Application of the Spin-Flip Dispersion Relations to the Minami Ambiguity for the Pion-Nucleon Scattering*

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The dispersion relations for the spin-flip amplitude of the pion-nucleon scattering in the forward direction have been applied to the Minami phase shifts obtained from the Fermi set in a calculation similar to that of Davidon and Goldberger for the Fermi and Yang phase shifts. The Minami phase shifts were found to be in disagreement with the requirements of the dispersion relations. The same calculation was also carried out for a new set of phase shifts, which is obtained by applying the Minami transformation to the Yang phase shifts. These phase shifts are also incomqatible with the dispersion relations.

I. INTRODUCTION

It has been recently shown by Davidon and Goldberger[1] and by Gilbert and Screaton[2] that the dispersion relations for the spin-flip amplitude[3] of the pion-nucleon scattering can be used to show that the Fermi set of phase shifts for the pion-nucleon scattering is in agreement with the requirements of causality, whereas the Yang set disagrees. In particular, it has been shown in reference 1 that the Fermi set of phase shifts satisfies a straight-line relationship involving the spin-flip amplitude which is required by the dispersion relations, whereas the Yang phase shifts do not. Furthermore this straight-line relationship can be extrapolated to give the renormalized unrationalized coupling constant f^2. The Fermi set of phase shifts yields $f^2=0.10$, which is in reasonable agreement with other determinations.[4] On the other hand, the Yang set of phase shifts yields a negative value of $f^2=-0.11$, which is not physically meaningful.

It has been pointed out by Davidon and Goldberger[1] that their calculation is essentially equivalent to the polarization experiment proposed by Fermi[5] to discriminate between the Fermi and the Yang phase shifts.

Although the Yang set of phase shifts was therefore essentially eliminated if one accepts the principle of causality, another general ambiguity, namely the Minami ambiguity in the phase shifts, remained.[6] The Minami phase shifts for a state of total isotopic spin T and total angular momentum J are obtained from the corresponding Fermi phase shifts by interchanging the phase shifts for $l=J-\frac{1}{2}$ and $l=J+\frac{1}{2}$, where l is the orbital angular momentum. In the following, the Minami phase shifts will be denoted by primes (δ'), whereas the Fermi phase shifts will be unprimed (δ). Thus for the $T=\frac{3}{2}$ state, we have

$$\delta'({}^2S_{\frac{1}{2}},\, T=\tfrac{3}{2})=\delta({}^2P_{\frac{1}{2}},\, T=\tfrac{3}{2})=\delta_{31}, \tag{1}$$

$$\delta'({}^2P_{\frac{1}{2}},\, T=\tfrac{3}{2})=\delta({}^2S_{\frac{1}{2}},\, T=\tfrac{3}{2})=\delta_{3}, \tag{2}$$

$$\delta'({}^2D_{\frac{3}{2}},\, T=\tfrac{3}{2})=\delta({}^2P_{\frac{3}{2}},\, T=\tfrac{3}{2})=\delta_{33}, \tag{3}$$

where δ_3 is the usual (Fermi set) s-wave phase shift for $T=\frac{3}{2}$, and δ_{31}, δ_{33} are the Fermi p-wave phase shifts for $T=\frac{3}{2}$, ${}^2P_{\frac{1}{2}}$ and ${}^2P_{\frac{3}{2}}$, respectively. The choice of the absolute sign taken in Eqs. (1)-(3) preserves the sense of the Coulomb interference for the transformed set of phase shifts.

It has been pointed out by Hayakawa, Kawaguchi, and Minami[7] that the only differences between the predictions of the Minami phase shifts and those of the original Fermi set appear in the polarization of the recoil nucleon.

One may also expect that the spin-flip dispersion relations will in fact distinguish between the Minami phase shifts and the Fermi phase shifts, since the polarization depends on the spin-flip amplitude.

Another possible set of phase shifts, which we will call the α' set, can be obtained by applying the Minami transformation to the Yang set.

In the present work, both the Minami phase shifts and the α' phase shifts have been tested against the requirements of the spin-flip dispersion relations and found to be in definite disagreement with them. Therefore one can conclude that the only set of phase shifts compatible with the causality requirements is the Fermi set.

Of course, one might remark here that the Minami phase shifts have been considered a physically less plausible set than the Fermi set, due to the fact that the momentum dependences of the various phase shifts

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[1] W. C. Davidon and M. L. Goldberger, Phys. Rev. **104**, 1119 (1956).
[2] W. Gilbert and G. R. Screaton, reported by A. Salam, *Proceedings of the CERN Symposium on High-Energy Accelerators and Pion Physics, Geneva, 1956* (European Organization of Nuclear Research, Geneva, 1956), Vol. II, p. 176. See also Phys. Rev. **104**, 1758 (1956).
[3] M. L. Goldberger, *Proceedings of the Sixth Annual Rochester Conference on High-Energy Physics* (Interscience Publishers, Inc., New York, 1956); R. Oehme, Phys. Rev. **100**, 1503 (1955); A. Salam, Nuovo cimento **3**, 424 (1956).
[4] G. F. Chew and F. E. Low, Phys. Rev. **101**, 1570 (1956); U. Haber-Schaim, Phys. Rev. **104**, 1113 (1956); S. J. Lindenbaum, *Annual Review of Nuclear Science* (Annual Reviews, Inc., Stanford, 1957), Vol. 7, p. 317.
[5] E. Fermi, Phys. Rev. **91**, 947 (1953).
[6] S. Minami, Progr. Theoret. Phys. (Japan) **11**, 213 (1954).
[7] Hayakawa, Kawaguchi, and Minami, Progr. Theoret. Phys. (Japan) **11**, 332 (1954); **12**, 355 (1954).

at low energies were not the usual ones expected for a short-range potential interaction, and also the fact that a d-wave resonance was hard to understand physically, whereas a p-wave resonance followed from general strong-coupling meson theory.

II. CALCULATIONS FOR THE MINAMI PHASE SHIFTS

The derivative of the spin-flip amplitude with respect to $\sin\theta$ at $\theta=0$ (θ=scattering angle in the center-of-mass system) is given by $(\hbar/\mu c)\bar{\eta}^2 a_{3,1}$, where

$$a_{3,1}=\sum_{l=1}^{\infty}\frac{l(l+1)}{4i\bar{\eta}^3}(e^{2i\delta_{l+}}-e^{2i\delta_{l-}})_{3,1}, \tag{4}$$

where $\bar{\eta}$ is the pion momentum in the center-of-mass system in units of μc (μ=pion mass), δ_{l+} and δ_{l-} are the phase shifts for orbital angular momentum l and total angular momentum $J=l+\frac{1}{2}$ and $l-\frac{1}{2}$, respectively; the subscripts 3 and 1 refer to total isotopic spin $T=\frac{3}{2}$ and $\frac{1}{2}$, respectively.

In the following, we shall neglect the small and unimportant contributions due to the $T=\frac{1}{2}$ state. For the Minami phase shifts, the spin-flip amplitude a_3 is given by

$$a_3(\delta')=(2i\bar{\eta}^3)^{-1}[1-e^{2i\delta_3}+3(1-e^{2i\delta_{33}})], \tag{5}$$

where we have used Eqs. (1)–(3), and it has been assumed that the d-wave phase shifts for the Fermi set are zero. For comparison with Eq. (5), a_3 for the Fermi phase shifts is given by

$$a_3(\delta)=(2i\bar{\eta}^3)^{-1}(e^{2i\delta_{33}}-e^{2i\delta_{31}}). \tag{6}$$

It has been shown[1] that the dispersion relations lead to the following equation:

$$y=f^2+Cx, \tag{7}$$

where x, y, and C are defined by

$$x\equiv\gamma(1+\gamma_B/\gamma)^{-1}, \tag{8}$$

$$y\equiv\tfrac{1}{2}x\{\mathrm{Re}(a_3)-\gamma[I_3(\gamma)+\tfrac{1}{3}I_3(-\gamma)]\}, \tag{9}$$

$$C\equiv\frac{1}{3\pi}\int_1^{\infty}d\gamma'\frac{\mathrm{Im}(a_3)}{\gamma'}, \tag{10}$$

where γ is the laboratory total energy of the pion in units μc^2, $\gamma_B\equiv\mu/(2M)$ (M=nucleon mass), and $I_3(\gamma)$ is the following principal-value integral:

$$I_3(\gamma)\equiv\frac{1}{\pi}P\int_1^{\infty}d\gamma'\frac{\mathrm{Im}(a_3)}{\gamma'(\gamma'-\gamma)}. \tag{11}$$

The values of y have been calculated, using Eq. (5) for $a_3(\delta')$. For the phase shift δ_3, we used Orear's formula,[8] $\delta_3=-0.11\bar{\eta}$. For δ_{33}, the expression obtained by Anderson[9] was employed:

$$\bar{\eta}^3\cot\delta_{33}=\frac{1+0.77\bar{\eta}^2}{0.248}\left(\frac{1.9427-\bar{\gamma}}{0.9427}\right), \tag{12}$$

where $\bar{\gamma}$ is the center-of-mass pion total energy in units μc^2.

It is useful to write $a_3(\delta')$ as the sum of two parts, $a_{3\alpha}(\delta')$ and $a_{3\beta}(\delta')$:

$$a_{3\alpha}(\delta')\equiv(2i\bar{\eta}^3)^{-1}(1-e^{2i\delta_3}), \tag{13}$$

$$a_{3\beta}(\delta')\equiv3(2i\bar{\eta}^3)^{-1}(1-e^{2i\delta_{33}}). \tag{14}$$

For zero kinetic energy T_π, both the real and imaginary parts of $a_{3\alpha}(\delta')$ diverge, as a result of the $\bar{\eta}^3$ denominator. Thus for $\gamma\to1$, $\bar{\eta}$ is given by

$$\bar{\eta}=[2(\gamma-1)]^{\frac{1}{2}}M/(M+\mu)=1.230(\gamma-1)^{\frac{1}{2}}. \quad (\gamma\to1) \tag{15}$$

With[8] $\delta_3=-0.11\bar{\eta}$, one obtains

$$\mathrm{Re}[a_{3\alpha}(\delta')]=0.0727(\gamma-1)^{-1}, \tag{16}$$

$$\mathrm{Im}[a_{3\alpha}(\delta')]=-0.0098(\gamma-1)^{-\frac{1}{2}}. \tag{17}$$

Since the divergence of $\mathrm{Im}[a_{3\alpha}(\delta')]$ is less strong than $(\gamma-1)^{-1}$, the integral $I_3(-\gamma)$ is obviously convergent, and a detailed argument shows that the principal-value integral $I_3(\gamma)$ is also convergent. However, y will, of course, diverge as a result of the divergence of $\mathrm{Re}[a_{3\alpha}(\delta')]$. It may be noted that there is no divergence for $a_{3\beta}(\delta')$, since $\delta_{33}\propto\bar{\eta}^3$ as $\bar{\eta}\to0$.

The integrals $I_3(\gamma)$ and $I_3(-\gamma)$ were evaluated by approximating the functions $\gamma^{-1}\,\mathrm{Im}[a_{3\alpha}(\delta')]$ and $\gamma^{-1}\,\mathrm{Im}[a_{3\beta}(\delta')]$ by a series of straight lines, generally with intervals of 0.2 in γ. However, in the region from $\gamma=1.0$ to 1.2, where $\gamma^{-1}\,\mathrm{Im}[a_{3\alpha}(\delta')]$ diverges, we used an approximation of the form $b(\gamma-1)^{-\frac{1}{2}}$, where b is a constant [see Eq. (17)]. At $\gamma=2.2$, $\mathrm{Im}[a_{3\beta}(\delta')]$ has its maximum and varies rapidly with γ, corresponding to the resonance behavior of δ_{33}. For this reason, intervals of γ of 0.1 were used between $\gamma=2.0$ and 2.4. The integrals were extended up to $\gamma=3.2$, corresponding to $T_\pi\approx300$ Mev. Above 300 Mev, the phase shifts δ_3 and δ_{33} are not well known at present. However, the contribution of the region $T_\pi>300$ Mev is expected to be small, since the integrand decreases essentially as γ^{-2} for large values of γ.

The upper curve in Fig. 1 shows the resulting values of y as a function of x. As discussed above, y becomes infinite at $\gamma=1$, which corresponds to $x=0.930$. Calculated values of y not shown in the figure are: $y=0.074$ at $x=1.030$ ($\gamma=1.1$) and $y=0.416$ at $x=0.980$ ($\gamma=1.05$). It is clear that this curve cannot be extrapolated to $x=0$ to give a finite value of f^2. Moreover, Eq. (7) predicts a linear behavior of y, with constant slope C, which is obviously not satisfied in Fig. 1. It can thus

[8] J. Orear, Phys. Rev. **96**, 176 (1954); **100**, 288 (1955).

[9] H. L. Anderson, *Proceedings of the Sixth Annual Rochester Conference on High-Energy Physics* (Interscience Publishers, Inc., New York, 1956).

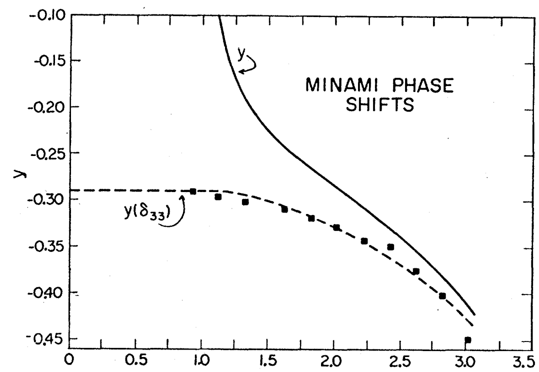

FIG. 1. Plot of y *vs* x for the Minami phase shifts. The solid curve represents y. The dashed curve marked $y(\delta_{33})$ gives the contribution to y due to the phase shift δ_{33} alone.

be concluded that the Minami phase shifts are incompatible with the requirements of the dispersion relations.

We note that the preceding results essentially depend on the fact that the Minami set gives a p phase shift $\delta'(^2P_{\frac{1}{2}}, T=\frac{3}{2})$ which behaves as $\bar{\eta}$ for $\bar{\eta}\rightarrow 0$, instead of $\bar{\eta}^3$ as is expected for a p wave scattered from a short-range potential. Since the dispersion relations were not derived from the assumption of a potential, it appears that they essentially predict that a p phase shift must increase at least as rapidly as $\bar{\eta}^3$ at low energies.

Since the $\bar{\eta}$ dependence of δ_3 is the determining factor in the preceding results, it is of interest to consider the evidence in favor of the Orear expression for δ_3 which has been used above. Several previous phase shift analyses of the experimental data from low energies to $\sim$200 Mev have yielded values of δ_3 generally compatible with the Orear expression.[10] Recently two experiments have also been performed at relatively low energies ($T_\pi\sim$20 Mev; $\bar{\eta}\sim$0.47), one with 10–35 Mev positive pions in a hydrogen diffusion cloud chamber, by Alston *et al.*[11] at Liverpool, the other with 19-Mev π^- in a hydrogen bubble chamber, by Nagle, Hildebrand, and Plano[12] in Chicago. The Liverpool results[11] give $\delta_3=-(0.13\pm0.035)\bar{\eta}$, which is in good agreement with Orear's prescription[8]: $\delta_3=-0.11\bar{\eta}$. The Chicago measurements[12] yield a value of $2\delta_1+\delta_3=(0.23\pm0.04)\bar{\eta}$, which is also in good agreement with Orear's value $0.21\bar{\eta}$ obtained by means of $\delta_1=0.16\bar{\eta}$. From these results, it appears that Orear's expressions for δ_3 and δ_1 have been adequately verified at low energies, at least down to $\sim$20 Mev ($\bar{\eta}\sim$0.47). Nevertheless, it does not seem to be absolutely ruled out that δ_3 could be proportional to $\bar{\eta}^3$ at very low energies, while going over to Orear's values at somewhat higher energies. From the point of view of the dispersion relations, this would have the consequence that Re$[a_{3\alpha}(\delta')]$ and hence y remain finite at $\gamma=1$ ($x=0.930$), and therefore the curve of y *vs* x could be extrapolated to $x=0$ to yield a finite value of f^2. However, even if this were the case, it is likely that the plot of y *vs* x would have a strong curvature and would not extrapolate to the correct value of f^2 ($=0.1$). In this connection, since the divergence of Re$[a_{3\alpha}(\delta')]$ at $\gamma=1$ masks the behavior of the part of y which is due to $a_{3\beta}(\delta')$ (arising from δ_{33}), it is of interest to obtain the y *vs* x plot derived from $a_{3\beta}(\delta')$ alone (by setting $\delta_3=0$). In this case, even without any detailed calculations, it can be seen from the following argument that Eq. (7) would predict a negative value of f^2, of the order of -3 times that given by the Fermi phase shifts.

The amplitude $a_{3\beta}(\delta')$ is given by Eq. (14). The phase shift δ_{31} involved in the Fermi amplitude $a_3(\delta)$ is quite small. Thus for the expression obtained by Anderson,[9]

$$\bar{\eta}^3 \cot\delta_{31}=-(0.0415-0.00775\bar{\eta}^2)^{-1}, \qquad (18)$$

$|\delta_{31}|$ has a maximum of 5.5° at $T_\pi=225$ Mev. In the approximation in which δ_{31} is neglected, a comparison of Eqs. (6) and (14) shows that

$$a_{3\beta}(\delta')=-3a_3(\delta). \qquad (19)$$

Since C, I_3, and y are directly proportional to a_3, we have

$$C'=-3C;\quad I_3'(\pm\gamma)=-3I_3(\pm\gamma);\quad y'=-3y, \qquad (20)$$

where the primed quantities refer to the Minami phase shifts, while the unprimed quantities refer to the Fermi set. Upon substituting (20) into Eq. (7), one obtains

$$f'^2=-3f^2, \qquad (21)$$

which gives $f'^2\approx-0.3$.

The actual plot of $y(\delta_{33})$ *vs* x is shown in Fig. 1 (dashed curve); here $y(\delta_{33})$ is the part of y which is due to δ_{33} alone $[a_{3\beta}(\delta')]$. It is seen that $y(\delta_{33})$ has a considerable curvature and extrapolates to $f^2=-0.29$, as expected from the previous arguments. Of course, it should be emphasized that $y(\delta_{33})$ is only a part of the complete y, and therefore no definite general conclusions can necessarily be drawn from $y(\delta_{33})$ alone. However, it is obvious that at the higher energies, y is almost entirely determined by the contribution of δ_{33} alone [i.e., $y\sim y(\delta_{33})$ for $\gamma\gtrsim 2$]. As we have already noted, the high-energy part of y [which is $\sim y(\delta_{33})$] has an intercept yielding a negative f^2. Hence it is clear that even if one varies the possible momentum dependence

[10] See Mukhin, Ozerov, Pontekorvo, Grigoriev, and Mitin, *Proceedings of the CERN Symposium on High-Energy Accelerators and Pion Physics, Geneva, 1956* (European Organization of Nuclear Research, Geneva, 1956), Vol. II, p. 204, and J. Orear, *Proceedings of the CERN Symposium on High-Energy Accelerators and Pion Physics, Geneva, 1956* (European Organization of Nuclear Research, Geneva, 1956), Vol. II, p. 233.

[11] Alston, Fidecaro, von Gierke, Evans, Newport, and Williams, *Proceedings of the CERN Symposium on High-Energy Accelerators and Pion Physics, Geneva, 1956* (European Organization of Nuclear Research, Geneva, 1956), Vol. II, p. 236.

[12] Nagle, Hildebrand, and Plano, *Proceedings of the CERN Symposium on High-Energy Accelerators and Pion Physics, Geneva, 1956* (European Organization of Nuclear Research, Geneva, 1956), Vol. II, p. 238.

of the δ_3 phase shift, in a manner compatible with the experimental data, one cannot obtain a straight line with an intercept giving the correct value of f^2. As was discussed above, with the use of the Orear phase shifts for δ_3, the resulting curve of y *vs* x (see Fig. 1) is in very marked disagreement with the dispersion relations.

In Fig. 1, the squares adjacent to the curve of $y(\delta_{33})$ indicate the order of magnitude of the uncertainties introduced by the numerical calculations of $\mathrm{Re}[a_{3\beta}(\delta')]$ and the integrals over $\mathrm{Im}[a_{3\beta}(\delta')]$. The uncertainties in the complete y (upper curve) are of the same order of magnitude.

It may be noted that the effect of the $T=\frac{1}{2}$ state (amplitude a_1), which was neglected by Davidon and Goldberger[1] in obtaining Eqs. (7)–(10), has been evaluated for the Minami phase shifts, using Orear's prescription of $\delta_1=0.16\bar{\eta}$. The contribution of the Fermi p-wave phase shift δ_{13} was neglected, since it is probably very small compared to that of δ_1, and since there is not much information on the values of δ_{13}. With the inclusion[1] of a_1, one still obtains an equation of the form of (7) for y *vs* x, provided that C and y are redefined as follows:

$$C\equiv\frac{1}{3\pi}\int_1^\infty d\gamma'\left[\frac{\mathrm{Im}(a_3)-\mathrm{Im}(a_1)}{\gamma'}\right], \quad (22)$$

$$y\equiv\tfrac{1}{2}x\{\mathrm{Re}(a_3)-\gamma[I_3(\gamma)+\tfrac{1}{3}I_3(-\gamma)+\tfrac{2}{3}I_1(-\gamma)]\}, \quad (23)$$

where

$$I_1(-\gamma)\equiv\frac{1}{\pi}\int_1^\infty d\gamma'\frac{\mathrm{Im}(a_1)}{\gamma'(\gamma'+\gamma)}. \quad (24)$$

Thus y differs from Eq. (9) by an extra term $\frac{2}{3}I_1(-\gamma)$ in the square bracket. The integral $I_1(-\gamma)$ of Eq. (24) was evaluated both for $\gamma=1$ and $\gamma=3$. The resulting change of y due to $I_1(-\gamma)$ is given by

$$\Delta y=-\tfrac{1}{3}\gamma x I_1(-\gamma), \quad (25)$$

and has the values $+0.0015$ at $\gamma=1$ and $+0.0080$ at $\gamma=3$. These corrections are of the order of the accuracy of the calculations (see Fig. 1) and can be safely neglected.

III. α' PHASE SHIFTS

As mentioned in the introduction, there exists a set of possible phase shifts which can be obtained by applying the Minami transformation[6] to the usual Yang phase shifts. These phase shifts will be called the α' set and are given by

$$\alpha'({}^2S_{\frac{1}{2}},\,T=\tfrac{3}{2})=\alpha_{31}=\chi-\delta_{31}, \quad (26)$$

$$\alpha'({}^2P_{\frac{1}{2}},\,T=\tfrac{3}{2})=\alpha_3=\delta_3, \quad (27)$$

$$\alpha'({}^2D_{\frac{3}{2}},\,T=\tfrac{3}{2})=\alpha_{33}=\chi-\delta_{33}, \quad (28)$$

where α_3, α_{31}, and α_{33} are the Yang phase shifts, which are in turn given in terms of the Fermi phase shifts,

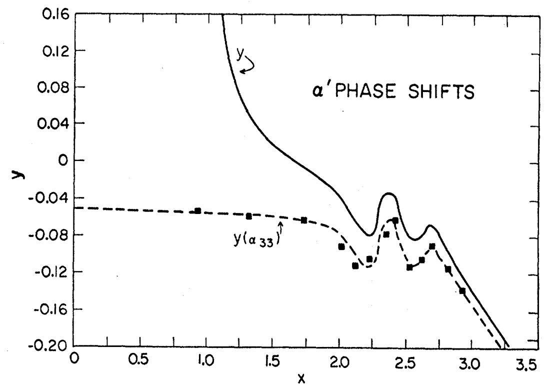

FIG. 2. Plot of y *vs* x for the α' phase shifts. The solid curve represents y. The dashed curve marked $y(\alpha_{33})$ gives the contribution to y due to the phase shift α_{33} alone.

δ_3, δ_{31}, δ_{33}, and the angle χ defined by

$$\tan\chi=\frac{2\sin2\delta_{33}+\sin2\delta_{31}}{2\cos2\delta_{33}+\cos2\delta_{31}}, \quad (29)$$

where the sign of $\sin\chi$ is the same as that of $2\sin2\delta_{33}+\sin2\delta_{31}$. Equation (26) shows that the large Yang phase shift α_{31} becomes an s-wave phase shift for the α' set, and α_{33} becomes a d-wave phase shift. Hence, in this set, there are both s-wave and d-wave resonances.

In a calculation similar to that of Sec. II for the Minami phase shifts, we have applied the spin-flip dispersion relations to the α' set. In similarity to Eq. (5), the spin-flip amplitude for the α' phase shifts is given by

$$a_3(\alpha')=(2i\bar{\eta}^3)^{-1}[1-e^{2i\delta_3}+3(1-e^{2i\alpha_{33}})], \quad (30)$$

which shows that the p-wave part (involving δ_3) is the same as for the Minami amplitude:

$$a_{3\alpha}(\alpha')=a_{3\alpha}(\delta'), \quad (31)$$

where $a_{3\alpha}(\delta')$ is given by Eq. (13). This result arises, of course, from the fact that the s-wave phase shift δ_3 is the same for the Fermi and the Yang solutions.

Figure 2 shows the results of the calculations. As for the Minami set, we have presented two curves. Using the values of Orear for δ_3, one obtains the solid curve, which represents the complete function y. Of course, the divergence of y at $x=0.930$ ($\gamma=1$) implies that the α' phase shifts are in disagreement with the dispersion relations. The dashed curve of Fig. 2 shows the part of y due to α_{33} alone $[y(\alpha_{33})]$, which would extrapolate to a negative value of f^2 $(=-0.052)$. As in Fig. 1, the squares adjacent to the curve of $y(\alpha_{33})$ indicate the order of magnitude of the uncertainties of the numerical calculations. Both curves have rapid variations (two maxima and two minima) in the region of the scattering resonance ($x=2.0$ to 2.8). This behavior is in disagreement with the straight-line relationship predicted by the dispersion relations. As discussed above, the reason for considering the part $y(\alpha_{33})$ is that the divergence of the complete y at $\gamma=1$ is due to the phase shift δ_3 alone. If δ_3 were proportional to $\bar{\eta}^3$ in a narrow region near $\bar{\eta}=0$ (while going over to $-0.11\bar{\eta}$ at higher energies), the divergence of y would be removed. However, as Fig. 2 shows, the spin-flip dispersion relations would still be violated by the α' phase shifts, because of the rapid variations of y near the resonance and the fact that a straight-line relationship with the correct coupling constant could not be obtained. Thus it can be concluded that the α' phase shifts are inconsistent with the spin-flip dispersion relations.

IV. CONCLUSIONS

We have applied the spin-flip dispersion relations[3] for the pion-nucleon scattering to the Minami phase shifts derived from the Fermi set and to a set of phase shifts (α') obtained by applying the Minami transformation[6] to the well-known Yang phase shifts. It has been shown that the Minami and the α' phase shifts both give a divergence (at $\gamma=1$) in the curve of y *vs* x, in definite disagreement with the straight-line behavior deduced from the dispersion relations. The divergence of y at $\gamma=1$ arises from the fact that δ_3 is proportional to $\bar{\eta}$ at low energies ($\delta_3=-0.11\bar{\eta}$).[8] In addition to the divergence, the curve of y *vs* x for the α' phase shifts has rapid variations in the region of the resonance. From these results, one can conclude that both the Minami and the α' phase shifts are incompatible with the dispersion relations.

The fact that the Yang, Minami, and α' phase shifts are all in very marked disagreement with the requirements of the spin-flip dispersion relations, while on the other hand the Fermi set is in agreement, makes it almost certain that the Fermi set is the only correct set and is also the unique solution for the pion-nucleon scattering at low energies up to $\sim$300 Mev.

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Decay of the Pi Meson

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A quantitative study of $\pi\rightarrow\mu+\nu$ decay is presented using the techniques of dispersion theory. The discussion is based on a model in which the decay occurs through pion disintegration into a nucleon-antinucleon pair, the latter annihilating via a Fermi interaction to produce the leptons. The weak vertex contains effectively both axial vector and pseudoscalar couplings even if one adopts the point of view of a universal axial vector and vector Fermi interaction. The pion-nucleon vertex which enters our model is also calculated using dispersion techniques. Under the assumption that this vertex is damped for large momentum transfers, we obtain a result for the pion lifetime largely independent of the detailed properties of the vertex and one which is in very close agreement with experiment. The precise prediction of our theory depends on the energy dependence of the complex phase shift for nucleon-antinucleon scattering in the $^1S_0$ isotopic triplet state.

I. INTRODUCTION

THE main interest in the problem of pion decay at the present time concerns the experimental absence of the modes $\pi\rightarrow e+\nu$ [1] and $\pi\rightarrow e+\nu+\gamma$.[2] Beyond this, however, one would also like to understand quantitatively the mechanism of the observed decay mode $\pi\rightarrow\mu+\nu$.

This process is customarily described in terms of virtual dissociation of the pion into a nucleon-antinucleon pair, the latter annihilating via the μ-capture Fermi interaction to produce the lepton pair. Only the axial vector and pseudoscalar Fermi couplings can contribute here. The former is of special relevance, since it and the vector coupling now appear to dominate in the other Fermi interactions: μ and β decay. Furthermore, a universal axial vector coupling would imply a suppression of $\pi\rightarrow e+\nu$ decay relative to $\pi\rightarrow\mu+\nu$ decay by a factor of $\sim 10^{-4}$.

It is possible that the physical picture described above has to be extended to include also Fermi couplings of hyperon pairs with leptons, although at the present time there is no experimental evidence for β decay of hyperons. In any case, if only to sharpen the problem, we want to see to what extent the simple picture based on an axial vector μ-capture coupling can be reconciled with the known rate for $\pi\rightarrow\mu+\nu$ decay.

It is necessary here to make precise what is meant by our assumption that the coupling is axial vector. What we assume is that the Fermi interaction Lagrangian contains only nonderivative axial vector (and vector) covariants. In the μ-capture reaction, however, the nucleons involved are surrounded by clouds of pions, pairs, etc. This means that the S-matrix element will in general contain terms which simulate Fermi interactions with derivative nucleon couplings. When reduced to the

[1] H. L. Anderson and C. M. G. Lattes, Nuovo cimento **6**, 1356 (1957).
[2] Cassels, Rigby, Wetherell, and Wormald, Proc. Phys. Soc. (London) **A70**, 729 (1957).