K_{e3} and $K_{\mu 3}$ modes may be made from Eq. (2) by neglecting m_e (and m_μ) compared to m_k. This gives the ratio (accurate to probably 30%)

$$w(K\rightarrow e+\nu+\pi)/w(K\rightarrow \mu+\nu+\pi)\sim 1,$$

which is quite consistent with experiment.[3]

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[1] Bogoliubov, Bilenky, and Logunov, Nuclear Phys. **5**, 383 (1958).
[2] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1956); E. C. G. Sudarshan and R. E. Marshak (to be published).
[3] For further discussion of the K_{e3} and $K_{\mu 3}$ decay modes, see A. Pais and S. B. Treiman, Phys. Rev. **105**, 1616 (1957); J. J. Sakurai, Nuovo cimento **5**, 649 (1958); J. J. Sakurai, Phys. Rev. **109**, 980 (1958).

Decays of the μ Meson in the Intermediate-Meson Theory*

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THE idea of a universal Fermi interaction has received some attention recently, partly as a result of various proposed symmetry principles[1] which restrict the form of the 4-fermion interaction, and partly because of a number of experiments[2] which indicate that the couplings which appear in the β decay are $V-A$, in agreement with those already found in the μ decay. It has further been suggested that this universality may come about through the interaction of a current with itself by exchange of a heavy charged boson,[1] which will be referred to as an intermediate meson. One advantage of such a mechanism is that direct 4-fermion interactions involving 4-charged or 4-neutral fermions cannot arise through the exchange of such a boson. These interactions could lead to unobserved decays like $\mu^\pm\rightarrow e^\pm+e^++e^-$ and $K^\pm\rightarrow\pi^\pm+\nu+\bar{\nu}$. However, as we shall see, the existence of the decay $\mu^\pm\rightarrow e^\pm+e^++e^-$ to a small extent, as an indirect process, is implied by the intermediate-boson hypothesis in its usual form.

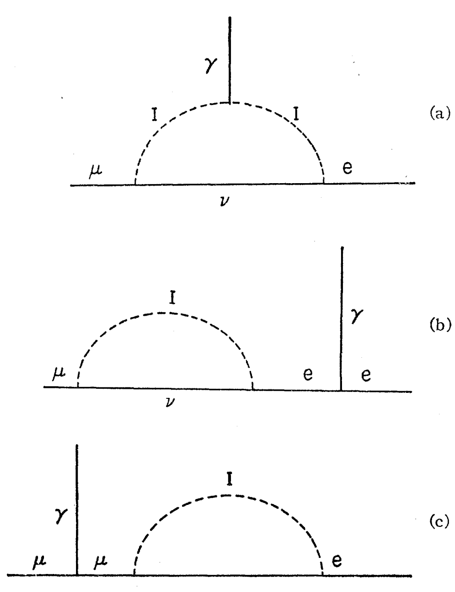

Fig. 1. Feynman diagrams for $\mu\rightarrow e+\gamma$ through an intermediate boson. *I* labels the intermediate boson field.

It is the purpose of this note to point out that the existence of such a heavy boson, with the properties required to give the known Fermi couplings, will itself lead to the occurrence of decays which are not found in nature, and which would not occur in any detectable amount if there are no intermediate bosons. Specifically, we consider the hypothetical decay $\mu\rightarrow e+\gamma$. This alternate decay mode of the μ has been looked for by Lokanathan and Steinberger,[3] who have found that the branching ratio for it compared to the ordinary μ decay,

$$\rho=R(\mu\rightarrow e+\gamma)/R(\mu\rightarrow e+\nu+\bar{\nu}),$$

is less than 2×10^{-5}.

If the intermediate meson exists and is coupled to the $\mu\nu$ and $e\nu$ pairs as has been suggested, then the decay $\mu\rightarrow e+\gamma$ can proceed by the following chain of virtual processes:

1. $\mu\rightarrow$intermediate meson and neutrino.
2. Intermediate meson$\rightarrow$intermediate meson+photon.
3. Intermediate meson+neutrino$\rightarrow$electron.

There are two similar chains in which the electron or μ meson emits the photons. The three Feynman graphs which represent this process in lowest order are given in Fig. 1.

The point to note[4] is that the coupling constant g_I for the interaction of the intermediate mesons with the fermions is proportional to the square root of the Fermi coupling constant G, and therefore the matrix element for the decay $\mu\rightarrow e+\gamma$ via the chain 1–3 will be of order Ge, whereas the matrix element for the ordinary μ decay is of order G, so that one would expect a measurable branching ratio for $\mu\rightarrow e+\gamma$. This is to be contrasted with the case when only the four-fermion couplings with 2 charged and 2 neutral fermions exist, where the matrix element for $\mu\rightarrow e+\gamma$ will involve at least G^2, and where the decay will therefore be very slow.

An evaluation of the graphs of Fig. 1 has been made for intermediate mesons of spin 1. The interaction of the intermediate mesons with leptons was taken as

$$g_I\{\bar{\psi}_\mu\gamma_\rho(1+\gamma_5)\psi_\nu\phi_\rho+\text{c.c.}\} \\ +g_I\{\bar{\psi}_e\gamma_\rho(1+\gamma_5)\psi_\nu\phi_\rho+\text{c.c.}\}, \quad (1)$$

where ρ is summed from 1 to 4, and ϕ_ρ are the 4-meson field operators. This interaction gives an effective 4-fermion coupling of the required $V-A$ form with correction terms which are very small provided that the heavy meson mass is large compared to the momentum transfers involved in the β and μ decays.

The diagrams 1(b) and 1(c) contribute only non-gauge-invariant terms of the form

$$\bar{\psi}_e\gamma_\rho(a+b\gamma_5)\psi_\mu A_\rho, \qquad (1)$$

which are exactly cancelled by similar terms coming from 1(a). The remaining terms coming from 1(a) are gauge invariant. The matrix element is

$$M=\bar{\psi}_e\sigma_{\alpha\beta}(1-\gamma_5)\psi_\mu F_{\alpha\beta}(g_I{}^2/m^2)(m_\mu e/96\pi^2)N, \quad (2)$$

provided $m\gtrsim 3m_\mu\gg m_e$, where m_μ is the μ-meson mass, m is the intermediate-meson mass, and N is a numerical factor. It may be seen that the coupling constant g_I and the heavy-meson mass m appear only in the combination $g_I{}^2/m^2$, which is the same combination appearing in the matrix element for the μ decay into electron and neutrinos, so that these parameters referring to the intermediate meson cancel in calculating the branching ratio ρ. This branching ratio is obtained in the usual way from the matrix element (2) and is

$$\rho=(\alpha/24\pi)N^2. \qquad (3)$$

The value of N is somewhat ambiguous because of the possibility of giving the intermediate meson an anomalous magnetic moment. For mesons with the "normal" magnetic moment of one magneton, the value of N diverges logarithmically. If a cutoff factor $\Lambda^2/(\Lambda^2+k^2)$ is introduced into the integrals[5] and Λ is taken to be the mass m of the intermediate meson, then N is 1. If the meson is given a magnetic moment of 2 magnetons, N is finite without a cutoff and again equal to 1. This value of $N=1$ gives $\rho=10^{-4}$, or 5 times the experimental upper limit. The cutoff parameter Λ^2 must be taken $<\frac{1}{4}\mu^2$ to reduce the branching ratio to be consistent with experiment.[6]

These results indicate that this kind of intermediate-meson theory is probably inconsistent with the experimental absence of the $\mu\rightarrow e+\gamma$ decay mode, whereas the universal couplings which the intermediate meson is supposed to generate are not. It would be interesting to try and obtain more precise experimental information about the branching ratios to see how serious the discrepancy is. Theories in which there are explicit selection rules forbidding $\mu\rightarrow e+\gamma$ and containing intermediate bosons are of course not ruled out.[7]

It may also be noted that the existence of an effective coupling of the kind in Eq. (2) between μ, e, and photon will imply the occurrence of the process $\mu^\pm\rightarrow e^\pm+e^++e^-$ by the internal conversion of the photon. The rate of this decay compared to the decay $\mu\rightarrow e+\gamma$ can be obtained from the result of Kroll and Wada,[8] and is

$$\frac{R(\mu^\pm\rightarrow e^\pm+e^++e^-)}{R(\mu^\pm\rightarrow e^\pm+\gamma)}\simeq\frac{2\alpha}{3\pi}\left[\ln\left(\frac{2m_\mu}{m_e}\right)-\frac{11}{6}\right]\simeq\frac{1}{150}.$$

When this is combined with the previous result, $\rho=10^{-4}$, it gives

$$R(\mu^\pm\rightarrow e^\pm+e^++e^-)/R(\mu^\pm\rightarrow e^\pm+\nu+\bar{\nu})\simeq 10^{-6}.$$

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[1] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958); E. C. G. Sudarshan and R. E. Marshak, Phys. Rev. **109**, 1860 (1958); J. J. Sakurai, Nuovo cimento **8**, 649 (1958).

[2] Goldhaber, Grodzins, and Sunyar, Phys. Rev. **109**, 1015 (1958); Hermannsfeldt, Maxson, Stehelin, and Allen, Phys. Rev. **107**, 641 (1957).

[3] S. Lokanathan and J. Steinberger, Phys. Rev. **98**, 240(A) (1955).

[4] See for example the discussion in R. P. Feynman and M. Gell-Mann, reference 1, where it is shown that, in our notation, $g_I{}^2/m^2=(\frac{1}{4}\sqrt{8})G$. This result will hold provided that nonlocal effects are sufficiently small that the Fermi coupling is described by the interaction of reference 1, $(\frac{1}{4}\sqrt{8})G\bar{\psi}_1\gamma_\mu(1+\gamma_5)\psi_2\bar{\psi}_3\gamma_\mu\times(1+\gamma_5)\psi_4$, which would mean that the intermediate meson mass is large compared to the momentum transfer involved.

[5] R. P. Feynman, Phys. Rev. **76**, 769 (1949).

[6] Intermediate mesons of spin zero, coupled to spinor fields directly or with derivatives, give effective scalar and pseudoscalar Fermi couplings, in disagreement with experiment. The branching ratio ρ for such intermediate mesons has been computed and found to be finite and again equal to 10^{-4}. The equality of the direct and derivative couplings here both for the effective Fermi couplings and for ρ follows from the equivalence theorem.

[7] Such selection rules were first proposed by E. J. Mahmoud and H. M. Konopinski, Phys. Rev. **92**, 1045 (1953). They essentially consist of giving opposite lepton numbers to μ^- and e^-. See also J. Schwinger, Ann. phys. **2**, 407 (1957). These theories require 4-component neutrinos to give the correct μ-decay spectrum.

[8] N. M. Kroll and W. Wada, Phys. Rev. **98**, 1355 (1955).

Errata

Electron Scattering by Polarized Nuclei, ROGER G. NEWTON [Phys. Rev. **109**, 2213 (1958)]. There is another change in Θ due to the fact that the components of $\boldsymbol{\mu}$ no longer commute and hence $\boldsymbol{\mu}\times\boldsymbol{\mu}\neq 0$. This gives rise to an additional term which depends on the electron polarization. The result is that Θ in (3) must be replaced by

$$\Theta_\pm=\Theta\pm s^{-1}\sin(\tfrac{1}{2}\theta)\cos(\mathbf{S},\mathbf{p}_f-\mathbf{p}_i) \qquad (7)$$

depending on whether it goes into σ_{++} or σ_{--}, and where Θ is given by (5).

It should be noticed that in consequence of (7) there is a term proportional to μ^2 which nevertheless changes sign when the nuclear spin is reversed. In other words, even though the electron interacts with the nuclear spin only via the nuclear magnetic moment, it is able to distinguish between the directions of the former and the latter. This at first sight surprising result is, of course, due to the conservation of angular momentum.