Neutron Production in Linear Deuterium Pinches*†

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Approximately 10^8 neutrons per discharge have been observed from magnetically self-constricted columns of deuterium plasma. These neutrons originated uniformly and simultaneously along a filament at the center of the discharge tube, but measurements of their kinetic energies showed that they were produced by a small group of deuterons with large axial velocities and were therefore not of thermonuclear origin. It is proposed that the deuterons were accelerated by axial electric fields created by the growth of $m=0$ ("sausage-type") instabilities.

I. INTRODUCTION

WORK with the "pinch effect" at the University of California Radiation Laboratory at Berkeley started in April, 1948 with equipment intended as a focusing device for the cyclotron beam.[1] The constriction of the gaseous discharge with high currents was noted and the importance of a symmetrical current return path determined. When Project Sherwood‡ was started, the information obtained from this experiment was applied to the problem of producing controlled thermonuclear energy, and the measurements described here were made in 1954.§ Because it was possible to show that this particular line of research could not lead to the useful production of power we abandoned the approach; the data presented here are, therefore, of a more qualitative nature than is desirable for a careful comparison between theory and experiment.

Several inert gases were used in the earliest studies, with particular emphasis upon helium because of its low atomic weight and the usefulness of the 4686 A ionized helium line for diagnostic purposes. However, the detection of neutrons from *d-d* reactions was recognized as a powerful tool for optimizing the operating conditions; therefore, as soon as there was spectroscopic evidence that possibly useful ion temperatures were being produced in helium discharges, the gas was changed to deuterium. Neutrons were detected and the average number per discharge was increased rapidly by adjusting the operating parameters.

A number of experiments was performed which gave the origin of the neutrons in space and time, and the effects of variations in circuit parameters, gas pressure, and impurities in the gas. Most of the measurements seemed consistent with a thermonuclear reaction at the time, except that the neutron yield rose to many orders of magnitude larger than our simple calculations predicted. In the end we were able to show by measurements of the neutron energies that the reactions did not occur in a high-temperature plasma, but rather that approximately 10^{14} deuterons per pulse were accelerated along the axis of the discharge tube to energies of about 2×10^5 electron volts, probably in very strong electric fields produced transiently by $m=0$ (sausage-type) instabilities.[2] In our later discharge tubes, approximately 10^8 neutrons were produced in each pulse. The yields presumably could be increased, but it can be shown that devices using directed beams of deuterons can yield no net power.[3]

II. THEORY OF THE DYNAMIC PINCH

The simple theory of the pinch mechanism has been widely discussed,[4–10] therefore in this section only a brief summary of some of the ideas is given, together with a derivation for the particular case in which the behavior is describable by shock hydrodynamics and the current through the gas is governed by parameters external to the pinch tube. When a longitudinal electric field is applied to a cylinder of low-pressure gas (a few hundred microns in our case), the gas is rapidly ionized to the point where it can be described as a good conductor. The rapidly increasing longitudinal current then flows mainly in a thin layer at the surface of the plasma and when the current becomes sufficiently large that the magnetic field pressure, $B^2/8\pi$, exceeds the gas pressure, NkT, the column collapses radially. A shock wave preceding the plasma boundary is reflected at the axis and reverses the direction of motion of the current

* This work was done under the auspices of the U. S. Atomic Energy Commission.

† A brief account of this and other related work has been published [O. A. Anderson *et al.*, Phys. Rev. **109**, 612 (1958)].

[1] W. K. H. Panofsky and W. R. Baker, Rev. Sci. Instr. **21**, 445 (1950).

‡ Project Sherwood is the code name given to the U. S. Atomic Energy Commission supported program aimed toward achieving controlled fusion for peacetime purposes.

§ The first dynamic pinch experiments in Berkeley were made for this purpose in 1952. The series of measurements described here began in late 1954 and continued through the following year.

[2] M. Kruskal and M. Schwarzchild, Proc. Roy. Soc. (London) **A223**, 348 (1954).

[3] R. F. Post, Revs. Modern Phys. **28**, 338 (1956). This article contains many other references.

[4] Burkhardt, Dunaway, Mather, Phillips, Sawyer, Stratton, Stovall, and Tuck, J. Appl. Phys. **28**, 519 (1957). Many recent references are given in this report.

[5] W. H. Bennett, Phys. Rev. **45**, 890 (1934).

[6] L. Tonks and W. Allis, Phys. Rev. **56**, 360 (1939).

[7] S. W. Cousins and A. A. Ware, Proc. Phys. Soc. (London) **A64**, 159 (1951).

[8] I. V. Kurchatov, Nucleonics **14**, No. 6, 36 (1956).

[9] M. A. Leontovich and S. M. Osovets, Atomnaya Energ. **1**, No. 3, 81 (1956) [translation J. Nuclear Energy **4**, 209 (1957)].

[10] J. A. Tuck, Gaseous Electronics Conference, Pittsburgh, October, 1956 (unpublished).

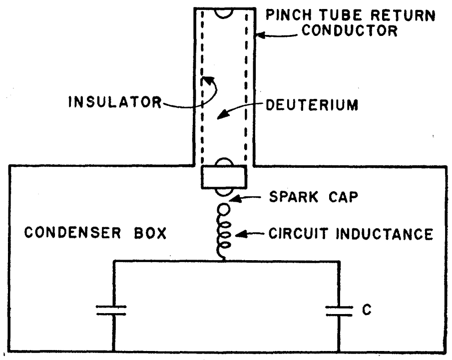

FIG. 1. Pinch circuit schematic diagram.

sheath.|| The plasma may contract and expand several times before instabilities destroy the ordered motion.

The original treatment of the radial motion of the pinch under the assumption of a perfectly conducting boundary was made by Rosenbluth, Garwin, and Rosenbluth for the particular case of constant voltage across the pinch.[11] The experimental conditions, unfortunately, are more frequently met by the assumption of a sinusoidal current that is primarily determined by an unavoidable, relatively large inductance in series with the pinch (Fig. 1).

This large external inductance essentially decouples the circuit equation, namely current *versus* time, from the hydrodynamic equation of pinch radius *versus* time, so that a relatively simple description of the pinch behavior results. The approximation of the hydrodynamic equation that fortunately fits the experimental conditions is that describing snowplow shocks. The snowplow model assumes that all the mass swept up by the shock is compressed in an infinitely thin layer immediately behind the shock so that the "piston" and the shock are the same interface. This model is probably a very good one because the neutral fraction inside the initial current-boundary layer may be quite large. This neutral fraction is ionized and pinched by the current layer. (For an initial density greater than 5×10^{15} the current layer is thick compared with neutral-ion scattering and thick compared with the neutral ionization time multiplied by the layer velocity.)

The contracting current sheath in the pinch corresponds to a shock of Mach 10 to 60, so that the hydrodynamics is always described by the "strong shock" limit. The characteristic limiting compression behind a plane parallel "strong shock," for example, is $\eta=(\gamma+1)/(\gamma-1)$, where γ is the usual ratio of specific heats. For a simple gas of elementary particles with three degrees of freedom, γ is 5/3 and the limiting compression is 4, which is a poor approximation to the snowplow limit; however, for a Mach-20 shock (10-ev temperature) progressing through cold deuterium, the specific heat of molecular breakup and the subsequent ionization are so large that the additional degrees of freedom result in a γ that closely approaches unity, with a consequent increase in the limiting compression. For monatomic argon, a shock compression of 8 has been observed for shocks strong enough to lead to a large fraction of ionization, and therefore it is reasonable to assume that with still stronger shocks in deuterium, where the kinetic-temperature energy per molecule equals the ionization binding energy, limiting compressions of perhaps 8 to 10 can be reached. This is just the limit for which the snowplow approximation is valid.

We shall derive an expression for the time required for the first radial compression according to the approximation that the discharge current varies sinusoidally with time. The equation of motion is based upon the principle that the time rate of change of momentum equals the force,

$$F_r=\frac{d}{dt}\left(M\frac{dr}{dt}\right). \tag{1}$$

Assume that the gas originally fills a cylinder of radius R, and that at some later time t the thin shell of current-carrying ionized material is at radius r, moving toward the axis of the cylinder and sweeping up the gas as it goes. The force on an element of area of the shell that is 1 centimeter high and $rd\theta$ wide is

$$F=-\frac{B^2}{8\pi}rd\theta=-\frac{I^2}{8\pi25r^2}rd\theta=-\frac{I^2d\theta}{200\pi r}, \tag{2}$$

where B is the magnetic field in gauss produced by the total current of I amperes. The mass in the shell is

$$M=\pi\rho(R^2-r^2)(d\theta/2\pi), \tag{3}$$

where $\rho=$ the initial density in g/cm^3. Then we have

$$-\frac{I^2}{200\pi r}=\frac{d}{dt}\left[\pi\rho(R^2-r^2)\frac{1}{2\pi}\frac{dr}{dt}\right]. \tag{4}$$

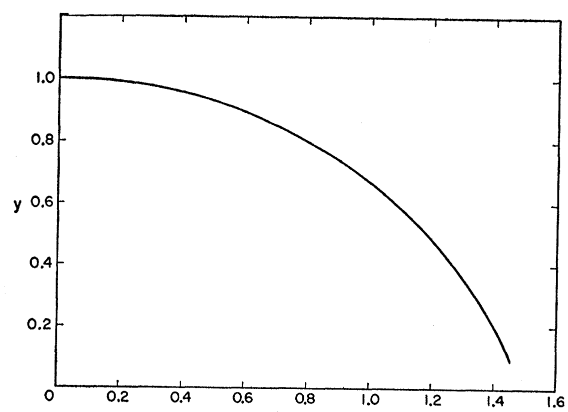

FIG. 2. Snowplow shock model solution [see Eq. (6)].

|| The time at which this occurs is called the "bounce" time throughout this paper.

[11] M. Rosenbluth, with R. Garwin and A. Rosenbluth, Los Alamos Report LA-1850, September, 1954 (unpublished).

In general, the first contraction of the pinch occurs in a time short enough that $I=I_0 \sin\omega t$ can be replaced by $I=I_0\omega t$:

$$\frac{d}{dt}\left[\frac{\rho}{2}(R^2-r^2)\frac{dr}{dt}\right]=-\frac{I_0^2\omega^2 t^2}{200\pi r}. \quad (5)$$

Let us define τ and y by

$$t=\left[\frac{I_0^2\omega^2}{100\pi\rho R^4}\right]^{-\frac{1}{4}}\tau, \quad (6)$$

and

$$r=yR:$$

Eq. (5) then reduces to

$$\frac{d}{d\tau}\left[(1-y^2)\frac{dy}{d\tau}\right]=-\frac{\tau^2}{y}. \quad (7)$$

The solution of this equation is shown in Fig. 2 for the boundary condition $y=1$ and $dy/d\tau=0$, when $\tau=0$. The minimum radius is $y=0.125$ if we assume that the probable fractional radius is $\frac{1}{8}$ of the full radius at first bounce time. The actual time to first bounce therefore is

$$t=1.43R\left[\frac{100\pi\rho}{I_0^2\omega^2}\right]=\frac{1.43R}{\omega}\left[\frac{100\pi\rho}{C^2V^2}\right]^{\frac{1}{4}} \text{ second}, \quad (8)$$

where V is the original voltage on the condenser of capacitance C.

This indicates that the pinch time scales as $\rho^{\frac{1}{4}}$ provided the pinch occurs in a time short compared with the quarter-cycle time of the circuit. As an example, for $\omega=0.516\times10^6$ sec^{-1}, $V=50\,000$ volts, $C=12\times10^{-6}$ farad, $R=3.8$ cm, and $\rho=3\times10^{-8}$ g/cm^3 at 170 microns pressure of D_2, we found the experimental time for the first compression: $t=0.68\times10^{-6}$ sec. The value given by Eq. (8) is 0.74×10^{-6} sec, which is in excellent agreement considering the simplifying assumptions used. This $\rho^{\frac{1}{4}}$ dependence was observed for deuterium pressures from 50 microns to 50 000 microns, giving strong support to the argument that nearly 100% of the gas is compressed. If some modest fraction of the initial deuterium were involved in the pinch at any particular pressure, it is unlikely that the same fraction would be maintained over such a wide pressure variation.

Next we attempt to place an upper limit on the number of neutrons that could be produced in one of these pinches from thermonuclear reactions. One-half of the reactions produce neutrons, so that the number of neutrons is $n=(\frac{1}{2}\times\frac{1}{2})N^2\langle\sigma v\rangle_{\mathrm{Av}}\times$(volume)$\times$(reaction time). It can be shown that the thermal relaxation times are short enough that the ions will be thermalized within the bounce time, and we can use the Maxwellian distribution values[3] of $\langle\sigma v\rangle_{\mathrm{Av}}$ given in Fig. 3. From an examination of the oscilloscope traces discussed in the next section it can be estimated that the pinch stays assembled for something like 4×10^{-8} second.

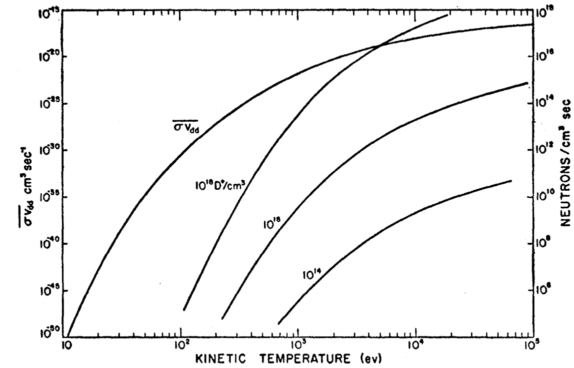

FIG. 3. $\langle\sigma v_{dd}\rangle_{\mathrm{Av}}$ for a Maxwellian distribution [calculated from Eq. (7) of reference 3], and neutrons per cm^3 *vs* density and temperature.

The minimum radius achieved during the snowplow shock is crucial to the yield calculation, but we have been unable to derive it theoretically in a satisfactory way. An argument was given earlier that a radial compression of 8 to 10 seemed likely; a similar conclusion can be reached from a consideration of the finite thickness of the snowplow sheath as measured with small probes that yield the magnetic fields and hence the current densities as functions of time and radius. Plausible results can also be obtained by an analysis of the inductance of the circuit from voltage signals, but these are of low precision. In the following discussion we assume a radial compression of 8 at the time of the first bounce.

The temperature of the pinched plasma is estimated from the radial speed and the relation whereby the mean energy after thermalization is given by

$$\bar{W}=\tfrac{3}{2}kT. \quad (9)$$

The radial velocity at the time of the first bounce is

$$\frac{dr}{dt}=R\left[\frac{I_0^2\omega^2}{100\pi\rho R^4}\right]^{\frac{1}{4}}\frac{dy}{d\tau}=\omega\left[\frac{C^2V^2}{100\pi\rho}\right]^{\frac{1}{4}}\frac{dy}{d\tau}, \quad (10)$$

and solution of Eq. (7) gives $(dy/d\tau)|_{y=0.125}=2.0$, so that for our numerical example we have

$$dr/dt=7.23\times10^6 dy/d\tau=1.45\times10^7 \text{ cm/sec}. \quad (11)$$

The directed kinetic energy of a deuteron is then obtained from $KE=\frac{1}{2}M_d(dr/dt)^2$. This energy calculation is not as accurate as the timing information because the maximum transfer from magnetic energy to particle energy occurs at small radius if the current is large, and in this case the variable inductance of the pinch becomes larger than the series inductance. For the experimental values in our example: $dr/dt=-1.45\times10^7$ cm/sec yields directed energy of 215 ev and a temperature of ≈140 ev. The electrons will have received little radial energy. This is an upper limit to the temperature because we have assumed that the current is linear with time, whereas the pinch inductance causes the current to dip at about the time of each bounce; the correct calculation would yield a value perhaps 25% lower. However, the temperature at the second bounce should be increased by an amount roughly proportional to the current increase, or about 20%, and the third bounce (at which time neutrons were usually observed) would be slightly hotter.

In addition, the particles in the strong snowplow shock have energy in the moving system because of the inelastic nature of collisions with the snowplow, which is approximately equally shared between ions and electrons. An additional ion energy of 15 ev is attributable to these inelastic collisions.

Finally, the effect of Joule heating must be included. From the known current and radius as functions of time, the effective thickness of the current sheath [about 0.8 cm (experimentally measured at Livermore)], and Spitzer's formula for resistivity with a transverse magnetic field,[12] an electron temperature of 20 ev is calculated for first bounce time.

During the first compression the electrons are heated by ion-electron collisions so that at the end of the first compression the electrons and ions each have temperatures of about 90 ev, although the ion-energy distribution is by no means Maxwellian. By the third compression the mean temperature has been raised perhaps 20%, as previously described, and the high-energy part of the Maxwell distribution has been fairly well populated. Because no heat losses have been included, an ion temperature of about 110 ev can be considered an upper limit.

The neutron yield from a tube 45 cm long, for example, can now be estimated:

$$N=\tfrac{1}{4}n^2\langle\sigma v\rangle_{\mathrm{Av}}\pi r^2 l\Delta t\approx 0.3. \tag{12}$$

Because the yield is approximately proportional to the twelfth power of the ion temperature in the energy region that we are considering, the above calculation may well be in error by an order of magnitude or more. However, a factor of 10^8 would be necessary to explain the experimental results, corresponding to an ion temperature of about 1 kilovolt.

We shall ascribe the neutron production to an instability; therefore a brief comment on such phenomena is in order. The instabilities of a pinch have been examined theoretically by Kruskal and Schwarzchild[2] and documented experimentally by Cousins and Ware,[7] Bostick, Combes, and Levine,[13] and Burkhardt *et al.*[4] The theoretical treatment considers the first-order perturbations of a sharp plasma–magnetic-field interface in radial geometry, and it is found that an initial random noise level of perturbation is described by the exponential growth of the amplitude of each of a set of normal modes. The normal modes are described by the axial wave number k and the azimuthal quantum number m such that any perturbation can be written as a Fourier sum $\sum_{m,k} A_{m,k}\exp(im\theta+ikz)$, and the exponential growth of the amplitude ξ of any given mode is described by $\xi=\xi_0\exp(im\theta+ikz+\omega t)$, where ω is determined by the dispersion relation. The mode associated with $m=0$ is the "sausage" or "necking-off" type of instability, while $m=1$ describes the kink-type instability. Higher m modes describe various degrees of fluting. The breakup of the pinch after the second or third bounce is usually ascribed to the very rapid nonlinear large-amplitude growth rate of the $m=0$ sausage-type instability necking off the pinch. The growth time is of the order of the radius divided by sound speed—hence the few bounces before breakup.

III. EXPERIMENTAL PROCEDURE

A. Apparatus

The pinch circuit was designed to have minimum inductance external to the pinch tube in order that the voltage applied to the pinch tube and the current growth would be maximized. There were two exceptions to this rule: only easily available condensers were used, and the pinch tubes were mounted outside the condenser box so that measuring apparatus could easily be placed

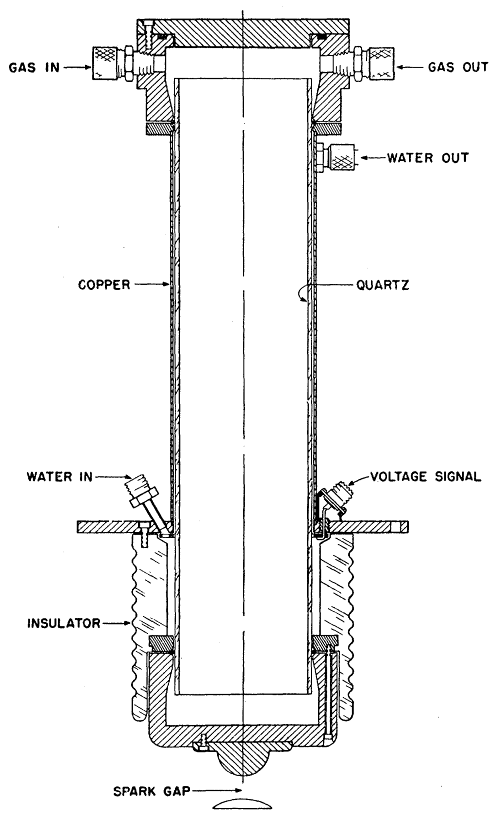

FIG. 4. Cross section of a pinch tube.

[12] Lyman Spitzer, Jr., *Physics of Fully Ionized Gases* (Interscience Publishers, Inc., New York, 1956).

[13] Bostick, Combes, and Levine, Investigations of the Properties of the Pinch Effect in an Ionized Gas Carrying a High Current, Tufts University, Medford, Massachusetts, Research Laboratory of Physical Electronics (January, 1956).

near the regions of interest. The arrangement is shown schematically in Fig. 1. The condenser bank contained 25 Cornell-Dubilier 0.5-microfarad 50-kilovolt condensers in parallel. The condenser circuit and spark gap had a total inductance of 2×10^{-7} henry.

Approximately thirty pinch tubes were built in connection with the experiments described in this paper. Figure 4 is a cross-sectional view of one of these tubes. The voltage across the tube was monitored by using the cooling water as a voltage divider, and the current was obtained from noninductive shunts. Pure gases were flushed through the system continuously to reduce the contaminant problem.

Typical data are shown in the tracings of Fig. 5. The curve labeled "discharge radius" is not quantitatively correct, but the behavior is inferred from an analysis of the voltage wave form. The voltage signal is $LdI/dt + IdL/dt$ plus a small resistive drop, and may be interpreted as follows:

A. The full condenser voltage is across the pinch tube before appreciable ionization takes place.

B. The gas is ionized and $V\approx LdI/dt+IR$.

C and D. The current sheath is moving toward the axis of the tube and the increase in voltage is given principally by IdL/dt.

E. The shock wave that preceded the boundary of the plasma has been reflected at the center of the tube and has reversed the direction of motion of the current sheath, changing the sign of the IdL/dt term.

F. IdL/dt is still negative, but the outward speed of the sheath is decreasing.

G. The sheath has made its maximum radial excursion and is about to move toward the axis once more.

H through L. Records of three more compressions. In some cases the peak voltages are many times the voltage originally applied to the tube. At times later than L (or sometimes K) the voltage trace consists of a very-high-frequency signal lasting for approximately 10^{-7} second, following which all detail disappears. Either

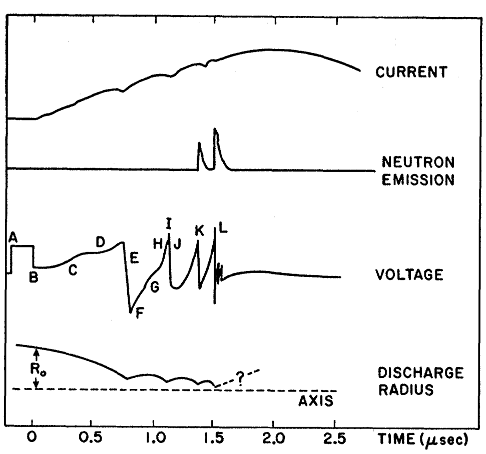

FIG. 5. Pinch signals. (Interpretation is given in text.)

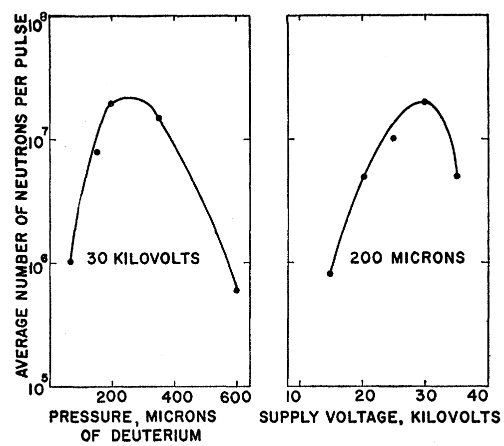

FIG. 6. Neutron yields *vs* voltage and pressure in 5-by 45-cm quartz tube.

instabilities fill the whole tube with plasma at this time or the insulator becomes conducting; in either case the interior of the pinch tube is isolated from the external circuits.

Pyrex tubing was used originally, but an examination of the voltage signals showed that the plasma radius remained constant for about $\frac{1}{2}$ microsecond longer than expected, possibly indicating that a considerable amount of material was being evolved from the insulator surface. When quartz was used the current sheath left the wall almost immediately and much more detail was visible on the voltage trace. With other parameters unchanged, 100 times as many neutrons were produced with the quartz insulators as with Pyrex.

The absolute numbers of neutrons per pulse were obtained with a long boron counter and calibrated Po-Be source, but for day-to-day use a lead-shielded, europium-activated lithium iodide crystal and photomultiplier assembly surrounded by 5 cm of paraffin was more convenient. The timing of the neutron bursts was obtained from proton recoils in small volumes of plastic scintillator. Several plastic scintillators were used to compare the timing and relative numbers of the neutrons from different parts of a discharge. The spatial origin of the neutrons was determined with plastic scintillators which were either imbedded in blocks of paraffin containing channels or hidden in the shadow of a scattering material. Neutron and gamma-ray energies were measured with nuclear emulsions.

IV. NEUTRON MEASUREMENTS PERTAINING TO THE PRODUCTION MECHANISM

The experimental observation of neutrons from a linear pinch preceded the theoretical analysis so that the yield per pulse, for example, although surprisingly large, did not appear to rule out a thermonuclear origin. In general, the early experiments seem to be reasonably consistent with thermonuclear production, whereas the later experiments conclusively excluded this possibility.

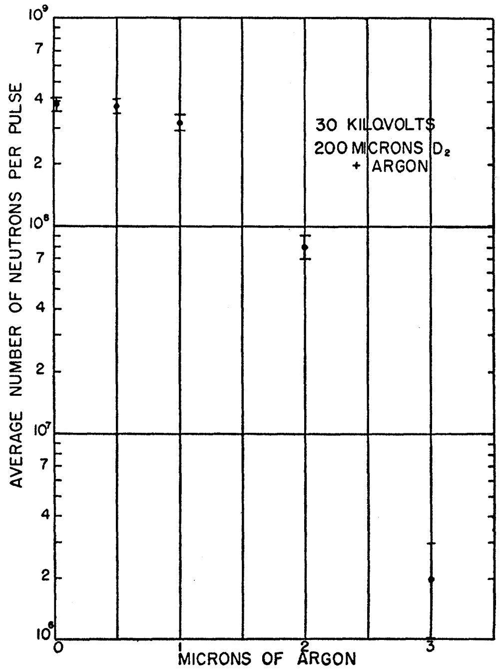

FIG. 7. Neutron yield from 5-by 90-cm quartz tube with 200-μ D_2+a partial pressure of argon.

A. Yield

The average number of neutrons per pulse was raised by several orders of magnitude during the course of these experiments by changing tube design and adjusting operating parameters, to a final volue of 4×10^8. The yields as functions of deuterium pressure and condenser voltage for a tube 45 cm long and 5.5 cm in diameter are shown in Fig. 6. The theory predicts an increase in yield with increasing condenser voltage; the decrease at high voltages is not understood. A theory that this might be due to poor ionization at high field strengths was apparently disproved by experiments in which the gas was preionized before the pinch current was applied, no change being observed. Larger-diameter pinch tubes gave maximum yields at higher voltages; e.g., quartz tubes 7.6 cm in diameter and 45 cm long had maxima at about 45 kilovolts.

B. Time of Neutron Emission

The neutron emission usually began at the time of the third maximum contraction of the plasma, although some tubes gave sizable yields on the second bounce. The shape of the emission curve is interesting in that it rose to a maximum in about 10^{-8} second and fell to zero in about 2×10^{-7} second. The emission at the time of maximum contraction and maximum energy in the pinch is consistent with a thermonuclear event, but the sudden rise in output probably can be explained only by invoking some sort of instability.

C. Quenching by Impurities and Axial Magnetic Fields

Impurities in the deuterium plasma could reduce the neutron output in several ways (to be discussed in Sec. V). The first experimental evidence pointing to the effects of impurities was the hundredfold increase in neutron production when the insulator was changed from Pyrex to quartz. Small percentages of other gases were then introduced; the effect of argon in our maximum-yield tube is shown in Fig. 7. The points shown are averages over many pulses, but the manner in which the shot-to-shot yield varied is also interesting. With pure deuterium the shot-to-shot variation was less than a factor of two. The individual pulse yields became increasingly erratic as argon was added; with 3 microns of argon, for example, a few of the pulses gave as many neutrons as from pure deuterium, while most of the pulse yields were down by a factor of one hundred or more. The effect of helium was much less pronounced, the output falling in approximately the same way as the number of deuterons in the plasma.

The yield was also considerably reduced when weak axial magnetic fields were applied to the pinch tube (Fig. 8). It will be shown later that this behavior is incompatible with a thermonuclear reaction unless the compressions are much higher than is thought possible.

D. Spatial Origin of the Neutrons

One of the first measurements was to determine whether neutrons were produced at one of the electrodes or along the axis. Lead-shielded plastic scintillator recoil

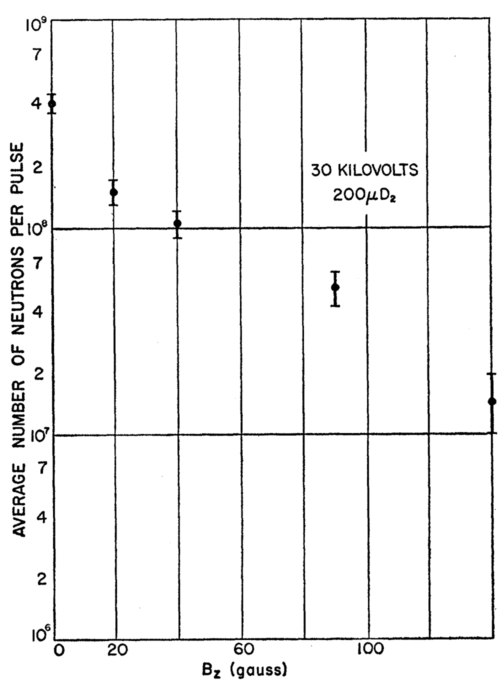

FIG. 8. Neutron yield from 5-by 90-cm quartz tube with 200-μ D_2 and an axial magnetic field.

detectors in paraffin collimators were moved along the axis while the total yield was monitored with a LiI crystal several feet away. In this manner it was shown that the output decreased slightly near the electrodes, excluding the bombardment of adsorbed deuterium as a production mechanism.

Because of the difficulty in collimating neutrons, measurements were also made on a long tube (90 cm by 5.5 cm) with small uncollimated plastic scintillator recoil detectors. The calculated relative pulse heights as the detector was moved along the tube axis agreed with the experimental points; the points were independent of the polarity of the applied voltage.

Experiments with collimators did not yield a precise determination of the radial extent of the neutron source, but it was easy to show that the neutrons did not originate at the walls, and in fact came from a cylinder not greater than 2 centimeters in diameter. It was then possible to proceed to a detector capable of better angular resolution, namely a plastic scintillator in the shadow of a thin wedge of scattering material (copper or tungsten), shown schematically in Fig. 9. The data, Fig. 10, indicated neutron production within a cylinder not more than 1 centimeter in diameter. Greater precision could not be obtained with a single detector because the constricted plasma was apparently displaced as much as 0.5 cm from the geometrical axis of the pinch tube in some of the discharges, perhaps because of kink instabilities.

E. Simultaneity of Neutron Emission Along the Tube Axis

Two recoil detectors were placed along the side of the 90 cm long tube, one of them at a fixed position as a

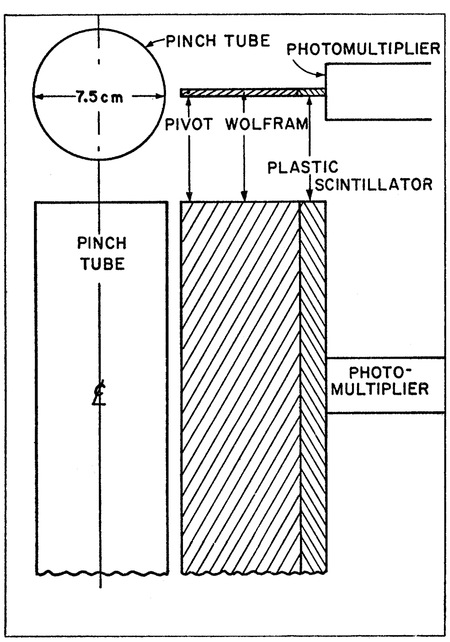

FIG. 9. Apparatus for radial neutron distribution measurement.

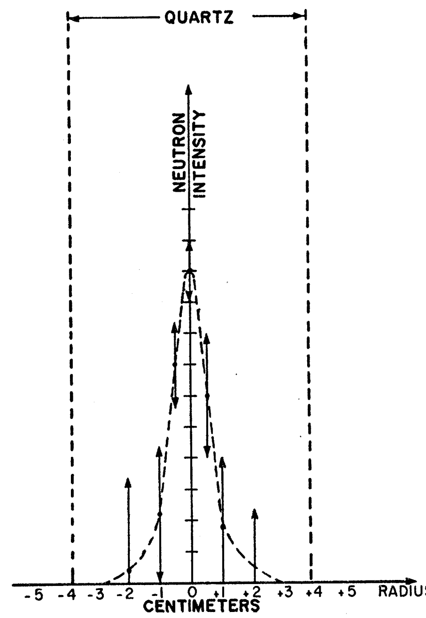

FIG. 10. Radial neutron distribution.

monitor and the other movable along the tube. The onsets of the neutron pulses were simultaneous to within 10^{-8} second at all points.

F. Neutron Energy Measurements

Although the rapid quenching of the neutron production by small amounts of impurities seemed to favor a thermonuclear reaction, the unexpectedly high yields forced a continued search for some other production mechanism. The voltage across the pinch tube at the time of neutron emission was about 10 kilovolts; if deuterons were accelerated to 10 or more kev in a particular direction, then a careful measurement of the neutron energies should disclose this fact. The energies expected from a directed deuteron beam striking a deuterium target are shown in Fig. 11.

Ilford C.2 emulsions were exposed at the sides and at 10 cm from each end of the pinch tube on the axis. An additional set was exposed with the applied voltage reversed in polarity. The tube used for these measurements contained a quartz insulator 46 cm long and 7.5 cm in diameter and was filled with 100 microns of deuterium. The condenser bank was charged to 35 kilovolts.

The recoil-proton histograms from neutrons emitted from the pinch tube at 0° and 180° to the direction of the applied voltage are shown in Fig. 12. Equivalent neutron and inferred deuteron energies are also shown. The spectra may be interpreted as due to 50- to 100-kilovolt deuterons directed along the tube axis which bombard deuterons at rest. In some of the cases the energies must have been at least 200 kev. From the 180° data it is clear that the thermonuclear contribution, if

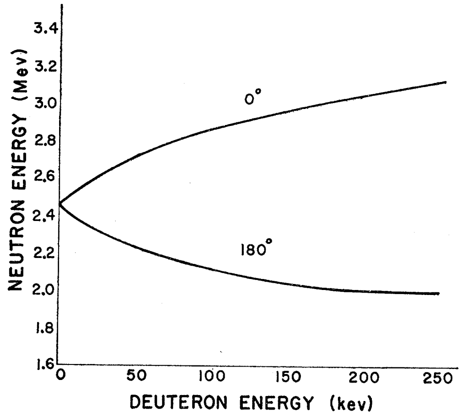

FIG. 11. Neutron energies from *d-d* reactions as a function of incident deuteron energy.

any, must have been quite small. The technique was checked by bombarding a deuterium-loaded target with deuterons from an accelerator.

G. High-Energy Electron Search

It seemed possible that under certain conditions high-energy electrons might be produced by the same mechanism as accelerated the deuterons. Newly poured Ilford G.5 electron-sensitive emulsions were exposed at the ends of the discharge tube with a thin electrode and scanned for electrons of 100 or more kilovolts. Some electrons of many Mev were detected in numbers statistically above background, but it is possible that the effect was spurious, and in any event the number of electrons accelerated to such energies was very small.

V. INTERPRETATION

The results of the experiment in which the neutrons from the dynamic pinch were observed are inconsistent with a thermonuclear origin. The proposed explanation of the pinch neutron phenomena that will be discussed here is dependent upon the formation of the $m=0$, or sausage-type, pinch instability mode. The rapid dynamic growth of this instability mode results in a rapid change of inductance of the pinch. The axial electric field across each instability resulting from this change in inductance accelerates a small group of deuterons to approximately 50 kev per instability. The collisions of these deuterons with the remainder of the deuterons of the pinch give rise to the observed neutron production.

The emphasis upon deciding for or against a possible thermonuclear origin is based upon the recognized principle that, in general, useful power cannot be obtained from a fusion reaction unless the ions are in approximately thermal equilibrium with one another. In other words, an accelerated deuterium or tritium "beam" incident upon a target cannot produce useful thermonuclear power because the irreversible energy loss of slowing down in the target is always greater than the reaction energy produced.[3] Even though these pinch experiments produce a relatively large number of neutrons, unless these neutrons were from a thermonuclear origin there would be no hope of extending the yield to power-producing value. It is therefore worth while to consider the evidence that argues against a thermonuclear origin.

(a) The first and primary evidence is the energy distribution of the recoil-proton tracks in photographic emulsion exposed at either end and at the sides of the linear pinch tube. These showed that in one axial direction the neutrons causing the recoils were of a higher mean energy than would come from a deuteron collision whose combined center of mass was at rest relative to the emulsion plates. The interpretation of this center-of-mass velocity is that a deuteron of mean axial kinetic energy of 50 kev strikes a deuteron at rest. The resulting nuclear reaction occurs with a moving center of mass, so that neutrons emitted in the forward direction have a higher energy than those directed backward.

The observed neutron-energy distribution corresponded to deuterons with a mean axial energy of 50 kev with a half-width of ± 50 kev. The neutrons originate from a small class of deuterons that are accelerated axially to a high velocity in the direction that would correspond to the applied electric field.

(b) A weak axial magnetic field of 50 to 100 gauss quenches the neutron production. The dynamics of the pinch[11] would indicate that a compression of 50 might be expected on the first bounce, increasing to 100 on the second bounce, which in turn agrees with density measurements made on a helium pinch by observing the Stark broadening of spectral lines. With a compression of 100, an initial axial field of 100 gauss trapped inside the pinch would be increased to 10 000 gauss, which is a trivial pressure compared with the pinch field of approximately 100 000 gauss. The effect of an internal axial field on a thermonuclear pinch would be dependent upon the internal H_z pressure relative to the external H_θ pressure. For these to be comparable we solve the equilibrium equation:

$$\begin{aligned} &H_z = H_\theta, \\ &H_z = R^2 H_{z0}/r^2, \quad H_{z0} = 100 \text{ gauss}, \\ &H_\theta = I/5r, \qquad I = 150\,000 \text{ amp}. \end{aligned} \tag{13}$$

Therefore

$$(R^2/r^2)100 = 150\,000/5r, \quad R = 3.75 \text{ cm}; \tag{14}$$

therefore

$$r = 0.047 \text{ cm}.$$

This is a considerably smaller radius than would be expected owing to the limitation of shock phenomena. It implies a compression of 6500, which is two orders of magnitude larger than measured for a helium pinch, and similarly unlikely for a deuterium pinch. In other words, the tenfold quenching of the neutron production by such a small amount of axial field suggests that the production phenomenon is critically associated with radii less than 0.05 cm.

(c) The neutron yield does not increase with increasing voltage as rapidly as would be expected of a thermonuclear process. The yield curve *versus* applied voltage actually decreases slightly at the higher voltages. This could be interpreted in terms of poorer initial ionization with resulting impaired pinch dynamics; however, the timing of the dynamic bounces agrees with the scaling from much lower voltages so that the ionization process must be sufficiently complete to trap the major fraction of the gas. The higher voltage gives an observed shorter bounce time and therefore should give a higher temperature. The fact that an actual decrease in yield was observed as the voltage was raised is inconsistent with a thermonuclear origin.

(d) The neutron yield is much greater than would be expected from a simple analysis of the dynamic pinch behavior.

The remaining experimental evidence supports a thermonuclear origin and must also be consistent with any other theory of neutron production.

(e) The quenching effect of impurities in a thermonuclear pinch should be proportional to the specific heat of the impurity relative to the thermal energy of single particles. The radiated bremsstrahlung power in these short times (0.1 μsec) is trivial. The specific heat of an impurity of atomic number Z is comprised of the energy needed to strip approximately Z electrons and give them each kT energy. Therefore the specific heat of an impurity relative to a deuteron atom is approximately $(2ZkT)/(2kT)=Z$. Therefore an argon atom should add 20 times the specific heat of a deuterium atom, so that 1% impurity of argon should cause a 20% effect on temperature. If the temperature were high enough to give the observed yield thermonuclearly, then a 20% change in temperature would be a 100% change in yield—in approximate agreement with an observed quenching factor of 4.

(f) Shot-to-shot variation of the neutron burst usually occurs at the third dynamic bounce of the pinch, at which time the shock should have reached the maximum temperature. (The radius-*versus*-time behavior of the pinch can be obtained qualitatively from the inductance behavior of the pinch as a circuit element.)

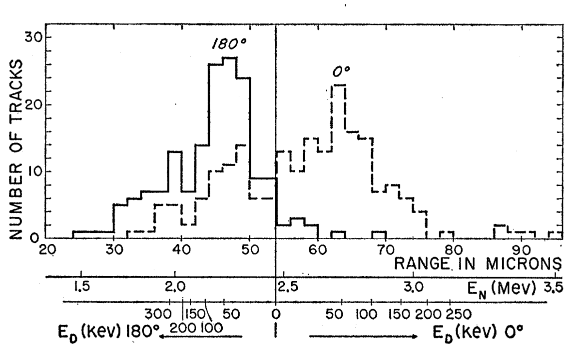

FIG. 12. Recoil proton histograms from $D(d,n)He^3$ neutrons emitted at 0° and 180° to the applied electric field.

(g) The initial voltage applied across the tube is 50 kilovolts or less, and at the time of neutron production is measured as between 10 and 20 kilovolts.

(h) The neutrons are produced uniformly along the axial length of the tube within a statistical magnitude of $\pm 15\%$ and with an axial resolution of $\pm 10\%$. This excludes the possibility of the neutrons' being generated by deuteron bombardment of deuterium occluded on one or both electrode surfaces.

(i) The radial distribution of neutron production agrees with a radially centered distribution of full width at half maximum of 1 cm diameter or less. The quartz tube diameter is 7.5 cm.

(j) The neutrons are produced simultaneously along the axial length of the tube to within 5% of the transit time of a 50-kev deuteron from one end of the tube to the other. This excludes the possibility of deuterons being accelerated by a sheath drop at one electrode, and traveling focused within the pinch, making collisions along the full axial length.

The proposed mechanism for neutron production depends upon the instability breakup of the pinch. The voltage between the electrodes of the pinch as a function of time depends upon the inductance of the pinch as a circuit element. If we assume axial uniformity so that the radius of the pinch is uniform with axial position, then the inductance measurements give the pinch radius as a function of time. The first bounce agrees with the concept that the pinch compresses to 1/7 to $\frac{1}{8}$ of the original tube diameter, hence the compression of 50. The second, and sometimes third, bounces are observable, but thereafter the circuit behavior is interpretable only in terms of an instability breaking up the pinch. After breakup (usually after what would appear as the third bounce), the circuit appears as if the current were flowing on the inside of the quartz tube wall. At very low condenser voltages, 20 kilovolts and below, there sometimes appears a very high-voltage spike (100 000 to 200 000 kilovolts) at the time of the second or third bounce. The above behavior is in agreement with the idea that after one or two bounces the inherent instabilities have had a chance to grow to sufficient magnitude to perturb seriously the inductive behavior of the pinch. When this rate of change of inductance is high enough, the resulting voltage can reach a value many times the applied voltage and may be large enough to break down (voltage-wise) the inside of the quartz wall—causing a surface current to flow that is large enough to electrically isolate the pinch from the external circuit. This is why the very high cumulative voltage during neutron production need not appear in the external circuit.

The instability mode that would grow fastest is the sausage type ($m=0$), where the plasma is "necked off" in one or more places, Fig. 13. This mode should grow

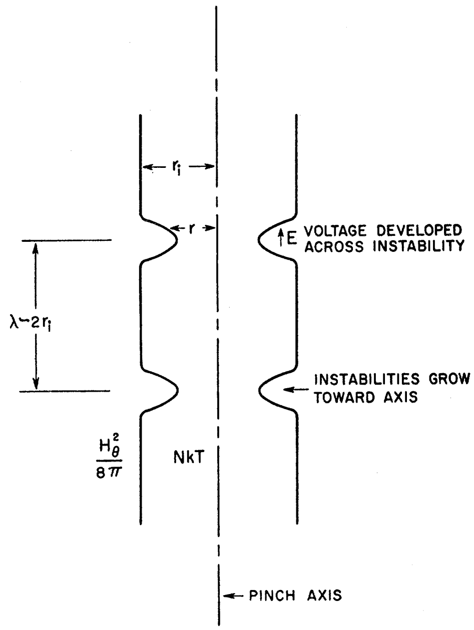

FIG. 13. Sausage instability with resulting electric field.

fastest at large amplitude because the rate of increase of pressure ($H_\theta^2/8\pi$) is more rapid for a small change in pinch radius r than is the differential pressure for the kink instability ($m \geqslant 1$) for a small amplitude. The rate of growth of a given wavelength instability is exponential for an amplitude that is small compared with both the wavelength λ and pinch radius r.

As the amplitude becomes larger, the troughs of magnetic field growing radially inward speed up because of the increased magnetic field pressure of the smaller radius pinch. The resultant necking off of the pinch causes an axial flow of plasma that expands the pinch immediately adjacent to the neck. This nonlinear coupling tends to space the instabilities regularly so that the configuration expected at the time of the second or third bounce is a series of $m=0$ instabilities regularly spaced along the length of the pinch. This is confirmed by time-resolved pictures taken at Los Alamos of the pinch at this time. The spacing corresponds to a wavelength equal to the pinch diameter at the second bounce, which is in agreement with the predicted structure of the nonlinear growth of the trough.

The shape of the neck or trough as it grows radially inward can be derived on the basis of a simplified hydrodynamical model. Once the initial instability has grown to an amplitude equal to $\frac{1}{4}$ the pinch radius, then the magnetic field pressure at the point of minimum radius is approximately twice as great as the plasma pressure—assuming that the original growth was from an equilibrium radius. For an overpressure of 2 or greater, the flow velocities are governed primarily by shock hydrodynamics, and therefore according to the snowplow model, which is sufficiently accurate for the approximation, the plasma–magnetic-field interface will then move at a velocity determined by the pressure difference. Since we are primarily interested in the instability shape at small radius, we shall assume that the magnetic field pressure is always much greater than the plasma pressure.

It is shown in the Appendix [Eq. (16A)] that the voltage per lobe (for the numerical example discussed in the theory section) is

$$V=I\frac{dL}{dt}=1.26\times10^3\frac{1}{r_f(1-2\Delta)^{\frac{1}{2}}}\times\left[-\ln\frac{R}{r_i}+1+\frac{\pi}{2}\left(\sqrt{\Delta}-\frac{1}{\sqrt{\Delta}}\right)-\frac{4\Delta}{3}\right],\quad(15)$$

where $r_i=0.375$ cm and $R=3.75$, $\Delta=r_f/r_i$. Therefore one has

$$V=\frac{1.26\times10^3}{r_f(1-2\Delta)^{\frac{1}{2}}}\left[-1.3+\frac{\pi}{2}\left(\sqrt{\Delta}-\frac{1}{\sqrt{\Delta}}\right)-\frac{4\Delta}{3}\right].\quad(16)$$

In order to attain 100 kev per gap, so that an ion making one cycloidal orbit across the gap will pick up $\frac{1}{2}$ this energy or 50 kev, one finds

$$r_f\simeq0.05\text{ cm}.$$

This is just the radius at which the calculated axial magnetic-field pressure of 100 gauss should prevent further instability growth and quench the neutrons.

The ions can cross the magnetic field between lobes by being accelerated by the electric field, because their Larmor radius is greater than the lobe spacing. The sides of the lobe are $\sim2r$ apart; the magnetic field is $30\,000/r$; the Larmor radius is

$$r_D=200\sqrt{E}/H,\quad\text{or}\quad r_D=200r\sqrt{E}/30\,000.\quad(17)$$

For an ion to cross the gap in a single cycloid orbit requires

$$1.4r_D=2r;$$

therefore

$$E\geqslant36\text{ kev}.$$

The range of an accelerated deuteron in the plasma of the lobes can be calculated from the slowing-down time τ. The time necessary for the deuteron to lose $1/e$ of its energy is[3]

$$\tau=1.1\times10^{12}(T^{\frac{3}{2}}/n_e).\quad(18)$$

T, the temperature in kev, is 0.1, n_e is the electron density and is 1.7×10^{18}. Therefore

$$\tau=1.1\times10^{12}(0.03/1.7\times10^{18})=2\times10^{-8}\text{ sec}.\quad(19)$$

The corresponding range for a 50-kev deuteron is 4 cm, which is too long by a factor of 4. Halving the mean electron temperature and doubling the density could easily account for the difference. However, the long range would indicate that a small class of particles could be accelerated from one instability to the next, giving much higher energies, and thereby explaining the few very high-energy deuterons observed. The same process would apply for electrons if it were not for the difficulty of their crossing the high magnetic fields.

A comparison can be made between the proposed acceleration mechanism and the experimental observations.

(a) A small class of high-energy axially accelerated deuterons should give rise to a forward-peaked neutron distribution. The large observed width of the distribution can be caused by the bending of the deuteron trajectories in the strong magnetic field of the instability.

(b) A weak axial magnetic field of 100 gauss should stabilize the $m=0$ mode of instability, for radii less than 0.05 cm. This is just the critical radius to which the instability must grow to create the accelerating voltages.

(c) The strong quenching by impurities cannot be properly explained. It might be associated with the ion-injection mechanism into the accelerating electric field. A boundary layer must exist between the magnetic field and plasma during the instability growth. The characteristics of such a layer are that the ions penetrate into the magnetic field further than the electrons. The resulting charge-separation electric field turns the ions around in a distance small compared with the ion Larmor radius. An ion of smaller $(Z_{\text{effective}})(e/M)$ would penetrate farther into the vacuum region, would feel more of the accelerating electric field, and might be preferentially accelerated in the instability breakup process. Therefore, a small number of impurity ions that have retained some orbit electrons might be preferentially accelerated and result in no neutrons. If this explanation is true, then a small addition of tritium would be expected to give a disproportionately higher neutron yield than the ratio of cross sections would indicate. This experiment has been tried at Los Alamos with negative results. It is also suggested that a very small impurity addition cools the electron temperature by inelastic excitation of bound atomic levels. If the electron temperature is lowered, the boundary layer becomes thicker and the dynamics at small radii is impaired.

(d) The neutron yield as a function of applied voltage might be expected to level out once the instability breakup voltages have reached 50 to 100 kev. Certainly rather wide fluctuations in yield might be expected for a statistical process of a small number of instabilities.

(e) The neutron yield from a thermonuclear process has been estimated in Sec. II. The expected yield from the instability breakup process can be estimated with a lower limit from a space-charge-limited current. (A discussion of it follows this list of comparisons.)

(f) The timing of the appearance of neutrons agrees with the time in the pinch history at which one would expect instabilities to have grown to large amplitude. The radius-*versus*-time behavior can be used to calculate the expected breakup voltages.

(g) For weak pinches (small currents) a sudden rise in voltage can be observed at the pinch dynamic time at which one would expect neutrons. However, at higher pinch currents the shorting effect of the insulator (quartz) wall must be invoked to explain the lack of observed external high voltages. The direction of acceleration is in agreement with the change of inductance of the known current.

(h), (i) Uniformity along the axis and a radially centered source are in agreement with the theory within the resolution of the experiments.

(j) The theory would predict that the instabilities should grow simultaneously within a time small compared with the time of the third bounce. The third-bounce time is of the order of $(5 \text{ to } 10)\times10^{-8}$ sec, so that the observed simultaneity of $\pm5\times10^{-8}$ is reasonable.

The accelerated current per lobe should be greater than the space-charge-limited value because there should exist both an ion and an electron density greater than the space-charge neutralizing value. An ion current of 1000 amp/cm^2 at 50 kev requires an electron density of only 3×10^{13} for neutralization, so that a strong electric field accelerates a very much greater current than the space-charge-limited value. Indeed, if the electron density outside the pinch is 0.2% of the value inside ($\sim1.7\times10^{18}$), it is possible that the whole current of the pinch can be carried space-charge-free by the ions. This might occur if the instability actually interrupted the electron current by the mechanism of increasing the resistance of the necked region.

The space-charge-limited current, in plane geometry, on the assumption that the magnetic field is strong enough to prevent electrons from being accelerated, is

$$i=\frac{4}{9}K_2\frac{2e}{M_n}\frac{V^{\frac{3}{2}}}{d^2}\ \text{amp/cm}^2. \tag{20}$$

Since we are interested in only an approximate lower limit, we neglect the logarithmic voltage gradient over the slot. Therefore we have

$$i=(0.56/r^2)\ \text{amp/cm}^2,$$

for

$$V=\text{voltage}=50\ \text{kv}, \tag{21}$$

for

$$d=\text{spacing}=r.$$

This current must be integrated over the surface of the slot:

$$i_t=\int_{r_0}^{r}\frac{0.56}{r^2}2\pi r dr=3.5\ \ln\frac{r_0}{r}=7.6\ \text{amp per lobe}. \tag{22}$$

The lobes are equally spaced along the length of the tube at $\lambda=2r_0=0.7$ cm. Therefore in a 70-cm tube there are 100 lobes, which gives a peak current of 760 amp.

The length of time that the instability configuration should exist is roughly the time for another instability to grow to an amplitude equal to the pinch radius r_0.

During this time of approximately 0.1 μsec, the original slot will be growing axially, giving roughly the same rate of change of inductance as during the radial growth. This time of 0.1 μsec is roughly $\frac{1}{2}$ the observed neutron pulse length, and $\frac{1}{2}$ the voltage spike width observed for weak pinches.

The number of accelerated deuterons becomes

$$N_{D\text{ac}}=760\times 6\times 10^{18}\times 10^{-7}=4.6\times 10^{14}. \qquad (23)$$

The number of deuterons that the accelerated ions see in slowing down in a relaxation length is $4\times 1.7\times 10^{18}=6.8\times 10^{18}$.

The reaction cross section $\sigma(d\text{-}d)_{50\text{ kev}}$ is 3.7×10^{-26} cm^2, of which one-half the reactions lead to neutrons, so that the expected neutron production becomes

$$\begin{aligned} n&=N_{D\text{ac}}\times N_D\times\frac{\sigma}{2}\\ &=4.6\times 10^{14}\times 6.8\times 10^{18}\times\frac{3.7}{2}\times 10^{-26}\\ &=6\times 10^{7}\text{ neutrons.} \qquad (24)\end{aligned}$$

This is the approximate yield observed from a tube of this length, although some yields as high as 10^8 neutrons have been recorded. If the mean deuteron energy is assumed lower than 50 kev, the reaction cross section and space-charge-limited current are so much lower that space-charge neutralization must be assumed in order to account for the observed yield.

Two additional theories of the neutron production have been proposed. The first, briefly referred to by Kurchatov at the Harwell Conference,[8] was one in which a Fermi-type acceleration was proposed in which a small class of ions was accelerated by reflection from moving random magnetic fields. If the field motion is random, then the ion motion must be random and no shift in the center-of-mass motion should be observed.

The theory proposed by James Tuck of Los Alamos invokes the $m=0$ mode of instability and accelerates just those deuterons that break through the sheath at the neck of the instability. The ions are accelerated to essentially the neck velocity, and then they are deflected into the axial direction by the magnetic field. By the snowplow hydrodynamic model, this requires an instability minimum radius of 0.01 cm, which is considerably smaller than considered here, but feasible.

ACKNOWLEDGMENTS

The work was performed under the personal encouragement of H. F. York and C. M. Van Atta. Jack Riedel (now at the Palmer Physical Laboratory, Princeton) was active in the early part of the work. The preliminary photographic emulsion scanning was performed by the nuclear emulsion group at Livermore under Steve White. John Howard did the spectrographic investigation of the helium pinch. We have been aided by many fruitful discussions with Marshal Rosenbluth of Los Alamos, and Edward Teller of Livermore. The strong support of the technician group at Berkeley made it possible to perform these many experiments.

APPENDIX. CALCULATION OF THE VOLTAGE ACROSS AN $m=0$ INSTABILITY

If an initial instability (Fig. 14) grows from an equilibrium pinch of radius r_i, then the velocity of every point on the r, z curve describing the interface shape is dependent upon the pressure $H^2/8\pi$. For snowplow hydrodynamics the force equals the time rate of change of momentum, so that to first approximation one has

$$P=\frac{H^2}{8\pi}=\rho u^2=\frac{I^2}{200\pi r^2}, \qquad (1\text{A})$$

where ρ = density, u = velocity normal to the shock, and I = pinch current. The radial shock velocity increases as the instability grows inward and a simple graphical integration gives the contours of growth of Fig. 14. The initial azimuthally symmetric perturbation is at $r=0.9r_i$ and the normal velocity at any point on the

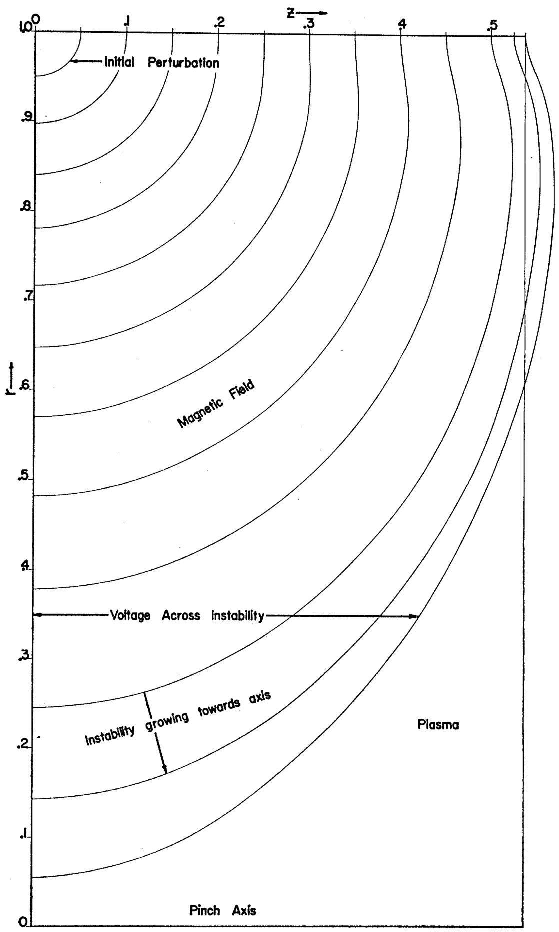

FIG. 14. Growth of sausage instability with resulting electric field.

interface becomes $r_i u_0/r$, where u_0 is the shock velocity at the initial pinch radius r_i. The shape at large growth can be approximated by the parabola

$$z^2=0.36r_i(r-r_f), \tag{2A}$$

where r_f= the minimum radius of the instability.

A voltage is developed between each instability lobe because of the rate of change of inductance:

$$V=(d/dt)(LI) \tag{3A}$$

$$=I(dL/dt)+L(dI/dt), \tag{4A}$$

where L= inductance, I= current.

The total voltage across the pinch is either the applied voltage, which is small at second-bounce time, or is less than this because the inside quartz wall has shorted on itself, in which case we have

$$I(dL/dt)=-L(dI/dt), \tag{5A}$$

or

$$L_1/L_2=I_2/I_1. \tag{6A}$$

Since even a large number of regularly spaced instabilities does not change the total inductance of the pinch by very much (∼30%), the current I during the instability growth tends to remain constant. Therefore the voltage across a given instability is

$$V=I(dL/dt), \tag{7A}$$

$$L=2\times10^{-9}\times2\int_{r_f}^{r_i-r_f}\ln\frac{R}{r}dz \text{ henries/cm}, \tag{8A}$$

where R= outside conductor radius, r= pinch radius, r_f= minimum instability radius, z= axial position, and r_i= initial pinch radius. But we have

$$z=0.6[r_i(r-r_f)]^{\frac{1}{2}}. \tag{9A}$$

Therefore

$$L=4\times10^{-9}\times0.3(r_i)^{\frac{1}{2}}\int_{r_f}^{r_i-r_f}\frac{\ln R/r}{(r-r_f)^{\frac{1}{2}}}dr. \tag{10A}$$

This can be integrated to give

$$L=1.2\times10^{-9}(r_i)^{\frac{1}{2}}2(r_i)^{\frac{1}{2}}\left(1-2\frac{r_f}{r_i}\right)^{\frac{1}{2}}\left[\ln\left(\frac{R/r_i}{1-r_f/r_i}\right)+2-\frac{2(r_f/r_i)^{\frac{1}{2}}}{(1-2r_f/r_i)^{\frac{1}{2}}}\tan^{-1}\left(\frac{1-2r_f/r_i}{r_f/r_i}\right)^{\frac{1}{2}}\right]. \tag{11A}$$

The time derivative of the inductance during instability growth becomes

$$\frac{dL}{dt}=1.2\times10^{-9}(r_i)^{\frac{1}{2}}\frac{dr_f}{dt}\times\frac{2}{(r_f)^{\frac{1}{2}}}\left(\frac{r_f/r_i}{1-2r_f/r_i}\right)^{\frac{1}{2}}\times\left\{-\ln\left(\frac{R/r_i}{1-r_f/r_i}\right)+\left(2-\frac{r_i}{r_f}\right)\times\left(\frac{r_f/r_i}{1-2r_f/r_i}\right)^{\frac{1}{2}}\tan^{-1}\left[\left(\frac{r_f/r_i}{1-2r_f/r_i}\right)^{-\frac{1}{2}}\right]\right\}, \tag{12A}$$

and if we make the approximation $r_f/r_i=\Delta\ll1$, then

$$\frac{dL}{dt}=1.2\times10^{-9}(r_i)^{\frac{1}{2}}2\left(\frac{\Delta}{1-2\Delta}\right)^{\frac{1}{2}}\frac{dr_f}{dt}\frac{1}{(r_f)^{\frac{1}{2}}}\times\left[-\ln\left(\frac{R}{r_i}\right)+1+\frac{\pi}{2}\left(\sqrt{\Delta}-\frac{1}{\sqrt{\Delta}}\right)-\tfrac{4}{3}\Delta\right]. \tag{13A}$$

The radial velocity of the neck of the instability, $dr_f/dt=u$, can be calculated from Eq. (1A),

$$\frac{dr_f}{dt}=\left(\frac{I^2}{200\pi\rho r\rho^2}\right)^{\frac{1}{2}}, \tag{14A}$$

where ρ= density in pinch (170 microns D_2, compressed 100-fold at second bounce time) $=3\times10^{-6}$ g/cm³, I= 150 000 amp.

Therefore

$$\frac{dr_f}{dt}=\frac{3.5\times10^6\text{ cm/sec}}{r_f}. \tag{15A}$$

From Eqs. (7A), (13A), and (15A), the voltage per lobe becomes

$$V=I\frac{dL}{dt}=1.26\times10^3\frac{1}{r_f(1-2\Delta)^{\frac{1}{2}}}\times\left[-\ln\frac{R}{r_i}+1+\frac{\pi}{2}\left(\sqrt{\Delta}-\frac{1}{\sqrt{\Delta}}\right)-\frac{4\Delta}{3}\right], \tag{16A}$$

where

$$r_i=0.375,\quad R=3.75.$$

From the experiment concerning the neutron quenching by an axial magnetic field we infer that $r_f\approx0.05$ cm, giving V= 120 000 volts.