energy directly, but only through its influence on the electronic kinetic stress against which the sound wave works. Interestingly enough, the proposed mechanism leads to conditions for maxima and minima which are just the reverse of those observed by MBG.

As has been noted, resonant ultrasonic attenuation can provide a tool for mapping the Fermi surface.[6] It presents neither the magnetic field strength problems of the de Haas-van Alphen effect nor the field penetration problems of microwave cyclotron resonance. These desirable qualities are shared with resonant boundary scattering of the electrons, which produces oscillations of the transport coefficients in H^{-1} rather than in H, and which has recently been detected in the electrical conductivity of sodium wires.[7] Doubtless, much desirable information can be obtained between the two effects.

The attenuation has been calculated for the three geometries of interest, but the "solutions" unfortunately require considerable numerical work. The $Y_{\parallel}$ case, for which the simplicity of the electronic motions is reflected in the simplicity of the solution, will be attempted first.

The writer is indebted to R. W. Morse and H. E. Bömmel for valuable discussions.

[1] H. E. Bömmel, Phys. Rev. **100**, 758 (1955).
[2] R. W. Morse and H. V. Bohm, *Proceedings of the Fifth International Conference on Low-Temperature Physics and Chemistry, Madison, Wisconsin, August 26–31, 1957* (to be published).
[3] Morse, Bohm, and Gavenda, Phys. Rev. **109**, 1394 (1958).
[4] M. S. Steinberg, Phys. Rev. **109**, 1486 (1958).
[5] M. S. Steinberg, Phys. Rev. **110**, 772 (1958).
[6] A. B. Pippard, Phil. Mag. **2**, 1147 (1957).
[7] J. Babiskin and P. G. Siebenmann, Phys. Rev. **107**, 1249 (1957).

Møller Scattering of Arbitrarily Polarized Electrons

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THE electron-electron cross section for arbitrarily polarized electrons has been evaluated. The covariant form of this cross section is

$$d\sigma = d\sigma_M + d\sigma_P$$

$$= 4r_0^2 \frac{m^2\beta_1'\epsilon_1' d\Omega}{[(p_1\cdot p_2)^2 - m^4]^{\frac{1}{2}}} \frac{d\epsilon_1'}{d(\epsilon_1'+\epsilon_2')}(X_M+X_P),$$

$$X_M = t^2[2m^4+2m^2(p_1\cdot p_1')+(p_1\cdot p_2)^2+(p_1\cdot p_2')^2]$$
$$+k^2[2m^4+2m^2(p_1\cdot p_2')+(p_1\cdot p_2)^2+(p_1\cdot p_1')^2]$$
$$+tk[4m^2(p_1\cdot p_2)+2(p_1\cdot p_2)^2],$$

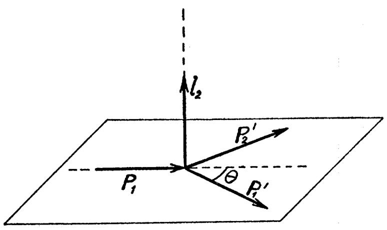

FIG. 1. Angular relations in the case of measurement of the transverse component of the polarization vector in the laboratory system.

$$\begin{aligned} X_P = &-m^2t^2[(S_1\cdot S_2)(p_1-p_1')^2+(S_1\cdot p_1')(S_2\cdot p_2')] \\ &-m^2k^2[(S_1\cdot S_2)(p_1-p_2')^2+(S_1\cdot p_2')(S_2\cdot p_1')] \\ &-tk[2(S_1\cdot p_2)(S_2\cdot p_1')(p_1\cdot p_1') \\ &+2(S_1\cdot p_1')(S_2\cdot p_1)(p_2\cdot p_1') \\ &-2(S_1\cdot p_1')(S_2\cdot p_1')(p_1\cdot p_2) \\ &-2(S_1\cdot S_2)(p_2\cdot p_1')(p_1\cdot p_1')+m^2(S_1\cdot p_2)(S_2\cdot p_1) \\ &+m^2(S_1\cdot p_2)(S_2\cdot p_1')+m^2(S_1\cdot p_1')(S_2\cdot p_1) \\ &-4m^2(S_1\cdot S_2)(p_1\cdot p_2)], \end{aligned} \tag{1}$$

where $p_1=(\mathbf{p}_1,\epsilon_1)$, $p_2=(\mathbf{p}_2,\epsilon_2)$ are initial and $p_1'=(\mathbf{p}_1',\epsilon_1')$, $p_2'=(\mathbf{p}_2',\epsilon_2')$ are final four-momenta of the electrons, and $\epsilon_i=m\gamma_i=m/(1-\beta_i^2)^{\frac{1}{2}}$, $t=1/[\sqrt{2}(p_1-p_1')^2]$, $k=1/[\sqrt{2}(p_1-p_2')^2]$. The four-pseudovector[1]

$$S=\left(\mathbf{l}+\frac{(\mathbf{l}\cdot\mathbf{p})}{m(\epsilon+m)}\mathbf{p}, \frac{(\mathbf{l}\cdot\mathbf{p})}{m}\right)$$

is the covariant polarization vector of the electron. $d\sigma_M$ is the Møller cross section without the polarization, and $d\sigma_P$ denotes the polarization-dependent part of the cross section.

In the particular case of longitudinal polarization discussed by Bincer,[2] our results are identical with his.

Let us discuss the possibility of using the Møller scattering for measuring the transverse component of the polarization vector $\mathbf{l}_1$. We can put, in the laboratory system,

$$\text{(a) } S_2=(\mathbf{l}_2,0), \quad \text{(b) } (S_2\cdot p_1)=(\mathbf{l}_2\cdot\mathbf{p}_1)=0, \\ \text{(c) } (S_2\cdot p_1')=(\mathbf{l}_2\cdot\mathbf{p}_1')=0. \tag{2}$$

The assumption (2a) means that the target electrons are at rest. Their polarization vector $\mathbf{l}_2$ can be fixed by the direction of the magnetic field (e.g., in the foil). Then the condition (2b) means that the polarization vector $\mathbf{l}_2$ of the target electrons is perpendicular to the beam of incident electrons. The meaning of (2c) is that the measurement of the momenta of scattered electrons $\mathbf{p}_1'$, $\mathbf{p}_2'$ ought to be performed in a plane perpendicular to the vector $\mathbf{l}_2$ (see Fig. 1).

Because of (2) the cross section has now the following much simpler form:

$$d\sigma = d\sigma_M + d\sigma_P = d\sigma_M + (S_1\cdot S_2)d\sigma_S, \tag{3}$$

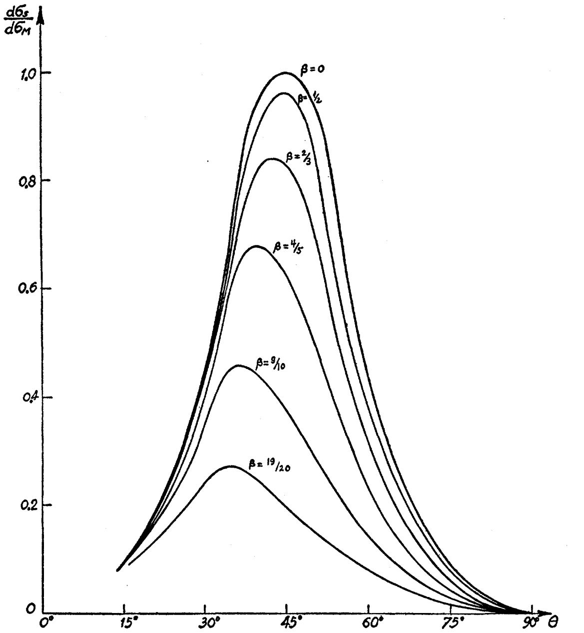

FIG. 2. The ratio $d\sigma_S/d\sigma_M$ against the scattering angle θ for different values of β.

where

$$d\sigma_S=4r_0^2\frac{m^2\beta_1'\epsilon_1'd\Omega}{[(p_1\cdot p_2)^2-m^4]^{\frac{1}{2}}}\frac{d\epsilon_1'}{d(\epsilon_1'+\epsilon_2')}\left[\frac{-m^2}{2(p_1-p_1')^2}-\frac{m^2}{2(p_1-p_2')^2}+\frac{(p_2\cdot p_1')(p_1\cdot p_1')+2m^2(p_1\cdot p_2)}{(p_1-p_1')^2(p_1-p_2')^2}\right]. \quad (4)$$

We see that by virtue of (2) the polarization vectors S_1 and S_2 enter the cross section only through the scalar product $S_1\cdot S_2$.

In order to find the best conditions for the measurement of the polarization vector, let us investigate the ratio

$$\kappa=d\sigma_P/d\sigma_M=(S_1\cdot S_2)d\sigma_S/d\sigma_M. \quad (5)$$

In the laboratory system we have now $(S_1\cdot S_2)=(\mathbf{l}_1\cdot\mathbf{l}_2)$ and the expression (5) becomes

$$\kappa=(\mathbf{l}_1\cdot\mathbf{l}_2)\times\frac{\frac{1}{4}+\frac{1-2\gamma}{(\gamma-1)^2(1-x^2)}}{\frac{\gamma^2}{(\gamma-1)^2}\left[\frac{4}{(1-x^2)^2}-\frac{3}{1-x^2}+\left(\frac{\gamma-1}{2\gamma}\right)^2\left(1+\frac{4}{1-x^2}\right)\right]}, \quad (6)$$

where

$$1-x^2=2(\gamma+1)\sin^2 2\theta/[2+(\gamma-1)\sin^2\theta]^2,$$

and θ is the angle between p_1 and $\mathbf{p}_1'$ in the laboratory system. In Fig. 2 the ratio $d\sigma_S/d\sigma_M$ is plotted against the angle of measurement for different values of β. The curves show strong maxima in the vicinity of $\theta=\frac{1}{4}\pi$, which with increasing β move towards smaller angles. Besides, one sees that the polarization effects increase with decreasing β. At any rate we see that the Møller cross section $d\sigma_M$ is never smaller than $|d\sigma_P|$.

To calculate the transverse component of the polarization vector $\mathbf{l}_1$, we find it convenient to introduce the ratio

$$\alpha=\frac{d\sigma_M+(S_1\cdot S_2)d\sigma_S}{d\sigma_M-(S_1\cdot S_2)d\sigma_S}=\frac{d\sigma_M+(\mathbf{l}_1\cdot\mathbf{l}_2)d\sigma_S}{d\sigma_M-(\mathbf{l}_1\cdot\mathbf{l}_2)d\sigma_S}, \quad (7)$$

which is the ratio of the cross sections for two opposite spin orientations of the target electrons under the assumption that both target and beam electrons are completely polarized. In the case of partial polarizations of the incident electrons and target electrons, the l's in (7) are to be replaced by

$$\mathbf{l}_1'=c_1\mathbf{l}_1,\quad \mathbf{l}_2'=c_2\mathbf{l}_2, \quad (8)$$

where c_1 and c_2 denote the degrees of polarization and the vectors $\mathbf{l}_1$ and $\mathbf{l}_2$ are unit vectors pointing in the directions of the average polarizations of the electrons. Then α is the value which can be found experimentally.

In the laboratory system the product $(\mathbf{l}_1'\cdot\mathbf{l}_2')$ can be expressed from (7) by the formula

$$(\mathbf{l}_1'\cdot\mathbf{l}_2')=c_1c_2(\mathbf{l}_1\cdot\mathbf{l}_2)=\left(\frac{\alpha-1}{\alpha+1}\right)\frac{d\sigma_M}{d\sigma_S}=\left(\frac{\alpha-1}{\alpha+1}\right)\times\frac{\frac{\gamma^2}{(\gamma-1^2)}\left[\frac{4}{(1-x^2)^2}-\frac{3}{1-x^2}+\left(\frac{\gamma-1}{2\gamma}\right)^2\left(1+\frac{4}{1-x^2}\right)\right]}{\frac{1}{4}+\frac{1-2\gamma}{(\gamma-1)^2(1-x^2)}}. \quad (9)$$

If the constants c_1 and c_2 are known, the transverse components of $\mathbf{l}_1'$ can be obtained from measurements of α for two independent directions of $\mathbf{l}_2'$.

A paper by Ford and Mullin[3] dealing with a similar problem has recently been published. The general expressions for the cross section given by these authors [formulas (4) and (5) of reference 3] differ in some respects[4] from our formula (1). Contrary to the statement made by Ford and Mullin (reference 3, p. 479, footnote 7) their results also differ from those of Bincer.[2,4] Moreover it can be shown that for some other particular cases, e.g.,

$$S_2\cdot p_1=0,\quad S_2\cdot p_1'=0,\quad \beta\to 0,\quad \theta=\tfrac{1}{4}\pi,\quad S_1\cdot S_2=1,$$

their cross section (4) and (5) becomes negative, showing that these equations are not correct.

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[1] H. A. Tolhoek, Revs. Modern Phys. **28**, 277 (1956).

[2] A. Bincer, Phys. Rev. **107**, 1434 (1957).

[3] G. W. Ford and C. J. Mullin, Phys. Rev. **108**, 477 (1957).

[4] *Note added in proof.*—We have been informed that the differences between our results and those of Ford and Mullin are due to two typographical errors in their paper. See G. W. Ford and C. J. Mullin, Phys. Rev. **110**, 1485(E) (1958) this issue.

Number of 3*d* Electrons in the Transition Metals*

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AT a recent conference, values were presented for the number of 3*d* electrons in Cu, Ni, Co, Fe, and Cr,[1] in the solid state as obtained from absolute measurements of the scattering factor for x-rays. Inasmuch as the results differ considerably from the well-known atomic configurations, they occasion some curiosity, as the authors have remarked.[1] Furthermore, the configurations advocated in reference 1 are difficult to reconcile with self-consistent calculations for the valence band-wave functions of the transition elements.[2] The interpretation of the experiments by Weiss and DeMarco rest on two assumptions. First, it is assumed that the contribution to the scattering by the "argon core" electrons may be assessed by means of a self-consistent field calculation for the free atom. Next, it is assumed that the extent of the 3*d* wave functions is greatly exceeded by that of the 4*p* and 4*s* so that the latter may be neglected. It is then the case that the scattering in excess of that for the "argon core" is due only to the 3*d* electrons. The last assumption may possibly be questioned and it would seem desirable to obtain some independent information on the number of 3*d* electrons per atom in the transition elements in the solid state. It is the purpose of this note to point out that some such information exists already in the literature and tends to favor the conventional configurations assigned to the atoms in the metal over those suggested by Weiss and DeMarco.

Several years ago Nilsson[3] observed that the excitation function for *K* series characteristic x-radiation from the transition elements exhibited a region fairly rich in structural detail over a few tens of volts from the excitation potential. A particular feature of interest here was termed by Nilsson the "*h* structure." It gives the appearance of a partially resolved "line" occurring slightly below the Fermi energy as determined from the results of Beeman and Friedman.[4] Nilsson interpreted this line as due to ionizing transitions in which the electron final state lies in the unfilled part of the 3*d* band. Its intensity would then be expected to vary with the filling of the 3*d* band and Nilsson observed this to be the case, assuming conventional configurations appropriate to the atoms in the metal.

A specific example may be taken from a comparison of Co and Fe. According to the assignments of Weiss and DeMarco, the line would be expected to be about four times as intense in Fe as in Co. The observed factor is approximately $\frac{4}{3}$ which would have been expected on the basis of conventional atomic configurations in the metals.

It is suggested that since in the scattering experiments there are no selection rules effective in sorting states of a special symmetry, those experiments yield only some measure of the total extent of the electron cloud. Distinction among symmetry types is more properly sought in spectroscopic measurements.

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* Supported by the Air Force Office of Scientific Research.

[1] R. J. Weiss and J. J. DeMarco, Revs. Modern Phys. **30**, 59 (1958).

[2] J. Callaway (private communication).

[3] A. Nilsson, Arkiv Fysik **6**, 513 (1953).

[4] W. W. Beeman and H. Friedman, Phys. Rev. **56**, 392 (1939).

μ-Mesonic Molecular Ions and Nuclear Catalysis*

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THE experimental observation[1,2] of the catalysis of nuclear reactions by μ mesons in liquid hydrogen[3] created interest at a number of laboratories and led to several independent theoretical investigations of the processes involved.[4] Although it soon became clear that this phenomenon could not lead to the production of useful power, some of the estimates of particular reaction rates were in wide disagreement, primarily because of the gross approximations made. Because of the intrinsic interest in the effect, and in order to obtain a more satisfactory understanding of these processes, we have made a detailed investigation of the molecular systems involved in these reactions.

It is assumed that nuclear catalysis proceeds through the following processes:

(1) The μ meson is slowed down by collisions in the liquid hydrogen and captured by a proton, forming a $(p\mu)$ atom.

(2) The $(p\mu)$ atom migrates and encounters a deuteron which captures the meson by exchange. Owing to the difference in reduced mass, this process releases 135 ev to the system.

(3) The $(d\mu)$ atom slows down by collisions and forms a $(p\mu d)^+$ molecular ion by a process of electron ejection.

(4) The nuclear reaction $p+d\rightarrow \mathrm{He}^3+\gamma$ (5.5 Mev) occurs. The γ ray may then eject the meson.