Coulomb Corrections to the Fermi Nuclear Matrix Element*

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The Fermi nuclear matrix elements for positron decay between two $T=1$ states can be evaluated exactly for charge-independent nuclear forces in the limit of nonrelativistic nucleon velocities to give $M_F=\sqrt{2}$. Sherr and Gerhart have used this fact to place limits on the Fierz interference terms for the scalar and vector β interactions by comparing the ft values for the O^{14}, Al^{26}, and Cl^{34} $0^+\rightarrow 0^+$ β decays. The equality of these ft values to within an experimental uncertainty of $\sim 10\%$ implies a limit for the contribution of the Fierz interference term to the ft value of 12%. Such a limit will be modified by the correction terms arising from the Coulomb interaction and from v^2/c^2 terms in the nucleon velocities. These Coulomb correction terms are calculated on the coupling model with harmonic-oscillator wave functions and are shown to affect the ratio of the ft values for Cl^{34} and O^{14} by less than 1.5%. The relativistic corrections to this ratio have been shown to be even smaller. Therefore the uncertainty in the theoretical value for the square of the Fermi nuclear matrix elements is much smaller than the present experimental uncertainty in the ft values.

I. INTRODUCTION

THE ft value for the β decay of a nucleus through a transition between $J=0^+$, $T=1$ corresponding states with positron emission is given by

$$ft=2\pi^3 \ln 2[\xi+\xi b\langle W^{-1}\rangle]^{-1},$$
$$\xi=\left|\int 1\right|^2[k^{-2}(|C_S|^2+|C_S'|^2)+(|C_V|^2+|C_V'|^2)],$$
$$\xi b=-2\gamma\left|\int 1\right|^2 \mathrm{Re}[k^{-1}(C_S C_V^*+C_V' C_V'^*)], \quad (1)$$
$$k=\int 1 \Big/ \int \beta,$$

in the notation of Rose,[1] of Lee and Yang,[2] and of Porter, Wagner, and Freedman.[3] Sherr and Gerhart[4,5] have pointed out that the ft values for several of these allowed favored transitions with different beta end points can be used to place a limit on the Fierz interference terms between the scalar and the vector interaction. If one assumes that nuclear forces are charge-independent, the Fermi nuclear matrix elements for positron decay in the nonrelativistic limit are given by

$$M_F=\int \Psi_f^*(T_\xi+iT_\eta)\Psi_i dV, \quad (2)$$

where T_ξ, T_η, and T_ζ are the components of the total isotopic-spin vector $\mathbf{T}$ for the nucleus. The component T_ζ has the eigenvalue $\langle T_\zeta\rangle=\frac{1}{2}(N-Z)$ where N is the number of neutrons and Z is the number of protons in the nucleus and $\langle \mathbf{T}^2\rangle=T(T+1)$. The values of M_F for positron decay between two $T=1$ states is given as $M_F=\sqrt{2}$. This result is not dependent on any specific model for the nucleus, nor even upon the particular nucleus. Using this result Gerhart has analyzed the experimental data for the $0^+\rightarrow 0^+$ decays of O^{14}, Al^{26}, and Cl^{34} and given a value for the interference term b of $b=0.00\pm 0.12$.

The value found for b is subject to any corrections to the nuclear matrix element which would vary from one nucleus to another. There are two types of corrections to be considered, (1) modification by Coulomb forces of the assumption of charge independence and (2) relativistic correction terms in the beta interaction. The relativistic correction terms have been found to have a very small effect on the ratio of the ft values for two $0^+\rightarrow 0^+$ transitions.[6] In this paper the Coulomb corrections will be considered.

II. ISOTOPIC SPIN IMPURITY AND DYNAMIC DISTORTION

One effect of the Coulomb interaction is to mix states of different total isotopic spin T. In the $O^{14}\rightarrow N^{14*}$ decay the initial state is the $J=0^+$, $T=1$ ground state of O^{14} and the final state is the first excited state at 2.31 Mev in N^{14} with $J=0^+$, $T=1$. To the ground state of O^{14} the Coulomb interaction will add components of excited $J=0^+$, $T=1$, 2 states. The first excited state of N^{14} will have $J=0^+$, $T=0$, 1, 2 components from excited states. Similarly in the decay $Cl^{34}\rightarrow S^{34}$ the $J=0^+$, $T=1$ ground state of Cl^{34} has $J=0^+$, $T=0$, 1, 2 components from excited states, while the S^{34} ground state of $J=0^+$, $T=1$ has $J=0^+$, $T=1$, 2 components. The introduction of $T=1$ isotopic-spin impurities into $T=0$ nuclear states has been calculated theoretically[7,8] and investigated experimentally by means of the selection rules on electric dipole transitions for nuclei

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[1] M. E. Rose, *Beta- and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (Interscience Publishers, Inc., New York, 1955), pp. 273–291.
[2] T. D. Lee and C. N. Yang, Phys. Rev. **104**, 254 (1956).
[3] Porter, Wagner, and Freedman, Phys. Rev. **107**, 135 (1957).
[4] J. B. Gerhart and R. Sherr, Bull. Am. Phys. Soc. Ser. II, **1**, 195 (1956).
[5] J. B. Gerhart, Phys. Rev. **109**, 897 (1958).
[6] W. M. MacDonald (to be published).
[7] W. M. MacDonald, Phys. Rev. **100**, 51 (1955).
[8] W. M. MacDonald, Phys. Rev. **101**, 271 (1956).

with $A \leqslant 20$.[9,10] These calculations must be extended to determine the impurity of a $T=1$ state arising from excited $T=0$ and $T=2$ states in the $A=14$ and $A=34$ isotopic multiplets.

A second effect of the Coulomb interaction is the dynamic distortion of the wave functions of the initial and final nucleus with a resultant diminution of the overlap of Ψ_f and $(T_\xi+iT_\eta)\Psi_i$. This distortion is produced by the mixing of states of same total angular momentum and isotopic spin, but with the amplitudes and phases of the admixed states different in the initial and final nucleus. An illustration of this effect is obtained by comparing the wave functions of a neutron and a proton outside a closed shell produced by a central potential $V(r)$. The Schrödinger equation is

$$\left\{-\frac{\hbar^2}{2M}\Delta+V(r)+g(r)(\tfrac{1}{2}-t_\zeta)\right\}\Psi=E\Psi, \tag{3}$$

where $g(r)$ is the Coulomb potential produced by the protons in closed shells. A suitable charge-independent unperturbed Hamiltonian is

$$H_0=-\frac{\hbar^2}{2M}\Delta+V(r)+\tfrac{1}{2}g(r), \tag{4}$$

with the perturbation $H_1=g(r)t_\zeta$. Eigenfunctions of H_0 will be

$$f(r)\begin{pmatrix}1\\0\end{pmatrix} \text{ for a neutron}, \quad f(r)\begin{pmatrix}0\\1\end{pmatrix} \text{ for a proton.}$$

We are neglecting distortion of the core, and therefore the perturbation cannot change the total isotopic-spin quantum number of $t=\frac{1}{2}$. The perturbation will change the radial wave function for a proton and it will no longer be true that $(t_\xi+it_\eta)\psi_{\text{proton}}=\psi_{\text{neutron}}$.

The total effect of these two types of mixing is easily found in the $O^{14}\rightarrow N^{14*}$ decay. Let $\Psi(TT_\zeta)$ denote eigenstates of T^2 and T_ζ and $\Psi^0(1,T_\zeta)$ be the principal component of the ground state of O^{14} and of the first excited state of N^{14}. As the result of the Coulomb interaction, one has

$$\Psi_f=a_0\Psi^0(1,0)+\sum_\nu[a_\nu^{(1)}\Psi^\nu(1,0)+a_\nu^{(2)}\Psi^\nu(2,0)+a_\nu^{(3)}\Psi^\nu(0,0)], \tag{5}$$

$$\Psi_i=b_0\Psi^0(1,-1)+\sum_\nu[b_\nu^{(1)}\Psi^\nu(1,-1)+b_\nu^{(2)}\Psi^\nu(2,-1)].$$

The Fermi matrix element given by Eq. (2) is

$$M_F=(a_0b_0+\textstyle\sum_\nu a_\nu^{(1)}b_\nu^{(1)})\sqrt{2}+\sum_\nu a_\nu^{(2)}b_\nu^{(2)}\sqrt{6}.$$

The normalization of Ψ_f and Ψ_i provides the relations

$$\begin{aligned} a_0^2+\textstyle\sum_\nu[(a_\nu^{(1)})^2+(a_\nu^{(2)})^2+(a_\nu^{(3)})^2]=1,\\ b_0^2+\textstyle\sum_\nu[(b_\nu^{(1)})^2+(b_\nu^{(2)})^2]=1. \end{aligned} \tag{6}$$

[9] W. M. MacDonald, Phys. Rev. **98**, 60 (1955).
[10] D. H. Wilkinson, Phil. Mag. **1**, 127 (1956).

The coefficients $a_\nu^{(i)}$ and $b_\nu^{(i)}$ are small compared to unity and we can expand in powers of these coefficients to obtain the result to lowest order

$$|M_F|^2=2(1-\delta),$$

$$\delta=\textstyle\sum[(a_\nu^{(1)}-b_\nu^{(1)})^2+(a_\nu^{(3)})^2+(a_\nu^{(2)})^2+(b_\nu^{(2)})^2-2\sqrt{3}a_\nu^{(2)}b_\nu^{(2)}]. \tag{7}$$

A precisely similar result holds for the matrix element of the Cl^{34} decay. Since ft is inversely proportional to $|M_F|^2$ the ft value is increased by a fraction δ over the value calculated on the assumption of charge independence. The form of δ is precisely what we expect. The $T=0$ components of the final state do not contribute to the matrix element and this portion of the wave function is effectively subtracted. On the other hand, the dynamic distortion reduces the matrix element only if the initial and final states are distorted differently, i.e., only if $a_1\neq b_1$. The effect of the $T=2$ components depends upon the relative size and phase of $a_\nu^{(2)}$ and $b_\nu^{(2)}$. The effect of the $T=2$ components will be shown to reduce, or even change the sign, of δ.

If δ did not change from one nucleus to another, the Coulomb interaction would not produce any difference in the ft values for $O^{14}\rightarrow N^{14*}$ and for $Cl^{34}\rightarrow S^{34}$. The corrections arising from the Coulomb interaction would amount only to a renormalization of the coupling constant and would have no effect upon the determination of the limits on the Fierz terms. Consequently we must calculate not merely the value of δ for say the $O^{14}\rightarrow N^{14*}$ decay, but the difference in the values of δ for the O^{14} and the Cl^{34} decays.

III. ISOTOPIC-SPIN IMPURITIES

The Coulomb potential can be written in the isotopic-spin formalism as

$$C=e^2\textstyle\sum_{i<j}(\frac{1}{2}-t_{\zeta i})(\frac{1}{2}-t_{\zeta j})r_{ij}^{-1}, \tag{8}$$

where $t_\zeta=\frac{1}{2}\tau_\zeta$ has eigenvalues $+\frac{1}{2}$ for a neutron state state and $(-\frac{1}{2})$ for a proton state. The Coulomb potential can be decomposed into irreducible tensors in isotopic-spin space. Since the component which is a scalar in isotopic spin can be included in a charge-independent nuclear Hamiltonian, only the vector and tensor components of C need be considered. The perturbation on the isotopic-spin states is therefore

$$\begin{aligned} \mathcal{C}&=V_0^{(1)}+V_0^{(2)},\\ V_0^{(1)}&=-(e/2)\textstyle\sum_{i<j}(t_{\zeta i}+t_{\zeta j})r_{ij}^{-1},\\ V_0^{(2)}&=e^2\textstyle\sum_{i<j}t_{\zeta i}t_{\zeta j}r_{ij}^{-1}. \end{aligned} \tag{9}$$

The two operators, the vector $V_0^{(1)}$ and the tensor $V_0^{(2)}$, satisfy selection rules in self-conjugate $(T_\zeta=0)$ nuclei

$$\begin{aligned} (\alpha' T\ T_\zeta=0|\,V_0^{(1)}\,|\alpha' T\ T_\zeta=0)=0,\\ (\alpha' T\pm1\ T_\zeta=0|\,V_0^{(2)}\,|\alpha' T\ T_\zeta=0)=0. \end{aligned} \tag{10}$$

In order to calculate the effect of $\mathcal{C}$ on the Fermi matrix elements, we must use a specific nuclear model which will provide wave functions for the initial and final states and for the excited states. Earlier calculations on the isotopic-spin impurities lead us to expect that the results are not sensitive to the particular model. We shall use the jj coupling model with harmonic-oscillator wave functions. On this model the ground states of O^{14} and N^{14} belong to the configuration $(1p_{\frac{1}{2}})^2$ with a core consisting of filled $1s_{\frac{1}{2}}$ and $1p_{\frac{3}{2}}$ shells. The ground states of Cl^{34} and S^{34} will belong to the $(1d_{\frac{3}{2}})^2$ configuration with a core consisting of filled $1s_{\frac{1}{2}}$, $1p_{\frac{3}{2}}$, $1p_{\frac{1}{2}}$, $1d_{\frac{5}{2}}$, and $2s_{\frac{1}{2}}$ shells.

In the construction of the wave functions with the quantum numbers J, M, T, and T_z we shall couple an antisymmetric wave function for the core nucleons to an antisymmetric wave function for the two nucleons outside the core. But we shall not antisymmetrize between these two sets of states. We shall be neglecting, therefore, the exchange integrals between these states. The two sets of states are in different shells, however, and the exchange integrals between them will be much smaller than the direct integrals.

The only excited states which will be considered in calculating the isotopic-spin impurity are those which can be formed by promoting a single nucleon from an (n,l,j) orbital to an $(n+1, l, j)$ orbital. The long-range character of the Coulomb potential causes the matrix elements between the members of the ground-state $T=1$ multiplet and such excited states to be much larger than matrix elements to excited states formed by changing the j or l value of a single nucleon.[7] Furthermore, the contributions from states of two-nucleon excitation are much smaller than contributions from states of single-nucleon excitation, especially in the $A=34$ triad. The states to be used in the calculations can be summarized as in Table I. The state A forms the lowest $T=1$ multiplet in the $A=14$ and $A=34$ multiplets, while states B, C, and E provide the $T=0$ impurity in A. These $T=0$ states appear only in the $T_z=0$ nuclei N^{14} and Cl^{34} and are mixed to the state A by the vector component of the perturbation $\mathcal{C}$. The state D gives rise to a $T=2$ impurity in the state A.

We shall denote the unperturbed single-nucleon states by $\varphi_{nlj}{}^m$. The excited-nucleon state with n replaced by $n+1$ will be denoted by $\psi_{nlj}{}^{m\tau}$. The construction of the states A, B, D, and E is given in Appendix I; state C can be shown to contribute a much smaller amount of isotopic-spin impurity than the other states and will not be considered further. The matrix element between the state A and a state of the type B formed by exciting an (nlj) nucleon from a filled shell is found for N^{14} to be

$$\langle B,nlj|\mathcal{C}|A\rangle=(-\tfrac{1}{2}e^2)(+\tfrac{2}{3})(2j+1)^{-\frac{1}{2}}\times\sum_{m,m'}\sum_{\alpha}\Big\{\int\psi_{nlj}{}^m(1)\varphi_\alpha{}^{m'}(2)r_{12}{}^{-1}(1-P_{12})\times\varphi_{nlj}{}^m(1)\varphi_\alpha{}^{m'}(2)+\tfrac{1}{2}\int\psi_{nlj}{}^m(1)\varphi_{1p\frac{1}{2}}{}^{m'}(2)\times r_{12}{}^{-1}\varphi_{nlj}{}^m(1)\varphi_{1p\frac{1}{2}}{}^{m'}(2)\Big\},\quad(11)$$

where α denotes a state of the filled shells and P_{12} exchanges the new nucleons. The sums on the magnetic quantum numbers are easily evaluated for the direct integrals using

$$\sum_m\varphi_{n'lj}{}^{*m}(1)\varphi_{nlj}{}^m(2)=[(2j+1)/4\pi]R_{n'l}(1)R_{nl}(2)r_1{}^{-2}.\quad(12)$$

The R_{nl} are the radial functions for the nucleon states. The exchange integral involving P_{12} is more complicated to sum. One can expand $r_{12}{}^{-1}$ in Legendre polynomials, thus obtaining the Slater expansion of the integral. One knows that for the Coulomb potential the successive terms in such an expansion decrease rapidly. We shall retain only the matrix element of the first spherically symmetric term and sum this.

$$\sum_{m,m'}\int\psi_{nlj}{}^m(1)\varphi_\alpha{}^{m'}(2)\frac{1}{r_>}\varphi_{nlj}{}^m(2)\varphi_\alpha{}^{m'}(1)=\delta_{l,l'}\delta_{j,j'}(2j+1)\int dr_1dr_2R_{n'l}(1)R_{nl}(1)R_{nl}{}^2(2)\frac{1}{r_>},\qquad r_>=\text{larger of } r_1,\ r_2.\quad(13)$$

Combining Eqs. (11), (12), and (13) we find

$$\langle B;nlj|\mathcal{C}|A\rangle=(-e^2/2)[(2j+1)/6]^{\frac{1}{2}}\beta(nlj),$$

where

$$\beta(nlj)=\sum_{n'l'j'}N(nlj;n'l'j')\mathcal{R}(nl;n'l')+\mathcal{R}(nl;1p),$$

$$N(nlj;n'l'j')=(2j'+1)\quad\text{if}\quad(nlj)\neq(n'l'j')$$
$$=2j\qquad\text{if}\quad(nlj)=(n'l'j'),\quad(14)$$

$$\mathcal{R}(nl;n'l')=\int dr_1dr_2d(\cos\theta_{12})R_{n+1,l}(1)\times R_{nl}(1)R_{n'l'}{}^2(2)r_{12}{}^{-1}.$$

In a similar fashion the matrix element to the state E

TABLE I. Composition of excited states considered.

Core state (JT)	Two-nucleon state (JT)	Total (JT)
(A) (0,0)	(0,1)	(0,1)
(B) excited (0,1)	(0,1)	(0,0)
(C) excited (1,0)	(1,0)	(0,0)
(D) excited (0,1)	(0,1)	(0,2)
(E) (0,0)	excited (0,0)	(0,0)

formed by exciting one of the $1p_{\frac{1}{2}}$ nucleons to a $2p_{\frac{1}{2}}$ nucleon state is calculated.

$$\langle E|\mathcal{C}|A\rangle=-\tfrac{1}{2}e^2\sum_{n'l'j'}(2j'+1)\mathcal{R}(1p;n'l')$$
$$=-\tfrac{1}{2}e^2\beta(1p_{\frac{1}{2}}). \quad (15)$$

Matrix elements between the state A and states of type D formed by exciting an (nlj) nucleon from a closed shell to a state $(n+1,lj)$ occur in $T_z=0,\pm1$ nuclei. We shall calculate the matrix elements of the vector and tensor components of in the $T_z=1$ nucleus (C^{14}).

$$\langle D;nlj|V_0^{(1)}|A\rangle=(e^2/2)[(2j+1)^{\frac{1}{2}}/2]\beta(nlj),$$
$$\langle D;nlj|V_0^{(2)}|A\rangle=-(e^2/2)[(2j+1)^{\frac{1}{2}}/4]\mathcal{R}(nl,1p). \quad (16)$$

From the Eckart-Wigner formula and the generalization by Racah[11] we find the matrix elements in a $T_z=0$ nucleus (N^{14}) to be

$$\langle D;nlj|V_0^{(1)}|A\rangle=(e^2/2)[(2j+1)/3]^{\frac{1}{2}}\beta(nlj),$$
$$\langle D;nlj|V_0^{(2)}|A\rangle=0. \quad (17)$$

The matrix elements in the $T_z=-1$ nucleus (O^{14}) are

$$\langle D;nlj|V_0^{(1)}|A\rangle=(e^2/2)[(2j+1)^{\frac{1}{2}}/2]\beta(nlj),$$
$$\langle D;nlj|V_0^{(2)}|A\rangle=(e^2/2)[(2j+1)^{\frac{1}{2}}/4]\mathcal{R}(nl,1p). \quad (18)$$

The corresponding matrix elements in Cl^{34} and S^{34} are easily found from Eqs. (14), (15), and (16) by replacing the $(1p)$ by $(1d)$ and extending the sum over filled states $1s_{\frac{1}{2}}$, $1p_{\frac{3}{2}}$, $1p_{\frac{1}{2}}$, $1d_{\frac{5}{2}}$, and $2s_{\frac{1}{2}}$.

The coefficients $a_\nu^{(3)}$, $a_\nu^{(2)}$, and $b_\nu^{(2)}$ which appear in Eq. (5) are now given in terms of the matrix elements.

$$a_\nu^{(3)}=(-e^2/2)[(2j+1)/6]^{\frac{1}{2}}\beta(nlj)(E_0-E_1)^{-1},$$
$$a_{1p_{\frac{1}{2}}}^{(3)}=(-e^2/2)\beta(1p_{\frac{1}{2}})(E_0-E_{1p_{\frac{1}{2}}})^{-1},\quad \nu=(nlj),$$
$$a_\nu^{(2)}=(e^2/2)[(2j+1)/3]^{\frac{1}{2}}\beta(nlj)(E_0-E_2)^{-1}, \quad (19)$$
$$b_\nu^{(2)}=(e^2/2)[(2j+1)^{\frac{1}{2}}/2][\beta(nlj)$$
$$+\tfrac{3}{2}\mathcal{R}(nl,1p)](E_0-E_2)^{-1}.$$

A corresponding set of formulas can be given for the coefficients in the expansion of the wave functions for the lowest 0^+, $T=1$ states of N^{14} and Cl^{34}. In the following set of equations $a_\nu^{(3)}$ and $a_\nu^{(2)}$ are coefficients in the Cl^{34} wave function; $b_\nu^{(2)}$ is a coefficient in the S^{34} wave functions.

$$a_\nu^{(3)}=(-e^2/2)[(2j+1)/6]^{\frac{1}{2}}\beta(nlj)(E_0-E_1)^{-1},$$
$$a_{1d}^{(3)}=(-e^2/2)\beta(nlj)(E_0-E_{1d})^{-1},$$
$$a_\nu^{(2)}=(e^2/2)[(2j+1)/3]^{\frac{1}{2}}\beta(nlj)(E_0-E_2)^{-1}, \quad (20)$$
$$b_\nu^{(2)}=(e^2/2)[(2j+1)^{\frac{1}{2}}/2][\beta(nlj)$$
$$+\tfrac{1}{2}\mathcal{R}(nlp)](E_0-E_2)^{-1}.$$

[11] E. P. Wigner, *Gruppentheorie* (Friedrich Vieweg und Sohn, Braunschweig, 1931); G. Racah, Phys. Rev. **62**, 438 (1942).

These coefficients will be evaluated by using harmonic-oscillator wave functions in the form given by Talmi.[12] The matrix elements will be presented in terms of the mean square radius R^2, which is given in terms of the parameter ν by $\nu=31/14R^2$ for N^{14} and by $\nu=99/34R^2$ for Cl^{34}. The numerical evaluation will be made, however, in terms of the parameter ν as determined by Talmi and Thieberger[13] from the binding energy of light nuclei.

The physical interpretation of these results is obvious. Each proton wave function is perturbed by interaction with the $(2j'+1)$ protons in every $(n'l'j')$, with the $2j$ other protons in the same shell, and with the single $(1p_{\frac{1}{2}})$ proton. In order to obtain an approximate expression for the isotopic-spin impurity we shall take the energy separation of the state A from all states of the types B and E to be equal to E_0-E_1. The separation of A from all the type-D states will be taken as E_0-E_2. The $T=0$ impurity in the first excited state of N^{14} is represented in Eq. (7) by the sum

$$\sum_\nu(a_\nu^{(2)})^2=(e^2/2)^2\{\tfrac{1}{6}\textstyle\sum_{nlj}(2j+1)\beta^2(nlj)$$
$$+\beta^2(1p_{\frac{1}{2}})\}(E_0-E_1)^{-2}. \quad (21)$$

The $T=2$ impurity of the state is

$$\sum_\nu(a_\nu^{(2)})^2=(e^2/2)^2(1/3)$$
$$\times\textstyle\sum_j(2j+1)\beta^2(nlj)(E_0-E_2)^{-2}. \quad (22)$$

Comparing these expressions with Eqs. (22), (42), (43), and (44) of reference 8, we see that if $E_2=E_1$, the total isotopic-spin impurity of this $T=1$ state is equal to the total $T=1$ impurity of a $T=0$ state calculated there. In the $T=1$ state, however, $\sim\frac{1}{3}$ of the impurity is from $T=0$ states and $\sim\frac{2}{3}$ is from $T=2$ states.

Although the effect of the $T=0$ impurity is to reduce the matrix element, Eqs. (7), (17), and (18) show that the $T=2$ impurity will increase the matrix element. The contribution of the single $1p$ proton to the matrix elements is much smaller than the perturbation produced by all the closed shell protons. Therefore the following approximate relations hold:

$$b_\nu^{(2)}\approx(\sqrt{3}/2)a_\nu^{(2)},$$
$$S_\nu=(a_\nu^{(2)})^2+(b_\nu^{(2)})^2-2\sqrt{3}a_\nu^{(2)}b_\nu^{(2)}\approx-(5/4)(a_\nu^{(2)})^2. \quad (23)$$

The approximate relation $(E_0-E_2)a_\nu^{(2)}\approx(E_0-E_1)\sqrt{2}a_\nu^{(3)}$ follows by comparing Eqs. (14) and (17) and gives the result that

$$S_\nu+T_\nu=(a_\nu^{(3)})^2+(a_\nu^{(2)})^2+(b_\nu^{(2)})^2-2\sqrt{3}a_\nu^{(2)}b_\nu^{(2)}$$
$$\approx-\left[\frac{5}{2}\left(\frac{E_0-E_1}{E_0-E_2}\right)^2-1\right](a_\nu^{(3)})^2,\quad T_\nu=(a_\nu^{(3)})^2. \quad (24)$$

[12] I. Talmi, Helv. Phys. Acta **25**, 185 (1952).

[13] I. Talmi and R. Thieberger, Phys. Rev. **103**, 719 (1956); the ν in their paper is one-half our ν.

The net effect of the $T=0$ and $T=2$ impurity upon the ft value is less than the effect of either impurity upon the ft value separately. The magnitude, and in fact the sign, of $(S_\nu+T_\nu)$ depends upon the ratio $(E_0-E_1)/(E_0-E_2)$. The ratio obviously satisfies $0\leqslant(E_0-E_1)/(E_0-E_2)<1$. The quantity $S_\nu+T_\nu$ then ranges from $[-\frac{3}{2}(a_\nu^{(3)})^2]$, through zero, to positive values. The total effect of the isotopic-spin impurity can be to increase or decrease the Fermi matrix element for β decay.

The coefficients given in Eqs. (19) and (20) were evaluated and are given in Table II. The sums on S and T in the $A=14$ and $A=34$ triads can be found.

$$A=14:$$
$$\sum_\nu(a_\nu^{(3)})^2=5.92(e^2/R)^2(E_0-E_1)^{-2},$$
$$\sum_\nu[(a_\nu^{(2)})^2+(b_\nu^{(2)})^2-2\sqrt{3}a_\nu^{(2)}b_\nu^{(2)}]=-6.95(e^2/R)^2(E_0-E_2)^{-2}, \tag{25}$$
$$A=34:$$
$$\sum_\nu(a_\nu^{(3)})^2=38.32(e^2/R)^2(E_0-E_1)^{-2},$$
$$\sum_\nu[(a_\nu^{(2)})^2+(b_\nu^{(2)})^2-2\sqrt{3}a_\nu^{(2)}b_\nu^{(2)}]=-83.94(e^2/R)^2(E_0-E_2)^{-2}. \tag{26}$$

To find δ and the total effect of the Coulomb interaction upon the ft values we shall now calculate the dynamic distortion, expressed by the terms $(a_\nu^{(1)}-b_\nu^{(1)})^2$ in Eq. (7).

IV. DYNAMIC DISTORTION

The dynamic distortion arises from the mixing of $T=1$ excited states to the $T=1$ states between which the beta decay takes place. The energy separation of these excited $T=1$ states from the lowest $T=1$ states will be the same in the nuclei between which the β decay takes place. We can write the difference in the matrix elements for O^{14} and N^{14} as

$$\epsilon_\nu=(E_0-E_3)(a_\nu^{(1)}-b_\nu^{(1)})=(\psi^\nu(1,-1),(\tfrac{1}{2}\mathcal{T}_-\mathcal{C}\mathcal{T}_+-\mathcal{C})\psi^0(1,-1)),$$

where

$$\mathcal{T}_\pm=T_\xi\pm iT_\eta. \tag{27}$$

In order to find the distortion coefficients ϵ_ν we need only calculate the matrix elements of a certain operator in the $T_\zeta=-1$ nucleus. When this operator operates on a $T=1$ state, the form of the operator is

$$\tfrac{1}{2}\mathcal{T}_-\mathcal{C}\mathcal{T}_+-\mathcal{C}=\{-\tfrac{1}{4}\textstyle\sum_{i<j}(t_i^-+t_j^-)r_{ij}^{-1}+\tfrac{1}{2}\sum_{i<j}(t_i^-t_\zeta+t_j^-t_{\zeta i})r_{ij}^{-1}\}\mathcal{T}_+. \tag{28}$$

We first consider perturbations produced by interaction with excited states formed by excitation of a single nucleon from an (nlj) state to an $(n+1, lj)$ state. Two kinds of states contribute most of the dynamic distortion. The first kind is formed by coupling a $J=0$, $T=1$ excited state of the nucleons originally in filled states to a $J=0$, $T=1$ state of the $(1p_{\frac{1}{2}})^2$ nucleons. The second kind of state is formed by coupling an unexcited $J=0^+$, $T=0$ state of the nucleons in filled states to a $J=0^+$, $T=1$ state of the $1p_{\frac{1}{2}}2p_{\frac{1}{2}}$ configuration.

The dynamic distortion coefficients ϵ_ν are then given by the equations

$$\epsilon_{nlj}=-(e^2/2)[(2j+1)/2]^{\frac{1}{2}}R(njl;1p) \quad \text{if} \quad (nlj)\neq(1p_{\frac{1}{2}}), \tag{29}$$
$$\epsilon_{1p_{\frac{1}{2}}}=-(e^2/2)2^{-\frac{1}{2}}\textstyle\sum_{n'l'j'}(2j'+1)R(1p,1p).$$

For N^{14} we find the results

$$a_{1s_{\frac{1}{2}}}^{(1)}-b_{1s_{\frac{1}{2}}}^{(1)}=-0.121(e^2/R)(E_0-E_3)^{-1},$$
$$a_{1p_{\frac{3}{2}}}^{(1)}-b_{1p_{\frac{3}{2}}}^{(1)}=-0.498(e^2/R)(E_0-E_3)^{-1},$$
$$a_{1p_{\frac{1}{2}}}^{(1)}-b_{1p_{\frac{1}{2}}}^{(1)}=-1.803(e^2/R)(E_0-E_3)^{-1}, \tag{30}$$
$$\textstyle\sum(a_\nu^{(1)}-b_\nu^{(1)})^2=1.34(e^2/R)^2(E_0-E_3)^{-2}.$$

We have used the same energy separation for all three states.

The calculation of the dynamic distortion in the $Cl^{34}\rightarrow S^{34}$ decay is slightly different. The dynamic distortion coefficient is given by

$$\epsilon_\nu=(E_0-E_3)(a_\nu^{(1)}-b_\nu^{(1)})=(\psi^\nu(1,1),(\tfrac{1}{2}\mathcal{T}_+\mathcal{C}\mathcal{T}_--\mathcal{C})\psi^0(1,1)). \tag{31}$$

The operator which appears here can be written as

$$\tfrac{1}{2}\mathcal{T}_+\mathcal{C}\mathcal{T}_--\mathcal{C}=\tfrac{1}{4}\textstyle\sum_{i<j}(t_i^++t_j^+)r_{ij}^{-1}+\tfrac{1}{2}\sum_{i<j}(t_i^+t_{\zeta j}+t_j^+t_{\zeta i})r_{ij}^{-1}. \tag{32}$$

TABLE II. Coefficients for isotopic-spin impurity.

		$A=14$	
$a_{1s_{\frac{1}{2}}}^{(3)}$	$-0.49(e^2/R)(E_0-E_1)^{-1}$		
$a_{1p_{\frac{3}{2}}}^{(3)}$	-1.50		
$a_{1p_{\frac{1}{2}}}^{(3)}$	-1.85		
$a_{1s_{\frac{1}{2}}}^{(2)}$	$0.69(e^2/R)(E_0-E_2)^{-1}$	$b_{1s_{\frac{1}{2}}}^{(2)}$	$0.74(e^2/R)(E_0-E_2)^{-1}$
$a_{1p_{\frac{3}{2}}}^{(2)}$	2.14	$b_{1p_{\frac{3}{2}}}^{(2)}$	2.48
$a_{1p_{\frac{1}{2}}}^{(2)}$	0	$b_{1p_{\frac{1}{2}}}^{(2)}$	0
		$A=34$	
$a_{1s_{\frac{1}{2}}}^{(3)}$	$-1.19(e^2/R)(E_0-E_1)^{-1}$		
$a_{1p_{\frac{3}{2}}}^{(3)}$	-2.82		
$a_{1p_{\frac{1}{2}}}^{(3)}$	-1.99		
$a_{1d_{\frac{5}{2}}}^{(3)}$	-4.10		
$a_{2s_{\frac{1}{2}}}^{(3)}$	-2.06		
$a_{1d_{\frac{3}{2}}}^{(3)}$	-4.10		
$a_{1s_{\frac{1}{2}}}^{(2)}$	$1.68(e^2/R)(E_0-E_2)^{-1}$	$b_{1s_{\frac{1}{2}}}^{(2)}$	$1.42(e^2/R)(E_0-E_2)^{-1}$
$a_{1p_{\frac{3}{2}}}^{(2)}$	3.97	$b_{1p_{\frac{3}{2}}}^{(2)}$	3.41
$a_{1p_{\frac{1}{2}}}^{(2)}$	2.82	$b_{1p_{\frac{1}{2}}}^{(2)}$	2.39
$a_{1d_{\frac{5}{2}}}^{(2)}$	5.80	$b_{1d_{\frac{5}{2}}}^{(2)}$	4.95
$a_{2s_{\frac{1}{2}}}^{(2)}$	2.92	$b_{2s_{\frac{1}{2}}}^{(2)}$	2.40

The same types of excited states are considered in calculating the dynamic distortion and the results are

$$\epsilon_{nlj}=(e^2/2)[(2j+1)/2]^{\frac{1}{2}}\Re(nlj;1d) \quad \text{if} \quad (nlj)\neq 1d_{\frac{1}{2}},$$
$$\epsilon_{1d\frac{3}{2}}=(e^2/2)2^{-\frac{1}{2}}[\textstyle\sum_{n'l'j'}(2j'+1)\Re(1d,n'j'l')+\frac{3}{4}\Re(1d,1d)]. \tag{33}$$

These coefficients are given in Table III, along with the sum $\sum_\nu(a_\nu^{(1)}-b_\nu^{(1)})^2$ in the approximation in which all the energy denominators are equal.

Equations (29) and (33) give the distortion, produced by the additional proton, of the wave functions of the protons in filled shells, and the distortion of the wave function for the additional proton by the protons in filled states. The dynamic distortion is a function of the number of protons, therefore, while isotopic-spin impurity depends upon the number of pairs of interacting protons. The importance of the dynamic distortion relative to the isotopic-spin impurity should decrease with increasing number of particles.

In addition to the dynamic distortion produced by the interaction of the $(J,T)=(0,1)$ states with these excited states we must consider the effect of the (0,1) state formed by exciting a $1p_{\frac{3}{2}}$ core nucleon to a $1p_{\frac{1}{2}}$ orbit. The orbital angular momentum is unchanged although the orientation of the spin is different. The matrix elements will not be as large as those already calculated, but the energy separation should be much less. In fact the separation of the single-particle levels is only ~ 6 Mev. In this case even a reasonable upper limit to the distortion can only be found by using the energy separation obtained from diagonalization of the energy matrix.

For N^{14} the wave functions of the ground state of O^{14} and the first excited state of N^{14} have been calculated in this way by Ferrell and Visscher working in LS coupling.[14] Only the interaction of states of the $(1p)^{10}$ configuration was considered, and therefore the wave functions give directly just the part of the dynamic distortion which we require. The contribution to δ is found to be 2.6×10^{-3}. This result will actually be found to exceed the contribution of the distortion arising from the other excited states of N^{14} by a factor of ~ 3. A similar calculation of the distortion in the $A=34$ states arising from the $1d_{\frac{5}{2}}\rightarrow 1d_{\frac{3}{2}}$ excitation has not been carried out. Here the isotopic-spin impurity should be much larger than the dynamic distortion, however, as has been pointed out.

TABLE III. Coefficients for dynamic distortion.

$A=14$	
$1S_{\frac{1}{2}}$	$-0.12(e^2/R)$
$1p_{\frac{3}{2}}$	-0.50
$1p_{\frac{1}{2}}$	-1.80
$\Sigma(a^{(1)}-b^{(1)})^2$	$3.51(e^2/R)^2(E_0-E_3)^{-2}$
$A=34$	
$1S_{\frac{1}{2}}$	$-0.069(e^2/R)$
$1p_{\frac{1}{2}}$	-0.13
$1p_{\frac{3}{2}}$	-0.10
$1d_{\frac{5}{2}}$	-0.20
$2S_{\frac{1}{2}}$	-0.32
$1d_{\frac{3}{2}}$	-3.51
$\Sigma(a^{(1)}-b^{(1)})^2$	$12.47(e^2/R)^2(E_0-E_3)^{-2}$

V. NET EFFECT OF COULOMB CORRECTIONS

We have calculated all the terms in δ, which gives the reduction of the Fermi nuclear beta-decay matrix by the Coulomb interaction. Only the excited states were considered which are believed to provide the principal contributions to the perturbation of the $J=0^+$, $T=1$ states between which the decay occurs. The contributions to δ from individual excited states do not interfere, and we have made the approximation of taking all the excited states characterized by the same isotopic spin to be the same (average) energy above the lowest $J=0^+$, $T=1$ state. Since the perturbations of the matrix element from states of different isotopic spin do interfere, we must consider the effect of different average energies of excitation for these states. The three different kinds of terms which appear in δ are summarized in Table IV.

The states $T=0$ and $T=1$ are probably thoroughly mixed in the spectrum at the excitation energy at which the states which perturb the lowest $J=0^+$, $T=0$ appear. A reasonable approximation is to take $E_0=E_1=\bar{E}$, and since the $T=0$ and $T=1$ states do not interfere destructively this is also safe. Since the matrix elements to the $T=2$ states are so large, the $T=2$ impurity actually changes the sign of δ if the energy E_2 is also taken as $\bar{E}$. The δ for this case is given and is seen to agree very well for $A=34$ with the conclusions of the simple arguments following Eq. (24). Since we shall certainly have $E_0-E_2>E_0-E_1$, however,

TABLE IV. Summary of Coulomb corrections to the Fermi nuclear matrix element.

$A=14$	
$T=0$ impurity	$\Sigma_\nu(a_\nu^{(3)})^2=5.92(e^2/R)^2(E_0-E_1)^{-2}$
$T=2$ impurity	$\Sigma_\nu[(a_\nu^{(2)})^2+(b_\nu^{(2)})^2-2\sqrt{3}a_\nu^{(2)}b_\nu^{(2)}]=-6.95(e^2/R)^2(E_0-E_2)^{-2}$
$T=1$ distortion	$\Sigma_\nu(a_\nu^{(1)}-b_\nu^{(1)})^2=3.51(e^2/R)^2(E_0-E_3)^{-2}$
Total effect for $E_1=E_2=E_3=E$	$\delta=2.49(e^2/R)^2(E_0-\bar{E})^2$
$A=34$	
$T=0$ impurity	$\Sigma_\nu(a_\nu^{(3)})^2=38.32(e^2/R)^2(E_0-E_1)^{-2}$
$T=2$ impurity	$\Sigma_\nu[(a_\nu^{(2)})^2+(b_\nu^{(2)})^2-2\sqrt{3}a_\nu^{(2)}b_\nu^{(2)}]=-83.94(e^2/R)^2(E_0-E_2)^{-2}$
$T=1$ distortion	$\Sigma_\nu(a_\nu^{(1)}-b_\nu^{(1)})^2=12.47(e^2/R)^2(E_0-E_3)^{-2}$
Total effect for $E_1=E_2=E_3=E$	$\delta=-33.15(e^2/R)^2(E_0-\bar{E})^{-2}$

[14] W. M. Visscher and R. A. Ferrell, Phys. Rev. **107**, 781 (1957).

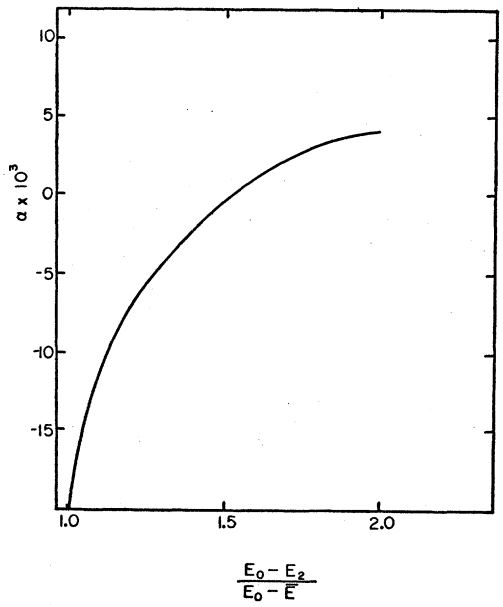

FIG. 1. The ratio $(ft)_{Cl^{34}}/(ft)_{N^{14}}=1+\alpha$ deviates from unity by α, which gives the effect of the Coulomb correction. The value of $\alpha=\delta(Cl^{34})-\delta(N^{14})$ is given as a function of $(E_0-E_2)/(E_0-\bar{E})$, which is taken for this purpose to be the same in the $A=14$ and $A=34$ nuclei.

the value of δ should be smaller in magnitude than this. In fact if E_0-E_2 is only $\sim 20\%$ larger than E_0-E_1 the value of $\delta\approx 0$.

The maximum amplitude of δ is given for $E_0=E_1=E_2=\bar{E}$. We shall take $E=2\hbar\nu/M$, the separation of the states $(n+1\,lj)$ and (nlj). The value of ν is taken from the work of Talmi and Thieberger on the binding energy of light nuclei.[13]

$$\begin{aligned} &A=14: \quad e^2(\nu/2\pi)^{\frac{1}{2}}=0.349 \text{ Mev}, \quad E=30.8 \text{ Mev}, \\ &A=34: \quad e^2(\nu/2\pi)^{\frac{1}{2}}=0.312 \text{ Mev}, \quad E=24.7 \text{ Mev}. \end{aligned} \tag{34}$$

The values of δ arising from these states are then

$$A=14: \quad \delta=9\times 10^{-4}; \quad A=34: \quad \delta=-1.2\times 10^{-2}. \tag{35}$$

To the δ for N^{14} we must add the contribution from the interaction of the $T=1$ states within the p shell giving $\delta=3.5\times 10^{-3}$. The correction to δ for $A=34$ from the $d_{\frac{3}{2}}\rightarrow d_{\frac{3}{2}}$ excitation is not included but would reduce the absolute value of δ (see Fig. 1).

The corrections to the Fermi matrix element arising from Coulomb interactions will change the ratio of the ft values for the $Cl^{34}\rightarrow S^{34}$ and $O^{14}\rightarrow C^{14}$ decays by an amount which is certainly less than $\sim 1.5\%$. This is less than the quoted experimental uncertainties. In Cl^{34} the quoted error is $\sim 3\%$.

VI. SUMMARY

The Coulomb interaction in nuclei produces a correction to the Fermi nuclear matrix element for β decay calculated for charge-independent nuclear forces. The correction arises from the introduction of isotopic-spin impurity into nuclear states and from a dynamic distortion of the wave function. The net effect is the result of interference between $T=2$ isotopic-spin impurities, which increase the matrix element, and $T=0$ impurities and dynamic distortion, which decrease the matrix element. The effect can be an increase or a decrease of the ft value calculated for charge-independent nuclear forces in the limit of nonrelativistic nucleon velocities. The magnitude of the correction to the ratio of the ft values for $Cl^{34}\rightarrow S^{34}$ and $O^{14}\rightarrow N^{14*}$ is less than about one percent. Since the corrections to this ratio from the (v^2/c^2) terms in the scalar matrix element are also less than one percent,[6] the limits on the Fierz interference terms are correctly given by the analysis of Gerhart as approximately 10%.

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To Professor E. P. Wigner I owe several interesting conversations on the effect of Coulomb forces on beta decay matrix elements, particularly on the effect of dynamic distortion.

APPENDIX I

The single-particle states will be denoted by $\varphi_{nlj}{}^{m,\tau}$, where $m=j_z$, and $\tau=0$, 1 represent proton and neutron isotopic-spin states, respectively. The Slater determinant of states φ_1, φ_2, $\cdots$ will be designated by $\{\varphi_1,\varphi_2\cdots,\varphi_n\}$. The state of the core nucleons will be designated by $\Psi_I(JTT_\xi)$; the state of the particles outside the core, by $\Psi_{II}(JTT_\xi)$. We shall give the states (A), (B), and (D) for N^{14}.

The state (A) is the $J=0^+$, $T=1$ state which lies lowest and predominates in the states between which the decay occurs.

$$\begin{aligned} &\Psi_I(000)=(12!)^{-\frac{1}{2}}\{\varphi_{1s\frac{1}{2}}{}^{-\frac{1}{2},0}\varphi_{1s\frac{1}{2}}{}^{-\frac{1}{2},1}\varphi_{1s\frac{1}{2}}{}^{\frac{1}{2},0}\cdots\}, \\ &\Psi_{II}(011)=2^{-\frac{1}{2}}\{\varphi_{1p\frac{1}{2}}{}^{-\frac{1}{2},1}\varphi_{1p\frac{1}{2}}{}^{\frac{1}{2},1}\}, \\ &\Psi(011)=\Psi_I(000)\Psi_{II}(011). \end{aligned} \tag{36}$$

The states of $T_\xi=0$, -1 are easily generated by applying the lowering operator $T_\xi-iT_\eta$.

The states (B) are formed by exciting a nucleon from a filled shell state $\varphi_{nlj}{}^m$ to a state $\psi_{nlj}{}^m=\varphi_{n+1lj}{}^m$.

$$\Psi_I{}^{nlj}(011)=(12!)^{-\frac{1}{2}}(2j+1)^{-\frac{1}{2}} \times\sum_{m=-j}^{j}\{\cdots\varphi_{nlj}{}^{m-1,1}\psi_{nlj}{}^{m,1}\varphi_{nlj}{}^{m,1}\varphi_{nlj}{}^{m+1,0}\cdots\}. \tag{37}$$

The nuclear state $\psi^{nlj}(010)$ is formed by vector coupling $\psi_I{}^{nlj}$ and $\psi_{II}(01T_\xi)$ to form a $T=0$ state. The states (D) $\psi^{nlj}(02T_\xi)$ can be formed by vector coupling the same two states to $T=2$ states.

The states (E) $\Psi^{nlj}(000)$ are easily formed by coupling $\psi_{\mathrm{I}}(000)$ to the state $\Psi_{\mathrm{II}}(100)$ given by the equation

$$\Psi_{\mathrm{II}}(000)=\tfrac{1}{2}[\{\varphi_{1p\frac{1}{2}}{}^{-\frac{1}{2},0}\psi_{1p\frac{1}{2}}{}^{\frac{1}{2},1}\}-\{\varphi_{1p\frac{1}{2}}{}^{-\frac{1}{2},1}\psi_{1p\frac{1}{2}}{}^{\frac{1}{2},0}\}-\{\varphi_{1p\frac{1}{2}}{}^{\frac{1}{2},0}\psi_{1p\frac{1}{2}}{}^{-\frac{1}{2},1}\}+\{\varphi_{1p\frac{1}{2}}{}^{\frac{1}{2},1}\psi_{1p\frac{1}{2}}{}^{-\frac{1}{2},0}\}]. \quad (38)$$

To calculate the states for $A=34$ we follow the same procedure as above. The states $\psi_{\mathrm{II}}(011)$ and $\psi_{\mathrm{II}}(000)$ have a different form for a $(1d_{\frac{3}{2}})^2$ configuration.

$$\Psi_{\mathrm{II}}(011)=\tfrac{1}{2}[\{\varphi_{1d\frac{3}{2}}{}^{-\frac{1}{2},1}\varphi_{1d\frac{3}{2}}{}^{\frac{1}{2},1}\}-\{\varphi_{1d\frac{3}{2}}{}^{-\frac{3}{2},1}\varphi_{1d\frac{3}{2}}{}^{\frac{3}{2},1}\}], \quad (39)$$

$$\Psi_{\mathrm{II}}(000)=8^{-\frac{1}{2}}[\{\varphi^{-\frac{1}{2},0}\psi^{\frac{1}{2},1}\}-\{\varphi^{-\frac{1}{2},1}\psi^{\frac{1}{2},0}\}-\{\varphi^{\frac{1}{2},0}\psi^{-\frac{1}{2},1}\}+\{\varphi^{\frac{1}{2},1}\psi^{-\frac{1}{2},0}\}-\{\varphi^{-\frac{3}{2},0}\psi^{\frac{3}{2},1}\}+\{\varphi^{-\frac{3}{2}}\psi^{\frac{3}{2},0}\}+\{\varphi^{\frac{3}{2},0}\psi^{-\frac{3}{2},1}\}-\{\varphi^{\frac{3}{2},1}\psi^{-\frac{3}{2},0}\}]. \quad (40)$$

The subscript $1d_{\frac{3}{2}}$ has been omitted from the single-particle wave functions in writing this equation.

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Decay of Er^{171}†

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The decay of Er^{171} (7.52 hours) has been studied with beta- and gamma-scintillation spectrometers, a magnetic lens spectrometer, and a 180° permanent-magnet spectrograph. A number of previously unobserved beta-ray and gamma-ray transitions were found. On the basis of coincidence studies, intensity data, internal conversion coefficients, and the measured transition energies, a consistent level scheme for Tm^{171} is proposed which has excited states at energies of 0.0051, 0.1167, 0.1291, 0.4251, 0.636, 0.688, 0.744, 0.921, and 1.008 Mev. The observed states are identified with appropriate orbitals of the Nilsson energy level diagram, and the experimental transition probabilities are examined in terms of the asymptotic selection rules for strongly deformed nuclei.

I. INTRODUCTION

ERBIUM-171 decays by negatron emission to energy levels of the odd-proton nuclide Tm^{171}. The ground state of Tm^{171} is beta unstable ($T_{\frac{1}{2}}=1.9$ yr), and there is at present no practical way of studying the excited states of this nucleus except by examining the radiations of Er^{171}. Several such studies have been reported, the most recent being that of Hatch and Boehm.[1] Their precision measurements confirmed, among other things, that the four lowest states of Tm^{171} form a $K=\frac{1}{2}$ anomalous rotational band, a conclusion which had been reached by earlier investigators[2,3] on the basis of less complete data. In all of the above studies, however, information was obtained only on the relatively low-energy (<0.5-Mev) transitions in Tm^{171}. From the estimated Er^{171} decay energy of ~1.5 Mev, it was evident that a further search for higher energy γ-ray transitions should be made.

In the present investigation, it has been established that the decay scheme of Er^{171} is much more complicated than previously proposed and involves a number of γ-ray transitions with energies >0.5 Mev. Although the observed energy levels of Tm^{171} above 0.5 Mev are quite weakly populated, sufficient data on these states have been obtained to make possible various tests of the predictions of current nuclear theory. In particular, certain features of the decay scheme can best be explained as resulting from operation of selection rules involving the asymptotic quantum numbers[4] used in describing the states of strongly deformed nuclei.

II. SOURCE PREPARATION

The Er^{171} source material was produced by thermal neutron bombardment of Er_2O_3 of natural isotopic abundance.[5] The only observed radiation from isotopes of erbium other than Er^{171} was the 0.33-Mev beta group of Er^{169} (9.4 day). A slight Dy^{165} contamination was detectable immediately after irradiation, but this activity did not interfere with the Er^{171} measurements because of its very different half-life (2.36 hr). Further chemical purification of the irradiated source material in an ion-exchange column failed to reveal the presence of any other contaminants; consequently, in most of the measurements described below, the Er_2O_3 was not purified after irradiation.

Sources for the lens spectrometer were mounted on Mylar backing 0.00025 inch thick. The source material was deposited on this backing by evaporating to dryness a drop of Zapon which had a small amount of irradiated Er_2O_3 in suspension.

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[1] E. N. Hatch and F. Boehm, Phys. Rev. **108**, 113 (1957).

[2] S. D. Koički and A. M. Koički, Bull. Inst. Nuclear Sci., "Boris Kidrich" (Belgrade) **6**, 1 (1956).

[3] S. A. E. Johansson, Phys. Rev. **105**, 189 (1957).

[4] S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **29**, No. 16 (1955).

[5] The Er_2O_3, of specified purity 99.9%, was obtained from F. H. Spedding, Iowa State College, Ames, Iowa.