Contribution of Three-Body Forces to the Binding of Hyperfragments*

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The potential energy of Λ in the hypertriton due to two- and three-body pion-exchange forces is estimated. The contributions from the two types of forces are found to be of the same order of magnitude, thus indicating that the three-body force produces the binding of Λ in the hypertriton. The role of the three-body force in the binding of other light hyperfragments is briefly discussed.

THE forces that give rise to the binding of Λ in nuclear matter have been attributed to the exchange of both π and K mesons.[1] Although, as has been pointed out by Henley,[2] the lowest order diagrams that contribute to pion-exchange Λ-nucleon and Λ-two-nucleon charge-independent forces are of the same order, only the effects of two-body forces have been considered in most treatments of the interaction between Λ and nucleons. The purpose of this note is to examine the role of three-body pion-exchange forces in the binding of hyperfragments.

We shall use the fixed-source pion-hyperon Hamiltonian for Λ and Σ with spin $\frac{1}{2}$ and same parity[3] to calculate the lowest order three-body potential between a Λ and two nucleons. Only diagrams in which at least one pion appears in all intermediate states will be considered. As has been shown in LR, the contribution to the Λ-nucleon potential from diagrams in which there are intermediate states with no pions present is considerably overestimated when the kinetic energy of the baryons in these states is neglected. The contribution from such diagrams has been included in LR by using a second order nondiagonal potential that converts a Λ into a Σ, and we shall therefore omit them entirely. The calculation of the potential is straightforward. We obtain for the contribution from the eight diagrams of the type given in Fig. 1:

$$\begin{aligned}V_{3B}=&-\mu(f^2/4\pi)(h^2/4\pi)(2/3\pi)\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\\&\times\{\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2[A_1(v,w)+A_2(v,w)\cos^2\theta]+A_3(v,w)S_{12,1}\\&+A_4(v,w)S_{12,2}+A_5(v,w)\sin^2\theta S_{12,3}+A_6(v,w)\\&\times\cos\theta[(v/w)S_{12,1}+(w/v)S_{12,2}-(x^2/vw)S_{12,4}]\}.\quad(1)\end{aligned}$$

Here, the $\boldsymbol{\tau}_i$ and $\boldsymbol{\sigma}_i$ are the isotopic spin and spin operators of the two nucleons; v, w, and x denote, respectively, the two Λ-nucleon and the nucleon-nucleon separations[4]; θ is the angle with vertex at the position of the Λ and sides along $\mathbf{v}$ and $\mathbf{w}$; the $S_{12,j}$ have the form of the usual tensor operator S_{12} for two nucleons, with $\mathbf{x}$ replaced by $\mathbf{x}_j$:

$$\begin{aligned}&\mathbf{x}_1=\mathbf{v},\\&\mathbf{x}_2=\mathbf{w},\\&\mathbf{x}_3=(\mathbf{v}\times\mathbf{w})/|\mathbf{v}\times\mathbf{w}|,\\&\mathbf{x}_4=\mathbf{x};\end{aligned}$$

and the A_k are given by:

$$A_1(v,w)=\frac{2e^{-w}(1-2w)}{w^3}K_0(v)-\frac{2e^{-w}(4+w+w^2)}{w^3v}K_1(v)+\text{Sym.},$$

$$A_2(v,w)=-\frac{e^{-w}(4-10w-w^2)}{w^3}K_0(v)+\frac{e^{-w}(16+2w+3w^2)}{w^3v}K_1(v)+\text{Sym.},$$

$$A_3(v,w)=\frac{e^{-w}(1-5w)}{2w^3}K_0(v)-\frac{e^{-v}(1-v)}{2v^3}K_0(w)-\frac{e^{-w}(4-2w-w^2)}{2w^3v}K_1(v)-\frac{e^{-v}(2+4v+v^2)}{2v^3w}K_1(w),$$

$$A_4(v,w)=A_3(w,v),$$

$$A_5(v,w)=\frac{e^{-w}(1-w)}{2w^3}K_0(v)-\frac{e^{-w}(4+2w+w^2)}{2w^3v}K_1(v)+\text{Sym.},$$

$$A_6(v,w)=-\frac{e^{-w}(2-8w-w^2)}{2w^3}K_0(v)+\frac{e^{-w}(8-2w+w^2)}{2w^3v}K_1(v)+\text{Sym.},$$

where the symbol "Sym." means that the expression is to be symmetrized with respect to v and w.

It is interesting to note that the potential is independent of the spin orientation of Λ. At large distances

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1 For example, see N. Dallaporta and F. Ferrari, Nuovo cimento **5**, 111 (1957); D. B. Lichtenberg and M. H. Ross, Phys. Rev. **107**, 1714 (1957).

2 E. M. Henley, Phys. Rev. **106**, 1083 (1957).

3 For example, see D. B. Lichtenberg and M. H. Ross, Phys. Rev. **107**, 1714 (1957). This paper will be referred to hereafter as LR.

4 We use the pion Compton wavelength as a length unit throughout this paper.

it is effectively the product of two two-body potentials each of which has unit range. For small distances it becomes singular, and we adopt repulsive cores for $v, w \leq 0.5$.

To compare the strengths of the two- and three-body forces, we shall estimate the potential energy of Λ in the hypertriton due to the nontensor part of each potential, neglecting the variation of the Λ wave function over the deuteron and the distortion of the deuteron. We denote by U_i the potential energy due to the two-body force when the Λ and nucleon are in the state with spin i; by U_{jk} the potential energy due to the three-body force when the two nucleons are in the state with isotopic spin j and spin k. We then have

$$\sum_i U_i = \int \psi^*(x) V_{2B,c}(v) \psi(x) d\tau, \tag{2a}$$

$$\sum_{j,k} U_{jk} = \int \psi^*(x) V_{3B,c}(v,w,\theta) \psi(x) d\tau, \tag{2b}$$

where $V_{2B,c}(v)$ is the central part of $V_{g\pi}$ as given by Dallaporta and Ferrari[1]; $V_{3B,c}(v,w,\theta)$ is the nontensor part of V_{3B} in Eq. (1); $\psi(x)$ is the normalized wave function of the deuteron; and $d\tau$ is a six-dimensional volume element.

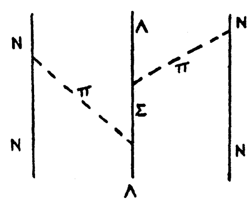

FIG. 1. Type of diagram contributing to lowest order three-body pion-exchange potential between Λ and two nucleons.

The integrand in Eq. (2b) decreases somewhat more slowly, for increasing values of both v and w, than the one in Eq. (2a). Thus, at least for this simply described hypertriton, $V_{3B,c}$ has a longer effective range than $V_{2B,c}$. We have evaluated the integrals in Eqs. (2) by expanding the integrands in products of power series in x, v, and w and integrating out from the repulsive cores. Using the values $(f^2/4\pi) = (h^2/4\pi) = 0.1$ for the coupling constants, we obtain

$$U_0 = -65 \text{ Mev},$$
$$U_1 = -34 \text{ Mev},$$
$$U_{10} = U_{01} = -17 \text{ Mev};$$

U_{00} and U_{11} are repulsive, but they play a role only when relative p-states (or higher odd states) of the two nucleons become effective. The uncertainty in the numerical values, arising from the approximations made in the evaluation of the integrals, is estimated to be less than fifteen percent.

For the hypertriton, U_{01} is to be compared with[5]

$$U_{2B,2} = \tfrac{1}{2}(3U_0 + U_1) = -115 \text{ Mev}. \tag{3}$$

Taking the results at their face value, we see that the contributions from the two types of forces are of the same order of magnitude, thus indicating that the three-body force produces the binding of Λ in $_\Lambda H^3$.[6] However, we have made only a crude attempt to approximate the structure of the hypertriton. The boundary condition at the cores has been ignored completely, an approximation which probably leads to an overestimate for the strength of both potentials. Also, we have neglected the tensor terms in Eq. (1), some of which may be important because they have nonvanishing matrix elements even for s-state nucleons. A rough estimate of the effect of these terms indicates that they increase the contribution to the potential energy from the three-body force.

Finally, we note that for $_\Lambda H^4$, $_\Lambda He^4$, and $_\Lambda He^5$, three-body forces contribute a higher fraction of the potential energy than they do for $_\Lambda H^3$, because for all these hyperfragments we have

$$U_{2B,n} \simeq nU_0, \tag{4a}$$

$$U_{3B,n} = \tfrac{1}{2}n(n-1)U_{01}, \tag{4b}$$

where n is the number of core nucleons. If the tensor terms, which contribute only for triplet-state nucleons, are included, Eq. (4b) has to be modified.

The results, which we believe to be at least qualitative, indicate that a more detailed analysis—including the three-body force—of the structure of the light hyperfragments is desirable in order to obtain quantitative information regarding the spin dependence of the two-body force.

ACKNOWLEDGMENT

I want to thank R. Gatto for bringing this problem to my attention.

[5] R. H. Dalitz, *Reports on Progress in Physics* (The Physical Society, London, 1957), Vol. 20, p. 163. The expression in Eq. (3) applies when the two-body Λ-nucleon force is more attractive in the singlet state. If the attraction is stronger in the triplet state, the expression of interest is $U_{2B,2} = 2U_1$.

[6] We assume that the contribution to the binding of Λ in the hypertriton due to the exchange of K mesons is of the same order of magnitude as (or smaller than) the contribution due to the exchange of pions.