up to its dehydration temperature, above 200°C. G_3S is entirely stable up to its Curie temperature,[8] 47°C; and G_3F is equally stable to its Curie temperature at 70°C.[9] G_2MCD alone among the three classes of compounds is unstable at room temperature, and increasingly less stable above.

A comparison of values of spontaneous polarizations and coercive fields for GASH and its vanadium analog, G_3S, G_3F, and G_2MCD, is given in Table I. G_3S is by far the most advantageous of the three. GASH has the disadvantage of unsymmetric hysteresis loops. The coercive field of G_2MCD is thirty times higher than that of G_3S, and the spontaneous polarization is half as high, but the hysteresis loops in both classes of crystals are nicely square and unbiased.

IV. ACKNOWLEDGMENTS

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Ferromagnetic Resonance Line Width in Yttrium Iron Garnet Single Crystals

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A study of ferromagnetic resonance line width in polished, single-crystal spheres of yttrium iron garnet is described. Wave-guide cavity perturbation techniques are used with samples 0.013 in. to 0.020 in. in diameter. An extremely narrow line width of 520 millioersteds (the full width) is observed at 9300 Mc/sec along the hard axis [100]. It is believed this is the narrowest resonance line yet reported on ferromagnetic materials. The line width is shown to be strongly influenced by the sample surface, with the line narrowing by over a factor of twenty as the polishing proceeds.

The line width at 3000 Mc/sec on the same sample is 530 millioersteds. The approximate invariance of the line width for more than a three-to-one change in frequency is compared with the predictions of some recent theories of ferromagnetic resonance line width. The invariance of line width with frequency is also used to conclude that $\lambda\omega$ and T_2 in the Landau-Lifshitz and Bloch-Bloembergen equations of motion, respectively, are approximately constant over this frequency range.

A STUDY of ferromagnetic resonance in single crystals of yttrium iron garnet (YIG) has been made at 9300 Mc/sec and 3000 Mc/sec using cavity perturbation techniques. The samples are polished spheres 0.013 in. to 0.020 in. in diameter, the same samples being used at both frequencies. At 9300 Mc/sec the samples are placed in the center (away from all walls) of a tunable TE_{108} transmission cavity with inside dimensions of $0.8\times0.9\times7.16$ in. At 3000 Mc/sec a tunable TE_{101} transmission cavity is used with inside dimensions of $0.7\times2.74\times2.74$ in., the samples being placed approximately six diameters off the wall. In both cases the samples are placed in an essentially uniform rf magnetic field.

Standard cavity perturbation techniques are used,[1] with the value of μ'' being computed from the change in transmission through the cavity when the applied dc field is moved from a reference point considerably above resonance to the point being measured. One important change in the usual technique was introduced, however, because of the narrowness of the resonance line. In obtaining ΔH, the usual procedure of shifting the frequency of the microwave source at each of the $\frac{1}{2}\mu_{\max}''$ points, to counteract the detuning effect of the sample, was found to yield erroneously narrowed lines. Thus for the data in this paper the cavity is manually retuned by tuning screws at each of these points, and the frequency of the frequency-stabilized source is never varied.

LINE WIDTH AT 9300 Mc/sec

A line width of 2.3 oe at 9300 Mc/sec has been reported previously[2] on YIG crystals grown at this laboratory. In a later batch and with more refined polishing techniques, an extremely narrow line width ΔH of 520 millioersteds (moe) has been observed at 9300 Mc/sec with the dc field along the hard axis [100]. (Fig. 1.) (ΔH here is the full width between $\frac{1}{2}\mu_{\max}''$ points.) The line widths along the medium and easy axes are slightly higher. It is believed this is the narrowest resonance line yet reported on ferromagnetic materials.[3] The quantities $4\pi M_s$ and K_1/M_s have also

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[1] Spencer, LeCraw, and Reggia, Proc. Inst. Radio Engrs. 44, 790 (1956).

[2] LeCraw, Spencer, and Porter, J. Appl. Phys. 29, 326 (1958).

[3] A line width on a YIG disk of 800 moe was reported by J. F. Dillon at the Philadelphia meeting of the American Physical Society, March, 1957. The line width he reported for YIG spheres was approximately 10 oe.

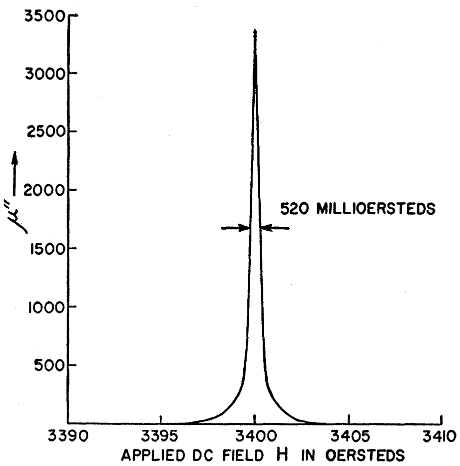

FIG. 1. Ferromagnetic resonance at 9300 Mc/sec in a 0.014-in. single-crystal sphere of yttrium iron garnet with the dc field along the hard axis [100]. The value of μ_{max}'' is 3380.

been measured on spheres from this batch and are 1750 gauss and 45 gauss, respectively.[4]

Because of the large values of μ'' in Fig. 1 and the implied large values of μ', the following comments are pertinent concerning "propagation" effects in spherical samples:

It is well known that the absorbed power P_m in a sphere (neglecting anisotropy) is a maximum at $\omega=\gamma H$, where H is the applied dc field. The μ'' in Fig. 1 is the imaginary part of a complex scalar defined by $\mu=1+4\pi m_x/h_x$, where h_x is the applied linearly-polarized rf field. It can be shown for spheres that P_m is proportional to μ''. However, this μ is not the permeability which determines the wavelength in the sample. It has been pointed out previously[5] that the intrinsic permeability (a tensor) which enters into Maxwell's equations and determines the wavelength in the sample, does *not* resonate at $\omega=\gamma H$ but at $\omega=\gamma H^i$, where H^i is the *internal* dc field and corresponds in this case to an applied dc field of $3400+\frac{4}{3}\pi M_s$ oe. Hence the intrinsic permeability components are small in the vicinity of the resonance shown in Fig. 1. In fact it can be shown that their real and imaginary parts, respectively, are approximately -2 and 0 in this region.[6] This important point has not always been fully recognized. Thus for an ϵ' of approximately 10, the wavelength in the sample is not greatly reduced, and propagation effects in spheres approximately 0.015 in. in diameter are negligible at 9300 Mc/sec.

[4] The value of $4\pi M_s$ was kindly measured for us by S. Foner of the Massachusetts Institute of Technology Lincoln Laboratory.

[5] A. D. Berk and B. A. Lengyel, Proc. Inst. Radio Engrs. **43**, 1587 (1955); see Eq. (30). Spencer, Ault, and LeCraw, Proc. Inst. Radio Engrs. **44**, 1311 (1956); see Figs. 5 and 6.

[6] See reference 1, Eq. (23). This was also pointed out by I. H. Solt in a paper at the Washington Conference on Magnetism and Magnetic Materials, November, 1957. For a detailed treatment of size effects in spheres, see J. E. Tompkins and E. G. Spencer, J. Appl. Phys. **28**, 969 (1957).

In the course of polishing the samples, the remarkable effect of the surface upon the line width became apparent. Figure 2 shows ΔH *vs* the mean grit size of the polishing papers used with the usual air-jet tumbling technique, and shows that in YIG the condition of the surface dominates the line width, at least for ΔH above about 500 moe. The effect can be explained qualitatively in terms of the numerous spin-wave modes which in spheres are degenerate with the uniform precession mode and which can be excited by small surface irregularities. These degeneracies do not occur in thin perpendicularly magnetized disks. Because of the above effect the line width varied, before final polishing, very nearly as the surface to volume ratio, with the smallest samples always having the widest lines (comparing samples with the same degree of polish). After final polishing, however, spheres from 0.013 in. to 0.020 in. in diameter yielded essentially the same line width.

An additional effect of the surface is that the direction of narrowest line width changes as the polishing proceeds. Down to 15 microns of mean grit size, the direction of narrowest line width is the easy axis [111]. Between 15 and 5 microns of mean grit size, the direction of narrowest line width shifts from the easy to the hard axis. This effect is not completely understood as yet.

LINE WIDTH AT 3000 Mc/sec

The measurement of line width at different frequencies is an important method of studying the validity of theories and models of the damping mechanisms in ferromagnetic resonance as well as the forms of the various equations of motion. In reference 2 the line width of YIG spheres was reported to be 2.3 oe at 9300 Mc/sec and 5.7 oe at 3000 Mc/sec. It was pointed out that this frequency dependence of line width is opposite to that observed in YIG spheres by Dillon[7] who reported 13 oe at 9300 Mc/sec and 31 oe at 24 000 Mc/sec.

The variation in line width of 2.3 oe to 5.7 oe from 9300 Mc/sec to 3000 Mc/sec appeared to be in approximate agreement ratio-wise with the variation predicted between these two frequencies by the line width theory in ferromagnetic insulators of Clogston *et al.*[8] The theory is based on the assumption of a random volume distribution of magnetic inhomogeneities with spatial variations of the order of atomic dimensions. (This assumed distribution is probably a satisfactory approximation in the various magnetic spinels.) After more refined polishing, however, the line width at 3000 Mc/sec on the same sphere as used in Fig. 1 narrowed to 530 moe along the easy axis, and slightly higher along the medium and hard axes. It is interesting to note that the shift in the direction of narrowest line

[7] J. F. Dillon, Jr., Phys. Rev. **105**, 759 (1957).

[8] Clogston, Suhl, Walker, and Anderson, J. Phys. Chem. Solids **1**, 129 (1956). See Eq. (29) together with Fig. 2.

width from the easy to the hard axis did not occur as the samples were polished, as was the case at 9300 Mc/sec.†

The fact that on the same sample the line width is 520 moe at 9300 Mc/sec and 530 moe at 3000 Mc/sec suggests that the model used by Clogston *et al.* probably does not apply in these samples. Similar reasoning may be used in connection with the line width calculations of Geschwind and Clogston[9] based on a random distribution of magnetic inhomogeneities with spatial variations large compared to atomic dimensions but small compared to sample dimensions. This theory also predicts a rather strong frequency dependence of line width from 9300 Mc/sec to 3000 Mc/sec. One is led then to propose that in these samples it is quite likely the surface (or possibly some as yet unidentified mechanism) which is still the dominant contributor to the line width.

The approximate invariance of the line width between these frequencies leads also to certain conclusions concerning the Landau-Lifshitz and Bloch-Bloembergen equations of motion given, respectively, in the following two equations:

$$\frac{d\mathbf{M}}{dt}=\gamma(\mathbf{M}\times\mathbf{H}^i)-\lambda\left[\frac{(\mathbf{H}^i\cdot\mathbf{M})\mathbf{M}}{M_s^2}-\mathbf{H}^i\right], \quad (1)$$

$$\left(\frac{d\mathbf{M}}{dt}\right)_{x,y}=\gamma(\mathbf{M}\times\mathbf{H}^i)_{x,y}-\frac{M_{x,y}}{T_2},$$
$$\left(\frac{d\mathbf{M}}{dt}\right)_{z}=\gamma(\mathbf{M}\times\mathbf{H}^i)_{z}-\frac{(M_z-M_s)}{T_1}, \quad (2)$$

where λ is a damping parameter, and T_1 and T_2 are the spin-lattice and spin-spin relaxation times, respectively. $\mathbf{H}^i$ is the total internal magnetic field.

Assuming small-amplitude rf fields, the damping parameter λ in Eq. (1) is given in terms of the line width by

$$\lambda=\gamma M_s\Delta H/(2H_{\mathrm{res}}), \quad (3)$$

where H_{res} is the applied dc field at resonance. There has been considerable speculation as to whether λ is an intrinsic constant of a ferromagnetic material and independent of the frequency of the rf driving source. Experimentally H_{res} changes by a factor greater than three from 9300 to 3000 Mc/sec while the line width remains essentially constant. Thus it appears that λ is not characteristic of the material alone and must be taken as approximately inversely proportional to frequency, if the Landau-Lifshitz equation is to be applied in this frequency range to these samples.

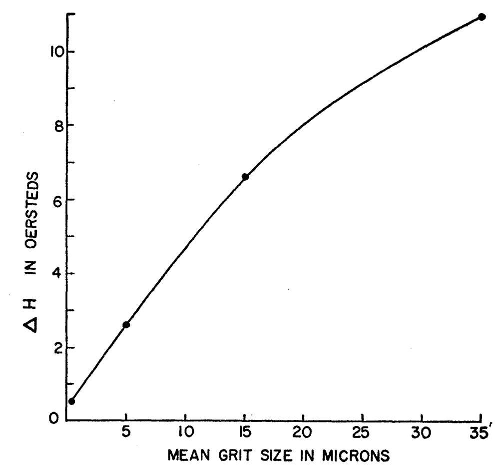

FIG. 2. Line width at 9300 Mc/sec *vs* the mean grit size of the polishing paper. These data were obtained on spheres 0.014 in. to 0.015 in. in diameter. The lowest two points are along [100] and the other two points are along [111], each point being measured along the direction of narrowest line width for the corresponding grit size.

The spin-spin relaxation time T_2 in Eq. (2) is given by

$$T_2=2/(\gamma\Delta H). \quad (4)$$

If one uses the Bloch-Bloembergen equation of motion, the assumption can then be made of an approximately constant spin-spin relaxation time over this frequency range. The value of T_2 corresponding to a 520 moe line width is 0.22 microsecond. The value of T_1 in Eq. (2) can also be expected to be at least 0.22 microsecond. A technique for measuring T_2 directly without using Eq. (4) or making a line width measurement has recently been reported.[10]

It is of interest to compare the predicted value of μ'' at resonance, obtained by solving Eqs. (1) and (2), with the measured value of 3380 in Fig. 1. Equations (1) and (2) both yield, for small amplitude rf fields,

$$\mu_{\mathrm{max}}''=4\pi M_s/\Delta H. \quad (5)$$

With the measured $4\pi M_s$ of 1750 gauss and ΔH of 520 moe, the predicted value is 3360, which is in very good agreement with the measured value.

A detailed study of this narrow resonance line as a function of temperature and magnitude of the exciting rf field is in progress, and should be highly informative from the viewpoint of critically examining a number of existing theories of ferromagnetic phenomena.

† *Note added in proof.*—Another point of interest at 3000 Mc/sec is that the critical rf field, h_{crit}, for onset of nonlinear effects is only 250 micro-oersteds. This very low value of h_{crit} is a consequence primarily of the 530 moe line width and is explained quantitatively by the theory of H. Suhl [J. Phys. Chem. Solids **1**, 209 (1957)] for the case of coincidence of the main and subsidiary resonances.

[9] S. Geschwind and A. M. Clogston, Phys. Rev. **108**, 49 (1957).

[10] E. G. Spencer and R. C. LeCraw, Bull. Am. Phys. Soc. Ser. II, **3**, 145 (1958).