which the latter investigators associated with thermal spikes. The metamictization of minerals is dominated by the behavior of the silica structure; but there are also present some effects which are related to the class of the refractory-ionic-metal oxides, and a phase separation seems to occur at least in some cases. Small amounts of stored energy are found on annealing metamictized minerals in calorimeters, and it is suggested here to be due mainly to chemical reaction between the separated phases and to recrystallization.

APPENDIX

The paper by Simon[42] appeared after the writing of the present paper. His data seem to agree with the interpretations given here.

The deposit of about the density of cristobalite found by Stewart[43] on the walls of a discharge tube was probably radiation-damaged vitreous silica.

The variability in the density of quartz found in nature has usually been attributed to impurities. That it may in part be due to radiation damage from cosmic rays and radioactive inclusions should be considered.

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This investigation would have been impossible but for the cooperation of the operating personnel of the many reactors in which irradiations were conducted. I am indebted to L. H. Fuchs for having arranged the procurement of some of the specimens and for having measured some of their properties prior to irradiation. All of the x-ray diffraction results were obtained by Stanley Siegel. F. S. Tomkins and M. Fred kindly made the comparator available for the expansion studies.

[42] I. Simon, J. Am. Ceram. Soc. **40**, 150 (1957).

[43] R. W. Stewart, Can. J. Research **26**, 230 (1948).

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Theory of an Experiment for Measuring the Mobility and Density of Carriers in the Space-Charge Region of a Semiconductor Surface

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The use of galvanomagnetic experiments to determine the mobility and density of carriers in the space-charge region of a semiconductor surface is considered. In part I an approximate model is used: it is a single crystal composed of two regions, a surface region of thickness of the order of a Debye length and a bulk region. Expressions for the resultant Hall coefficient are given for three experimental configurations by use of circuit analysis. The sensitivity of each configuration is derived, and by also considering experimental desirability, one is selected for study. It has the magnetic field perpendicular to the surface and the Hall voltages of surface and bulk are in parallel. Changes in Hall voltages of 1 to 50% are expected by using ambients to change the surface potential.

In part II the model is assumed to be a single crystal with continuous variation of the potential in the direction perpendicular to the surface. Rigorous expressions are derived for the Hall coefficient and magnetoresistance by use of the one-dimensional Boltzmann equation. A feature of the derivation is its independence of a specific model of the surface region. The resulting expressions contain surface densities and mobilities which can be evaluated from experimental data of Hall coefficient and conductivity. Conversely, the expressions can be used with theory based on specific surface models to predict values of conductivity, Hall coefficient, and magnetoresistance.

INTRODUCTION

THE interpretation of many experiments relating to the surface region of a semiconductor depends upon a knowledge of the density and mobility of carriers in the space-charge region as a function of surface potential.[1] The density of carriers has been calculated by solving Poisson's equation.[2,3] A theoretical study of the mobility of carriers in the space-charge region has been made by Schrieffer.[4] His study indicated that the mobility is a function of the surface potential and, for commonly obtained values of the surface potential, is lower than the bulk value. This general picture has been successful in interpreting a large number of experiments, including measurement of conductivity, field effect, and capacity.[1] However, a more direct measurement of the density and mobility of carriers in the space-charge region is desired to establish firmly the basic picture of the semiconductor surface and because of intrinsic interest in the scattering process.

[1] For a review of the current state of surface physics and an extensive bibliography see R. H. Kingston, J. Appl. Phys. **27**, 101 (1956); and *Semiconductor Surface Physics*, edited by R. H. Kingston (University of Pennsylvania Press, Philadelphia, 1957).

[2] R. H. Kingston and S. F. Neustadter, J. Appl. Phys. **26**, 718 (1955).

[3] C. G. B. Garrett and W. H. Brattain, Phys. Rev. **99**, 376 (1955).

[4] J. R. Schrieffer, Phys. Rev. **97**, 641 (1955).

Combined Hall coefficient and conductivity measurements have been most useful for studying the mobility and density of carriers in bulk crystals. However, several problems are encountered in using these experiments to study surface conduction. The first is that of separating the surface contribution from the bulk contribution to the Hall coefficient.[5] The second is that of sensitivity—it is known that bulk Hall voltages are an order of magnitude smaller than conductivity voltages, and because of the geometrical factor, it can be expected that the surface contribution will be small compared to the bulk contribution. Even if magnetic fields large enough to make the Hall field comparable to the longitudinal field could be applied, it is desirable to have the Hall field small in order to keep the effect linear.

I. GENERAL DESIGN OF THE EXPERIMENT

It was decided to consider only experimental methods susceptible to rigorous theoretical analysis. We therefore consider single crystals of large surface area and of small thickness. The properties of such crystals can be assumed to vary perpendicular to the surface (z direction) but not parallel to the surface. There are three independent ways that the magnetic field, current, and Hall probes can be oriented, as shown in Fig. 1. The mobility and Hall coefficient will in general be tensor quantities at points close to the surface, so that the three configurations will probably yield quite different results. However, all three can in principle be analyzed by solution of a one-dimensional Boltzmann equation. We shall consider each of them to determine which appears to offer the best possibility for a successful experiment.

A first consideration is to estimate the magnitude of the surface contribution to the observed Hall coefficient. To get a qualitative estimate of this, we first adopt a simple model and use an approximate method of analysis. We consider the sample to be composed of two regions (see Fig. 1), a bulk region of thickness d_b, and a surface region of thickness of the order of a Debye length, $d_s \cong L_D$, with

$$L_D = (\kappa\epsilon_0 kT/2q^2 n_i)^{\frac{1}{2}} = 0.67\times10^{-4} \text{ cm} \quad \text{for germanium.} \quad (1)$$

We assume that Hall voltages are induced in the two regions as if they were isolated from each other. Then the two regions are connected to each other either in parallel [Figs. 1(a), 1(c)] or series [Fig. 1(b)]. Circuit theory is used to derive an expression for the resultant Hall coefficient in terms of the surface and bulk Hall coefficients and conductivities.

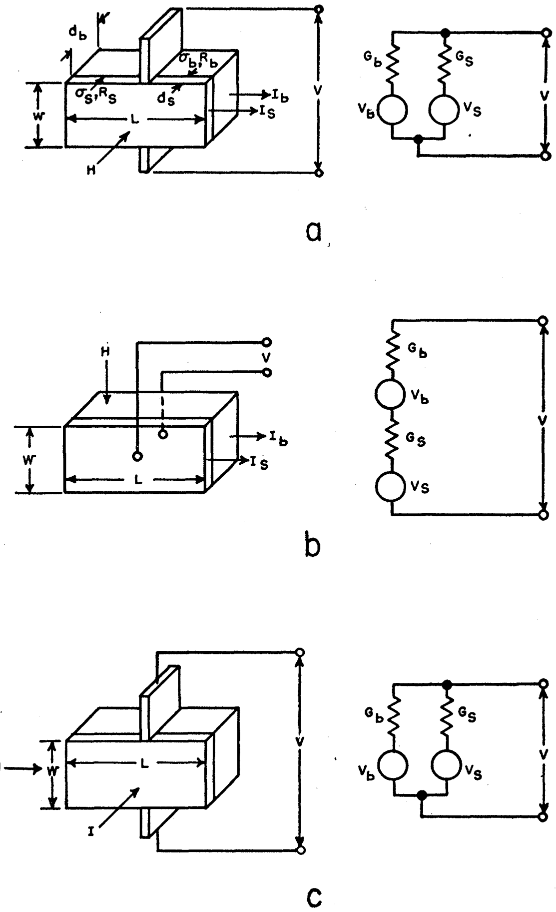

FIG. 1. Possible configurations for Hall measurements. (a) Currents in parallel and Hall voltages in parallel. (b) Currents in parallel and Hall voltages in series. (c) Currents in series and Hall voltages in parallel.

This approach is approximate in that it does not consider the potential to vary continuously in the z direction, nor does it consider all possible circulating currents. For example, in Figs. 1(a) and 1(c) circulating currents perpendicular to the surface are neglected. We use this method to estimate the sensitivity and to discuss some of the experimental problems associated with each configuration. The analysis enables us to select one approach [Fig. 1(a)] as appearing to be the most promising for study.

In Part II we then analyze configuration 1(a) by use of the Boltzmann equation. That analysis drops the two-region assumption and considers the potential to vary continuously in the direction perpendicular to the surface. It also treats the circulating-current problem rigorously. The Boltzmann analysis leads to the same expression for the Hall coefficient as the circuit approach, although this is not true for the magnetoresistance.

[5] In special cases the effects of the bulk can be neglected; an example is the recent work of Frederikse, Hosler, and Roberts, Phys. Rev. **103**, 67 (1956). They studied galvanomagnetic properties of magnesium stannide at helium temperatures and concluded that the bulk contributions were frozen out at this temperature; thus they observed surface effects directly. Our analysis is for the general case when the bulk contribution is not negligible.

A. Expressions for the Hall Coefficient by Two-Region Circuit Analysis

We now give the expressions for the Hall coefficient using the models of Figs. 1(a), 1(b), 1(c), and circuit theory. These expressions have been used in the study of inhomogeneous semiconductors. However, we sketch their derivation because they do not seem to appear in the literature.

1. Magnetic field perpendicular to broad area of sample, Hall voltages of surface and bulk acting in parallel, and bias currents in parallel.—The first configuration and its equivalent curcuit are shown in Fig. 1(a). The individual Hall voltages and currents are

$$V_b=I_bR_bH/d_b, \qquad V_s=I_sR_sH/d_s,$$
$$I_b=\sigma_bd_bI/(\sigma_bd_b+\sigma_sd_s), \quad I_s=\sigma_sd_sI/(\sigma_bd_b+\sigma_sd_s),$$
$$I=I_b+I_s, \qquad d=d_s+d_b,$$

where the subscripts b and s refer to the bulk and surface, respectively, σ is conductivity, R is Hall coefficient, and H is the magnetic field. The open-circuit Hall voltage is

$$V=IHR/d=(V_bG_b+V_sG_s)/(G_b+G_s),$$

where the conductances are given by

$$G_b=\sigma_bd_bL/w, \quad G_s=\sigma_sd_sL/w.$$

Solving, one finds for the Hall coefficient and conductivity of the total sample,

$$R=\frac{d(R_b\sigma_b^2d_b+R_s\sigma_s^2d_s)}{(\sigma_bd_b+\sigma_sd_s)^2}, \tag{2}$$

$$\sigma=\frac{(\sigma_bd_b+\sigma_sd_s)}{d}. \tag{3}$$

This configuration is attractive from the experimental point of view; no contacts need be made to the surface area, gaseous ambients or electric fields can be used to vary the surface conditions, and ordinary dc or low-frequency techniques can be used for the measurements. The side-arm technique is known to yield low noise,[6] since no current flows through the contacts; thus small changes in the Hall voltage can be detected. Furthermore, since the magnetic field causes the electrons to have orbits in the plane of the surface, one expects the conductivity mobility not to be seriously changed by the magnetic field. However, the shunting effect of the bulk is in the ratio of $\sigma_s^2d_s/\sigma_b^2d_b$ for the Hall coefficient, so it is of interest to consider other arrangements which may offer more sensitivity.

2. Magnetic field parallel to broad area of sample, Hall voltages of surface and bulk acting in series, and bias currents in parallel.—In this case the Hall probes are placed on the front and rear surfaces; thus the individual Hall voltages are now added in series [see Fig. 1(b)]:

$$V=V_b+V_s=IRH/w, \qquad I=I_b+I_s,$$
$$V_b=I_bR_bH/w, \qquad V_s=I_sR_sH/w,$$
$$I_b=\frac{\sigma_bd_bI}{\sigma_bd_b+\sigma_sd_s}, \qquad I_s=\frac{\sigma_sd_sI}{\sigma_bd_b+\sigma_sd_s}.$$

Solving, one finds

$$R=\frac{R_b\sigma_bd_b+R_s\sigma_sd_s}{\sigma_bd_b+\sigma_sd_s}, \tag{4}$$

$$\sigma=\frac{(\sigma_bd_b+\sigma_sd_s)}{d}. \tag{5}$$

In this case the shunting effect of the bulk is σ_bd_b/σ_sd_s. However, a disadvantage is that it requires attaching leads to the surface under study or the use of capacitive Hall probes. Also the magnetic field causes orbits perpendicular to the surface, thus altering the conductivity mobility to some extent. Presumably this could be interpreted in the theory, so in itself it is not a compelling reason not to use this setup.

3. Magnetic field parallel to large area of sample; Hall voltages of surface and bulk acting in parallel, and bias currents in series.—For this case we have, from Fig. 1(c),

$$V_b=IR_bH/L, \qquad V_s=IR_sH/L,$$
$$G_b=\sigma_bLd_b/w, \qquad G_s=\sigma_sLd_s/w,$$
$$V=\frac{V_bG_b+V_sG_s}{G_b+G_s}=\frac{IHR}{L}.$$

Solving, one finds

$$R=\frac{R_b\sigma_bd_b+R_s\sigma_sd_s}{\sigma_bd_b+\sigma_sd_s}, \tag{6}$$

$$\sigma=\frac{d}{(d_b/\sigma_b)+(d_s/\sigma_s)}. \tag{7}$$

In this case the shunting effect of the bulk is σ_bd_b/σ_sd_s. However, as for case two, it has the disadvantage of requiring leads attached to the surface under study, or the use of capacitive coupling. As in case 1(b), the magnetic field causes orbits perpendicular to the surface.

B. Sensitivity Analysis—Small-Signal Range

The sensitivity required for detecting a small change in the Hall coefficient due to a change in the surface potential is evaluated for the three cases as follows. A change in surface potential will cause a change in σ_s and R_s, while σ_b and R_b will remain unchanged. Thus

$$\Delta R=(\partial R/\partial\sigma_s)\Delta\sigma_s+(\partial R/\partial R_s)\Delta R_s. \tag{8}$$

Considering Fig. 1(a) we substitute Eq. (2) into Eq.

[6] H. C. Montgomery, Bell System Tech. J. **31**, 950 (1952); Frances L. Lummis and R. L. Petritz, Phys. Rev. **105**, 502 (1957).

(8) and find to first order

$$\Delta R=\frac{dd_s[2\sigma_b d_b(R_s\sigma_s-R_b\sigma_b)\Delta\sigma_s+(\sigma_b d_b+\sigma_s d_s)\sigma_s^2\Delta R_s]}{(\sigma_b d_b+\sigma_s d_s)^3}. \quad (9)$$

By a similar analysis we find for Figs. 1(b) and 1(c), using Eqs. (4) and (6),

$$\Delta R=\frac{d_s[(R_s\sigma_s-R_b\sigma_b)d_b\Delta\sigma_s+(\sigma_b d_b+\sigma_s d_s)\sigma_s\Delta R_s]}{(\sigma_b d_b+\sigma_s d_s)^2}. \quad (10)$$

Equations (9) and (10) can be used to evaluate the sensitivity for arbitrary initial surface potentials. The sensitivity of configuration 1(a) is in general different from that of 1(b) and 1(c). However, these expressions simplify greatly when one assumes the initial condition of the energy bands to be flat, that is, the surface potential, ϕ_s, to be equal to ϕ_b (bulk potential):

$$R_s(\phi_s=\phi_b)=R_b, \quad \sigma_s(\phi_s=\phi_b)=\sigma_b. \quad (11)$$

Substituting the relations of Eq. (11) into Eqs. (9) and (10), we find for all three cases

$$\frac{\Delta R}{R_b}=\frac{d_s}{d}\frac{\Delta R_s}{R_b}. \quad (12)$$

This shows that for small changes in surface potential around the condition of flat energy bands, all three configurations have the same sensitivity. Furthermore, ΔR is proportional only to the change in surface Hall coefficient (ΔR_s); the change in surface conductivity does not enter directly.

In order to obtain an estimate of the sensitivity required, we compare ΔR with the change in conductivity. Using Eqs. (3), (5), and (7), we find

$$\Delta\sigma=\frac{d_s}{d}\Delta\sigma_s, \quad [\text{Figs. 1(a), 1(b)}];$$
$$\Delta\sigma=\frac{\sigma_b^2 dd_s\Delta\sigma_s}{(\sigma_s d_b+\sigma_b d_s)^2}, \quad [1(\text{c})]. \quad (13)$$

Assuming again for the initial condition that the bands are flat, we have for all three cases

$$\frac{\Delta\sigma}{\sigma_b}=\frac{d_s\Delta\sigma_s}{d\sigma_b}. \quad (14)$$

Comparison of Eqs. (12) and (14) shows that the geometrical factor, d_s/d, is the same for the Hall coefficient as for the conductivity.

To estimate the absolute magnitude of $\Delta R/R_b$ and to compare it with $\Delta\sigma/\sigma_b$, we consider a one-carrier model and small deviations from $\phi_s=\phi_b$:

$$\sigma_s=qn_s\mu_s, \quad R_s=c/n_s q, \quad c\cong 1,$$
$$\Delta n_s=n_s(\phi_s\neq\phi_b)-n_b, \quad \Delta\mu_s=\mu_s(\phi_s\neq\phi_b)-\mu_b, \quad (15)$$
$$n_s(\phi_s=\phi_b)=n_b, \quad \mu_s(\phi_s=\phi_b)=\mu_b,$$

$$\frac{\Delta\sigma_s}{\sigma_b}=\frac{\Delta n_s}{n_b}+\frac{\Delta\mu_s}{\mu_b}, \quad \left|\frac{\Delta R_s}{R_b}\right|=\left|\frac{\Delta n_s}{n_b}\right|. \quad (16)$$

Substituting Eqs. (16) into Eqs. (12) and (14), we find

$$\left|\frac{\Delta R}{R_b}\right|=\frac{d_s}{d}\left|\frac{\Delta n_s}{n_b}\right|, \quad (17)$$

$$\frac{\Delta\sigma}{\sigma_b}=\frac{d_s}{d}\left(\frac{\Delta n_s}{n_b}+\frac{\Delta\mu_s}{\mu_b}\right). \quad (18)$$

Neglecting changes in mobility, we see that

$$\left|\frac{\Delta R}{R_b}\right|\cong\frac{\Delta\sigma}{\sigma_b}\cong\frac{d_s}{d}\left|\frac{\Delta n_s}{n_b}\right|. \quad (19)$$

Thus one can expect the same order of magnitude changes in $\Delta R/R_b$ as in $\Delta\sigma/\sigma_b$.

The Bardeen-Brattain gas ambient cycle[7] enables one to obtain $0.01\leqslant\Delta\sigma/\sigma_b\leqslant 0.5$, in thin ($d\cong 1$ mil) high-resistivity germanium. This indicates that our experimental problem is to measure changes in R of the order of 1% to 50%. Further sensitivity can be attained by decreasing the sample thickness and by reducing the bulk density of carriers (e.g., by making the sample intrinsic or by cooling).

C. Selection of Experimental Approach

The above analysis shows that all three configurations have the same sensitivity for small changes in surface potential about $\phi_s=\phi_b$. Therefore, the selection is reduced to considerations of experimental technique. The first method, Fig. 1(a) is chosen because it embodies the features described in Sec. A.

In conclusion, the configuration shown in Fig. 1(a) offers a good possibility for studying surface Hall effect. The sensitivity needed is approximately that required in conductivity measurements.

We have found it possible to carry out such measurements on high-resistivity germanium at room temperature, using the configurations of Fig. 1(a) and a modified Bardeen-Brattain technique for varying the surface potential.[8,9] The results are in good agreement with the estimates of this paper.

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[7] W. H. Brattain and J. Bardeen, Bell System Tech. J. **82**, 1 (1953).

[8] A preliminary report of the experimental and theoretical work has been given: J. N. Zemel and R. L. Petritz, Bull. Am. Phys. Soc. Ser. II, **2**, 131 (1957).

[9] J. N. Zemel and R. L. Petritz, Phys. Rev. **110**, 1263 (1958), following paper.

II. BOLTZMANN EQUATION AND EFFECTIVE MOBILITY ANALYSIS OF CONDUCTIVITY, HALL COEFFICIENT AND MAGNETO-RESISTANCE

The two-region circuit analysis used in Part I involved simplifying approximations in regard to circulating currents and the variation of the potential perpendicular to the surface. In Sec. A below, we analyze the configuration of Fig. 1(a) by use of a one-dimensional Boltzmann equation. We drop the two-region assumption and allow the potential to vary continuously in the z direction. This allows for a rigorous derivation of the expressions for the Hall coefficient and magnetoresistance. In Part B we show that the z dependence can be eliminated by integration to define effective surface densities and mobilities. The use of the resulting expression for analyzing experimental data is discussed in Sec. C. An interesting feature of the derivation is that a specific model of the surface is not assumed.

A. Solution of z Dependence of Hall Coefficient and Magnetoresistance

We first require an expression for superposing the z dependence of the Hall coefficient and magnetoresistance, analogous to the well-known expression for the conductivity,

$$\sigma=(1/d)\int_0^d \sigma(z)dz. \tag{20}$$

The previous solution for the Hall coefficient [Eq. (2)] obtained by circuit analysis generalizes to continuous variation in the z direction as[10]

$$R=d\int_0^d R(z)\sigma^2(z)dz \bigg/ \left[\int_0^d \sigma(z)dz\right]^2. \tag{21}$$

Presumably one should use the multicarrier formulas for the local Hall coefficient, $R(z)$, and the local conductivity, $\sigma(z)$, in Eq. (21). However, this point was not considered in the circuit analysis since the problem of circulating currents was not adequately covered. To show that this is the correct procedure and to derive an analogous expression for the magnetoresistance, we use the Boltzmann equation. Assuming spherical energy surfaces and a magnetic field, H, in the z direction, we have

$$-\frac{q}{m}\left(\mathbf{E}+\frac{1}{c}\mathbf{v}\times\mathbf{H}\right)\mathrm{grad}_{\mathbf{v}}f+\mathbf{v}\cdot\mathrm{grad}_{\mathbf{r}}f=-\frac{f-f_0}{\tau}, \tag{22}$$

$$E_z=-\partial\phi/\partial z,\quad \mathbf{H}=H\mathbf{k}.$$

The solution to this is written as

$$f=f_0+f_1(\mathbf{v},z),\quad f_0=ce^{-[\epsilon-q\phi(z)]/kT}.$$

[10] J. N. Zemel (private communication).

The function $f_1(\mathbf{v},z)$ is determined by solving Eq. (22) subject to appropriate boundary conditions for surface scattering. The current densities are then found by use of

$$j_x(z)=\iiint d\mathbf{v}\; v_x^2 f_1(\mathbf{v},z),$$

$$j_y(z)=\iiint d\mathbf{v}\; v_y^2 f_1(\mathbf{v},z).$$

For small magnetic fields, j_x and j_y can be written formally as

$$j_x(z)=|q|\textstyle\sum_k n_k(z)\langle\mu_k(z)\rangle E_x+\sum_k q_k n_k(z)\langle\mu_k^2(z)\rangle HE_y - |q|\sum_k n_k(z)\langle\mu_k^3(z)\rangle H^2E_x, \tag{23}$$

$$j_y(z)=|q|\textstyle\sum_k n_k(z)\langle\mu_k(z)\rangle E_y-\sum_k q_k n_k(z)\langle\mu_k^2(z)\rangle HE_x - |q|\sum_k n_k(z)\langle\mu_k^3(z)\rangle H^2E_y, \tag{24}$$

where $n_k(z)$ is the density of the kth specimen (holes or electrons) and $\langle\mu_k^m(z)\rangle$ are appropriate averages over momentum space; q_k is positive for holes, negative for electrons. The above expressions are quite general, being a result of the spherical energy surfaces, the small-magnetic-field approximation, and the linearity of the problem, i.e., the carriers contribute additively to the current.

In the bulk or interior of the crystal where the surface boundary condition is negligible,

$$\langle\mu_{kb}^m\rangle=\frac{q}{m_k}\frac{\langle\epsilon\tau^m\rangle}{\langle\epsilon\rangle} =\frac{q}{m_k}\iiint d\mathbf{v}\epsilon\tau^m(\epsilon)f_b\bigg/\iiint d\mathbf{v}\epsilon f_b, \tag{25}$$

where $\tau(\epsilon)$ is the relaxation time of the bulk scattering process, ϵ is the energy, and f_b is the bulk distribution function.

In the region near the surface, $\langle\mu_k^m(z)\rangle$ will differ from $\langle\mu_{kb}^m\rangle$ because of surface scattering. We shall not derive explicit expressions for $\langle\mu_k^m(z)\rangle$; Schrieffer has done essentially this for $\langle\mu_k(z)\rangle$ for two models of the surface region. Instead we shall continue our analysis keeping $\langle\mu_k^m(z)\rangle$ as formal averages over momentum space.

We next derive expressions for the Hall coefficient and magnetoresistance from Eqs. (23) and (24). The total currents for a sample of thickness d are

$$I_x=(w/L)\int_0^d j_x(z)dz, \tag{26}$$

$$I_y=(L/w)\int_0^d j_y(z)dz=0, \tag{27}$$

where the boundary condition is that the total current in the y direction is zero. This boundary condition takes into account all circulating currents, including those due to having more than one kind of carrier, and those due to the z dependence of the problem.

We can take E_x and E_y from under the integral sign because they are not functions of z (one-dimensional nature of the problem and curl $\mathbf{E}=0$). Substituting Eqs. (23) and (24) into Eqs. (26) and (27) and solving, we find for the conductivity and Hall coefficient (to order H)

$$\sigma=\frac{I_x}{E_x dw}=\frac{1}{d}|q|\sum_k\int_0^d n_k(z)\langle\mu_k(z)\rangle dz, \tag{28}$$

$$R=\frac{E_y dw}{HI_x}=d\sum_k\int_0^d q_k n_k(z)\langle\mu_k^2(z)\rangle dz\Big/\left[|q|\sum_k\int_0^d n_k(z)\langle\mu_k(z)\rangle dz\right]^2. \tag{29}$$

We note that if the local Hall coefficient and conductivity are defined as

$$R(z)=\textstyle\sum_k q_k n_d(z)\langle\mu_k^2(z)\rangle/[|q|\sum_k n_k(z)\langle\mu_k(z)\rangle]^2, \tag{30}$$

$$\sigma(z)=|q|\textstyle\sum_k n_k(z)\langle\mu_k(z)\rangle, \tag{31}$$

we can write Eq. (29) as Eq. (21), in agreement with the result derived by circuit analysis.

Similarly, we find for the magnetoresistance, to order H^2,

$$\frac{\Delta\rho}{\rho H^2}=\frac{|\Delta\sigma|}{\sigma_0 H^2}=\frac{I_x^0-I_x}{I_x^0 H^2}=\frac{\sum_k\int_0^d n_k(z)\langle\mu_k^3(z)\rangle dz}{\sum_k\int_0^d n_k(z)\langle\mu_k(z)\rangle dz}-\frac{\left[\sum_k\int_0^d q_k n_k(z)\langle\mu_k^2(z)\rangle dz\right]^2}{\left[|q|\sum_k\int_0^d n_k(z)\langle\mu_k(z)\rangle dz\right]^2}. \tag{32}$$

Whereas we were able to derive the Hall coefficient by circuit analysis, we have not been able to do so for the magnetoresistance.

B. Effective Mobility Formalism

The interpretation and analysis of surface conductance data has been simplified by the concept of the effective mobility of carriers in the space-charge region.[4] This enables one to calculate the total conductance as a function of surface potential for samples of arbitrary shape and composition by a simple superposition of bulk and surface contributions. We shall show that a similar method can be used for the Hall coefficient and magnetoresistance with corresponding usefulness. This transformation is carried out in the Appendix and the results are summarized in the following sections.

1. Conductivity.—We find that the one-carrier conductivity [Eq. (28)] can be expressed [Appendix, Sec. 1, Eq. (A-12)] as

$$\begin{gathered}\sigma d/q=n_b\langle\mu_b\rangle d_b+n_s\langle\mu_s\rangle d_s+\Delta n_s\langle\mu_c\rangle d_s,\\ d=d_b+d_s,\quad n_s=n_b+\Delta n_s,\end{gathered} \tag{33}$$

where n_b is the bulk density of carriers, $\langle\mu_b\rangle$ is the bulk "conductivity" mobility [defined in Eq. (25) with $m=1$], d is the total thickness, d_s is the effective thickness of the surface region and is of the order of the Debye length [Eq. (1)], n_s is the effective total density of surface carriers [Eqs. (A-5) and (A-11)], $\langle\mu_s\rangle$ is the effective "conductivity" mobility of surface carriers [Eqs. (A-4) and (A-10)], and $\langle\mu_c\rangle$ is a correlation mobility [Eq. (A-8)]. The first and second terms represent the bulk and surface contributions respectively, while the third term is a correlation term that has been ignored in previous treatments.[4] It is included for completeness but can be expected to be small and probably negligible [see discussion following Eq. (A-8)].

It is useful for calculations to subtract out the bulk conductance of the sample. This is most conveniently done when the bands are flat:

$$\sigma(\phi_s=\phi_b)=\sigma_b=qn_b\langle\mu_b\rangle. \tag{34}$$

Thus,

$$(d/qd_s)(\sigma-\sigma_b)=\Delta n_s\langle\mu_s\rangle-n_b(\langle\mu_b\rangle-\langle\mu_s\rangle)+\Delta n_s\langle\mu_c\rangle. \tag{35}$$

This can be interpreted as follows. The first term represents the change in conductance due to excess surface carriers of effective mobility $\langle\mu_s\rangle$. The second term represents a decrease in conductance due to a reduction in mobility of charge that already is present in the surface region. Finally, the third term represents a spatial correlation between $\Delta n(z)$ and $\Delta\mu(z)$. Calculations [2–4] show that Δn_s changes much faster than the effective mobility; therefore, we can expect that in high-resistivity materials the first term is the only one of importance. In low-resistivity materials (n_b large), it may be necessary to include the second term. The third term can probably be ignored in all practical cases, although further study is required to justify this.

When three carriers are involved, as in the case of germanium and silicon, we find

$$\begin{aligned}(d/qd_s)&(\sigma-\sigma_b)\\ &=\{\Delta n_s\langle\mu_{ns}\rangle+[\Delta p_s/(1+r)](\langle\mu_{2s}\rangle+r\langle\mu_{3s}\rangle)\}\\ &\quad-\{n_b(\langle\mu_{nb}\rangle-\langle\mu_{ns}\rangle)+[p_b/(1+r)]\\ &\quad\times[(\langle\mu_{2b}\rangle-\langle\mu_{2s}\rangle)+r(\langle\mu_{3b}\rangle-\langle\mu_{3s}\rangle)]\}\\ &\quad+\{\Delta n_s\langle\mu_{nc}\rangle+[\Delta p_s/(1+r)](\langle\mu_{2c}\rangle+r\langle\mu_{3c}\rangle)\},\end{aligned} \tag{36}$$

$$\sigma_b=q(n_b\langle\mu_{nb}\rangle+p_b\langle\mu_{pb}\rangle),\quad \langle\mu_{pb}\rangle=\frac{\langle\mu_{2b}\rangle+r\langle\mu_{3b}\rangle}{1+r}, \tag{37}$$

where $p_b = n_2 + n_3$ is the total bulk density of holes, $r = n_3/n_2$ is the fraction of light to heavy holes, and subscripts 2 and 3 refer to heavy and light holes, respectively.

2. Hall coefficient.—The expression for the Hall coefficient [Eq. (29)] is transformed into an effective mobility formalism in the Appendix [Sec. 2, Eq. (A-15)], where we find for the one-carrier case

$$-R\sigma^2 d/q = n_b\langle\mu_b{}^2\rangle d_b + n_s\langle\mu_s{}^2\rangle d_s + \Delta n_s\langle\mu_c{}^2\rangle d_s, \quad (38)$$

where $\langle\mu_b{}^2\rangle$ is the bulk "Hall" mobility [Eq. (25) with $m=2$], $\langle\mu_s{}^2\rangle$ is the effective "Hall" mobility of surface carriers [Eq. (A-16)], and $\langle\mu_c{}^2\rangle$ is the correlation "Hall" mobility [Eq. (A-17)]. Note that $\langle\mu_b{}^2\rangle$ is not the Hall mobility as normally denoted by $\mu_H = R\sigma$, but rather $\langle\mu_b{}^2\rangle = \mu_H\langle\mu_b\rangle$. We note that except for the correlation term, Eq. (38) is the same as Eq. (2) used in the two-region analysis.

For calculational purposes the appropriate bulk term to subtract off is

$$-R_b\sigma_b{}^2/q = n_b\langle\mu_b{}^2\rangle. \quad (39)$$

Thus

$$-(d/qd_s)(R\sigma^2 - R_b\sigma_b{}^2) = \Delta n_s\langle\mu_s{}^2\rangle - n_b(\langle\mu_b{}^2\rangle - \langle\mu_s{}^2\rangle) + \Delta n_s\langle\mu_c{}^2\rangle. \quad (40)$$

The terms are analogous to those appearing in the conductance [Eq. (35)].

The generalization of Eq. (40) to the three-carrier model leads to

$$\begin{aligned}-(d/qd_s)(R\sigma^2 - R_b\sigma_b{}^2) &= \{\Delta n_s\langle\mu_{ns}{}^2\rangle - [\Delta p_s/(1+r)](\langle\mu_{2s}{}^2\rangle + r\langle\mu_{3s}{}^2\rangle)\} \\ &\quad - \{n_b(\langle\mu_{nb}{}^2\rangle - \langle\mu_{ns}{}^2\rangle) - [p_b/(1+r)] \\ &\quad \times[(\langle\mu_{2b}{}^2\rangle - \langle\mu_{2s}{}^2\rangle) + r(\langle\mu_{3b}{}^2\rangle - \langle\mu_{3s}{}^2\rangle)]\} \\ &\quad + \{\Delta n_s\langle\mu_{nc}{}^2\rangle - [\Delta p_s/(1+r)] \\ &\quad \times(\langle\mu_{2c}{}^2\rangle + r\langle\mu_{3c}{}^2\rangle)\}, \quad (41)\end{aligned}$$

$$-R_b\sigma_b{}^2 = q\{n_b\langle\mu_{nb}{}^2\rangle - [p_b/(1+r)] \times(\langle\mu_{2b}{}^2\rangle + r\langle\mu_{3b}{}^2\rangle)\}. \quad (42)$$

3. Magnetoresistance.—The expression for the magnetoresistance [Eq. (32)] is transformed into an effective mobility formalism in the Appendix [Sec. 3, Eq. (A-21)], where we find for the one-carrier case

$$\frac{d\sigma}{q}\left[\frac{\Delta\rho}{\rho H^2} + (R\sigma)^2\right] = n_b\langle\mu_b{}^3\rangle d_b + n_s\langle\mu_s{}^3\rangle d_s + \Delta n_s\langle\mu_c{}^3\rangle d_s, \quad (43)$$

where $\langle\mu_b{}^3\rangle$ is the bulk "magnetoresistance" mobility [Eq. (25) with $m=3$], $\langle\mu_s{}^3\rangle$ is the effective "magnetoresistance" mobility of surface carriers [Eq. (A-23)], and $\langle\mu_c{}^3\rangle$ is the correlation "magnetoresistance" mobility [Eq. (A-24)].

For calculation purposes it is again useful to subtract a bulk quantity; the appropriate one in this case is

$$(\sigma_b/q)[(\Delta\rho/\rho H^2)_b + (R_b\sigma_b)^2] = n_b\langle\mu_b{}^3\rangle. \quad (44)$$

Subtracting Eq. (44) from Eq. (43), we have

$$\frac{d}{qd_s}\left\{\sigma\left[\frac{\Delta\rho}{\rho H^2} + (R\sigma)^2\right] - \sigma_b\left[\left(\frac{\Delta\rho}{\rho H^2}\right)_b + (R_b\sigma_b)^2\right]\right\} = \Delta n_s\langle\mu_s{}^3\rangle - n_b(\langle\mu_b{}^3\rangle - \langle\mu_s{}^3\rangle) + \Delta n_s\langle\mu_c{}^3\rangle. \quad (45)$$

The terms are analogous to those appearing in the conductance [Eq. (35)]. Generalizing to three carriers, we find

$$\begin{aligned}\frac{d}{qd_s}&\left\{\sigma\left[\frac{\Delta\rho}{\rho H^2} + (R\sigma)^2\right] - \sigma_b\left[\left(\frac{\Delta\rho}{\rho H^2}\right)_b + (R_b\sigma_b)^2\right]\right\} \\ &= \{\Delta n_s\langle\mu_{ns}{}^3\rangle + [\Delta p_s/(1+r)](\langle\mu_{2s}{}^3\rangle + r\langle\mu_{3s}{}^3\rangle)\} \\ &\quad - \{n_b(\langle\mu_{nb}{}^3\rangle - \langle\mu_{ns}{}^3\rangle) + [p_b/(1+r)] \\ &\quad \times[(\langle\mu_{2b}{}^3\rangle - \langle\mu_{2s}{}^3\rangle) + r(\langle\mu_{3b}{}^3\rangle - \langle\mu_{3s}{}^3\rangle)]\} \\ &\quad + \{\Delta n_s\langle\mu_{nc}{}^3\rangle + [\Delta p_s/(1+r)](\langle\mu_{2c}{}^3\rangle + r\langle\mu_{3c}{}^3\rangle)\}, \quad (46)\end{aligned}$$

$$\frac{\sigma_b}{q}\left[\left(\frac{\Delta\rho}{\rho H^2}\right)_b + (R_b\sigma_b)^2\right] = n_b\langle\mu_{nb}{}^3\rangle + [p_b/(1+r)](\langle\mu_{2b}{}^3\rangle + r\langle\mu_{3b}{}^3\rangle). \quad (47)$$

C. Discussion

Equations (36), (41), and (46) give the conductivity, Hall coefficient, and magnetoresistance in terms of carrier densities and effective mobilities. There are several ways that these expressions can be used. One is to consider the surface carrier densities and effective mobilities as unknowns and from experimental data to find values for them. This is possible because of the generality of the derivation. In writing down Eqs. (23) and (24) we assumed only spherical energy surfaces, linearity of the currents in carrier density, and small magnetic fields. Our next assumption was that the carrier density and mobility deviated from the bulk values only near the surface [Eq. (A-2)]. The analysis then followed rigorously to the derivation of Eqs. (36), (41), and (46). Thus, these solutions have general validity for surface problems. The restriction to spherical energy surfaces can be removed by generalizing Eqs. (23) and (24) to tensor form.

To illustrate their use in finding values of surface carrier densities and mobilities from experimental data, suppose that the bulk and surface are sufficiently n type so that holes can be neglected. We also neglect the correlation term. Then Eqs. (36) and (41) reduce to

$$(d/qd_s)(\sigma - \sigma_b) = \Delta n_s\langle\mu_{ns}\rangle - n_b(\langle\mu_{nb}\rangle - \langle\mu_{ns}\rangle), \quad (48)$$

$$-(d/qd_s)(R\sigma^2 - R_b\sigma_b{}^2) = \Delta n_s\langle\mu_{ns}{}^2\rangle - n_b(\langle\mu_{nb}{}^2\rangle - \langle\mu_{ns}{}^2\rangle). \quad (49)$$

The unknown quantities in these equations are Δn_s, $\langle\mu_{ns}\rangle$, $\langle\mu_{ns}^2\rangle$ and d_s. The quantities $(\sigma-\sigma_b)$ and $(R\sigma^2-R_b\sigma_b^2)$ are determined from experiment; the surface potential being changed by a suitable means (gas ambient, field effect, etc.). The bulk quantities are assumed to be known. If one assumes a relation between $\langle\mu_{ns}\rangle$ and $\langle\mu_{ns}^2\rangle$, the values of $\Delta n_s d_s$ and $\langle\mu_{ns}\rangle$ can be uniquely determined from the experimental values of R and σ. The surface potential does not enter such an analysis. If $\langle\mu_{ns}\rangle$ and $\langle\mu_{ns}^2\rangle$ are considered independent, two relations among three quantities are obtained. When more than one carrier must be considered, one must assume relationships between carrier mobilities and relationships between carrier densities to obtain unique answers.

Another usage of Eqs. (36), (41), and (46) is in connection with theoretical expressions for the surface quantities. For the Poisson model of the surface, the densities Δn_s and Δp_s, are found by solution of Poisson's equation and are tabulated[2,3] as functions of u_s and u_b, where $u=q\phi/kT$; d_s is equal to the Debye length [Eq. (1)]. The effective surface mobilities can in principle be found by a solution of the Boltzmann equation subject to appropriate boundary conditions. Schrieffer has done this for the "conductivity" mobility $\langle\mu_s\rangle$, assuming an energy-independent collision time and spherical energy surfaces.[4] He obtained curves of $\langle\mu_s\rangle$ for both the Poisson and the linear potential models of the space-charge region. Recently, Zemel has extended Schrieffer's solution to include galvanomagnetic effects.[11] Zemel finds an explicit expression for $\langle\mu_s^2\rangle$ for the linear potential model, but the assumption of constant collision time is not adequate to determine the "magnetoresistance" mobility, $\langle\mu_s^3\rangle$.

The correlation terms in Eqs. (36), (41), and (46) can also be evaluated from a solution of the Boltzmann equation with appropriate boundary conditions. While the correlation contribution should be small, it would be of interest to prove this rigorously.

Use of theoretical expressions for the carrier densities and mobilities allows for the calculation of σ, R, and $\Delta\rho/\rho$ as a function of surface potential. The surface potential can then be eliminated and one has explicit relations between R, σ, and $\Delta\rho/\rho$. The uniqueness of these relations is a feature of this method of studying the conduction process in the space-charge region of a semiconductor surface. Other methods generally introduce additional unknowns; for example, field-effect experiments depend upon the number of surface charges immobilized in surface states, which is usually not known.

While we have considered only two methods of using Eqs. (36), (41), and (46) in connection with experiments, it is clear that other methods are possible. For example, in order to study the sensitivity of the experiment for the determination of surface mobilities as a function of surface potential, we have calculated curves of R, σ, and $\Delta\rho/\rho H^2$ *versus* u_s. We first used surface mobilities equal to bulk mobilities; then we used surface mobilities as given by Schrieffer.[4] The carrier densities were taken from theory.[2,3] These curves check out the results of the small-signal analysis given in Part I, and provide a theoretical basis for interpreting galvanomagnetic experiments on semiconductor surfaces. They are presented and discussed with the experimental data in the following paper.[9]

[11] J. N. Zemel, Bull. Am. Phys. Soc. Ser. II, 3, 105 (1958).

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APPENDIX. TRANSFORMATION TO EFFECTIVE MOBILITY FORMALISM

We now transform the expressions of Eqs. (28), (29), and (32) into an effective mobility formalism. We consider first the conductivity, to exhibit the general method and to derive more complete expressions than those appearing in the literature.

1. Conductivity.—Considering one carrier and dropping unnecessary subscripts, we have from Eq. (28)

$$\sigma d/q=\int_0^d n(z)\langle\mu(z)\rangle dz. \tag{A-1}$$

We write

$$n(z)=n_b+\Delta n(z),\quad \langle\mu(z)\rangle=\langle\mu_b\rangle+\Delta\langle\mu(z)\rangle, \tag{A-2}$$

where $\langle\mu_b\rangle$ is the bulk "conductivity" mobility [defined in Eq. (25) with $m=1$], n_b is the bulk carrier density, $\Delta n(z)$ and $\Delta\langle\mu(z)\rangle$ differ from zero only in the space-charge region. The effective thickness of the space-charge region is denoted by d_s. In general, d_s is of the order of the Debye thickness, L_D, but its specific value will depend on the model. For example, in the Poisson field $d_s=L_D$, but in the case of the linear potential model d_s may differ from L_D. The brackets $\langle\ \rangle$ denote averages over velocity space. Substituting Eq. (A-2) into Eq. (A-1), we find

$$\sigma d/q=n_b\langle\mu_b\rangle d+n_b\int_0^d\Delta\langle\mu(z)\rangle dz+\langle\mu_b\rangle\int_0^d\Delta n(z)dz+\int_0^d\Delta n(z)\Delta\langle\mu(z)\rangle dz. \tag{A-3}$$

Now the three integrands are nonzero only in the space-charge region. We therefore define three space averages:

$$[\Delta\langle\mu\rangle]_{\text{Av}}=(1/d_s)\int_0^d \Delta\langle\mu(z)\rangle dz, \qquad \text{(A-4)}$$

$$\Delta n_s=(1/d_s)\int_0^d \Delta n(z)dz, \quad \text{effective excess density of surface carriers,} \qquad \text{(A-5)}$$

$$[\Delta n\Delta\langle\mu\rangle]_{\text{Av}}=(1/d_s)\int_0^d \Delta n(z)\Delta\langle\mu(z)\rangle dz. \qquad \text{(A-6)}$$

Substituting Eqs. (A-4), (A-5), (A-6) into Eq. (A-3), we have

$$\sigma d/q=n_b\langle\mu_b\rangle d+n_b[\Delta\langle\mu\rangle]_{\text{Av}}d_s+\Delta n_s\langle\mu_b\rangle d_s +[\Delta n\Delta\langle\mu\rangle]_{\text{Av}}d_s. \qquad \text{(A-7)}$$

We define a correlation mobility

$$\langle\mu_c\rangle=\{[\Delta n\Delta\langle\mu\rangle]_{\text{Av}}-\Delta n_s[\Delta\langle\mu\rangle]_{\text{Av}}\}/\Delta n_s. \qquad \text{(A-8)}$$

$\langle\mu_c\rangle$ represents a spatial correlation between $\Delta n(z)$ and $\Delta\mu(z)$; it is zero when the surface and bulk potentials are equal, but otherwise will not in general be zero. Since $\langle\mu_c\rangle$ depends on the difference of two spatial averages we would expect it to be small, but direct evaluation by means of explicit solution of the Boltzmann equation is necessary for rigorous justification of this.

Substituting Eq. (A-8) into Eq. (A-7), we have

$$\sigma d/q=n_b\langle\mu_b\rangle d+n_b[\Delta\langle\mu\rangle]_{\text{Av}}d_s+\Delta n_s\langle\mu_b\rangle d_s +\Delta n_s\langle\mu_c\rangle d_s+\Delta n_s[\Delta\langle\mu\rangle]_{\text{Av}}d_s. \qquad \text{(A-9)}$$

We now define the effective surface mobility and density as

$$\langle\mu_s\rangle=\langle\mu_b\rangle+[\Delta\langle\mu\rangle]_{\text{Av}}, \qquad \text{(A-10)}$$

$$n_s=n_b+\Delta n_s, \qquad \text{(A-11)}$$

where in general we expect $\langle\mu_s\rangle\leqslant\langle\mu_b\rangle$. Substituting Eqs. (A-10) and (A-11) into Eq. (A-9), we have

$$\sigma d/q=n_b\langle\mu_b\rangle d_b+n_s\langle\mu_s\rangle d_s+\Delta n_s\langle\mu_c\rangle d_s, \qquad \text{(A-12)}$$

where $d=d_b+d_s$.

Thus we have expressed the total conductance in terms of a bulk contribution $n_b\langle\mu_b\rangle d_b$, a surface contribution $n_s\langle\mu_s\rangle d_s$, and a correlation term $\Delta n_s\langle\mu_c\rangle d_s$. Except for small effects due to the correlation mobility not being identically zero, the effective surface mobility defined by Eq. (A-10) is defined in the same way as the effective surface mobility of Schrieffer.[4] Note that $\langle\mu_s\rangle$, $\langle\mu_c\rangle$, n_s, and Δn_s all involve a spatial average as well as a momentum average. The momentum average is explicitly indicated by the brackets $\langle\ \rangle$, while the spatial average is indicated by the subscript s or c.

2. Hall Coefficient.—We now transform Eq. (29) into an effective-mobility formalism. Considering one carrier, we have

$$-R\sigma^2 d/q=\int_0^d n(z)\langle\mu^2(z)\rangle dz. \qquad \text{(A-13)}$$

We write

$$\langle\mu^2(z)\rangle=\langle\mu_b^2\rangle+\Delta\langle\mu^2(z)\rangle, \qquad \text{(A-14)}$$

where $\langle\mu_b^2\rangle$ is the bulk "Hall" mobility defined by Eq. (25) with $m=2$. Substituting Eq. (A-14) into Eq. (A-13), by an analysis similar to that for conductivity, we find

$$-R\sigma^2 d/q=n_b\langle\mu_b^2\rangle d_b+n_s\langle\mu_s^2\rangle d_s+\Delta n_s\langle\mu_c^2\rangle d_s, \qquad \text{(A-15)}$$

where

$$\langle\mu_s^2\rangle=\langle\mu_b^2\rangle+[\Delta\langle\mu^2\rangle]_{\text{Av}}, \quad \text{effective surface "Hall" mobility;} \qquad \text{(A-16)}$$

$$\langle\mu_c^2\rangle=\{[\Delta n\Delta\langle\mu^2\rangle]_{\text{Av}}-\Delta n_s[\Delta\langle\mu^2\rangle]_{\text{Av}}\}/\Delta n_s, \quad \text{correlation "Hall" mobility;} \qquad \text{(A-17)}$$

$$[\Delta\langle\mu^2\rangle]_{\text{Av}}=(1/d_s)\int_0^d \Delta\langle\mu^2(z)\rangle dz, \qquad \text{(A-18)}$$

$$[\Delta n\Delta\langle\mu^2\rangle]_{\text{Av}}=(1/d_s)\int_0^d \Delta n(z)\Delta\langle\mu^2(z)\rangle dz. \qquad \text{(A-19)}$$

3. Magnetoresistance.—Considering one carrier, we have from Eq. (32)

$$\frac{\sigma d}{q}\left[\frac{\Delta\rho}{\rho H^2}+(R\sigma)^2\right]=\int_0^d n(z)\langle\mu^3(z)\rangle dz. \qquad \text{(A-20)}$$

We find, by an analysis similar to that for σ and R, that

$$\frac{\sigma d}{q}\left[\frac{\Delta\rho}{\rho H^2}+(R\sigma)^2\right]=n_b\langle\mu_b^3\rangle d_b+n_s\langle\mu_s^3\rangle d_s +\Delta n_s\langle\mu_c^3\rangle d_s, \qquad \text{(A-21)}$$

where

$$\langle\mu^3(z)\rangle=\langle\mu_b^3\rangle+\Delta\langle\mu^3(z)\rangle, \qquad \text{(A-22)}$$

$$\langle\mu_s^3\rangle=\langle\mu_b^3\rangle+[\Delta\langle\mu^3\rangle]_{\text{Av}}, \quad \text{effective surface "magnetoresistance" mobility,} \qquad \text{(A-23)}$$

$$\langle\mu_c^3\rangle=\{[\Delta n\Delta\langle\mu^3\rangle]_{\text{Av}}-\Delta n_s[\Delta\langle\mu^3\rangle]_{\text{Av}}\}/\Delta n_s, \quad \text{correlation "magnetoresistance" mobility;} \qquad \text{(A-24)}$$

$$[\langle\Delta\mu^3\rangle]_{\text{Av}}=(1/d_s)\int_0^d \Delta\langle\mu^3(z)\rangle dz, \qquad \text{(A-25)}$$

$$[\Delta n\Delta\langle\mu^3\rangle]_{\text{Av}}=(1/d_s)\int_0^d \Delta n(z)\Delta\langle\mu^3(z)\rangle dz, \qquad \text{(A-26)}$$

and $\langle\mu_b^3\rangle$ is the bulk "magnetoresistance" mobility, defined by Eq. (25) with $m=3$.