are characterized by the apparent disappearance of large fractions of the incident energy and by a large number of events with no visible secondary mesons.

The present work is involved with an investigation of inelastic pion-nucleon events at the higher pion energy of 1.5 Bev. At this energy, secondary pion production was expected to occur much more frequently than below 1 Bev, and with the information concerning elementary pion-nucleon interactions at 1.5 Bev which had recently become available,[2,3] it was hoped that a test of the applicability of the nuclear cascade process to these events could be made.

DETAILS OF EXPOSURE

Strips of 400-μ Ilford G5 emulsion were exposed in the manner previously described[4] to the 1.5-Bev π^- beam of the Brookhaven Cosmotron. The momentum of the pions, measured by Cool, Madansky, and Piccioni[5] for the same beam, was 1.63 Bev/c, corresponding to an energy of 1.5 Bev. We have measured by multiple scattering the energies of the pions traversing our emulsions and have obtained a consistent value of (1.4 ± 0.2) Bev. For the present investigation, we accept the value of 1.5 Bev for the pion energy.

The average density of pion tracks throughout our emulsions was approximately 1.7×10^4 tracks per cm^2. The beam tracks were very flat in the emulsion strips and several centimeters of track normally remained in one strip. The angular spread of the beam was very narrow $(\pm1.3°)$ so that incident pions were readily recognizable. The μ-meson contamination in the beam was only 5%,[5] and the electron contamination was less than 1%.

DETAILS OF SCANNING

A total of 2.19 cm^3 of emulsion was area scanned for stars, 716 of which were identified as pion-produced and 740 were neutron-produced. For the measured pion flux, the apparent mean free path for star production by a 1.5-Bev pion was thus (51 ± 3) cm.

From the composition of Ilford G5 emulsion, if one takes the nuclear radius to be equal to $1.3\times10^{-13}\ A^{\frac{1}{3}}$ cm, the mean free path assuming the nuclear cross section equal to the geometrical cross section would be 31 cm. If a transparency factor for nuclear matter is calculated from the known total cross section for 1.5-Bev pions, the mean free path of 31 cm is increased to 38 cm. By along-the-track scanning Walker and Crussard[2] observed a mean free path of (35 ± 1) cm for 1.5-Bev pion interactions in emulsion.

The disparity between the apparent mean free path obtained in area scanning and the mean free path obtained by along-the-track scanning is certainly attributable to the inefficiency of area scanning. In area scanning, the zero-prong stars and the small-angle diffraction scatterings by nuclei are almost entirely missed. In addition, the efficiency for observing stars with only a single gray or black prong is only of the order of 50%.[1]

Unfortunately, information on the abundance of zero- and one-prong stars produced by 1.5-Bev pions is not available. However, we have estimated our scanning inefficiency from the extensive scanning data of Blau and Oliver[1] for a pion energy of 0.75 Bev. At this energy the mean free path in emulsion was found to be (38 ± 2) cm, which is close to the value of (35 ± 1) cm found by Walker and Crussard at 1.5 Bev. If we allow for the estimated muon contamination at 750 Mev of 6% and if we assume that all zero-prong stars and half of the stars with one prong would have been missed, the mean free path that would have been observed in area scanning at 750 Mev is (51 ± 3) cm. This is precisely the same value that we observe in our area scanning at 1.5 Bev. On this basis, then, we may assume that our sample of 716 pion stars represents, in accordance with Blau and Oliver's data, 75% of the total number of 1.5-Bev pion inelastic events in the scanned volume.

PRELIMINARY STAR CLASSIFICATION

Of the 716 meson stars found by scanning, a sample of 472 was chosen for analysis on the basis of satisfying the following requirements: (1) that the star was not so close to either of the emulsion surfaces that steeply dipping tracks might have escaped detection, (2) that the event was produced by a pion within the angular divergence of the beam, and (3) that the star was not located near the edge of the plate where possible large distortion of the emulsion might have affected the measurements.

In the first analysis of the 472 stars, all tracks from the stars were grain counted approximately and were classified on this basis either as protons or pions. Grain densities, g, were made relative to that of the 1.5-Bev beam pions. The normalized grain density of tracks was specified by $g^*=g/g_0$, i.e., relative to the ionization plateau value, g_0, which was taken to be 3% above the 1.5-Bev pion grain density. In this preliminary classification of outgoing pion and proton tracks from stars, a track was adjudged to be a pion if its normalized grain density, g^*, was less than 1.4. Otherwise the track was regarded as a proton. Some gray or

[1] M. Blau and M. Caulton, Phys. Rev. **96**, 150 (1954), M. Blau and A. R. Oliver, Phys. Rev. **102**, 489 (1956). A summary of the literature and general conclusions reached in earlier work with pions at energies lower than 0.5 Bev are given in these papers.

[2] W. D. Walker and J. Crussard, Phys. Rev. **98**, 1416 (1955); Walker, Hushfar, and Shephard, Phys. Rev. **104**, 526 (1956); Walker, Crussard, and Koshiba, Phys. Rev. **95**, 852 (1954).

[3] Eisberg, Fowler, Lea, Shephard, Shutt, Thorndike, and Wittemore, Phys. Rev. **97**, 797 (1955).

[4] Hill, Salant, Widgoff, Osborne, Pevsner, Ritson, Crussard, and Walker, Phys. Rev. **101**, 1127 (1956).

[5] Cool, Madansky, and Piccioni, Phys. Rev. **93**, 249 (1954); Cool, Piccioni, and Clark, Phys. Rev. **103**, 1082 (1956).

black tracks were immediately classified as pions because of their relatively large scattering which was obvious visually. From the 472 stars selected for analysis, 301 tracks were identified as pions and the remainder were classified as protons.

At 1.5-Bev incident pion energy, it is improbable, although kinematically possible, that a recoil proton in a pion-nucleon collision should acquire sufficient energy to be below the ionization plateau. For a nucleon Fermi energy of 25 Mev, recoil protons at or below the ionization plateau are limited to a forward cone of half-angle 27°. We have thus immediately identified tracks with $g^* \leqslant 1.0$ outside this cone as pions. However, to be certain of distinguishing relativistic protons from pions, multiple-scattering measurements were made on an appreciable sample of the fast tracks.

SCATTERING ANALYSIS OF TRACKS

Since most tracks having $g^*>2$ could be visually identified as either protons or pions, a sample of tracks with $g^* \leqslant 2$ was selected for scattering analysis. Only relatively flat tracks were scattered. Tracks having a length of at least 1 mm per emulsion strip were selected. The average length of a scattered track was 3.5 mm. Of 470 tracks with $g^* \leqslant 2$, a sample of 169 tracks was scattered. Of these 169 tracks, 118 were identified as pions, 49 as protons, one as a deuteron, and one as a K particle. The lone K particle escaped from the emulsion stack without interacting and after traversing 38 mm. In other experiments[3] two heavy mesons have been found per 700 meson endings for a pion bombardment at 1.5 Bev. Though not identical experiments, the finding of one K particle for 300 π mesons is in agreement with the earlier experiment.

Of the 169 tracks scattered, 113 had $g^* \leqslant 1.4$, and of these, the appreciable number of 22 were identified as fast protons. Of the same 169 tracks scattered, 56 had $2.0 \geqslant g^* \geqslant 1.4$, and again the appreciable number of 28 of these were identified as pions. It is clear therefore that pions are by no means restricted to grain densities less than $g^*=1.4$, i.e., energies greater than 70 Mev. It is interesting to note, however, that the use of a grain density cutoff of approximately $g^*=1.4$ leads to about as many protons being identified as pions, as pions are identified as protons. This is a rather important conclusion since it supports the classification of tracks which, because of unsuitability for multiple scattering, are identified on the basis of grain density alone, and it also confirms the value of the cutoff grain density which has been used for identifying pions from protons in many high-energy cosmic ray studies.

ANALYSIS OF LOW-ENERGY PIONS

Since multiple scattering measurements were made only on tracks for which the normalized grain density was less than 2.0, it is likely that some low-energy pions of less than 30 Mev ($g^*>2$) were overlooked in the preliminary grain-count analysis. In the group of 472 stars, only 15 pions out of the final total of 309 were identified on the basis of their obvious visual scattering or because they ended in characteristic pion interactions such as a σ-star or $\pi-\mu$ decay. In order to estimate the efficiency for detecting low-energy pions, a random sample of 30 stars was selected and all tracks, including the steep ones, with $g^*>2.0$ were identified by tracing through the emulsion stack and then measuring scattering *versus* ionization, scattering *versus* range, ionization *versus* range, or ionization *versus* distance along each track. For the 30 stars, with a total of 160 tracks with ionization greater than twice plateau, two additional low-energy pions were located. It is thus possible that as many as 30 low-energy pions in the group of 472 stars were overlooked.

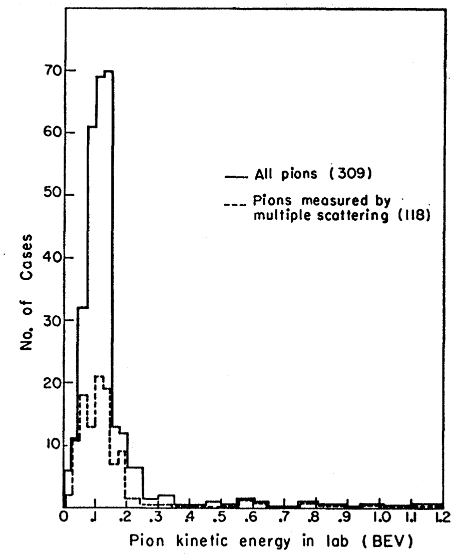

FIG. 1. Experimental energy spectrum of charged pions from stars produced by 1.5-Bev incident pions. The areas under the histograms are normalized to the total numbers of events considered on the basis of 25-Mev bins.

ENERGY DISTRIBUTION OF PIONS

The energy spectrum of charged pions from stars produced by 1.5-Bev pions is given in Fig. 1. This figure contains two histograms, one for the whole group of 309 pions (these exclude the estimated group of 30 pions which had energies of less than 30 Mev and which may have been overlooked), and another dotted histogram of 118 pions, the energies of which were determined from combined measurements of grain density and multiple scattering. It should be noted that the energies of the highest-energy pions (>0.5 Bev) were all measured by multiple scattering. The reason for this, of course, is that the very high-energy pions are all collimated in a narrow forward cone in the direction of the incident pion. Since the incident pion beam lies very closely in the plane of the emulsion, it

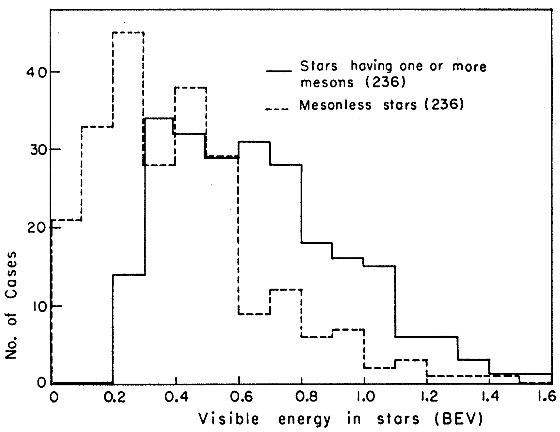

FIG. 2. Distribution of visible energy in stars produced by 1.5-Bev incident pions.

follows that the high-energy pions are generally suitable for scattering measurements. It should also be noted that the two histograms of Fig. 1 follow one another closely in the energy region below 0.5 Bev. This, we believe, indicates that the group of 118 tracks which was selected for multiple-scattering measurements represents a good sample of all the 309 pion tracks identified from grain-count measurements alone.

RELIABILITY OF ENERGY DETERMINATIONS

The specifications of energy values in Fig. 1 are based mainly on grain-density measurements referred to the available grain density *versus* energy data for G5 emulsions.[6]

Of the 309 observed pion tracks, only 91 have grain densities at or below the ionization plateau for which the pion energy is 140 Mev. Since there is no ambiguity in the energy specification of a pion having $g^*>1.0$, we believe that there is little doubt of the correctness of the measured energies of at least two-thirds of the total number of observed pions.

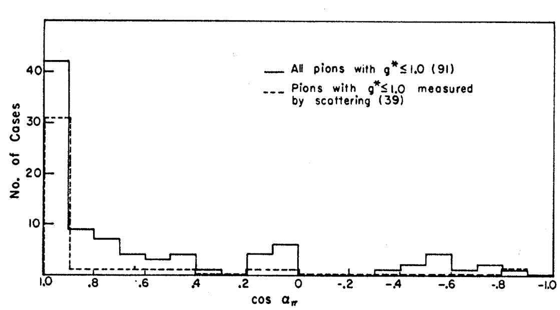

FIG. 3. Angular distribution of the more energetic ($\geq$140 Mev) pions from stars produced by 1.5-Bev incident pions. The angle α_π is the angle, in the laboratory system, of the secondary pion to the direction of the incident pion.

[6] M. Blau, Phys. Rev. **75**, 279 (1949), and unpublished data; A. Husain and E. Pickup, Phys. Rev. **98**, 136 (1955) and unpublished data; J. R. Fleming and J. J. Lord, Phys. Rev. **92**, 511 (1954).

Ionization measurements for tracks near or below the plateau were made by grain or blob counting generally to an accuracy of 4% standard error. The ionization for these tracks was measured relative to pion beam tracks in the same region and at the same level in the emulsion. A pion track of grain density equal to the plateau value and measured to 4% statistical accuracy would have an energy of 140_{-10}^{+50} Mev. Because of the slow variation of ionization with energy below the plateau ionization value, a given standard error in the grain density measurement leads to a larger uncertainty of the energy of those tracks with $g^*<1$.

For the 91 tracks having grain densities at or below the plateau value, accurate energy specifications could be made only if scattering measurements showed on which side of the minimum of the ionization-*versus*-energy curve the energy was located. In 39 out of the 91 cases, statistically acceptable multiple-scattering measurements were possible. The minimum in the ionization-*versus*-energy curve for pions occurs at approximately 450 Mev, and 26 of the 39 cases scattered were identified as being on the lower energy side of the minimum.

The problem of assigning accurate energy values to the remaining 52 tracks which had grain densities at or below the pleateau value and which were unscatterable was a difficult one. Many of these tracks were so steeply dipping that only approximate grain counts could be obtained from short track lengths in a number of plates. A large number were traced through successive emulsions, care being taken to standardize the grain density against beam tracks in the same area and at various levels in each emulsion. These tracks were also more susceptible to systematic errors in determining dip angles and eclipsing of grains along the lengths of steep tracks.

Of the 52 unscattered tracks with $g^*\leqslant 1.0$, 26 had $g^*\geqslant 0.99$ and their energies were thus below 150 Mev. Of the other 26 tracks above 150 Mev, 6 had $0.99>g^*>0.97$ and 20 were measured with $g^*<0.97$. For these 20 tracks, two assignments of energy were possible, *viz.*, to the high- or low-energy side of the ionization-*versus*-energy minimum. Since it is unlikely that the steeply dipping pions would have energies as high as the forward cone group of 39 which were scattered and for which approximately two-thirds had energies less than 450 Mev, we have arbitrarily assumed that

TABLE I. Frequencies of stars containing different numbers of meson secondaries.

Meson	Number per star	Number of stars	Percentage of total cases
π	0	237	50 $\pm$3.3
π	1	169	36 $\pm$2.8
π	2	57	12 $\pm$1.6
π	3	8	1.7$\pm$0.6
π	4	1	0.2$\pm$0.2
K	1	1	0.2$\pm$0.2

all of the 20 uncertain tracks had energies less than or equal to 450 Mev. This assumption, though unsubstantiated, is probably not greatly in error, since only of the order of a third of the 20 pions might have had energies greater than 450 Mev.

To sum up, we would point out that the possibility of missing 10% of the pions with energies less than 30 Mev and the difficulty of making accurate energy determinations on ~20% of the tracks above 150 Mev, will not greatly affect the form of the observed spectrum. The very marked peak which occurs between 100 and 150 Mev contains 41% of the pions. Three-fourths of all pions are in the energy range 50–150 Mev for which the energy determinations are straight forward.

CHARACTERISTICS OF STARS

As shown in Table I, 50% of all stars have no visible pion secondaries. Distributions of heavy prongs in stars containing different numbers of π-meson secondaries are given in Table II. Stars with mesons have, on the average, fewer heavy-particle prongs and more visible energy in the prongs than the mesonless stars. The mean number of heavy-particle prongs from mesonless stars is 7.5 ± 0.2. The mean number of nonmeson prongs from stars showing one or more charged mesons is 5.6 ± 0.2. To obtain an estimate of the visible energy in stars, every heavy-particle prong was assigned an energy corresponding to that for a proton of the observed grain density plus a binding energy of 8 Mev. The energy assigned to a pion was its kinetic energy plus 140 Mev. The mean visible energy for stars without mesons is (390 ± 30) Mev and for meson stars is (660 ± 50) Mev. The energy distributions for meson and mesonless stars are given in Fig. 2. The distribution for stars without visible mesons is similar to that for meson stars except that the distribution is shifted by the mean energy difference of 270 Mev. This can probably be explained by assuming that a π^0 with approximately 130-Mev kinetic energy is emitted in a large fraction of these events.

ANGULAR DISTRIBUTION OF PIONS

Angular distributions of pions with $g^*\leqslant 1$ and $g^*\geqslant 1$ are shown in Figs. 3 and 4, respectively. Both histograms show large forward-backward asymmetries with respect to laboratory angle, and the asymmetry is larger for the higher energy group of pions. It can be seen in Fig. 3 that most of the high-energy pions in the forward direction were scattered. It is also clear from Fig. 4 that the scattered pions represented a good sample of all the pions observed.

TABLE II. Distributions of heavy prongs in stars containing different numbers of meson secondaries.

$N_\pi \backslash N_h$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
0	0	11	17	34	22	28	20	27	13	11	9	12	11	11	4	5	2
1	11	11	31	24	20	13	14	14	7	7	6	2	5	2	1	1	0
2	0	6	10	8	10	9	3	1	1	3	3	3	0	0	0	0	0
3	0	2	1	2	1	0	1	0	0	1	0	0	0	0	0	0	0
4	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

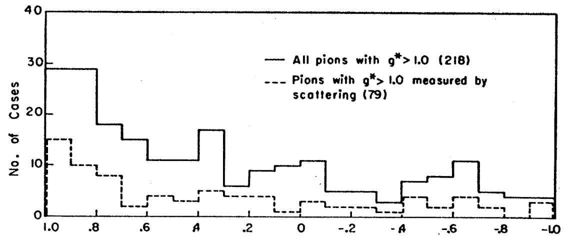

FIG. 4. Angular distribution of the less energetic (<140 Mev) pions from stars produced by 1.5-Bev incident pions. The angle α_π is the angle, in the laboratory system, of the secondary pion to the direction of the incident pion.

For the three angular intervals: 0°–60°, 60°–120°, and 120°–180°, the energy distributions of pions are shown in Fig. 5. The peak in the energy spectrum between 100 and 150 Mev is strikingly reproduced in each of the angular intervals. It will also be noticed that, with the exception of a small fraction of the highest energy pions in the forward angular interval, the energy distribution in all angular intervals is very closely the same. This seems to strongly suggest that the pions in the 100–150 Mev peak are secondary pions, which, since they do not show the influence of the forward center-of-mass motion, arise from processes largely uncoupled from the initial encounter of the incident pion.

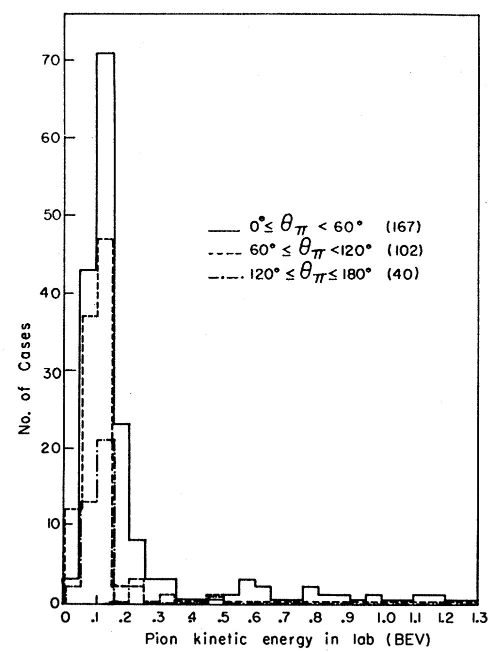

FIG. 5. Energy distributions of pions, from 1.5-Bev pion stars, for three different angular intervals: 0° to 60°, 60° to 120° and 120° to 180°. Angles are measured in the laboratory system to the direction of the incident pion.

COMPARISON OF RESULTS WITH OTHER EXPERIMENTS AT LOWER PION ENERGIES

The percentage of stars with charged meson secondaries is greater at 1.5 Bev than in comparable experiments on pion-produced stars at lower energies. At 210 Mev, Morrish[7] observed that $(27\pm7)\%$ of stars showed a charged pion escaping. At 500 Mev Blau and Caulton[1] observed that $(40\pm6)\%$ of the stars had one or more secondary pions, and at 750 Mev Blau and Oliver[2] found approximately the same value, $(43\pm4)\%$. At 1.5 Bev we find that $(50\pm3)\%$ of the stars have one or more charged meson secondaries.

Although our results are certainly in line with the other experiments at lower pion energies, it is striking that the multiplicity of secondary pions even at 1.5 Bev is so low. The mean number of charged secondary mesons observed per star is 0.66. If we include an estimated 30 pions probably missed because of the lower efficiency for detecting pions below 30 Mev, the mean number of charged pions per star is 0.72 ± 0.05. This value is for those stars having at least one black or gray prong. A very approximate estimate of the multiplicity for all stars may be obtained in the following way. In our experiments (Table II), we have observed that the multiplicity of the 11 one-prong stars is unity. If we assume, as seems a reasonable figure, that the multiplicity of zero-prong stars is unity and that of the other one-prong stars, which were missed, is likewise unity, then the inclusion of the 25% of stars which were overlooked in scanning increases the multiplicity to 0.8. Blau and Oliver found at 750-Mev incident pion energy that the mean number of secondary pions per star was approximately 0.43.

In Fig. 6 the low-energy portion of the secondary pion spectrum observed below 0.45 Bev in our experiment is compared with that given by Blau and Caulton.[1]

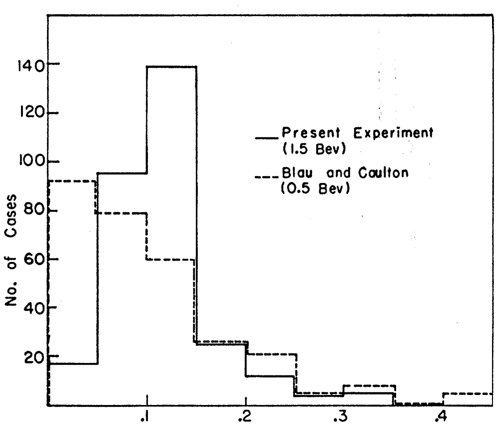

FIG. 6. Comparison of pion energy spectra from stars produced by 1.5-Bev incident pions and 0.5-Bev pions.

[7] A. H. Morrish, Phil. Mag. **45**, 47 (1954); Phys. Rev. **90**, 674 (1953).

(Both curves are normalized to the same total number of events below 0.45 Bev.) When one considers the large change in the incident pion energy from 0.5 Bev in the case of Blau and Caulton's experiment to 1.5 Bev in the present experiment, there is a surprisingly little difference in the secondary pion spectra, especially in the energy region above 0.15 Bev. The average pion energy in our experiment at 1.5-Bev incident pion energy is 0.15 Bev, and at 0.5-Bev incident pion energy Blau and Caulton observed an average pion energy of 0.11 Bev. There is very strong evidence therefore that, in high-energy pion interactions with nuclei, most of the incident pion energy is not spent in the production of secondary pions. As pointed out in the introduction, the evidence from earlier work already strongly suggested the "disappearance" of large fractions of pion energies in high-energy pion-nucleus interactions.

TABLE III. Results of the Los Alamos cascade calculation.

1. General features of the calculation	
Number of cascades	452
Number of transparencies	64
Number of inelastic events	388
Transparency	0.14 ± 0.02
Mean residual excitation energy per cascade	(247 ± 25) Mev
Mean number of protons struck per cascade	7.7 ± 0.1
Mean number of neutrons struck per cascade	8.9 ± 0.2

2. Number of particles escaping per cascade

Particle	Mean number per cascade
π^+	0.20 ± 0.02
π^-	0.91 ± 0.05
π^0	0.88 ± 0.05
p	2.52 ± 0.08
n	4.34 ± 0.11

3. Frequency of emission of charged pions per cascade

Number of pions	Number of cascades	Percent of total events
0	90	23.2 ± 2.4
1	177	45.6 ± 3.4
2	110	28.4 ± 2.7
3	11	2.8 ± 0.8

Concerning the total energies which are apparent in pion stars, a similar conclusion relative to the disappearance of energy is also reached. In our experiment with 1.5-Bev incident pions, the mean visible energy for stars with secondary mesons is (660 ± 50) Mev, of which the average total meson energy (rest energy plus kinetic energy) is 290 Mev. If we assume that the average energy taken off by neutrons exceeds the average proton energy evolved by 30%, the estimated energy per star is 1080 Mev. Even if it were assumed that on the average as many π^0's were emitted as charged pions and with an average energy equal to that of the charged pions, there is still an imbalance of approximately 270 Mev.§ Similar energy deficiencies

§ *Note added in proof.*—However, if a value for the neutron-proton emission ratio equal to 4.34/2.52 is taken from the Los in the pion stars produced by lower energy pions were observed by Blau and co-workers. Blau and Oliver observed stars of mean prong number equal to 4, whereas for stars of 900-Mev total excitation, as in their experiment, the anticipated number of prongs was 9.

COMPARISON WITH A MONTE CARLO CASCADE CALCULATION

Nuclear cascade calculations for high-energy pions incident on nuclei have been performed by Metropolis *et al.*[8] on the Los Alamos MANIAC computer.[9] In this model the particles were assumed to interact with individual nucleons according to the observed cross sections for free particles. Production of up to two mesons was considered according to an isotropic model in pion-nucleon and nucleon-nucleon collisions. Absorption of a pion was assumed to take place on two nucleons. The identity, energy, and direction of particles emitted per cascade were recorded, together with the excitation energy of the residual nucleus. The evaporation of the residual nucleus was not treated. For comparison with the results of the present experiment, a calculation was made of 452 cascades of 1.5-Bev negative pions incident on a $_{44}Ru^{100}$ nucleus, taken as a representative nucleus of AgBr emulsion.

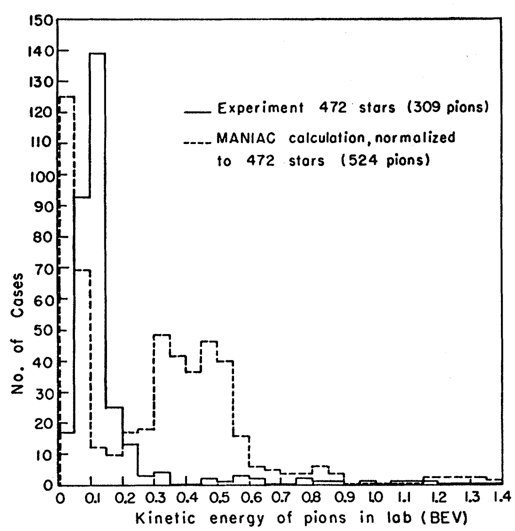

FIG. 7. Comparison of experimental and calculated pion energy spectra from stars produced by 1.5-Bev incident pions.

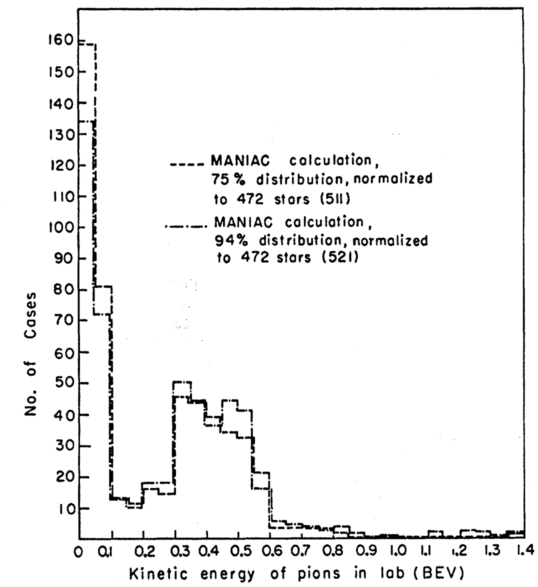

FIG. 8. Calculated pion energy spectra after modification for experimental scanning efficiencies. The pions are from stars produced by 1.5-Bev incident pions.

Results of the Los Alamos cascade calculations are shown in Table III. The average number of charged pions per cascade is 1.11 ± 0.05, as compared with 0.72 ± 0.05 obtained in our experiment. A histogram of the results of the calculation for the energy distribution of charged pions is shown in Fig. 7, compared with the experimental distribution, normalized to the same number of stars. The calculated pion spectrum exhibits two distinct peaks—one at low energy in the interval 0–50 Mev and a broad maximum at 450 Mev. The experimental spectrum has a single maximum‖ intermediate between these peaks.

As discussed in an earlier section, the employment of area scanning makes it likely that a number of small stars with zero or one gray or black prong have been missed. It is important to determine how the scanning inefficiency will disturb the comparison between the calculated and experimental distributions. The simplest, though not the most logical, way in which this can be done is to modify the calculated distribution by a scanning efficiency factor.

Alamos Monte Carlo calculations (see Table III), excellent energy balance is obtained. The agreement of the observed and calculated proton energy distributions (see Fig. 9) gives support to the use of the calculated ratio. The most likely explanation of the energy *losses* in pion stars seems to lie in the abundant emission of neutrons.

[8] Bivins, Metropolis, Storm, Turkevich, Miller, and Friedlander, Bull. Am. Phys. Soc. Ser. II, **2**, 63 (1957); Friedlander, Miller, Metropolis, Bivins, and Storm, Bull. Am. Phys. Soc. Ser. II, **2**, 63 (1957); and private communication. See also the following two papers: N. Metropolis *et al.*, Phys. Rev. **109**, 185, 204 (1958).

[9] The Los Alamos calculations for 1.5-Bev incident pions should be regarded as preliminary. Although they have been found to give good agreement with experiment at 500 Mev,[8] above this energy only two broad energy bins have been used to cover the entire region to 1.5 Bev. Furthermore, many of the experimental constants, especially in regard to secondary pion production, were of a preliminary nature when the program was set up in 1956. We are indebted to Dr. N. Metropolis, and the group working on the Monte Carlo problem, for allowing us to quote the results of their cascades at this stage of the work.

‖ *Note added in proof.*—This peak in the region of 150 Mev is undoubtedly associated with the decay of the $\frac{3}{2}$, $\frac{3}{2}$ isobar. This process has been discussed in are earlier paper on a Monte Carlo calculation of pion production by pions on hydrogen: Crew, Hill, and Lavatelli, Phys. Rev. **106**, 1051 (1957). See also the analysis by Lindenbaum and Sternheimer, Phys. Rev. **106**, 1107 (1957).

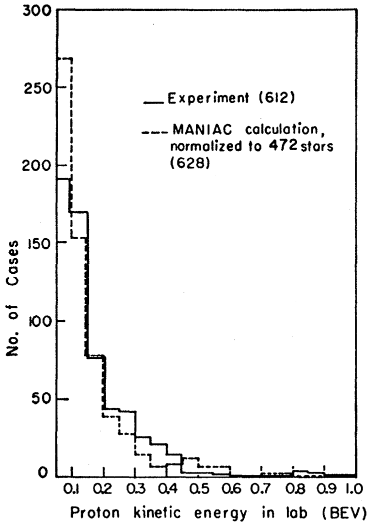

FIG. 9. Comparison of experimental and calculated energy spectra of protons of energies greater than 50 Mev. The protons are from stars produced by 1.5-Bev incident pions.

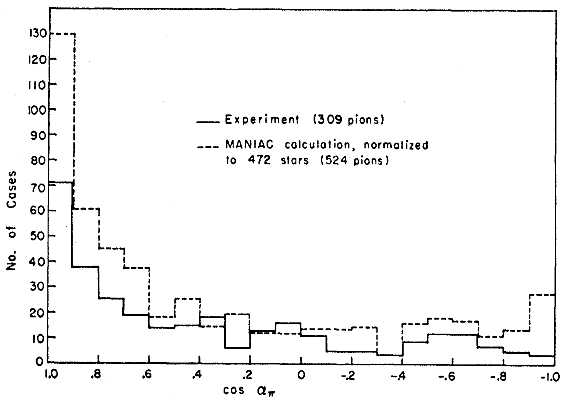

FIG. 10. Comparison of experimental and calculated angular distributions of charged pions from stars produced by 1.5-Bev incident pions. Angle α_π is the angle, in the laboratory system, of the secondary pion to the direction of the incident pion.

In the calculation, 31 cascades have zero gray or black prongs and 66 have but one. These correspond to 25% of the total inelastic events. These will not all correspond to zero and one prong stars, because the evaporation of the excited residual nucleus has not been treated and additional heavy prongs may be expected in some cases. On the average the zero- and one-prong cascades have 140-Mev excitation energy. According to Le Couteur's calculations on the evaporation model,[10] we would expect an average of 1.8 additional protons per cascade.

In the experiment it is believed that approximately 25% of the inelastic events were not found in area scanning. If we assume the worst possible case for the calculation in which all of the zero- and one-prong cascades were missed, then we may compare the distribution for the remaining 75% of the stars with the experimental results. The modified histogram for this energy distribution is shown in Fig. 8, normalized to the number of stars experimentally observed, 472.

It is believed that the efficiency for finding one-prong events is approximately 50%. This would reduce the number expected to be missed to 16% of the total number of cascades. The number of observed zero- and one-prong stars will also be reduced considerably by taking into account the evaporation prongs. If we estimate that cascades with an excitation energy, E^*, less than 50 Mev emit no protons, and cascades with 50 Mev$<E^*<$100 Mev emit a single proton, and if we allow a 50% scanning efficiency for one-prong events, the expected number of zero- and one-prong events missed, according to the cascade calculations, becomes only 6%. The distribution for 94% over-all efficiency is also plotted in Fig. 8, normalized to 472 stars.

The histograms of the energy spectrum of pions in Figs. 7 and 8 for the cascade calculation are thus only slightly altered from their original form when allowance is made for the inefficiency of area scanning. It is true that the small stars, with zero or one gray or black track, contain a larger fraction of the higher energy pions than do the stars comprising the entire group in the calculation. This is as expected, since the small stars are those in which pions scattered or produced in the primary collision make few or no further collisions before escaping. However, the scanning inefficiency factor is totally inadequate to bring the number of 0.2- to 0.6-Bev pions in the calculated spectrum into line with the experimental spectrum. The mean multiplicity of secondary charged mesons is very nearly the same in the total and reduced histograms.

A histogram of the calculated proton energy spectrum above 50 Mev is shown in Fig. 9 and this is in agreement with experiment. The calculated angular distribution of charged pions from all cascades is shown in Fig. 10. The calculated distribution shows a much greater forward peaking than that found experimentally. This is consistent with the larger number of high-energy pions appearing in the cascade energy spectrum.

ACKNOWLEDGMENTS

We wish to acknowledge our indebtedness to Dr. E. O. Salant and the emulsion group at the Brookhaven National Laboratory for their assistance in exposing and scanning the emulsions. To Dr. F. T. Gardner of our own emulsion group we are indebted for his assistance in initially analyzing the stars. To Dr. N. Metropolis, Mr. R. Bivins, and Mr. M. Storm of the Los Alamos Theoretical Division, and to Dr. A. Turkevich of the University of Chicago, we are indebted for their discussions of the nuclear cascade calculations and for supplying us with the complete information of the cascades used in the comparison with our experimental observations. We also wish to thank our colleague Dr. L. S. Lavatelli for many helpful discussions, especially of the Monte Carlo calculations.

[10] K. J. Le Couteur, Proc. Phys. Soc. (London) **A63**, 259 (1950).