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Excitation of Spin Waves in a Ferromagnet by a Uniform rf Field*

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It is possible to excite exchange and magnetostatic spin waves in a ferromagnet by a uniform rf field, provided that spins on the surface of the specimen experience anisotropy interactions different from those acting on spins in the interior. Modes with an odd number of half-wavelengths should be excited in a flat plate. The definition of what is meant by a different anisotropy interaction is worked out and is a rather lenient condition. Experiments which would determine the exchange energy constant should be possible using sufficiently thin platelets of single crystals having parallel faces. It is perhaps not unlikely that the theory may account for the observation by Waring and Jarrett of a large number of resonance peaks in $NiMnO_3$.

THE purpose of this note is to demonstrate the possibility of exciting by a uniform rf field both exchange and magnetostatic spin waves in a ferromagnetic insulator. The excitation of spin waves by an *inhomogeneous* rf field[1] is now well recognized, but it has been believed that a uniform or homogeneous rf field would excite only the uniform precession mode in which the magnetization precesses as a whole. Indeed, if all ferromagnetic atoms in the specimen experience identical exchange, Zeeman, and anisotropy interactions, then one may prove rigorously that only the uniform mode is excited by a uniform rf field. We must recognize, however, that atoms on the surface of the specimen are in a special position, particularly with respect to the interactions leading to magnetocrystalline anisotropy.[2] The local symmetry of a spin at the surface is always lower than the symmetry of a spin in the interior, and we must expect that terms in the energy which vanish by symmetry at interior points will not vanish at the surface. In short, we expect the effective anisotropy field acting on a spin at the surface to be larger than that acting on a spin in the interior. We discuss, for simplicity, a geometry in which only the exchange energy contributes to the wave energy. Magnetostatic modes may be treated along the same lines and will exhibit similar effects.

Although the proportion of surface atoms in an actual specimen may be less than 1:10^4 of the total number of atoms, yet a modest surface anisotropy can have a far-reaching effect on the transition probabilities for the excitation of spin waves in the microwave region. The essential point is simple: the surface anisotropy acts to pin down the surface spins; if one thinks of a line of length L with the origin on one end, the modes will tend to have the form $\sin(p\pi z/L)$, where p is an integer. The modes of odd p will have a nonvanishing interaction with a uniform rf field, because the instantaneous transverse magnetic moment does not sum to zero over the line.

We now consider the question of the strength of the surface anisotropy required for the end spins to act as fixed rather than as free. The solution of the analogous problem for an elastic string in mechanical vibration is given by Rayleigh.[3] The motion of the interior spins is given by the usual classical equation

$$\partial\mathbf{S}/\partial t=\mathfrak{D}\mathbf{S}\times\nabla^2\mathbf{S}+\gamma\mathbf{S}\times\mathbf{H}, \tag{1}$$

provided $ka\ll 1$, where $k=2\pi/\lambda$ is the wave vector and a is the interatomic spacing. On an atomic model,

$$\mathfrak{D}=2Ja^2/\hbar, \tag{2}$$

where J is the exchange integral. If the average magnetization is parallel to the line and in the z direction, the solutions of Eq. (1) for small amplitudes are of the form

$$S_x,\ S_y\sim e^{i\omega t}e^{\pm ikz}, \tag{3}$$

with

$$\omega=\mathfrak{D}Sk^2+\gamma H_0; \tag{4}$$

here H_0 is the static magnetic field in the z direction.

Let us examine in detail on an atomic model the motion of the end spins $m=1$ and N in a line of N spins along the z axis. We suppose that the end spins experience a surface anisotropy field H_a which we choose for simplicity to be parallel to the line and to the static magnetic field H_0. For convenience we neglect anisotropy and magnetostatic effects in the interior of the line. We have for $m=1$ the equation

$$\partial\mathbf{S}_1/\partial t=(2J/\hbar)\mathbf{S}_1\times\mathbf{S}_2+\gamma\mathbf{S}_1\times(\mathbf{H}_0+\mathbf{H}_a). \tag{5}$$

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[1] R. L. White and I. H. Solt, Phys. Rev. **104**, 56 (1956); J. F. Dillon, Jr., Bull. Am. Phys. Soc. Ser. II, **1**, 125 (1956); J. E. Mercereau and R. P. Feynman, Phys. Rev. **104**, 63 (1956); L. R. Walker, Phys. Rev. **105**, 390 (1957). Excitation by an inhomogeneous field is often known as the White-Solt effect.

[2] L. Néel, Compt. rend. **237**, 1468 (1953); J. phys. radium **15**, 225 (1954). Besides the special situation of the surface with respect to magnetocrystalline anisotropy, it is possible that oxide or impurity layers on the surface will alter the exchange interaction drastically, which will also place the surface spins in a special environment. The production of an antiferromagnetic cobalt oxide layer on cobalt particles has been reported by W. H. Meiklejohn and C. P. Bean, Phys. Rev. **102**, 1413 (1956). We shall not discuss this situation explicitly as it is likely to be similar to the situation analyzed in our present paper.

[3] Lord Rayleigh, *Theory of Sound* (reprinted by Dover Publications, New York, 1945), Vol. I, p. 200.

The x-component equation for small amplitudes is

$$\partial S_1^x/\partial t = (2J/\hbar)(S_1^y - S_2^y)S + \gamma S_1^y(H_0 + H_a). \quad (6)$$

We write, with a as the lattice constant,

$$S_2^y = S_1^y + a\frac{\partial S_1^y}{\partial z} + \frac{a^2}{2}\frac{\partial^2 S_1^y}{\partial z^2} + \cdots, \quad (7)$$

so that, with $\omega_1 = \gamma(H_0 + H_a)$,

$$\frac{\partial S_1^x}{\partial t} \cong -\frac{2JS}{\hbar}\left[a\frac{\partial S_1^y}{\partial z} + \frac{a^2}{2}\frac{\partial^2 S_1^y}{\partial z^2}\right] + \omega_1 S_1^y; \quad (8)$$

$$\frac{\partial S_1^y}{\partial t} \cong \frac{2JS}{\hbar}\left[a\frac{\partial S_1^x}{\partial z} + \frac{a^2}{2}\frac{\partial^2 S_1^x}{\partial z^2}\right] - \omega_1 S_1^x. \quad (9)$$

Combining (8) and (9), we have

$$\partial^2 S_1^x/\partial t^2 = -(2JS/\hbar)^2 \mathfrak{L}^2 S_1^x + (4\omega_1 JS/\hbar)\mathfrak{L} S_1^x - \omega_1^2 S_1^x. \quad (10)$$

where

$$\mathfrak{L} \equiv a\frac{\partial}{\partial z} + \frac{a^2}{2}\frac{\partial^2}{\partial z^2}. \quad (11)$$

If we take as the solution in the interior of the line

$$S^x = e^{i\omega t}(\alpha \sin kz + \beta \cos kz), \quad (12)$$

we have from (10) at $z=0$, with $\omega_e = 2JS/\hbar$,

$$-\beta\omega^2 = \omega_e^2[\alpha(ka)^3 + \beta(ka)^2 - \tfrac{1}{4}\beta(ka)^4] + 2\omega_e\omega_1[\alpha ka - \tfrac{1}{2}\beta(ka)^2] - \beta\omega_1^2. \quad (13)$$

We find

$$\frac{\beta}{\alpha} = \frac{\omega_e^2(ka)^3 + 2\omega_e\omega_1(ka)}{\omega_1^2 - \omega^2 - \omega_e^2(ka)^2 + \frac{1}{4}\omega_e^2(ka)^4 + \omega_e\omega_1(ka)^2}. \quad (14)$$

We consider (14) in the limit $ka \ll 1$ and $\omega_e \gg \omega_1 \gg \omega$; $\omega_1 \gg \omega_e ka$. We have

$$\frac{\beta}{\alpha} \approx \frac{2\omega_e ka}{\omega_1} \ll 1, \quad (15)$$

and the ends behave as if they were fixed. The relation (15) is quite plausible. For other limiting cases we should work with Eq. (14); the behavior of β/α is not particularly simple in the general case.

The normal solutions of the equation of motion must be symmetric or antisymmetric with respect to reflection in the center of the line ($z = L/2$), because the Hamiltonian is symmetric under the reflection operation. In particular we must have

$$\alpha \sin kL + \beta \cos kL = \pm\beta. \quad (16)$$

If $\alpha \gg \beta$, then $\sin kL$ must be $\ll 1$ and the wave vectors are given approximately by

$$k = p\pi/L, \quad (17)$$

where p is any integer.

The inequality (15) is not difficult to satisfy. The anisotropy energy of a surface atom may be of the order of 10^{-2} to 10^{-1} of the isotropic exchange energy. Even for a specimen as thin as 3×10^{-4} cm, for which $L/a \approx 10^4$, we have

$$\frac{\beta}{\alpha} \approx 2\pi p(10^{-2} \text{ to } 10^{-3}) \ll 1, \quad (18)$$

as long as p is less than 10 or 100, according to the material.

We now consider the magnitude of the excitation of the several spin-wave modes by the uniform rf field $H_x = h_0 e^{i\omega t}$. We shall assume that the spins at the ends of the line are effectively pinned down by the surface anisotropy. We look for solutions of the form

$$S_x = e^{i\omega t}\sum_p a_p \sin k_p z, \quad (19)$$

where $k_p = p\pi/L$. The equations of motion are

$$\partial S_x/\partial t = -\mathfrak{L}S_y; \quad (20)$$

$$\partial S_y/\partial t = \mathfrak{L}S_x + \gamma S H_x; \quad (21)$$

where now

$$\mathfrak{L} = \mathfrak{D}S(\partial^2/\partial z^2) - \omega_0. \quad (22)$$

Combining (20) and (21), we have

$$\partial^2 S_x/\partial t^2 = -\mathfrak{L}^2 S_x + \omega_0\gamma S H_x. \quad (23)$$

On substituting (19) in (23),

$$\sum_p a_p(\omega_p^2 - \omega^2)\sin k_p z = \gamma S\omega_0 h_0, \quad (24)$$

with the notation

$$\omega_p = \mathfrak{D}S k_p^2 + \omega_0. \quad (25)$$

We multiply both sides of (24) by $\sin k_m z$ and integrate over z between 0 and L, finding

$$a_m = \frac{4\gamma S\omega_0 h_0}{\pi m}\left(\frac{1}{\omega_m^2 - \omega^2}\right), \quad (26)$$

for m odd; a_m vanishes for m even. Thus the selection rule for excitation by a uniform rf field is that the number of half wavelengths should be odd. In a suitable *inhomogeneous* field the even modes will also be excited, usually less strongly than the odd modes. Eddy current effects in thin metallic films may possibly permit even mode excitation.

In a resonance experiment at constant frequency ω we pass through a finite series of resonances on varying ω_0 from zero to ω; that is, we have spin-wave resonances at discrete static field intensities between 0 and ω/γ. The last statement must be translated appropriately if volume anisotropy and demagnetizing effects are present. At constant frequency the effective oscillator strength of the spin waves at resonance is proportional to

$$\frac{\omega_0}{m}=\frac{\omega-(\mathfrak{D}S\pi^2m^2/L^2)}{m}, \qquad (27)$$

so that the oscillator strength decreases as m increases; the numerator is always positive. Under the special condition of a large negative uniaxial anisotropy, the decrease with increasing m is less marked and it may be shown that $a_m \propto m^{-1}$ is a good approximation in this case, with H_x and H_0 perpendicular to the crystal axis.

The separation of successive spin-wave modes is

$$\Delta\omega=\omega_{m+1}-\omega_{m-1}=4\pi^2\mathfrak{D}Sm/L^2 = 8\pi^2JSm(a/L)^2\hbar^{-1}. \qquad (28)$$

For $S=1$, $J\approx 3\times 10^{-14}$ ergs, and $a/L\approx 10^{-4}$, we have

$$\Delta\omega\approx 3\times 10^7 m, \qquad (29)$$

or

$$\Delta H\approx 2m \text{ oersteds}. \qquad (30)$$

It should be possible in *thin single crystals* with faces quite accurately parallel to resolve a considerable number of spin-wave resonances, particularly for $m>10$, if the individual resonance lines are not too wide. If the crystal is too thick (say over 0.01 or 0.1 mm) the spin-wave structure may usually be obscured by the line width, but the resulting apparent single line may be slightly skewed by the spin-wave substructure.

We consider briefly the situation in three dimensions, taking for convenience the normal to the slab to be the z axis. We look for solutions of the form

$$S_x=e^{i\omega t}\sum a(klm)\sin(k\pi x/L_x) \times\sin(l\pi y/L_y)\sin(m\pi z/L_z), \qquad (31)$$

where k, l, m are odd integers. In (22) and (23) we replace $\partial^2/\partial z^2$ by ∇^2; then

$$\omega_p=\mathfrak{D}S\pi^2\left(\frac{k^2}{L_x^2}+\frac{l^2}{L_y^2}+\frac{m^2}{L_z^2}\right). \qquad (32)$$

The Fourier component of the driving field is

$$h_0\int\sin(k\pi x/L_x)\sin(l\pi y/L_y)\sin(m\pi z/L_z)dxdydz = (8L_xL_yL_zh_0/\pi^3)(1/klm). \qquad (33)$$

Now k, l will only have a significant effect (given usual relaxation frequencies) on the resonant frequency ω_p if they are at least of the order of $L_{x,y}/L_z$ in size, but $L_{x,y}/L_z\sim 10^2$ or more for a reasonable slab. From (33) we see that the intensity of split-off satellites will be down by ~ 100 or more from the intensity of the resonance corresponding to k, $l\approx 1$.

Waring and Jarrett[4] have reported microwave resonance observations on a thin crystal of $NiMnO_3$, a ferromagnetic compound having the ilmenite structure. Their results at 24 000 Mc/sec show a large number of resonance peaks spaced systematically; the results suggested the present interpretation, although not enough work has been done to make possible a detailed comparison of their observations with the present theory. Nickel manganite is highly anisotropic, having negative uniaxial anisotropy, and there is also an anisotropy in the basal plane. Until the anisotropy constants are determined accurately and the role of the twinning which occurs understood, it will not be possible to undertake a detailed analysis. Other materials, however, should show spin-wave resonances if the specimens are prepared properly. The observation of exchange spin-wave resonances should be a very good method for the determination of the exchange interaction constants. In a noncubic crystal there may be different exchange constants for different directions of spin wave propagation and polarization.

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[4] R. K. Waring and H. S. Jarrett, (private communication).