of the electron losses considerably less peaked (θ^{-2}). Ferrell[14] assumed that the difference between the observed distribution of the characteristic losses and that calculated from the plasma theory is due to "contamination" of the characteristic loss by the no-loss peak. Evidence obtained previously by us[15] does not seem to bear out this assumption. Ritchie[16] has calculated I_c/I_{LL} *versus* foil thickness assuming the angular distribution given by plasma theory and a detector with an acceptance of 1 milliradian. He finds considerably better agreement with the experiment than shown here. Although our explanation of the peaking as a diffraction phenomenon may be incorrect we feel that the peaking itself is a well-established experimental fact. The most exact comparison with the theory would be somewhere between the extremes represented by the differential and integral cross section. We feel that the experimental facts support our choice of the integral cross section for comparison. The shaded area is a measurement of the internal consistency of our data and is not intended to imply an accuracy. Known sources of possible error in thickness and measured intensity could give uncertainties of 50% in the region below 200 A.

Since neither the predicted value of low-lying loss nor its dependence on grain size is confirmed, it must be concluded that the 6.3-ev loss of aluminum is not a "lowered" plasma loss.

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Elastic and Piezoelectric Constants of Alpha-Quartz

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The elastic and piezoelectric constants of alpha-quartz have been determined by the resonance (previously known as dynamic) method employing the extensional mode of bars and the contour-extensional mode I (Lamé mode) of square plates. It is believed that this approach results in very accurate values.

THE elastic and piezoelectric constants of alpha-quartz have been determined by several methods, e.g., the static method,[1] the resonance (previously known as dynamic) method,[2–5] the ultrasonic method,[6] and by means of light diffraction.[7]

By employing the resonance method, the elastic and piezoelectric constants are derived from measured values of the series resonance frequency, the motional capacitance, and the dimensions of appropriately cut and excited specimens. The constants to be determined and the quantities measured are related by the mathematical solution for the frequency and the displacement of the mode of motion for the specimens considered. Extensional modes of narrow bars and contour modes of square plates are determined by the elastic compliances s_{pq} and the piezoelectric strain constants d_{jp}. The thickness modes of thin plates lead to the elastic stiffnesses c_{pq} and the piezoelectric stress constants e_{jp}. If one set of constants, for example s_{pq} and d_{jp}, is known, the corresponding c_{pq} and e_{jp} can be calculated.

The resonance method is considered as a simple and very reliable means for determination of the elastic and piezoelectric constants as the measurements can be performed with a high degree of accuracy provided sufficiently large crystals are available. Furthermore, the conversion of these measurements into the material constants is accomplished by means of a rigorous mathematical solution when the extensional mode of bars and contour-extensional mode I (Lamé mode) of square plates are used. For other modes, close approximations are known. The resonance method is described in detail by Bechmann and Ayers[8] and has been adopted as an Institute of Radio Engineers standard.[9]

The results should be independent of the modes of motion used. Measurements have shown that the values

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[8] R. Bechmann and S. Ayers, *Piezoelectricity* (Her Majesty's Stationery Office, London, 1957), Rept. No. 4, Selected Engineering Reports, Post Office Research Station.
[9] Inst. Radio Engrs., Proc. Inst. Radio Engrs. **46**, 764 (1958).

for the elastic constants obtained from thickness modes and those from contour modes differ slightly. Because of the rigorous mathematical solutions for the frequency equation for both modes of bars and plates mentioned, it is assumed that the values for the coefficients s_{pq} and d_{jp} are the more accurate ones.

The complete set of elastic constants for quartz can be determined by the use of at least four differently oriented bars $(xyt)\psi$, giving the elastic compliances

$$s_{11},\quad s_{33},\quad s_{14},\quad s_{44}+2s_{13},$$

and by use of at least three differently oriented square plates $(yxlt)\theta$ 45°, giving the elastic compliances

$$s_{44},\quad s_{66}=2(s_{11}-s_{12}),\quad s_{14}.$$

Both piezoelectric strain constants d_{11}, d_{14} can be obtained independently from these bars and plates.

TABLE I. Elastic compliances of alpha-quartz in 10^{-12} meter^2 newton^{-1}.

s_{pq}	$s_{pq}^{E\sigma}$	$s_{pq}^D-s_{pq}^E$	$s_{pq}^{\sigma *}-s_{pq}^{\theta *}$
s_{11}	12.77	−0.134	−0.028
s_{33}	9.60	0	−0.008
s_{12}	−1.79	0.134	−0.028
s_{13}	−1.22	0	−0.016
s_{44}	20.04	−0.0132	0
s_{66}	29.12	−0.536	0
s_{14}	4.50	−0.042	0

TABLE II. Elastic stiffnesses of alpha-quartz in 10^9 newton meter^{-2}.

c_{pq}	$c_{pq}^{E\sigma}$	$c_{pq}^D-c_{pq}^E$	$c_{pq}^{\sigma *}-c_{pq}^{\theta *}$
c_{11}	86.74	0.746	0.288
c_{33}	107.2	0	0.193
c_{12}	6.99	−0.746	0.288
c_{13}	11.91	0	0.236
c_{44}	57.94	0.0415	0
c_{66}	39.88	0.746	0
c_{14}	−17.91	−0.177	0

Measurements have been made on bars and plates during the last few years and a high degree of reproducibility has been achieved. Since this method has been described previously,[8,9] no details are given here. Cuts with orientations giving a reduced frequency-temperature variation are particularly suitable. The specimens have been carefully prepared and plated with very thin layers. The influence of both resonator thickness and plating thickness was taken into consideration by progressively reducing the thickness of each, thus permitting extrapolation to zero. The plate dimensions were measured precisely and the specimens mounted with two pins at the nodal point in the center of the plate.

The elastic compliances s_{pq} and the piezoelectric strain coefficients d_{jp} at 20°C are listed in Tables I and III. These values are averages obtained from measurements on a large number of differently oriented specimens. The elastic stiffnesses c_{pq} and the piezoelectric stress constants e_{jp}, calculated from s_{pq} and d_{jp}, are given in Tables II and III. The superscripts E and D in Tables I and II refer to constant field and constant displacement (short circuit and open circuit conditions). The superscript σ indicates that the values are measured under adiabatic conditions when the entropy σ is constant. In these tables the differences between the adiabatic (σ) and isothermal (θ) values are added. In addition, the superscript * refers to constant E or D.

TABLE III. Piezoelectric strain and stress constants of alpha-quartz.

Strain	Stress
$d_{11}=2.31\times10^{-12}$ coulomb newton^{-1}	$e_{11}=0.171$ coulomb meter^{-2}
$d_{14}=0.727\times10^{-12}$ coulomb newton^{-1}	$e_{14}=-0.0406$ coulomb meter^{-2}
$g_{11}=0.0578$ meter^2 coulomb^{-1}	$h_{11}=4.36\times10^9$ newton coulomb^{-1}
$g_{14}=0.0182$ meter^2 coulomb^{-1}	$h_{14}=-1.04\times10^9$ newton coulomb^{-1}

TABLE IV. Dielectric permittivities and impermeabilities of alpha-quartz.

Permittivity (10^{-12} farad meter^{-1})			Impermeability (10^9 meter farad^{-1})		
ϵ_{ij}	ϵ_{ij}^T	$\epsilon_{ij}^S-\epsilon_{ij}^T$	β_{ij}	β_{ij}^T	$\beta_{ij}^S-\beta_{ij}^T$
ϵ_{11}	39.97	−0.76	β_{11}	25.02	0.485
ϵ_{33}	41.03	0	β_{33}	24.37	0

The piezoelectric strain constants g_{jp} and the piezoelectric stress constants h_{jp}, both related to constant displacement conditions, calculated from d_{jp}, e_{jp} and the dielectric constants, are also shown in Table III. Table IV contains the values for the dielectric constants ϵ_{ij} and the dielectric impermeabilities β_{ij} at constant stress (T) and the difference between these values at constant strain (S) and constant stress (T).[10]

All values in Tables I to IV are expressed in rationalized mks units. The signs of the elastic and piezoelectric constants refer to left-handed quartz and are chosen in accordance with the IRE 1949 Standards.[11]

For both right and left quartz, s_{14} is positive and c_{14} is negative. For right quartz, d_{11}, d_{14}, and e_{11} are negative; e_{14} is positive. For left quartz, d_{11}, d_{14}, and e_{11} are positive; e_{14} is negative.

By using the resonance method, the elastic and piezoelectric constants of various kinds of synthetic quartz have been determined. The results will be published at a later date.

[10] W. G. Cady, *Piezoelectricity* (McGraw-Hill Book Company, Inc., New York, 1946).

[11] Inst. Radio Engrs., Proc. Inst. Radio Engrs. **37**, 1378 (1949).