Note on Relations between Baryon-Meson Coupling Constants

A. PAIS
Institute for Advanced Study, Princeton, New Jersey
(Received May 7, 1958)

IN a previous paper[1] it has been shown that the existence of certain baryon-meson coupling constant relations leads to directly verifiable connections between observables. The relations discussed state in essence that the Σ and Λ particles are coupled with the same strength, whether to π or to K mesons. In the notation of I this is expressed by

$$G_2=G_3, \quad F_1=F_2, \quad F_3=F_4, \tag{1}$$

or

$$G_2=-G_3, \quad F_1=-F_2, \quad F_3=-F_4. \tag{2}$$

Connections were then established between the various amplitudes A for Σ or Λ production in π-nucleon collisions, namely (in obvious notation)

$$A_\Lambda \approx -A_0, \tag{3}$$

$$A_0\sqrt{2} \approx -A_-. \tag{4}$$

The near equality signs refer to the neglect of $\delta \equiv (M_\Sigma - M_\Lambda)/M_\Lambda$ in making these statements. [This quantity is not much bigger than the ratio $\{m(\pi^+) - m(\pi^0)\}/m(\pi^0)$. Thus one should not be much worse off in the present case than in neglecting π-mass differences in meson phenomena, as long as one is not too close to threshold.]

For a further study of the strong interactions it seems interesting to know whether any other relations between the constants could possibly lead to practically useful connections between the production amplitudes. Here one has in mind connections stronger than the triangle relations between the Σ amplitudes which are already implied by charge independence. Thus one may envisage either or both of the following relations:

$$A_\Lambda + \alpha_1 A_0 + \alpha_2 A_- \approx 0, \tag{5}$$

$$A_0 + \alpha_3 A_- \approx 0, \tag{6}$$

where α_1, α_2, α_3 are numbers. The relations (3) and (4) are special cases of (5) and (6). It is the purpose of this note to state the following theorem:

In order to establish nontrivial equations of the type (5) and/or (6), the relations (1) or (2) are not only sufficient, but also necessary.

In other words, the relations (1) and (2) exhaust the possibility of establishing stronger connections than those following from charge independence. Because of the complexity of the interactions this result is perhaps not obvious at once, but it is not very surprising: for *only* when the conditions (1) or (2) are met does one deal with a stronger invariance (four-dimensional rotation group) for *all* baryon-meson couplings than charge independence. (As was noted in I, it is not necessary to have stronger relations than (1) or (2) and further mass degeneracies[2] to establish this four-invariance.)

It is clear that the above statement does not mean that relations like (5) or (6) and other than (3), (4) could not be established. What is implied is that further dynamical arguments would be needed to achieve this.

It has been noted[3] that for the case $G_3=0$ the interaction allows for an additional substitution invariance if $\delta \approx 0$ and if the F relations (1) or (2) are satisfied. Unfortunately this has no physical consequences as the substitution in question does not leave the free-field terms and the anticommutation relations invariant.

To prove the theorem it is legitimate to consider an expansion in Feynman diagrams. One groups together in one term all contributions where particles of the same multiplet occur in an equivalent way virtually, the Σ and Λ states being considered to belong to one multiplet. It suffices to consider the following terms: the two second-order ones (crossed and uncrossed term), and the fourth-order terms obtained from these two by inserting a virtual π-line, of which there are six.[4] Now one must require the relations (5) and/or (6) to hold for each type of term. After having shown that the cases in which one or more of the constants occurring in Eq. (1) vanish do not lead to a desired relation, one then finds that the consistency conditions $(F_1G_2-F_2G_3)=0$, $G_2^2=G_3^2$ must be met. This completes the argument apart from the cascade couplings F_3, F_4. Inserting these in simple ways one gets $F_1F_3-F_2F_4=0$ as a further necessary condition. Thus one is led to the set (1) or (2). Note that the above statement is independent of whether or not other strong interactions (such as quadrilinear ones) are present.

A similar result holds true for K-nucleon scattering. Also for this case a result was found in I which is stronger than the triangle relations for this type of scattering: it was shown that if (1) or (2) is valid, the charge exchange cross section vanishes in the approximation $\delta=0$. It can again be seen that the necessary and sufficient conditions for obtaining a result stronger than the K-nucleon triangle are the relations (1) or (2). The study of K^- capture leads to the same conclusion.

The relations (1) or (2) also have consequences for photoproduction of K particles. From the definition of the electric charge operator given in I, it follows that the quantum numbers S_1, S_2 introduced in the previous paper remain good also in the presence of electromagnetic interactions. Hence it is easily seen that

$$A_\Lambda{}^\gamma \approx -A_0{}^\gamma,$$

where the A^γ are the amplitudes for the reactions $\gamma+p \rightarrow \Lambda+K^+$, Σ^0+K^+, respectively. The above theorem also applies to photoproduction.

Finally it may be reiterated that this as well as the previous work are based on the assumptions that Σ and Λ have the same spin, that the (Σ,Λ) parity is even, that charge independence in the conventional sense holds true, and that the baryon spectrum is complete.

The author is grateful to Professor Niels Bohr for the hospitality extended to him at the Institute for Theoretical Physics, Copenhagen, where this work was done. He is indebted to Nordita (Nordisk Institut for Teoretisk Atomfysik) for a grant.

[1] A. Pais, Phys. Rev. **110**, 574 (1958), quoted here as I.
[2] The symmetries and degeneracies considered by M. Gell-Mann, Phys. Rev. **106**, 1296 (1957) and J. Schwinger, Ann. phys. **2**, 407 (1957) in their treatment of the π couplings, actually correspond to an invariance with respect to the direct product of three unitary unimodular groups.
[3] d'Espagnat, Prentki, and Salam, Nuclear Phys. **3**, 446 (1957), Eq. (2.1).
[4] Apart from the self-energy terms. Not all six terms are needed for the argument.

Decay of *K* Mesons as a Test of the Universal Fermi Interaction*

F. ZACHARIASEN
Stanford University, Stanford, California
(Received March 10, 1958)

On the basis of current experimental evidence, there is no distinction between μ mesons and electrons other than a difference in rest mass. Electromagnetic interactions are identical since electrons and μ mesons have the same charge, and the hypothesis of a universal Fermi interaction, which asserts that they couple in exactly the same way in the weak interactions, has received considerable support by comparing β-decay phenomena with the decay of free μ mesons and μ capture in nuclei. An at least equally fruitful series of experiments for reassuring one about the validity of the universal interaction idea is the analysis of the decays of π and K mesons into electrons and μ mesons. The details of such individual decays are not easily obtained theoretically, since they presumably involve strongly coupled virtual particles; however, the ratio of decay rates into electron modes and μ-meson modes may often be calculated exactly with no assumptions beyond saying that the weak interaction is local and may be treated in first order, and that the weak interaction is the same for electrons and μ mesons.[1]

If the universal coupling is taken as $V\pm A$, such analyses predict theoretically that in most cases the electron decay mode should be far less prevalent than the μ-meson mode, and (qualitatively at least) experiments support this conclusion. The only experimental situation in which the electron decay mode and the μ-meson decay mode are both actually observed is in the K_{e3} and $K_{\mu 3}$ decays of K^+ mesons. It would seem to be valuable, therefore, to use this process as a further test of the universal interaction hypothesis.

Decays into electrons are observed in the process

$$K^+\rightarrow e^+ + \nu + \pi^0.$$

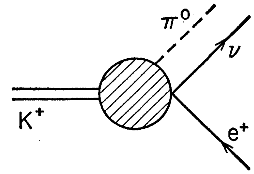

FIG. 1. Diagram representing the decay $K^+\rightarrow e^+ + \nu + \pi^0$.

This is to be compared with the decay

$$K^+\rightarrow \mu^+ + \nu + \pi^0$$

which is also observed. With the $V\pm A$ coupling hypothesis,[2] Lorentz invariance arguments give the following for the form of the electron-decay matrix element (see Fig. 1):

$$M(K\rightarrow e+\nu+\pi) = (\bar{u}_{p_\nu}|fp_K(1\pm i\gamma_5)+gm_e(1\mp i\gamma_5)|v_{p_e}). \quad (1)$$

Here f and g are functions of $p_K\cdot p_\pi/m_K$, m_K, and m_π, where p_k and p_π are the 4-momenta of the K meson and pion, respectively. If the K decays from rest one has $p_k\cdot p_\pi/m_k=E_\pi$, the energy of the pion. The Dirac spinors u_{p_ν} and v_{p_e} describe a neutrino of momentum p_ν and a positron of momentum p_e, respectively. If time-reversal invariance holds, f and g are real functions.

The decay rate then is given by

$$\frac{d^2w(K\rightarrow e+\nu+\pi)}{dE_\pi dE_e}=\frac{1}{(2\pi)^3}\frac{1}{4m_K} \times\{f^2m_K^2(4E_eE_\nu-m_K^2-m_\pi^2+m_e^2+2m_KE_\pi) + g^2m_e^2(m_K^2+m_\pi^2-m_e^2-2m_KE_\pi) - 2fgm_em_K(2m_eE_\nu)\}. \quad (2)$$

Here E_e and E_ν are the positron and neutrino energies; we of course have

$$m_K=E_e+E_\nu+E_\pi. \quad (3)$$

On the assumption of a universal $V\pm A$ interaction, the decay rate for the process $K\rightarrow\mu+\nu+\pi$ is given by the same expression with the index e replaced by the index μ. Since the functions f and g depend *only* on E_π, and since they are the same in both the e- and μ-decay modes, it is possible to test the universal coupling hypothesis by measuring the μ-decay rate at a fixed value of E_π for two values of E_μ and thus determining f and g. Then the e-decay spectrum for the same value of E_π may be predicted uniquely and compared with experiment. A violation of this predicted spectrum by the experimental results must imply a violation of the assumption of a $V\pm A$ universal coupling.

At the present time, unfortunately, the experimental information is too meager to do this, but it is to be hoped that the data will exist before too long.

An estimate of the ratio of the total lifetime of the