Variation of Hall Mobility of Carriers in Nondegenerate Semiconductors with Electric Field

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The study of Hall effect along with conductivity at high electric fields will shed more light on the behavior of carriers at such fields. This can be done by little modification of existing apparatus.

In this paper the authors have obtained an expression for Hall mobility applicable in a large range of fields and non-Maxwellian distribution of velocities of carriers. This expression, together with the velocity distribution of carriers in the presence of the field, obtained by Yamashita and Watanabe, has been used for the theoretical investigation of the variation of Hall mobility with electric field.

INTRODUCTION

So far the measurement of electrical conductivity has been the only experimental approach to the study of the velocity distribution of carriers in a semiconductor at high electric fields. In interpreting the data it is usually assumed that the number of carriers does not change with electric field, an assumption probably true but not proved. If we apply a small magnetic field and study the Hall effect we can get more data to compare with theory. Moreover, the product of Hall coefficient, velocity of light, and conductivity is equal to Hall mobility and is independent of the number of carriers. It may be added that the study of the Hall effect involves very little modification of the apparatus.

In this communication the authors have obtained an expression for Hall mobility applicable in a good range of fields. Utilizing this expression and the velocity distribution obtained by Yamashita and Watanabe[1] by solving the Bloch integral equation to second order of approximation, the authors have studied the variation of Hall mobility with electric field.

MOBILITY

Using Conwell's[2] notation and Eq. (8) of her treatment, one has

$$dj=\frac{2}{(2\pi)^3}\frac{q^2}{m}E\tau\frac{df}{dk}k^3\cos^2\theta\,\sin\theta d\theta d\varphi dk. \qquad (1A)$$

If we apply a small magnetic field H so that terms involving second and higher orders of (qH/mc) are negligible, Eq. (1A) remains unchanged and[3]

$$dJ=(qH/mc)\tau dj, \qquad (1B)$$

where J is the Hall current.

Proceeding in the same way as Conwell[2] from Eqs. (1A) and (1B), we obtain

$$\mu=\frac{q}{3m}\left\langle\frac{1}{v^2}\frac{d}{dv}(\tau v^3)\right\rangle, \qquad (2)$$

$$\mu'=\frac{q}{m}\left\langle\frac{1}{v^2}\frac{d}{dv}(\tau^2 v^3)\right\rangle \Big/ \left\langle\frac{1}{v^2}\frac{d}{dv}(\tau v^3)\right\rangle, \qquad (3)$$

where μ and μ' are the drift and Hall mobilities, respectively, and v the carrier velocity.

For a Maxwellian distribution, Eqs. (2) and (3) are equivalent to the expressions given by Shockley.[3]

DISTRIBUTION FUNCTION

Yamashita and Watanabe[1] have obtained the velocity distribution of carriers in the presence of an electric field by solving the Bloch integral equation for nonpolar semiconductors to the second order of approximation. They found the following velocity distribution in the presence of a field E:

$$N(x)dx=Ax^2(x^2+y)^y\exp(-x^2)dx, \qquad (4)$$

where $x=v(m/2kT)^{\frac{1}{2}}$, $N(x)dx$ is the number of carriers in the range x to $x+dx$, m is the effective mass of electrons, k is the Boltzmann constant, T is crystal temperature, μ_0 is the zero-field drift mobility, c is the velocity of sound in the crystal, and $y=(3\pi/16)\times(\mu_0{}^2E^2/c^2)$. Equation (4) was obtained by neglecting the scattering of carriers by optical modes.

MOBILITY IN PRESENCE OF FIELD

The time of relaxation of a carrier due to scattering by acoustical modes is given by[3]

$$\tau=l/x, \qquad (5)$$

where l is constant at a given temperature. Using expressions (2), (3), (4), and (5), we obtain the drift and Hall mobilities:

$$\frac{\mu}{\mu_0}=\left(\frac{\pi^{\frac{1}{2}}}{4}\right)\left(\frac{I}{\eta}\right), \qquad (6)$$

[1] J. Yamashita and N. Watanabe, Progr. Theoret. Phys. (Japan) **12**, 443 (1954).
[2] E. Conwell, Phys. Rev. **88**, 1379 (1952).
[3] W. Shockley, *Electrons and Holes in Semiconductors* (D. Van Nostrand Company, Inc., New York, 1950).

and

$$\frac{\mu'}{\mu_0}=\left(\frac{3\pi^{\frac{1}{2}}}{4}\right)\left(\frac{\mathcal{J}}{I}\right), \tag{7}$$

where

$$n=\int_0^\infty x^2(x^2+y)^y\exp(-x^2)dx,$$

$$I=2\int_0^\infty x(x^2+y)^y\exp(-x^2)dx,$$

$$\mathcal{J}=\int_0^\infty (x^2+y)^y\exp(-x^2)dx.$$

For integral values of y the integrals can be evaluated by expansion of $(x^2+y)^y$ and remembering that

$$\int_0^\infty z^n e^{-z}dz=\Pi(n),$$

where Π is a tabulated function.[4]

For nonintegral values of y the integrals have to be evaluated numerically.

LOW-FIELD MOBILITY

When the electric field is low, y is small and we may put

$$(x^2+y)^y=1+y\ln(x^2).$$

Remembering that

$$\int_0^\infty z^n e^{-z}\ln(z)dz=\Pi(n)\times\Psi(n),$$

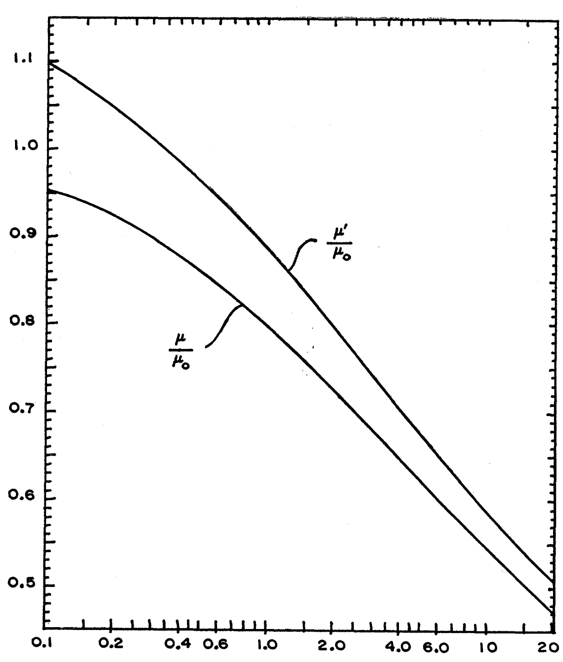

FIG. 1. Variation of μ/μ_0 and μ'/μ_0 with y.

[4] E. Jahnke and F. Emde, *Tables of Functions* (Dover Publications, New York, 1945), fourth edition, p. 9.

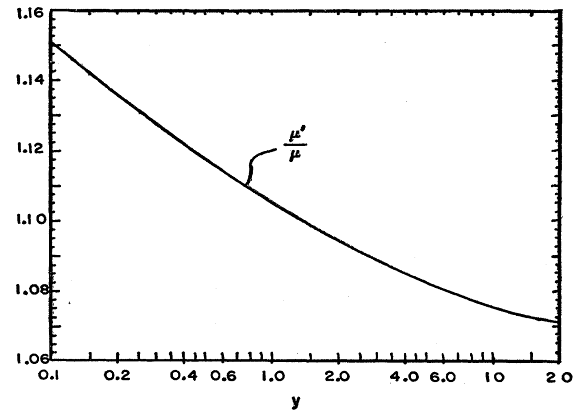

FIG. 2. Variation of μ'/μ with y.

where Ψ is also a tabulated function,[4] we may write in the low-field case

$$n=\tfrac{1}{2}\int_0^\infty z^{\frac{1}{2}}(1+y\ln z)e^{-z}dz=\tfrac{1}{2}\Pi(\tfrac{1}{2})\{1+y\Psi(\tfrac{1}{2})\},$$

$$I=\int_0^\infty (1+y\ln z)e^{-z}dz=\Pi(0)\{1+y\Psi(0)\},$$

$$\mathcal{J}=\tfrac{1}{2}\int_0^\infty z^{-\frac{1}{2}}(1+y\ln z)e^{-z}dz=\tfrac{1}{2}\Pi(-\tfrac{1}{2})\{1+y\Psi(-\tfrac{1}{2})\}.$$

Hence, from Eqs. (6) and (7) we get

$$\mu/\mu_0=1-0.6137y, \tag{8}$$

$$\mu'/\mu_0=\tfrac{3}{8}\pi\{1-1.3863y\}, \tag{9}$$

and

$$\mu'/\mu=\tfrac{3}{8}\pi(1-0.7726y). \tag{10}$$

Equation (8) has already been derived by Sodha.[5]

HIGH-FIELD MOBILITY

For high fields, y is large and Yamashita and Watanabe[1] have pointed out that

$$(x^2+y)^y\exp(-x^2)\propto\exp(-x^4/2y).$$

Hence we may write

$$n=\int_0^\infty x^2\exp(-x^4/2y)dx=2^{-2}(2y)^{\frac{3}{4}}\Pi(-\tfrac{1}{4}),$$

$$I=\int_0^\infty 2x\exp(-x^4/2y)dx=2^{-1}(2y)^{\frac{1}{2}}\Pi(-\tfrac{1}{2}),$$

$$\mathcal{J}=\int_0^\infty \exp(-x^4/2y)dx=2^{-2}(2y)^{\frac{1}{4}}\Pi(-\tfrac{3}{4}).$$

[5] M. S. Sodha, Phys. Rev. **108**, 1375 (1957).

Thus at high fields we get

$$\mu/\mu_0 = 1.078 y^{-\frac{1}{4}}, \quad (11)$$

$$\mu'/\mu_0 = 1.143 y^{-\frac{1}{4}}, \quad (12)$$

$$\mu'/\mu = 1.061. \quad (13)$$

CALCULATIONS

The authors have calculated μ/μ_0 and μ'/μ_0 for $y=1$, 2, 3, 4, 5, 6, 7, 8, 9, 10, and 20 by exact integration and for $y=0.1$, 0.2, 0.3, 0.5, 0.8, and 1.0 by numerical integration. The values for $y=1$ obtained by numerical integration and exact integration are found to be in agreement to the third decimal place.

The high-field approximation gives an error of 7% in the values of μ/μ_0 and μ'/μ_0 at $y=20$. The values of μ/μ_0 and μ'/μ_0 obtained by the low-field approximation are 0.939 and 1.020, compared to 0.953 and 1.098 given by the numerical integration for $y=0.1$. Thus the high-field and low-field approximations are justified in the ranges $20<y<\infty$ and $0<y<0.1$ respectively.

The results are graphically illustrated by Figs. 1 and 2.

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Barkhausen Pulses in Barium Titanate

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A study has been made of the Barkhausen pulses that occur during polarization reversal in single crystals of barium titanate. By both pulse counting and oscillographic techniques, the pulse shapes and in particular their heights and rise times have been studied as a function of the crystal thickness and the applied field strength. The pulse shape represents an initial rapid increase in the volume of the region switched followed by a slower relaxational type of growth, the latter being described by a time constant of 5 to 6 μsec. The pulse heights increase with the crystal thickness and linearly with the applied field while they are practically independent of temperature between room temperature and 94°C. The relaxation time is essentially independent of the crystal thickness, of the applied field, and of the pulse height. The total number of pulses in a given crystal is independent of the field and temperature. In crystals 5×10^{-3} cm thick, the average volume corresponding to a pulse is 10^{-11} cm^3 while the total volume represented by all the pulses is less than one percent of the crystal volume between the electrodes. Individual pulses occur quite independently of each other and of their surroundings.

These observations are not consistent with the usual jerky domain-wall motion models for the generation of Barkhausen pulses. It is concluded that the eventual size and shape of the rapidly switching region represented by a Barkhausen pulse are mainly determined by the crystal thickness and the condition that the depolarizing field within the region must not exceed the applied field. This criterion is successful in accounting for some of the features of the pulses if the region is assumed to be spike-shaped and extending more or less through the crystal thickness, in particular, the average pulse size and its dependence on the field. These deductions suggest that the Barkhausen pulses could represent the nucleation and initial stages of growth of new spike-shaped domains extending along the *c* axis and that the fixed number of pulses given by a crystal would then indicate a definite number of nucleating sites on the crystal surfaces. Under certain conditions a spike-shaped critical nucleus is consistent with the empirically determined nucleation probability factor, $\exp(-\alpha/E)$, where E is the applied field strength.

To account for the polarization reversal in the remainder of the crystal it is presumed that, after their formation, the spikes expand radially (sideways) in all directions. By using this model the rate of polarization reversal as a function of time can be formulated, assuming that the radial wall velocity is proportional to the field and the nucleations occur randomly. Satisfactory agreement with experiment is obtained at low fields if it is assumed that the expanding domains stop short of overrunning adjacent nucleating sites. Relaxing this restriction for higher fields again leads to good agreement with experiment. Also, the observed dependence of the switching time and the maximum current on the applied field is predicted by using certain approximations.

INTRODUCTION

FROM the investigations made particularly by Merz[1] and Little[2] using electrical and optical techniques has evolved the following description of the sequence by which the reversal of the polarization of barium titanate is accomplished: spike- or wedge-shaped domains are nucleated at the surfaces of the crystals and grow in the forwards direction, that is, along the *c* axis. Merz concluded that there was negligible sideways expansion of these domains but this is contradicted to some extent by Little's work, though the latter used a quite different orientation of the field with respect to the crystal axes and the direction of the spontaneous polarization.

When the polarization state is slowly reversed many

[1] W. J. Merz, Phys. Rev. **95**, 690 (1954).
[2] E. Little, Phys. Rev. **98**, 978 (1955).