Intrinsic Quadrupole Moment and the Resonance Width of Photonuclear Reactions*

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The correlation between nuclear deformation and the resonance width of photonuclear reactions is discussed. The general trend of the nuclear deformation is in good agreement with the predictions of Marumori, Suekane, and Yamamoto, and the existence of the correlation is clarified. The calculation assuming the splitting of the resonance and a constant width for spherical nuclei agrees qualitatively with experiments, but quantitatively the calculated values are too low, especially for strongly deformed nuclei. For light nuclei the quantatitive calculation has little meaning, but a similar correlation is found for them. The mass-number dependence of the resonance width is discussed qualitatively; it turns out that the variation of this width arises from various causes and is not a simple function of mass number. A discussion of the nuclear shape in the highly excited state (~20 Mev) is given, and it is shown that this nuclear shape may not be very different from the shape in the ground state.

I. INTRODUCTION

PHOTONUCLEAR reactions have been studied extensively for the past several years. The observed mass-number dependence of the resonance energy is in agreement with the theoretical predictions[1,2] of Goldhaber and Teller[3] (hereafter denoted as GT) or of Steinwedel, Jensen, and Jensen[4] (hereafter denoted as SJJ). The systematics of the resonance width has also been studied experimentally,[1,2] and it has been found that the values of the resonance width are small at magic numbers and large in the intermediate regions.[2] Since this behavior is quite similar to that of the nuclear quadrupole moment, it was proposed by the author[5] and independently by Danos[6] that this fluctuation of the width might be explained by nuclear deformation. If we assume that the resonance splits into two parts due to the deformation, we can calculate the value of this splitting from the quadrupole moment. The calculated results[5] agreed fairly well with experiments. However, in this calculation we should have used intrinsic quadrupole moments obtained from Coulomb excitation instead of the quadrupole moment from spectroscopic experiments. Therefore the results of the previous note[5] should be taken rather qualitatively.

Recently, many experiments on Coulomb excitation have been performed,[7] giving us sufficient data to study the systematic variation of nuclear deformation. Moreover, the resonance width has now been measured for rare-earth nuclei.[8] These nuclei have very large resonance widths and their cross-section curves show the possible existence of a splitting into two peaks.

Therefore we have extended the calculations of our previous note[5] using intrinsic quadrupole moments, and the collective model of GT[3] or SJJ.[4] This type of calculation of splitting has also been performed[9] by using the independent-particle model as discussed by Wilkinson.[10]

In Sec. II we summarize the experimental data on intrinsic quadrupole moments. Our values agree fairly well with the calculations of surface rigidity by Marumori, Suekane, and Yamamoto.[11] In Sec. III we investigate the correlation between nuclear deformation and the resonance width. In Sec. IV the calculation of the splitting is carried out and the results are compared with experiments. In Sec. V we give a qualitative discussion about light nuclei ($Z<21$), for which the quantitative calculation has little meaning. In Sec. VI we comment concerning the mass-number dependence of the resonance width. In Sec. VII the validity of our model is discussed qualitatively. In the last section we summarize our results.

II. NUCLEAR SHAPE AND INTRINSIC QUADRUPOLE MOMENT

Bohr and Mottelson[12] showed that the spectroscopic quadrupole moment, Q, was given by

$$Q=Q_p+Q_s, \tag{1}$$

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[10] D. H. Wilkinson, *Proceedings of the Glasgow Conference on Nuclear and Meson Physics* (Pergamon Press, London, 1955), p. 161.
[11] Marumori, Suekane, and Yamamoto, Progr. Theoret. Phys. (Japan) **15**, 582, 584 (1956); **16**, 320 (1956).
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where Q_p is the quadrupole moment due to the particle outside a core and Q_s is the quadrupole moment due to the surface deformation. Furthermore it was shown[12] that

$$Q_s = P(x)Q_0, \tag{2}$$

where Q_0 is the intrinsic quadrupole moment, which is related to the "classical shape" of the nucleus. The projection operator P is a function of x, the parameter of the coupling strength between particles and the surface. $P(x)$ is given by Bohr and Mottleson for weak coupling (w), intermediate coupling (i), and strong coupling (s). Therefore, if we know Q and the value of x, we can calculate Q_0. We can also obtain Q_0 directly from the experiments on Coulomb excitation.

If we do not have any data on Coulomb excitation for even-even nuclei, we shall make use of the data on the first excited states[13] of such nuclei to calculate the value of Q_0. However, it is well known that the value of Q_0 obtained from the energy of the first excited level is several times larger than the value of Q_0 from Coulomb excitation. Therefore we normalize the former to nuclei for which the data on Coulomb excitation are known, and determine the approximate value for other nuclei. The values obtained in this way are marked with a superscript "e" in Table I.

The value of Q_0 determined from Q depends upon x. The coupling strength x can be determined by the following method: we introduce a quantity, $F(E2)$, defined by the ratio of the observed electric quadrupole transition probability, $B(E2)_{\text{obs}}$, to the single-particle value, namely

$$F(E2) \equiv B(E2)_{\text{obs}}/B(E2)_{\text{s.p.}}. \tag{3}$$

For $B(E2)_{\text{s.p.}}$ we use the following estimate[7]:

$$B(E2)_{\text{s.p.}} = 3\times 10^{-5} A^{4/3} \times 10^{-48}\ \text{cm}^4. \tag{4}$$

We define the strength of the coupling in the following way:

(1) Weak coupling (w):

$$Q_{\text{obs}} \cong Q_{\text{s.p.}};\quad \text{or}\quad F(E2) \cong 1.$$

(2) Intermediate coupling (i):

$$Q_{\text{obs}} = 2\text{—}3Q_{\text{s.p.}},\quad \text{or}\quad Q_{\text{obs}} \cong Q_{\text{conf}},\quad \text{or}\quad 1 < F(E2) \lesssim 10,$$

or

$$\mu_{\text{obs}} \neq \mu_{\text{s.p.}}\quad \text{but}\quad \mu_{\text{obs}} \cong \mu_{\text{conf}}.$$

Here μ is the magnetic moment and "conf" means the values obtained by configuration mixing.[11,14,15]

(3) Strong coupling (s):

$$Q_{\text{obs}} \gg Q_{\text{s.p.}},\quad \text{or}\quad Q_{\text{obs}} \neq Q_{\text{conf}},\quad \text{or}\quad F(E2) \gg 10,$$

or

$$\mu_{\text{obs}} \neq \mu_{\text{conf}}.$$

Of course, these criteria are rather arbitrary.

Finally, we calculate the nuclear eccentricity, here defined by[16]

$$e = (R_1 - R_2)/R_0, \tag{5}$$

where R_1 and R_2 are the longer and shorter axes of a spheroidal nucleus, respectively, and R_0 is the radius of a spherical nucleus with the same volume. The relation between Q_0 and e is

$$e = 5Q_0/(4ZR_0^2). \tag{6}$$

Throughout this paper we shall take $R_0 = (1.5)^{\frac{1}{2}} A^{\frac{1}{3}} \times 10^{-13}$ cm.

The results for stable nuclei ($Z \geqq 21$) are listed in Table I. Light nuclei will be discussed in Sec. V.

III. CORRELATION BETWEEN NUCLEAR ECCENTRICITY AND RESONANCE WIDTH OF PHOTONUCLEAR REACTIONS

In order to show that there exists a correlation between the nuclear eccentricity and the experimental resonance full width at half-maximum of the cross section for photon absorption, we plot these two quantities in Fig. 1.

Since the deformation of the nucleus seems to be affected more strongly by neutrons than by protons,[13] we draw the graph as a function of neutron number.

It was shown by Morinaga[17] and Johansson[18] that the (γ,p) reaction was sometimes several times to ten times larger than the (γ,n) reaction for nuclei of $A \lesssim 40$. Therefore, for such light nuclei the approximation of taking only the width of the reaction which emits neutrons might not be good. However, for the heavier nuclei shown in Fig. 1, this approximation is expected to be fairly good.

The experimental uncertainty of the width is assumed to be ± 1 Mev; while the uncertainty in e is estimated to be from 20 to 30% of its value.

IV. SPLITTING OF THE RESONANCE DUE TO DEFORMATION

(a) Approximate Calculation

Figure 1 clearly shows that a correlation exists between Q_0 and the resonance width Γ. The discrepancies at neutron number $N < 50$ might be due to the contribution of the (γ,p) reaction.

This correlation can be explained by the splitting of the resonance. The resonance energy of dipole vibration (hereafter denoted as E_0) decreases with increasing

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[16] This definition of eccentricity is different from the previous one (reference 5). In reference 5, $\epsilon = (R_1^2 - R_2^2)^{\frac{1}{2}}/R_1$. The relation between these two definitions is: $\epsilon^2 = e(6+e)/(3+4e)$.
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[18] S. A. E. Johansson, Phys. Rev. **97**, 1186 (1955).

TABLE I. Various quantities related to nuclear deformation. Items enclosed in parentheses represent assumed or uncertain values. Column 1: Nucleus studied. Column 2: ground state spin, I. Column 3: observed value Q_{obs} of the spectroscopic quadrupole moment. The data are taken from the review article of Blin-Stoyle,[a] unless otherwise stated. Column 4: single-particle value $Q_{s.p.}$ of the spectroscopic quadrupole moment. Column 5: the value Q_{conf} calculated from configuration mixing.[b,c] Column 6: intrinsic quadrupole moment Q_0 calculated from Q_{obs} for the case of intermediate coupling. Column 7: intrinsic quadrupole moment Q_0 calculated from Q_{obs} for the case of strong coupling. Column 8: intrinsic quadrupole moment Q_0 from experiments on Coulomb excitation. Column 9: enhancement factor $F(E2)$ defined by Eqs. (3) and (4). Column 10: single-particle value $\mu_{s.n.}$ of magnetic moment. Column 11: calculated value μ_{conf} of magnetic moment using configuration mixing.[c] Column 12: observed value μ_{obs} of magnetic moment taken from the table of Blin-Stoyle.[a] Column 13: coupling strength between extra particles and the surface determined as weak (w), intermediate (i), or strong (s) by the criteria in the text. Column 14: nuclear eccentricity e calculated by Eq. (6). Q_0 is taken from the Coulomb excitation measurements, if available. Otherwise Q_0 is taken from the spectroscopic value Q_{obs} with the coupling strength listed in column 13.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
			$Q(10^{-24}$ cm$^2)$			$Q_0(10^{-24}$ cm$^2)$				μ			
Nucleus	I	obs	s.p.	conf	inter.	strong	Coul.	$F(E2)$	s.p.	conf	obs	Coupling	e
$_{21}Sc_{24}^{45}$	7/2								5.79	4.74	4.76	s	
$_{22}Ti_{24}^{46}$	0	...	...	...			0.75[d]	11	...	...	...	s	0.23
Ti_{25}^{47}	5/2		...				(>0.92[d])	210			−0.8	s	(>0.28)
Ti_{26}^{48}	0		...	...			0.56[d]	6.0	...	...	...	i	(0.17)
Ti_{27}^{49}	7/2	...							−1.91	−0.58	−1.1	s	
Ti_{28}^{50}	0	...	...	...			(0.52[e])	(4.9)	...	...	...	i	(0.16)
$_{23}V_{27}^{50}$	(6)							27				s	
V_{28}^{51}	7/2	0.3±0.2	−0.02	−0.03	0.42	0.68	(>0.39)	34	5.79	5.02	5.15	i or s	(0.18) (s)
$_{24}Cr_{26}^{50}$	0	...	...	...					...	...	...		
Cr_{28}^{52}	0	...	...	...			(0.76[e])	(13)	...	...	...	i	(0.19)
Cr_{29}^{53}	3/2		...						−1.91	−0.49	−0.47	i	
Cr_{30}^{54}	0	...	...	...			(1.00[e])	(16)	...	...	...	s	(0.23)
$_{25}Mn_{30}^{55}$	5/2	0.3±0.15[f]	0.08	...	0.35	0.76	1.26[d]	320	4.13		3.47	s	0.29
$_{26}Fe_{28}^{54}$	0	...	...	...			(0.77[e])	(12)	...	...	...		(0.16)
Fe_{30}^{56}	0	...	...	...			1.00[d]	15	...	...	...	s	0.21
Fe_{31}^{57}	1/2[g]	...	...	...			>1.00[d]	15	−1.91	+0.4	0.05	s	0.21
Fe_{32}^{58}	0	...	...	...			(1.02[e])	(15)	...	...	...	s	(0.21)
$_{27}Co_{32}^{59}$	7/2	0.5±0.2	0.08	0.19	0.55	0.99			5.79	4.10	4.65	i or s	0.10 (i)
				[0.30][h]									0.19 (s)
$_{28}Ni_{30}^{58}$	0	...	...				(0.73[e])	(11)	...	...	...	i	(0.15)
Ni_{32}^{60}	0	...	...	...			(0.77[e])	(12)	...	...	...	i	(0.16)
Ni_{33}^{61}	(3/2)		...						−1.91	−0.03	~0	i	
Ni_{34}^{62}	0	...	...	...			(0.76[e])	(11)	...	...	...	i	(0.16)
Ni_{36}^{64}	0	...	...	...			(0.78[e])	(13)	...	...	...	i or s	(0.16)
$_{29}Cu_{34}^{63}$	3/2	−0.16	−0.06	−0.11	−0.17	−0.90	(−1.03[d])	6.5	3.79	2.17	2.23	i	(0.03)
Cu_{36}^{65}	3/2	−0.15	−0.06	−0.11	−0.15	−0.84	(−0.96[d])	5.5	3.79	2.30	2.38	i	0.03
$_{30}Zn_{34}^{64}$	0	...	...	...			1.04[d]	14	...	...	...	s	0.19
Zn_{36}^{66}	0	...	...	...			0.93[d]	11	...	...	...	s	0.16
Zn_{37}^{67}	5/2	...	...					18	1.37	0.81	0.88	s	
Zn_{38}^{68}	0	...	...	...			(1.00[e])	(14)	...	...	...	s	0.17
Zn_{40}^{70}	0	...	...	...					...	...	...		
$_{31}Ga_{38}^{69}$	3/2	0.23	0.05	0.15	0.25	0.98			3.79	1.58	2.02	s	0.16
				[0.20][h]									
Ga_{40}^{71}	3/2	0.15	0.05	0.15	0.13	0.50			3.79	1.82	2.56	s	0.08
				[0.14][h]									
$_{32}Ge_{38}^{70}$	0	...		...			−0.99[d]	10	...	...	...	s	0.15
Ge_{40}^{72}	0	...	...				−1.26[d]	18	...	...	...	s	0.19
Ge_{41}^{73}	9/2	−0.2	...	−0.43	−0.22	−0.37	>−1.49[d]	450	−1.91	−1.72	−0.88	s	>0.21
Ge_{42}^{74}	0	...	...				−1.58[d]	27	...	...	...	s	0.23
Ge_{44}^{76}	0	...	...				−1.52[d]	24	...	...	...	s	0.23
$_{33}As_{42}^{75}$	3/2	0.32	0.06	0.18	0.36	1.46	(>1.20)[d]	90	3.79	2.21	1.44	s	>0.17
										1.54			
$_{34}Se_{40}^{74}$	0	...	...	...			1.45[d]	23	...	...	...	s	0.17
Se_{42}^{76}	0	...	...	...			2.07[d]	44	...	...	...	s	0.27
Se_{43}^{77}	1/2	...	...	...			(>2.08)[d]	43	0.64	...	0.53	s	>0.27
Se_{44}^{78}	0	...	...	...			1.89[d]	36	...	...	...	s	0.24
Se_{46}^{80}	0	...	...	...			1.52[d]	22	...	...	...	s	0.20
Se_{48}^{82}	0	...	...	...			0.75[d]	5.2	...	...	...	i	0.09
$_{35}Br_{44}^{79}$	3/2	0.33	0.06	0.19	0.41	1.92	(>1.62)[i]		3.79	2.55	2.11	s	0.16
										1.91			
Br_{46}^{81}	3/2	0.28	0.06	0.19	0.34	1.62	(>1.81)[i]		3.79	2.52	2.27	s	0.13
										1.91			
$_{36}Kr_{42}^{78}$	0	...	...	...			2.9[i]	83	...	...	...	s	0.36
Kr_{44}^{80}	0	...	...	...			1.7[i]	29	...	...	...	s	0.21
Kr_{46}^{82}	0	...	...	...			0.9[i]	8.8	...	...	...	i	<0.12
Kr_{47}^{83}	9/2	0.15	...	0.28	0.19	0.28			−1.91	−0.83	−0.97	i	0.02
Kr_{48}^{84}	0	...	...	...			0.75[i]	5.6	...	...	...	i	<0.08
Kr_{50}^{86}	0	...	...	...									
$_{37}Rb_{48}^{85}$	5/2	0.31[k]	0.07	...	0.34	0.66			0.86	1.32	1.35	i	0.04
Rb_{50}^{87}	3/2	0.15[k]	0.05	[0.17][h]	(0.14)	0.38			3.79	2.79	2.75	i	0.02
$_{38}Sr_{46}^{84}$	0	...	...	...					...	...	...		
Sr_{48}^{86}	0	...	...	...			(0.76[e])	(5.1)	...	...	...	i	(<0.08)
Sr_{49}^{87}	9/2		...	[0.14][h]					−1.91	−0.68	−1.1		

TABLE I.—*Continued.*

(1)	(2)	(3)	(4) $Q(10^{-24}$ cm$^2)$	(5)	(6)	(7) $Q_0(10^{-24}$ cm$^2)$	(8)	(9)	(10)	(11) μ	(12)	(13)	(14)
Nucleus	I	obs	s.p.	conf	inter.	strong	Coul.	$F(E2)$	s.p.	conf	obs	Coupling	e
Sr_{50}^{88}	0	...	...	...			(0.58[e])	(3.0)	...	...	...	i	(<0.05)
$_{39}Y_{50}^{89}$	1/2	...	...	...					−0.26	...	−0.14		
$_{40}Zr_{50}^{90}$	0	...	...	...			(0.55[e])	(2.8)	...	...	...	w	(<0.05)
Zr_{51}^{91}	5/2		...	[−0.083][h]					−1.91	−0.8	−1.9[l]	w	
Zr_{52}^{92}	0	...	...	...			0.9[m]	6.0	...	...	...	i	<0.09
Zr_{54}^{94}	0	...	...	...					...	...	...	i	
Zr_{56}^{96}	0	...	...	...					...	...	...		
$_{41}Nb_{52}^{93}$	9/2	−0.2[n]	−0.13	−0.33	(0.12)	−0.24			6.79	6.60 5.71	6.16	(i or w)	$\gtrsim$0.01
$_{42}Mo_{50}^{92}$	0	...	...	...			(0.71[e])	(4.1)	...	...	...	s	(<0.06)
Mo_{52}^{94}	0	...	...	...			1.70[d]	23	...	...	...	s	0.17
Mo_{53}^{95}	5/2		...				(>1.43[d])	95	−1.91	−0.35 −1.08	−0.91	s	(>0.13)
Mo_{54}^{96}	0	...	...	...			1.76[d]	23	...	...	...	s	0.17
Mo_{55}^{97}	5/2		...						−1.91	+0.05 −0.65	−0.93	s	
Mo_{56}^{98}	0	...	...	...			1.64[d]	20	...	...	...	s	0.16
Mo_{58}^{100}	0	...	...	...			2.57[d]	47	...	...	...	s	0.24
$_{44}Ru_{52}^{96}$	0	...	...	...					...	...	...		
Ru_{54}^{98}	0	...	...	...					...	...	...		
Ru_{55}^{99}	5/2		...				(1.45)[d]	70	−1.91			s	0.12
Ru_{56}^{100}	0	...	...	...			1.73[d]	22	...	...	...	s	0.15
Ru_{57}^{101}	5/2		...				(>1.94)[d]	(23)	−1.91			s	0.16
Ru_{58}^{102}	0	...	...	...			2.51[d]	44	...	...	...	s	0.21
Ru_{60}^{104}	0	...	...	...			3.22[d]	71	...	...	...	s	0.27
$_{45}Rh_{58}^{103}$	1/2	...	...	...			2.3[o]		−0.26		−0.10	s	0.20
$_{46}Pd_{56}^{102}$	0	...	...	...			...		...		...		
Pd_{58}^{104}	0	...	...	...			2.1[d]	31	...	...	...	s	0.17
Pd_{59}^{105}	5/2	...	...				(2.0)[p]		−1.91	−0.45	−0.57	s	0.17
Pd_{60}^{106}	0	...	...	...			2.4[d]	39				s	0.19
Pd_{62}^{108}	0	...	...	...			2.8[d]	51	...	...	...	s	0.23
Pd_{64}^{110}	0	...	...	...			3.2[d]	60	...	...	...	s	0.27
$_{47}Ag_{60}^{107}$	1/2	...	...				2.0[q]	24	−0.26	...	−0.11	s	0.16
Ag_{62}^{109}	1/2	...	...	...			2.2[q]	28	−0.26		−0.13	s	0.17
$_{48}Cd_{58}^{106}$	0	...	...	...					...	...	...		
Cd_{60}^{108}	0	...	...	...					...	...	...		
Cd_{62}^{110}	0	...	...	...			2.02[d]	31	...	...	...	s	0.15
Cd_{63}^{111}	1/2	...	...	...			2.2[d]	30	−1.91	−0.49	−0.59	s	0.16
Cd_{64}^{112}	0	...	...	...			2.14[d]	28	...	...	...	s	0.16
Cd_{65}^{113}	1/2	...	...	...			2.9[d]	70	−1.91	−0.77	−0.62	s	0.21
Cd_{66}^{114}	0	...	...	...			2.35[d]	33	...	...	...	s	0.17
Cd_{68}^{116}	0	...	...	...			2.49[d]	36	...	...	...	s	0.19
$_{49}In_{64}^{113}$	9/2	1.14	0.16	0.41	1.10	1.93			6.79	5.62	5.49	s	0.14
In_{66}^{115}	9/2	1.16	0.16	0.42	1.12	1.97			6.79	5.59	5.50	s	0.14
$_{50}Sn_{62}^{112}$	0	...	...	...									
Sn_{64}^{114}	0	...	...	...									
Sn_{65}^{115}	1/2	...	...	...					−1.91	−0.73	−0.92	i	
Sn_{66}^{116}	0	...	...	...			1.32[m]	10	...	...	...	i	<0.09
Sn_{67}^{117}	1/2	...	...	...					−1.91	−1.18 −0.50	−1.00	i	
Sn_{68}^{118}	0	...	...	...			1.37[m]	11	...	...	...	i	<0.09
Sn_{69}^{119}	1/2	...	...	...					−1.91	−0.95	−1.05	i	0.09
Sn_{70}^{120}	0	...	...	...			1.41[m]	11	...	...	...	i	0.11
Sn_{72}^{122}	0	...	...	...			1.42[m]	11	...	...	...	i	0.11
Sn_{74}^{124}	0	...	...	...			1.38[m]	10	...	...	...	i	0.09
$_{51}Sb_{70}^{121}$	5/2	−0.5	−0.13	−0.26	−0.58	−1.27			4.79	3.49	3.36	s	0.08
Sb_{72}^{123}	7/2	−0.7	−0.15	−0.39	−0.73	−1.35			1.72	2.49	2.55	s	0.08
$_{52}Te_{68}^{120}$	0	...	...	...			2.35[d]	31	...	...	...	s	0.15
Te_{70}^{122}	0	...	...	...			2.17[d]	26	...	...	...	s	0.13
Te_{71}^{123}	1/2	...	...	...			(0.7)[q]		−1.91	−0.82	−0.74	(i)	(0.04)
Te_{72}^{124}	0	...	...	...			1.97[d]	21	...	...	...	s	0.12
Te_{73}^{125}	1/2	...	...	...			2.1[q]		−1.91	−0.60	−0.89	s	0.16
Te_{74}^{126}	0	...	...	...			1.79[d]	17	...	...	...	s	0.11
Te_{76}^{128}	0	...	...	...			1.67[d]	15	...	...	...	s	0.09
Te_{78}^{130}	0	...	...	...			1.61[d]	13	...	...	...	s	0.09
$_{53}I_{74}^{127}$	5/2	−0.69	−0.14	−0.31	−0.97	−1.80			4.79	3.04	2.81	s	0.10
$_{54}Xe_{70}^{124}$	0	...	...	...					...	...	...		
Xe_{72}^{126}	0	...	...	...			(2.48[e])	(32)	...	...	...		
Xe_{74}^{128}	0	...	...	...			(2.33[e])	(28)	...	...	...	s	0.13
Xe_{76}^{129}	1/2		...	...					−1.91	+0.14 −1.10 −0.46	−0.78	s	

TABLE I.—*Continued.*

(1) Nucleus	(2) I	(3) $Q(10^{-24}$ cm$^2)$ obs	(4) $Q(10^{-24}$ cm$^2)$ s.p.	(5) conf	(6) inter.	(7) $Q_0(10^{-24}$ cm$^2)$ strong	(8) Coul.	(9) $F(E2)$	(10) μ s.p.	(11) μ conf	(12) μ obs	(13) Coupling	(14) e
$_{54}Xe_{76}^{130}$	0	...	...	...			(2.07[e])	(22)	...	...	...	s	0.12
$_{54}Xe_{77}^{131}$	3/2	−0.12	...	−0.26	−0.20	−0.72			1.15	0.48 0.70	0.70	s	(0.11) assumed
$_{54}Xe_{78}^{132}$	0	...	...	...			(1.84[e])	(17)	...	...	...	s	0.11
$_{54}Xe_{80}^{134}$	0	...	...	...			(1.49[e])	(11)	...	...	...	s	0.08
$_{54}Xe_{82}^{136}$	0	...	...	...			(1.29[e])	(8)		...	...	i	0.07
$_{55}Cs_{78}^{133}$	7/2	−0.003	−0.11	...					1.72	2.10 2.42 2.75	2.58	(w)	~0
$_{56}Ba_{74}^{130}$	0	...	...	...					...	...	...		
$_{56}Ba_{76}^{132}$	0	...	...	...			(1.52[e])	(11)	...	...	...	s	0.09
$_{56}Ba_{78}^{134}$	0	...	...	...			(1.70[e])	(14)	...	...	...	s	0.11
$_{56}Ba_{79}^{135}$	3/2		...	...					1.15	0.94	0.83	(s)	0.12
$_{56}Ba_{80}^{136}$	0	...	...	...			(1.60[e])	(12)	...	...	...	s	0.09
$_{56}Ba_{81}^{137}$	3/2		...						1.15	0.95	0.94	i or s	
$_{56}Ba_{82}^{138}$	0	...	...	...			(1.26[e])	(7)	...	...	...	i	0.08
$_{57}La_{81}^{138}$	5	(0.7)[r]											(0.05)
$_{57}La_{82}^{139}$	7/2	0.23[f]	0.16	0.44					1.72	1.88 2.19	2.78	w	~0
$_{58}Ce_{78}^{136}$	0	...	...	...					...	...	...		
$_{58}Ce_{80}^{138}$	0	...	...	...					...	...	...		
$_{58}Ce_{82}^{140}$	0	...	...	...			(1.19[e])	(6)	...	...	...	i	0.07
$_{58}Ce_{84}^{142}$	0	...	...	...					...	...	...		
$_{59}Pr_{82}^{141}$	5/2	−0.01[f]	−0.14	−0.30 [−0.35]					4.79	4.53 3.95	4.0	(w)	(~0)
$_{60}Nd_{82}^{142}$	0	...	...	...		...	0.88[s]	3.4	...	...	...	i	0.04
$_{60}Nd_{83}^{143}$	7/2	~1	...	...	1.4	2.1			−1.91	−0.84	−1.0	i	0.08
$_{60}Nd_{84}^{144}$	0	...	...				1.33[s]	7.7				i	0.07
$_{60}Nd_{85}^{145}$	7/2	~1	...		1.4	2.1			−1.91	−1.05 −0.64	−0.65	i	0.08
$_{60}Nd_{86}^{146}$	0	...	...	...			1.58[s]	11	...	...	...	i	0.08
$_{60}Nd_{88}^{148}$	0	...	...	...			2.63[t]	29	...	...	...	s	0.13
$_{60}Nd_{90}^{150}$	0	...	...	...			4.80[t]	96	...	...	...	s	0.24
$_{62}Sm_{82}^{144}$	0	...	...	...					...	...	...		
$_{62}Sm_{85}^{147}$	7/2	0.72	...	...	1.03	1.54			−1.91	−1.01 −0.60	−0.76	(s)	0.08
$_{62}Sm_{86}^{148}$	0	...	...	...			2.24[i]	27	...	...	...	s	0.11
$_{62}Sm_{87}^{149}$	7/2	0.72	...		1.03	1.54			−1.91	−1.21 −0.81	−0.64	s	0.08
$_{62}Sm_{88}^{150}$	0	...	...	...			3.13[i]	41	...	...	...	s	0.15
$_{62}Sm_{90}^{152}$	0	...	...	...			5.57[i]	127	...	...	...	s	0.27
$_{62}Sm_{92}^{154}$	0	...	...	...			6.71[i]	182	...	...	...	s	0.32
$_{63}Eu_{88}^{151}$	5/2	1.2	0.16	0.36		3.4			4.79	3.6	3.6	s	0.15
$_{63}Eu_{90}^{153}$	5/2	2.5	0.16			7.1	7.1[u]		4.79	1.6		s	0.32
$_{64}Gd_{88}^{152}$	0	...	...	...			(4.0[e])	(110)	...	...	...	s	0.17
$_{64}Gd_{90}^{154}$	0	...	...	...			6.5[u]	179	...	...	...	s	0.29
$_{64}Gd_{91}^{155}$	3/2	1.1[v]	...	...		6.9	6.8[u]				−0.31	s	0.29
$_{64}Gd_{92}^{156}$	0	...	...				7.1[w]	200	...	...	...	s	0.32
$_{64}Gd_{93}^{157}$	3/2	1.0[v]	...			6.3	6.2[u]				−0.38	s	0.28
$_{64}Gd_{94}^{158}$	0	...	...				7.7[u]	231	...	...	...	s	0.35
$_{64}Gd_{96}^{160}$	0	...	...	...			9.7[u]	361	...	...	...	s	0.44
$_{65}Tb_{94}^{159}$	3/2						8.7[u]		0.12		1.5±0.4	s	0.37
$_{66}Dy_{90}^{156}$	0	...	...	...					...	...	...	s	
$_{66}Dy_{92}^{158}$	0	...	...	...					...	...	...	s	
$_{66}Dy_{94}^{160}$	0	...	...	...			7.1[x]	193	...	...	...	s	0.29
$_{66}Dy_{95}^{161}$	(3/2) (5/2)											s	
$_{66}Dy_{96}^{162}$	0	...	...	...			7.9[x]	236	...	...	...	s	0.34
$_{66}Dy_{97}^{163}$	(3/2) **(5/2)**											s	
$_{66}Dy_{98}^{164}$	0	...	...	...			9.2[x]	315	...	...	...	s	0.39
$_{67}Ho_{98}^{165}$	7/2	2	0.18			4.2	7.6[u]	213				s	0.31
$_{68}Er_{94}^{162}$	0	...	...	...					...	...	...	s	
$_{68}Er_{96}^{164}$	0	...	...	...			7.3[x]	198	...	...	...	s	0.29
$_{68}Er_{98}^{166}$	0	...	...	...			(7.3[e])	(195)	...	...	...	s	0.29
$_{68}Er_{99}^{167}$	7/2	10.2 (3.5)[f]	...	0.70		21.9 (7.5)			−1.91		−0.5	s	(0.29)
$_{68}Er_{100}^{168}$	0	...	...	...			(7.55[y])[s]	(208)	...	...	...	s	(0.29)[y]
$_{68}Er_{102}^{170}$	0	...	...	...					...	...	...	s	
$_{69}Tm_{100}^{169}$	1/2	...	...				(6.16)[s]		2.79		−0.2	s	(0.24)
$_{70}Yb_{98}^{168}$	0	...	...	...					...	...	...	s	
$_{70}Yb_{100}^{170}$	0	...	...	...			7.1[x]	(178)	...	...	...	s	0.27

TABLE I.—*Continued.*

(1) Nucleus	(2) I	(3) $Q(10^{-24}$ cm$^2)$ obs	(4) $Q(10^{-24}$ cm$^2)$ s.p.	(5) $Q(10^{-24}$ cm$^2)$ conf	(6) inter.	(7) $Q_0(10^{-24}$ cm$^2)$ strong	(8) Coul.	(9) $F(E2)$	(10) μ s.p.	(11) μ conf	(12) μ obs	(13) Coupling	(14) ϵ
Yb_{101}^{171}	1/2	...	...	...					0.64	0.5	0.45	s	
Yb_{102}^{172}	0	...	...	...					...	...	...	s	
Yb_{103}^{173}	5/2	3.9	...	...		10.9			1.37		−0.7	s	
Yb_{104}^{174}	0	...	...	...			(6.93[y])[s]		+ ...		...	s	(0.25)[y]
Yb_{106}^{176}	0	...	...	...					...	...	...	s	
$_{71}Lu_{104}^{175}$	7/2	3.9[s]	0.18	0.74		8.1	8.5[u]		1.72	2.4	2.9	s	0.32
Lu_{105}^{176}	≥7	8.0				(∼10)					4.2	s	(0.37)
$_{72}Hf_{102}^{174}$	0	...	...	...					...	...	...	s	
Hf_{104}^{176}	0	...	...	...			7.9[aa]		...	...	...	s	0.29
Hf_{105}^{177}	7/2	3.0	...	...		6.4	8.9[u]		−1.91	...		s	0.33
Hf_{106}^{178}	0	...	...	...			7.9[bb]		...	...	...	s	0.29
Hf_{107}^{179}	9/2	3.0	...			5.5	8.3[u]					s	0.31
Hf_{108}^{180}	0	...	...	...			7.9[aa]		...	...	...	s	0.29
$_{73}Ta_{108}^{181}$	7/2	2.7	0.20	0.65		5.6	6.7[bb]		1.72	2.6	2.1	s	0.22
$_{74}W_{106}^{180}$	0	...	...	...			(8.0[e])	210	...	...	...	s	(0.28)
W_{108}^{182}	0	...	...	...			7.6[x]		...	...	...	s	0.27
W_{109}^{183}	1/2	...	...	...			(6.9[bb,cc])		0.64	...	0.1	s	(0.24)
W_{110}^{184}	0	...	...	...			6.7[x]		...	...	...	s	0.24
W_{112}^{186}	0	...	...	...			5.9[x]	109	...	...	...	s	0.21
$_{75}Re_{110}^{185}$	5/2	2.8	0.18	0.39	5.0	7.7	4.7[bb]		4.79	3.19	3.17	s	0.16
Re_{112}^{187}	5/2	2.6	0.18	0.40	4.6	7.1	4.3[bb]		4.79	3.17	3.20	s	0.15
$_{76}Os_{108}^{184}$	0	...	...	...					...	...	...	s	
Os_{110}^{186}	0	...	...	...			(5.6[e])	(104)	...	...	...	s	0.19
Os_{111}^{187}	1/2	...	...	...								s	
Os_{112}^{188}	0	...	...	...			(5.3[e])		...	...	...	s	0.18
Os_{113}^{189}	3/2	0.6[s]	...		1.0	3.7					0.7	s	0.12
Os_{114}^{190}	0	...	...	...			(5.0[e])		...	...	...	s	0.17
Os_{116}^{192}	0	...	...	...			(3.9[e])	(45)	...	...	...	s	0.13
$_{77}Ir_{114}^{191}$	3/2	1.0±0.5	0.14	0.40	1.5	5.9	5.3[dd]		0.12	0.44	0.17	s	0.17
Ir_{116}^{193}	3/2	1.0±0.5	0.14	0.40	1.5	5.9	3.4[dd]		0.12	0.44	0.17	s	0.11
$_{78}Pt_{112}^{190}$	0	...	...						...	...	...		
Pt_{114}^{192}	0	...	...	...			(2.52[e])	(19)	...	...	...	i or s	0.08
Pt_{116}^{194}	0	...	...	...			2.4[bb]	17	...	...	...	i	0.08
Pt_{117}^{195}	1/2	...	...	...			3.2[bb]		0.64		0.61	i	0.11
Pt_{118}^{196}	0	...	...	...			1.7[bb]	8.7	...	...	...	i	0.05
Pt_{120}^{198}	0	...	...	...			1.4[bb]	5.8	...	...	...	i	0.03
$_{79}Au_{118}^{197}$	3/2	0.6	0.14	0.29	0.8	3.5	2.6[aa]	18	0.12	0.45 0.29	0.14	i or s	0.08
$_{80}Hg_{116}^{196}$	0	...	...				(2.2[e])		...	...	...	i	0.07
Hg_{118}^{198}	0	...	...	...			2.2[dd]	14	...	...	...	i	0.07
Hg_{119}^{199}	1/2	...	...	...	...		1.6[dd]		0.64		0.50	i	0.05
Hg_{120}^{200}	0	...	...	...			2.4[dd]		...	...	...	i	0.08
Hg_{121}^{201}	3/2	0.45[n]	...	...	0.75	2.7			−1.91	−0.51	−0.56	i or s	0.07
Hg_{122}^{202}	0	...	...	...			2.2[dd]		...	...	...	i	0.07
Hg_{124}^{204}	0	...	...	...					...	...	...	i	
$_{81}Tl_{122}^{203}$	1/2	...	...	...			1.6[aa]		2.79	1.44	1.61	i	0.04
Tl_{124}^{205}	1/2	...	...	...			1.4[aa]		2.79	1.43	1.63	i	0.04
$_{82}Pb_{122}^{204}$	0	...	...	...			1.5[aa]		...	...	...	i	0.04
Pb_{124}^{206}	0	...	...	...			1.02[aa]		...	...	...	i	0.03
Pb_{125}^{207}	1/2	...	...	...			0.69[aa]		0.64		0.59	w	0.01
Pb_{126}^{208}	0	...	...	...			(∼0)		...	...	...		∼0
$_{83}Bi_{126}^{209}$	9/2	−0.4	−0.30	−0.53 [−0.55][h]					2.62	3.30	4.08	w	∼0
$_{90}Th_{142}^{232}$	0	...	...	...			5.7[dd]	76	...	...	...	s	0.13
$_{92}U_{142}^{234}$	0	...	...	...			(6.3[e])	(91)	...	...	...	s	0.15
U_{143}^{235}	7/2	(∼8)	...		(8.45)	(17)	9.7[s]				−0.8	s	0.23
U_{146}^{238}	0	...	...	...			6.9[dd]	108	...	...	...	s	0.16

[a] R. J. Blin-Stoyle, Revs. Modern Phys. **28**, 75 (1956).
[b] See reference 11.
[c] See reference 14.
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[g] G. Trumpy, Nature **176**, 507 (1955).
[h] Values enclosed in square brackets are taken from reference 11.
[i] Wolicki, Fagg, and Geer, Phys. Rev. **105**, 238 (1957).
[j] G. F. Pieper and N. P. Heydenburg, Bull. Am. Phys. Soc. Ser. II, **2**, 69 (1957).
[k] Senitzky, Rabi, and Perl, Phys. Rev. **98**, 1537 (1955).
[l] K. Murakawa, Phys. Rev. **100**, 1369 (1955).
[m] P. H. Stelson and F. K. McGowan, Bull. Am. Phys. Soc. Ser. II, **2**, 69 (1957).
[n] K. Murakawa, Phys. Rev. **98**, 1285 (1955).
[o] N. P. Heydenburg and G. M. Temmer, Phys. Rev. **95**, 861 (1954).
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[r] P. B. Sogo and C. D. Jeffries, Phys. Rev. **99**, 613 (1955). However, they obtained the ratio of Q between La^{138} and La^{139} and, assuming that $Q(La^{139})=0.9\times10^{-24}$ cm^2, they reported $Q(La^{138})=2.7\times10^{-24}$ cm^2. Here we take the recent data for $Q(La^{139})$ (reference f); therefore $Q(La^{138})=0.7\times10^{-24}$ cm^2.
[s] See reference 7.
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[x] E. D. Klema and R. K. Osborn, Phys. Rev. **103**, 833 (1956).
[y] Values for natural isotopes.
[z] K. Murakawa and T. Kamei, Phys. Rev. **105**, 671 (1957); Gerold Luhrs, Z. Physik **141**, 486 (1955).
[aa] P. H. Stelson and F. K. McGowan, Phys. Rev. **99**, 112 (1955).
[bb] McClelland, Mark, and Goodman, Phys. Rev. **97**, 1191 (1955).
[cc] Value taken from reference bb and normalized to W^{182}.
[dd] Davis, Divatia, Lind, and Moffat, Phys. Rev. **103**, 1801 (1956).

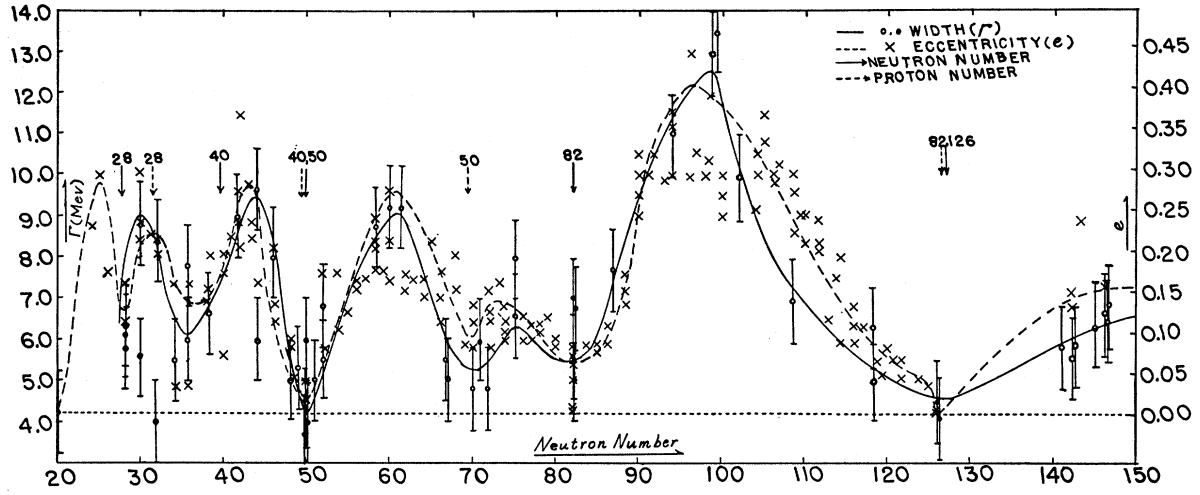

FIG. 1. Correlation between the nuclear eccentricity e and the photonuclear resonance width Γ. Crosses are eccentricities listed in Table I. Open circles are observed resonance widths for nonmagic nuclei; closed circles are for magic nuclei. (See Table IV.) Dashed and solid curves are for eccentricity and for resonance width respectively. Arrows indicate the magic numbers: the solid one is for neutrons and the dashed one is for protons. The dotted straight line indicates the intrinsic width, Γ_0, which is taken in this paper as 4.2 Mev. The uncertainty of the resonance width is assumed to be 1 Mev; the uncertainty of the eccentricity is from 20 to 30% of its value.

mass number, and its dependence is

$$\begin{aligned} &E_0 \propto R_0^{-n} \propto A^{-n/3}, \\ &n=\tfrac{1}{2}, \quad \text{GT model}^3 \\ &=1, \quad \text{SJJ model}^4 \\ &=\tfrac{1}{2}\text{—}1, \quad \text{experiments.}^{1,2,19} \end{aligned} \tag{7}$$

If the nucleus is a spheroid, the resonance splits into two parts; we apply the GT model[3] to a prolate[20] nucleus. The two frequencies E_1 and E_2, corresponding to oscillations along the longer and shorter axes respectively, are

$$\begin{aligned} E_1&=\left(\frac{\lceil 2\pi R_2^2\rho\varphi\hbar^2}{\epsilon(2\pi/3)R_1R_2^2\rho m}\right)^{\frac{1}{2}}\cong 40R_1^{-\frac{1}{2}}, \\ E_2&=\left(\frac{2\pi R_1R_2\rho\varphi\hbar^2}{\epsilon(2\pi/3)R_1R_2^2\rho m}\right)^{\frac{1}{2}}\cong 40R_2^{-\frac{1}{2}}, \end{aligned} \tag{8}$$

where all notations except R_1 and R_2 are the same as those in reference 3.

The value of the splitting, ΔE, is given by

$$\Delta E=E_2-E_1=40R_0^{-\frac{1}{2}}(\tfrac{1}{2}e)=\tfrac{1}{2}eE_0. \tag{9}$$

Equation (9) holds for the GT model, if the deformation is not very large.

More generally, if we assume $E_0 \propto R_0^{-n}$, we might expect *approximately*

$$\Delta E/E_0=ne. \tag{10}$$

[19] E. G. Fuller and E. Hayward, Phys. Rev. **101**, 692 (1956).

[20] The calculation for an oblate nucleus is quite similar to that described here.

(b) Hydrodynamical Calculation

We now examine whether Eq. (10) is exact in the SJJ model.[4] According to the SJJ model,[4] the density of nucleons is

$$\begin{aligned} \rho_p&=\rho_p{}^0+\eta(\mathbf{r},t), \\ \rho_n&=\rho_n{}^0-\eta(\mathbf{r},t). \end{aligned} \tag{11}$$

Here, $\rho_p{}^0$ ($\rho_n{}^0$) is the original density of protons (neutrons) and η is the change in the density. If we put $\eta=\eta_0(r)e^{i\omega t}$, η_0 satisfies the Helmholtz equation

$$\Delta\eta_0+k^2\eta=0. \tag{12}$$

Here, k is the wave number of dipole vibration. For a spherical nucleus, SJJ use the boundary condition $(\partial\eta_0/\partial r)_{R_0}=0$.

For a spheroidal nucleus, we write Eq. (12) in spheroidal coordinates[21] and take the radial part

$$\frac{d}{d\xi}\left[(\xi^2-1)\frac{dJ}{d\xi}\right]-\left[A-h^2\xi^2+\frac{m^2}{\xi^2-1}\right]J=0, \tag{13}$$

where ξ is a variable which is related to the shape of the spheroid. A and m are constants, and $h=\frac{1}{2}ak$, where a is the distance between foci of the spheroid, given by $a=2(5Q_0/2Z)^{\frac{1}{2}}$.

The boundary condition for J is

$$(dJ/d\xi)_{\xi=z}=0, \tag{14}$$

where $z=1/\epsilon$. (ϵ is the eccentricity used in the previous note.[16])

[21] P. M. Morse and H. Feshbach, *Methods of Theoretical Physics* (McGraw-Hill Book Company, Inc., New York, 1953), pp. 1502–1505.

TABLE II. Results of hydrodynamical calculation of the splitting of the resonance. The second column is the value calculated from Eq. (10) by putting $n=1$. The third column is the value obtained from hydrodynamical calculation. The fourth column is the ratio of these two values.

Nucleus	e	$(\Delta E/E_0)_H$	r
$_{53}I_{74}^{127}$	0.10	0.09	0.90
$_{73}Ta_{108}^{181}$	0.22	0.18	0.82
$_{25}Mn_{30}^{55}$	0.29	0.24	0.81

The solution of Eq. (13) is given by [21]

$$J(\xi)=je_{lm}(h,\xi)=\frac{(l-m)!}{(l+m)!}\left(\frac{\xi^2-1}{\xi^2}\right)^{m/2}\cdot\sum{}'\, i^{n+m-l}d_n(h|ml)\frac{(n+2m)!}{n!}j_{n+m}(h\xi), \quad (15)$$

where the prime over the summation indicates that only even values of n are included if $(l-m)$ is even and only odd values of n are included if $(l-m)$ is odd. The quantity $d_n(h|ml)$ is the coefficient of expansion and is tabulated by Stratton *et al.*[22] The difference of eigenvalues between $je_{10}(h,z)$ and $je_{11}(h,z)$ gives the value of the splitting.

Numerical calculations for three nuclei are shown in Table II.

The ratio n of the value of the hydrodynamical calculation of $\Delta E/E_0$ to that of Eq. (10) with $n=1$ is about 1–e; therefore we shall use the following formula:

$$(\Delta E/E_0)_{SJJ}=e(1-e). \quad (16)$$

(c) Comparison with Experiments

In Eqs. (9) and (16), we use the experimental values e and E_0 to obtain the value of ΔE. If we further assume that the widths of the two split resonances remain the same as Γ_0 for a spherical nucleus, we find the value of the width Γ of a deformed nucleus:

$$\Gamma=\Gamma_0+\Delta\Gamma. \quad (17)$$

$\Delta\Gamma$ is the increase of the width due to deformation and is now approximated by $\Delta\Gamma\cong\Delta E$.

We determine Γ_0 from the observed resonance widths for spherical nuclei. Table III shows that these widths for spherical nuclei are about 4 to 5 Mev. Therefore we tentatively assume that

$$\Gamma_0=4.2 \text{ Mev}. \quad (18)$$

This we shall call the *intrinsic width*. Using Table I and Eqs. (9), (16), (17), and (18), we calculate the value of the width Γ. The results are listed in Table IV. The quantity σ_m is the maximum cross section for neutron emission.

TABLE III. The widths of spherically symmetric nuclei.

Nucleus	Z	N	e	Γ_{obs} (Mev)
Ca^{40}	20	20	~0	4.2
Sr^{88}	38	50	(0.05)[a]	4.0
Y^{89}	39	50		3.8
Zr^{90}	40	50	~0	4.3
Zr^{91}	40	51		5.0[c]
Zr^{92}	40	52	<0.09[b]	5.5
Pb^{208}	82	126	~0	4.5[d]
Bi^{209}	83	126	~0	4.1

[a] This value is obtained from the first excited level; therefore, it should not be considered accurate.
[b] Average value for Zr^{92} and Zr^{94}. The value for Zr^{92} may be less than this.
[c] The value of Q_0 for this isotope is unknown, but a recent experiment shows that the magnetic moment is very close to the single-particle value. This seems to indicate that the deformation is very small (see Table I, reference 1).
[d] The value for the natural isotopes. The main contribution is supposed to be from Pb^{206}, Pb^{207}, and Pb^{208} and the eccentricities are 0.02, 0.01, and about 0, respectively.

As seen in Table IV, the agreement with experiment is fair, if we consider the uncertainties of Γ_{calc} and Γ_{obs}. (For the former it would be about ±0.5 Mev and for the latter it is usually about ±1 Mev.) The experimental variation of σ_m/A is also suggestive, because the narrower width Γ_{obs} usually corresponds to the larger σ_m/A. [Sometimes, however, we find disagreements, which might be due to the effects of (γ,p) reactions, or of neutron multiplicity.] The relation between Γ_{obs} and σ_m/A can be explained by the sum rule[23] that the integrated cross section is proportional to A. Since $\int\sigma dE\sim\sigma_m\Gamma$, small Γ corresponds to large σ_m/A.

Table IV shows that for most of the nuclei, $\Gamma_{calc}<\Gamma_{obs}$. For some strongly deformed nuclei the disagreement is outside the uncertainties estimated above. Therefore we arrive at the following conclusion.

If the nucleus is not deformed strongly, we might expect that our present calculation assuming the splitting of the resonance can explain the experimental results. However, at least for strongly deformed nuclei, we cannot explain the experimental data by using only the idea of splitting. We must take into account the broadening of the intrinsic width Γ_0 itself.

From Table IV the fluctuation of Γ_0 might be about 0.5—1 Mev if the nuclei are not so strongly deformed.

Note added in proof.—New measurements of Γ are in good agreement with our calculation: e.g., Eu^{153} has a larger width than Eu^{151} and their cross sections have different shapes (cf. their deformations in Table I). The author is indebted to Dr. Katz and Dr. Cook for sending him their experimental data.

V. LIGHT NUCLEI ($Z<21$)

In the above discussion we omitted light nuclei, since the collective model may not apply so well to them and the contribution of the (γ,p) reaction is usually sig-

[22] Stratton, Morse, Chu, and Hutner, *Elliptic Cylinder and Spheroidal Wave Functions* (John Wiley and Sons, Inc., New York, 1941); Stratton, Morse, Chu, Little, and Corbató, *Spheroidal Wave Functions* (The Technology Press of Massachusetts Institute of Technology and John Wiley and Sons, Inc., New York, 1956).

[23] J. S. Levinger and H. A. Bethe, Phys. Rev. 78, 115 (1950).

TABLE IV. Comparison of calculated value of the width and experiments. (The intrinsic width, Γ_0, is assumed to be 4.2 Mev.)

Nucleus	ϵ	E_0(obs)	$\Delta\Gamma$ GT	$\Delta\Gamma$ SJJ	Γ_{calc} GT	Γ_{calc} SJJ	Γ_{obs}	σ_m/A (mb)	Reference
$_{20}Ca_{20}^{40}$	~0	19.3	~0	~0	(4.2)	(4.2)	4.2	(0.4)	a
$_{23}V_{28}^{51}$	0.11 (*i*)	18.7	1.0	1.8	5.2	6.0	5.8	1.7	b
	0.18 (*s*)		1.7	2.8	5.9	7.0			
$_{24}Cr_{28}^{52}$	(<0.19[e])	17.5	(<1.7)	(<2.7)	(<5.9)	(<6.9)	5.8	2.0	d
$_{25}Mn_{30}^{55}$	0.29	18.4	2.7	3.8	6.9	8.0	8.8	1.8	b
$_{26}Fe_{28}^{54}$	(0.16[e])	18.7	(1.5)	(2.5)	(5.7)	(6.7)	6.3 or 6.9	1.2	a or e
Fe_{30}^{56}	0.21	18.0	1.9	3.0	6.1	7.2	6.1	1.3	a
$_{27}Co_{32}^{59}$	0.10 (*i*)	16.9	0.8	1.4	5.0	5.6	5.4	2.2	a
	0.19 (*s*)	17.3	1.6	2.6	5.8	6.8	8.4	1.3	b
$_{28}Ni_{30}^{58}$	(<0.15[e])	18.5	(<1.4)	(<2.4)	(<5.6)	(<6.6)	5.6	0.9	a
Ni_{32}^{60}	(<0.16[e])	16.0	(<1.3)	(<2.2)	(<5.5)	(<6.4)	4.0	1.6	d
$_{29}Cu_{34}^{63}$	0.03	18.1	0.5	1.0	4.7	5.2	5.5[f]	1.6	...
Cu_{36}^{65}	0.03	18.6	0.6	1.1	4.8	5.3	(6.0)	2.3	d
$_{30}Zn_{34}^{64}$	$\lesssim$0.19	18.7	$\lesssim$1.8	$\lesssim$2.8	$\lesssim$6.0	$\lesssim$7.0	7.9 or 6.0	1.9	a or g
$_{32}Ge_{38}^{70}$	0.15	20.0	1.5	2.6	5.7	6.8	($\gtrsim$6.5)[h]	1.8	g
Ge_{44}^{76}	0.23	18.9	2.2	3.4	6.4	7.6	($\gtrsim\gtrsim$9.5)[h]	3.2	g
$_{33}As_{42}^{75}$	>0.17	17.3	>1.5	>2.4	>5.7	>6.6	9.0	0.8	b
$_{35}Br_{44}^{79}$	0.16	18.0	1.4	2.4	5.6	6.6	6.0	2.8	a
Br_{46}^{81}	0.13	18.0	1.2	2.0	5.4	6.2	8.0	1.6	a
$_{37}Rb_{50}^{87}$	0.02	17.5	0.2	0.4	4.4	4.6	6.0	2.6	a
$_{38}Sr_{48}^{86}$	(<0.08[e])	15.9	<0.6	<1.3	<4.8	<5.5	5.0	1.9	i
Sr_{49}^{87}	(<0.08)	15.8	<0.6	<1.2	<4.8	<5.4	5.3	1.7	i
Sr_{50}^{88}	(<0.05)	16.3	<0.4	<0.8	<4.6	<5.0	4.0	2.3	i
$_{39}Y_{50}^{89}$	(~0)	16.3	(~0)	(~0)	(4.2)	(4.2)	3.8	2.2	i
$_{40}Zr_{50}^{90}$	(~0)	15.8	(~0)	(~0)	(4.2)	(4.2)	4.3	2.2	i
Zr_{51}^{91}	($\lesssim$0.06)	16.5	($\lesssim$0.5)	($\lesssim$1.0)	($\lesssim$4.7)	($\lesssim$5.2)	5.0	2.2	i
Zr_{52}^{92}	(<0.09)	16.9	(<0.8)	(<1.5)	(<5.0)	(<5.7)	5.5	2.1	i
$_{41}Nb_{52}^{93}$	0.01	17.0	0.1	0.2	4.3	4.4	6.1 or 6.8	2.1	a or b
$_{42}Mo_{50}^{92}$	(<0.06)	18.7	(<0.6)	(<1.2)	(<4.8)	(<5.4)	6.0	1.5	a
$_{45}Rh_{58}^{103}$	0.20	16.5	1.7	2.7	5.9	6.9	8.9	2.0	b
$_{47}Ag_{60}^{107}$	0.16	16.3	1.3	2.2	5.5	6.4	9.2	1.9	a
Ag_{62}^{109}	0.17	16.5	1.4	2.3	5.6	6.5	9.2	2.9	a
$_{49}In_{66}^{115}$	0.14	15.0	1.1	1.8	5.3	6.0	5.5 or ~5.0	3.7	a or j
$_{50}Sn$[k]	<0.10	17.0	<0.9	<1.5	<5.1	<5.7	6.0	2.4	l
$_{51}Sb_{70}^{121}$	0.08	14.8	0.6	1.2	4.8	5.4	(4.8)	5.6	a
Sb_{72}^{123}	0.08	14.8	0.6	1.2	4.8	5.4	(4.8)	2.9	a
$_{53}I_{74}^{127}$	0.10	16.5	0.8	1.5	5.0	5.7	8.0	3.0	l
		15.2					6.6	3.5	a
$_{57}La_{82}^{139}$	~0	13.8	~0	~0	~4.2	~4.2	5.7	~4.3	b
		15.5					7.0	2.7	l
$_{58}Ce$[k]	(~0.05)	16.0	(0.8)	(1.6)	(5.0)	(5.8)	5.0	3.0	l
$_{62}Sm_{82}^{144}$	(~0.10)	17.3	~0.9	~1.6	~5.1	~5.8	6.8	(0.9)	g
$_{62}Sm$[k]	(~0.15)	16.0	~1.2	~2.1	~5.4	~6.3	7.5	2.7	l
$_{65}Tb_{94}^{159}$	0.37	16.5	3.1	4.0	7.3	8.2	8.0	2.4	l
$_{67}Ho_{98}^{165}$	0.31	15.0	2.3	3.3	6.5	7.5	13.0	1.9	l
$_{68}Er$[k]	0.29	17.0	2.5	3.5	6.7	7.7	13.5	2.1	l
$_{70}Yb$[k]	0.27	16.0	2.2	3.2	6.4	7.4	10.0	2.6	l
$_{73}Ta_{108}^{181}$	0.22	16.5	1.8	2.8	6.0	7.0	7.0	3.4	l
$_{79}Au_{118}^{197}$	$\lesssim\lesssim$0.08	13.0	$\lesssim\lesssim$0.5	$\lesssim\lesssim$1.0	$\lesssim\lesssim$4.7	$\lesssim\lesssim$5.2	5.0 or 6.3	3.0	l or a
$_{82}Pb$[k]	$\gtrsim\gtrsim$0	14.0	$\gtrsim\gtrsim$0	$\gtrsim\gtrsim$0	$\gtrsim\gtrsim$4.2	$\gtrsim\gtrsim$4.2	4.5	3.8	l
$_{83}Bi_{126}^{209}$	~0	13.2	~0	~0	~4.2	~4.2	4.1	3.0	b
$_{90}Th_{142}^{232}$	0.13	14.5	0.9	1.6	5.1	5.8	5.6 or 6.0	3.5	m or n
$_{92}U_{141}^{233}$	(0.15)	14.0	1.1	1.8	5.3	6.0	6.0	7.2	n
U_{146}^{238}	0.16	13.8	1.1	1.9	5.3	6.1	6.6	4.1	b
$_{94}Pu_{145}^{239}$	(0.17)	13.6	1.1	1.9	5.3	6.1	6.3	6.6	n

[a] See reference 1.
[b] See reference 2.
[c] Data calculated from the first excited states.
[d] J. Goldemberg and L. Katz, Can. J. Phys. **32**, 49 (1954).
[e] J. H. Carber and K. H. Lokan, Australian J. Phys. **10**, 312 (1957).
[f] The average value of the results of several authors.
[g] deSonza Santos, Goldemberg, Pierona, Silva, Borello, Villaca, and Lopes, Acad. Brasil. Cienc. **27**, 437 (1955).
[h] These values were not listed in the original paper (reference g), since their experiments did not cover high energy and they could not measure the half-width. The values listed in the table were obtained by linear extrapolation of their cross-section curves, so the actual values of the widths would be somewhat larger than these.
[i] P. F. Yergin and B. P. Fabricand, Phys. Rev. **104**, 1334 (1956).
[j] Bogdankevich, Lazareva, and Nikolaev, Zhur. Eksptl. i Teoret. Fiz. **31**, 405 (1956) [translation: Soviet Phys. JETP **4**, 320 (1957)].
[k] In reference 8 the experiments were performed for natural isotopes.
[l] See reference 8.
[m] Lazareva, Gavrilov, Valuev, Zatsepina, and Stavinsky, *Proceedings of the Conference of the Academy of Science of the U. S. S. R. on the Peaceful Uses of Atomic Energy, Moscow, July 1–5, 1955*, Session of the Division of Physical and Mathematical Sciences (Akademiia Nauk, S.S.S.R., Moscow, 1955) [English translation by Consultants Bureau, New York, 1955], p. 217.
[n] Katz, McNeill, LeBlanc, and Brown, Can. J. Phys. **35**, 470 (1957).

nificant. However, qualitatively the same tendency is expected to appear also for light nuclei.

The cross section of Be^9 has three peaks and one of them is regarded as a giant resonance. The latter has a large width (7 to 8 Mev),[24] which might be due to the nonsphericity of this nucleus. C^{13} has a similar cross section,[25] which could be explained in the same way.

The measurement of fine structure in the cross

[24] R. Nathans and J. Halpern, Phys. Rev. **92**, 940 (1953).

[25] Cook, Penfold, and Telegdi, Phys. Rev. **104**, 554 (1956); B. C. Cook, Phys. Rev. **106**, 300 (1957).

TABLE V. The energies of the first excited states of even-even nuclei of $A \leqq 40$ and their resonance widths.

Nucleus	ΔE_1 (Mev)[a]	Γ (Mev)	Reference
$_6C_6^{12}$	4.44	4.2	b
$_8O_8^{16}$	6.05	3.4	b
$_{10}Ne_{10}^{20}$	1.63	6.6	c
$_{12}Mg_{12}^{24}$	1.38	7∼8	d
$_{12}Mg_{14}^{26}$	1.84	2.5 (γ,n)	e,f
		2.5 (γ,p)	e,f
$_{14}Si_{14}^{28}$	1.8	3.5	g
$_{16}S_{16}^{32}$	2.25	4.5	g
$_{16}S_{18}^{34}$	2.13	4.0	g
$_{18}A_{22}^{40}$	1.46	8.5	c
$_{20}Ca_{20}^{40}$	3.8	4.2	g

[a] ΔE_1 is taken from reference 13.
[b] J. H. Carber and K. H. Lokan, Australian J. Phys. **10**, 312 (1957).
[c] See reference 35.
[d] R. Nathans and P. F. Yergin, Phys. Rev. **98**, 1296 (1955).
[e] Katz, Haslam, Goldemberg, and Taylor, Can. J. Phys. **32**, 580 (1954).
[f] However for Mg^{26} there is another experiment, according to which the cross section is quite different. Therefore the values listed here are doubtful [P. F. Yergin, Phys. Rev. **104**, 1340 (1956)].
[g] See reference 1.

section[26] shows that the number of breaks for Li^7 or F^{19} is less than that for C^{12} or O^{16}. This also might be due to the nonsphericity of Li^7 or F^{19}.

$_{11}Na_{12}^{23}$ is a famous exception to the shell model and its Q is fairly large. Its Γ, 6.0 Mev,[1] is also rather large for this region. $_{10}Ne_{11}^{21}$ is also an exception to the shell model, which might be related to the large resonance width of Ne^{20}. (See the later discussion and Table V.) $_{13}Al_{14}^{27}$ has a large Q. The width for neutron emission from Al is not so large,[1] but measurements[19] of the (γ,γ) cross section show that the total resonance width might be large.

According to the analysis of magnetic moments by Bohr and Mottelson,[12] P^{31} is deformed much more strongly than F^{19}. The Γ of F^{19} is fairly large, 5.6 Mev,[27] showing that it is deformed; while for P^{31} Γ has the large value of 10.2 Mev,[2] which is consistent with the prediction of Bohr and Mottelson that P^{31} has about the same deformation as rare-earth nuclei. Si^{29} may be similar to P^{31}.

For even-even nuclei Table V lists the energies of the first excited states,[13] ΔE_1, together with the widths. Small values of ΔE_1 correspond to large deformations, and therefore to large widths. This relation proves to be true except for Mg^{26} and Si^{28}. For Mg^{26} the discrepancy is explained in a footnote to Table V. For Si^{28} it may be due to the contribution of the (γ,p) reaction. It is interesting to compare A^{40} and Ca^{40}. They have the same mass number, but the widths are quite different, in good agreement with the difference in values of ΔE_1.

VI. COMMENT ON THE MASS-NUMBER DEPENDENCE OF THE RESONANCE WIDTH

The resonance width Γ has been believed[1,2] to decrease with increasing mass number, A. Several authors tried to explain this trend, but all of them failed except Wildermuth.[28,29] However, as we have seen, the actual data on the resonance widths are not so simple. Clearly a strong fluctuation is observed. For spherical nuclei, the width Γ_0 does not decrease with A, but is rather constant. Γ may even increase with A, as suggested by the following argument.

One of the reasons for damping of the resonance is the collision of the particles, as Wildermuth has already discussed.[30] The probability of collision between particles belonging to different levels is, roughly speaking, inversely proportional to the level spacing. Therefore, the higher the level density the larger is the intrinsic width. Using this idea, we could explain the discrepancy between the observed values and the calculated values in Table IV, because the level density becomes higher for nonmagic nuclei.[31] However, according to this idea the intrinsic width must increase with A, because the level density increases with A. In fact, the calculations of intrinsic widths by Fujii-Takagi[32] and Fujita[33] show a tendency to increase with increasing A.

Another important reason for the damping of the resonance is the coupling of dipole vibration to surface oscillation. Preliminary calculations of Reifman[34] and Soga *et al.*[35] show that this is a decreasing function of A. Therefore we arrive at the following conclusion.

The mass-number dependence of the resonance width may be a superposition of two competing tendencies, which results in a roughly constant intrinsic width, Γ_0, for spherical nuclei (about 4 to 5 Mev). In addition to this, the fluctuation of Γ_0 due to the change of level density between closed shells and the increase of the apparent width due to the splitting, $\Delta\Gamma$, are superimposed. The resultant width, Γ, shows a very complicated behavior.

VII. VALIDITY OF THE MODEL

In the above analysis we always used the value of Q_0 in the ground state. We now consider whether the nuclear shape will be the same in the highly excited state reached by photon absorption.

For this purpose we evaluate the value of the amplitude of dipole vibration. For the GT model,[3] the value of the amplitude, ξ, is

$$E_0=\tfrac{1}{2}M\omega^2\xi^2=\frac{3A^{\frac{2}{3}}\varphi}{4\epsilon r_0}\xi^2. \tag{19}$$

[26] Montalbetti, Katz, Haslam, Horsley, and Cameron, Phys. Rev. **95**, 464 (1954); J. Goldemberg and L. Katz, Phys. Rev. **95**, 471 (1954).
[27] Ferguson, Halpern, Nathans, and Yergin, Phys. Rev. **95**, 776 (1954).
[28] K. Wildermuth, Z. Naturforsch. **10a**, 447 (1955).
[29] K. Wildermuth and H. Wittern, Z. Naturforsch. **12a**, 39 (1957).
[30] The validity of his treatment is not too clear, but if we are satisfied with a rough estimate of the intrinsic width, it might be justified. (Concerning this point the author is greatly indebted to Dr. Wildermuth for private communications.)
[31] This point is also suggested by J. Fujita (private communication).
[32] S. Fujii and S. Takagi, Progr. Theoret. Phys. (Japan) **14**, 405 (1955).
[33] J. Fujita, Progr. Theoret. Phys. (Japan) **16**, 112 (1956).
[34] A. Reifman, Z. Naturforsch. **8a**, 505 (1953)
[35] Soga, Iishima, and Nogami (private communication).

Then we find

$$\xi/R_0=1.5A^{-\frac{1}{3}}. \quad (20)$$

This is about 1/13 for $A=50$, 1/23 for $A=100$, and 1/35 for $A=200$. These results agree approximately with those of Fujii and Takagi[32] using a more exact model.

In the SJJ model,[4] the value of ξ is obviously zero. Both models agree fairly well with experiment, showing that the nuclear shape may be about the same in the ground state and in the excited state.

We now compare the combination of high excitation energy with small change in nuclear shape, with other examples of nuclear deformations, for instance surface vibration or fission. In the latter cases the deformation is carried out without changing the nucleon density. In the former case protons are separated from neutrons, which requires a considerable amount of energy. Moreover, all protons are pushed to one side of the nucleus, which is energetically unfavorable. Therefore, in photon absorption the energy used for deformation of the nucleus may be very small, so that the nuclear shape would be conserved approximately.

However, as seen in Table I, for nuclei of spin 9/2 (neutron configuration $g_{9/2}$) the situation is somewhat different. For Ge^{73}, Q_0 obtained from the spectroscopic Q is quite different from Q_0 obtained by Coulomb excitation. For Kr^{83} and Nb^{93} the same results might be expected. For proton configurations this tendency is not so strong (see $In^{113,115}$), which supports the conclusion that the nuclear shape is determined mainly by neutrons.[13] The value of Γ for Ge^{73} is very large, and for Sr^{87} and Nb^{93} it is somewhat larger than the neighboring values.

From the viewpoint of nuclear structure it is expected that these nuclei have a small Q in the ground state, but that the $E2$ transition probability is fairly large so that Q_0 obtained from Coulomb excitation is large.[36] In other words, the shape of the excited state is different from that of the ground state. This might explain the discrepancy between Γ_{obs} and Γ_{calc} of Table IV for these strongly deformed nuclei which have $g_{9/2}$ configurations of neutrons.

Therefore we can say that the nuclear shape may not change appreciably even in highly excited states reached by photon absorption, in contrast to the large change of shape in fission; this is an essential difference between the usual one-fluid model and the two-fluid model used in dipole vibration. However, for special nuclei (such as the $g_{9/2}$ neutron shell) where the surface tension is very weak, it might be possible that the nuclear shape will change appreciably even in relatively low excitation.

[36] The author is greatly indebted to K. W. Ford and S. A. Moszkowski for their explanation of this point.

VIII. SUMMARY AND CONCLUSIONS

Finally, we draw the following conclusions:

(1) The experimental data on nuclear deformation agree fairly well with the calculation of Marumori, Suekane, and Yamamoto,[11] with few exceptions.

(2) There is little doubt as to the existence of a strong correlation between the nuclear deformation and the resonance width of a photonuclear reaction.

(3) The calculation assuming splitting of the resonance and assuming constant intrinsic width, 4.2 Mev, gives results which agree qualitatively with experiments, but quantitatively the calculated values are too low. We must take into account also the change (with A) of the intrinsic width itself.

(4) The mass-number dependence of the resonance width is very complicated, though for spherical nuclei it is roughly constant.

(5) The nuclear shape may not change seriously even in highly excited states. This could be explained if we take into account the difference between the one-fluid model and the two-fluid model.

(6) From the above analysis we see that the resonance width of a photonuclear reaction has a strong relation to the nuclear structure. A careful investigation of the former in connection with the latter may give us some information on the relation of nuclear structure to nuclear reactions.

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