WAVE–LENGTHS OF THE TUNGSTEN X–RAY SPECTRUM.

By Elmer Dershem.

Introduction.

SINCE the X-ray spectra of practically all the available elements had been studied by one investigator or another with results which did not very closely agree and which in general comprised only a few of the principal or most prominent lines, it seemed wise to begin the present investigation with a view to determining more completely and accurately than heretofore the number of lines and their wave-lengths in the spectrum of at least one element. The element most easily tested and the one whose spectrum would be of the greatest value in the X-ray analysis of crystals was tungsten on account of its use as the anticathode of the Coolidge tube, the only type of tube which could be used during the long intervals of time necessary to secure spectral photographs, if the conditions required for the greatest resolving power and the greatest accuracy of measurement were complied with.

The photographic method was chosen for this work in preference to an ionization chamber and electrometer because in the latter method the intensity of the reflected beam must be great enough to give a continuous effect on the electrometer while the photographic plate gives a summation of the intensity of the reflected beam over a time that may be made so very much longer that weak lines have an opportunity to appear.

We shall now consider the factors affecting the accuracy of measurement and resolving power of an instrument using a crystal as a diffraction grating for X-rays. Resolving power is, as usual, defined as the ratio of a wave-length to the smallest difference which may exist between this and a neighboring wave-length and yet have the instrument show that the two waves are separate and not identical. A consideration of these factors will then show that the conditions for the best resolving power are those which lead to a decrease in intensity and would make impossible the securing of sufficient intensity to affect an electrometer under the necessary conditions of a narrow source, great distance from the crystal to the detector and a thin crystal which means less intensity because there are fewer reflecting planes. The theory will also show that the position of the central maximum of the reflected beam is not the true criterion by which the wave-length must be determined but it is instead the outer edge, which when corrections are made for the width of the source, gives the true measurement. The impossibility of measuring anything other than the central maximum with an ionization chamber eliminates this as a possible accurate method and leaves the photographic plate as the only recourse.

RESOLVING POWER OF A CRYSTAL USED AS A DIFFRACTION GRATING FOR X-RAYS.

In this discussion the assumption will be made that the slit, or source, is the same distance from the crystal as is the photographic plate. In this case, as shown by Bragg,[1] the amount of surface of the crystal exposed to the X-rays makes no difference in the sharpness of the lines since the same wave-length is always reflected to the same point on the plate. This will not be true if the atomic planes are not parallel. In reality the cleavage surfaces of crystals are quite noticeably warped and it is desirable to limit the surface of the crystal exposed to the rays by means of a narrow slit between lead blocks placed close to the crystal even though it does cause a decrease in intensity. It will also be assumed that the crystal is thin enough that the rays may penetrate entirely through the crystal and be reflected from the planes on the back side and again traversing the crystal to reach the photographic plate.

With these assumptions as to conditions which may be easily obtained in practice, the question to be determined is, What difference of wave-length is necessary that it may be possible to separate two waves of nearly the same length?

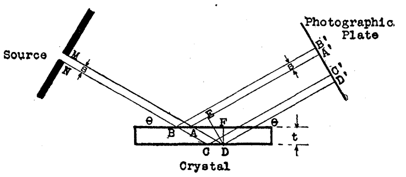

Fig. 1.

Let the source be a slit of width s at a distance r from the crystal, Fig. 1. Assume that the crystal is in a position to reflect some particular wave-length where $n\lambda = 2d \sin \theta$, in which n is the order of the spectrum, λ the wave-length, d the grating constant or distance between the atomic planes and θ the angle between the incident rays and the crystal surface. Then a ray coming from the side M of the slit may be reflected at A to A' on the photographic plate and a ray from the side N must strike the crystal at the same angle and consequently be reflected at the point B to the point B'. It is easily seen that the reflected rays AA' and BB' are at the same distance apart as the incident rays MA and NB. Hence due to the slit alone a single wave-length would cause a line on the photographic plate the same width as the slit.

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[1] Bragg and Bragg, X-Rays and Crystal Structure, G. Bell and Sons, London, 1915.

Considering next the question of the variation of the width of image with the thickness of the crystal, let DE be drawn perpendicular to AA'. Then DE is the width of the reflected beam due to the penetration into the crystal. Let $DF = t$ be the thickness of the crystal. Then $t = AD \sin\theta$ and

$$AD = \frac{DE}{\sin 2\theta}.$$

Then by substitution

$$t = \frac{DE \sin\theta}{\sin 2\theta} = \frac{DE \sin\theta}{2 \sin\theta \cos\theta} = \frac{DE}{2\cos\theta}.$$

Whence $DE = 2t \cos\theta$.

Since DE is the width of beam due to penetration into the crystal the total width of beam is $s + 2t\cos\theta$, in which s is the width of the slit, or source, t the thickness of the crystal and θ the angle which the incident ray makes with the crystal.

Then $s + 2t\cos\theta$ is the width of the line on the photographic plate. In order to resolve two lines of nearly the same wave-length it is necessary that their images on the plate should not overlap or, in other words, that the centers of their images must be further apart than the width of beam, $s + 2t\cos\theta$.

Assume two wave-lengths, λ and $\lambda + \Delta\lambda$. To find how small $\Delta\lambda$ may be and these wave-lengths still be clearly resolved on the plate. Using the formula $n\lambda = 2d\sin\theta$ let λ take on a small increment $\Delta\lambda$ and θ the corresponding increment $\Delta\theta$. Then by differentiation we have $n\Delta\lambda = 2d\cos\theta\Delta\theta$. This is justified in practice by the fact that $\Delta\theta$ is small in comparison to θ.

According to the above if the crystal is in a position to reflect a wave of length λ it must rotate through an angle $\Delta\theta$ in order to reflect a wave of length $\lambda + \Delta\lambda$ and since the reflected ray rotates twice as fast as the crystal the reflected ray must rotate through the angle $2\Delta\theta$. (See Fig. 2.)

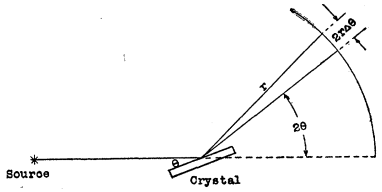

Fig. 2.

If the distance of the crystal from the plate is r then the displacement of the beam along the plate when the reflecting angle is changed from θ to $\theta + \Delta\theta$ is $2r\Delta\theta$. In order that rays reflected at these angles be separated it is necessary that this distance, $2r\Delta\theta$ be greater than the width of beam $s + 2t\cos\theta$.

$$2r\Delta\theta > s + 2t\cos\theta.$$

But

$$\Delta\theta = \frac{n\Delta\lambda}{2d \cos\theta}$$

and by substitution

$$\frac{2nr\Delta\lambda}{2d\cos\theta} > s + 2t\cos\theta$$

$$\Delta\lambda > \frac{d\cos\theta}{nr}(s + 2t\cos\theta).$$

$\Delta\lambda$ is then the smallest difference between the lengths of two waves that is permissible if the images due to these waves are to be separated on the plate. However the images must be separated by a slightly greater distance in order to leave a clear space between them. Just how much space is necessary for this is not a mathematical problem but a question which must be answered by experience. Probably but little need be added to $\Delta\lambda$ on this account. Neglecting for the time being the question of the necessary space between lines it may be of interest to determine the resolving power under the best conditions that were obtained with the apparatus used in the present work. For example, taking the line in the central part of the L spectrum having a wavelength of 1.241×10^{-8} cm. the experimental values of the quantities contained in the above formula were:

$$s = 0.032 \text{ cm.},$$

$$t = 0.019 \text{ cm.},$$

$$d = 2.814 \times 10^{-8} \text{ cm.},$$

$$r = 62 \text{ cm.},$$

$$\cos\theta = 0.977,$$

$$n = 1.$$

Substituting in the inequality

$$\frac{d\cos\theta}{nr}(s + 2t\cos\theta)$$

the above values of the quantities gives

$$\Delta\lambda > 0.00375 \times 10^{-8} \text{ cm.}$$

Since resolving power is defined as $\lambda/\Delta\lambda$ and for this case λ is 1.241×10^{-8} cm. we have

$$\frac{\lambda}{\Delta\lambda} < \frac{1.241}{.00375},$$

$$\frac{\lambda}{\Delta\lambda} < 331.$$

The resolving power in this case was less than 331, although the experimental values of the width of slit, thickness of crystal and distance to the plate were so chosen as to give the greatest possible resolving power consistent with the necessary requirement of retaining sufficient intensity in the reflected beam to affect the photographic plate in an exposure of a reasonable duration.

It is apparent that the ways in which the resolving power may be increased are to use a higher order than the first, to narrow the source, to decrease the thickness of the crystal and to increase the distance between the crystal and the plate. To do any one of these things tends to decrease the intensity and make necessary a longer exposure and this is not altogether desirable, as it gives the latent image an opportunity to spread and blurr the image and also increases the liability to fogging of the plate due to stray radiation. An increase of distance from crystal to plate decreases the intensity by absorption in the air and this may be a factor of considerable importance in working with the longer wave-lengths. Therefore at present it would not seem possible to so greatly increase the resolving power of a crystal used as a diffraction grating for X-rays as to make it at all comparable to the resolving powers of the grating or echelon used for ordinary light.

From this theory it may be seen, by reference to Fig. 1, that the true angle of reflection must be determined by measuring the position of the outer edge or most deviated portion of the spectral line and subtracting one half of the width of the source from this. This will eliminate any error of measurement due to penetration into the crystal but the crystal must be thin if two nearly equal wave-lengths are to be separated.

Methods of Applying the Theories Concerning Resolving Power.

Since it was the object in this work to make as accurate measurements of the wave-lengths as possible the apparatus and methods of using it will be described somewhat in detail.

The previous theory requiring the use of a thin crystal, the following method of securing and mounting one was adopted. A crystal of rock salt having a perfect cleavage face of about one square centimeter area was chosen and this was fastened face down onto a glass surface by the use of a wax especially prepared for the purpose by mixing Canada balsam and hard sealing wax in such proportions as would give a wax that was hard and tough at ordinary temperatures but which became a thin liquid when slightly heated. After the crystal was firmly cemented to the glass by pressing the two together while warm with a small quantity of wax between and allowing them to cool, the crystal was ground away until a thickness of not more than 0.019 cm. remained. It was found by experience that attempts to make the crystal thinner than this resulted in causing the crystal to crack and become useless.

The measurements of the position of the lines on the photographic plates were made with a Societa Genevoise dividing engine which was guaranteed by the makers to be accurate to 0.01 mm. in a total length of 40 cm.

To check against possible variations in the pitch of the screw the plates were measured a number of times and each time the setting was changed so that the measurement would be made by a different part of the screw. However the principal object of repeating the measurements was to compensate for the errors of setting by securing a number of readings and averaging the results.

A number of different methods of securing accurate settings of the dividing engine were tried and the one giving the most consistent results was the following. An achromatic combination lens of 1½ inches diameter was placed in a tube 22 inches long. Two parallel hairs were placed at one end of the tube and brought very close to the photographic plate so that the parallel hairs and the spectral line on the plate should be practically in one conjugate focal plane of the instrument at the same time. The spectral line and the parallel hairs were then viewed through a peep hole at the other end of the tube which was near the other conjugate focus of the lens. Owing to the great length of the tube as compared to the distance between the parallel hairs and the photographic plate there was very little parallax and owing to the large diameter of the lens the field of view was large enough to avoid to a considerable extent the loss of contrast that comes from magnifying a small section of surface which shades gradually from one portion to another. It is this difficulty that makes it impossible to use the ordinary microscope having a small objective. To secure proper illumination the apparatus was placed so that the observer looked through the plate toward a clear sky.

Whenever two objects are very close together they appear to blend into one, especially if the edges are not sharp and clearly defined. Owing to this effect as the photographic line approaches the parallel hairs of the microscope it blends with them while not really coinciding with them. To avoid as far as possible, the inaccuracies due to this effect, small dots were made with the point of a needle as nearly as possible along the outer edge of the line and it was then possible while the line was in the field of view of the microscope and yet not too close to the parallel hairs to choose the particular dot which most nearly denoted the position of the edge of the line and then take the measurement when this dot came exactly between the parallel hairs.

Description of Apparatus Used in Securing the X-Ray Spectrum.

The apparatus used in this work can perhaps best be described by referring to the isometric drawing of the framework, Fig. 3.

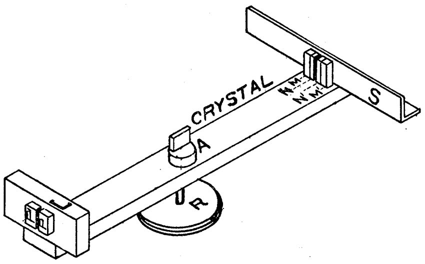

Fig. 3.

The mechanism was enclosed in a box lined with sheet lead $\frac{1}{8}$ inch thick in order to cut out stray radiation, but for simplicity this is not shown in the drawing. The crystal was mounted on the rotating axis *A* which was fitted with adjustable bearings such that this axis could be made truly vertical with respect to the horizontal plane of the instrument. Between the source and the crystal, as close as possible to the latter, a vertical lead plate $\frac{1}{2}$ inch thick was placed. This is not shown in the drawing. The area of crystal surface upon which the X-rays might strike was limited by a slot 3 mm. wide cut through the center of this plate.

One end of the framework of cast iron and steel carried the block of lead *L* which was about 2 inches thick and of sufficient area to subtend a solid angle at the anticathode of the X-ray tube greater than that subtended by the photographic plate and in this way served to protect the plate from the direct radiation of the tube. The previously mentioned lead-lined box enclosing the apparatus served to protect the plate from the radiation reflected from the walls of the room. A slot about 3/16 inches wide was cut through the center of this block of lead and this slot was covered by the two lead plates or jaws *P* and *P'* which had their inner surfaces plane polished and which could be set at any distance apart by means of gauges placed between their upper and lower edges. The slot or space between these two surfaces could then be considered as the source of the X-rays, since it was sufficiently close to the focal spot of the target that this spot subtended a larger angle at the slit than did the crystal, the latter being comparatively far away.

The other end of the framework carried a bar of angle steel, the vertical surface *S* of which was planed true and then set accurately at right angles to the line joining the center of the source and the center of the rotating axis on which the crystal was mounted. The photographic plate was placed in a light-proof envelope and clamped tightly to this surface and since the distance of the surface from the center of rotation of the crystal could be accurately determined by means of a bar of adjustable length which could later be measured on the dividing engine, it was possible to determine the distance of the film from the center of rotation by subtracting the thickness of the plate and the paper back of the plate from the measured length of the bar.

The mechanism for holding the crystal is shown in Fig. 4. One side of the shaft *A* was plane surfaced as was also the block of brass *F* and these could be firmly clamped together by the two screws *H* and *K*. These surfaces could then be separated and placed together at will, always fitting together in the same position. The block *F* carried the block *E* attached to it by three screws in such a way that the surface *BC* could be adjusted to the desired plane and then locked there by the pressure of the screw *I*. With the shaft set in its bearings the upper and lower parts of the surface *BC* were adjusted until when viewed through a microscope both the upper and lower edges remained in the axis of rotation as the shaft was rotated. Then this surface *BC* would contain the axis of rotation and by pressing a crystal surface against this face plate and waxing firmly from behind, the crystal surface would also contain the axis of rotation. The face plate could then be removed by taking out the screws *H* and *K* and the crystal would be left properly mounted.

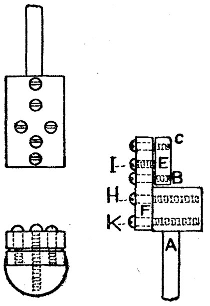

Fig. 4.

The axis *A* was made perpendicular to the framework by first placing a piece of silvered glass in the position of the crystal and adjusting the bearings until the image of a straight horizontal line drawn along the middle of the surface *S* was projected back onto the line at all points as the axis was rotated. When these adjustments were made it was assured that the axis of the shaft bearing the crystal was perpendicular to the horizontal plane of the instrument and that whenever a crystal face was placed against the removable face plate its surface would also contain the axis of rotation. The only other adjustment was to set the apparatus as a whole so that the slot between the jaws *P* and *P′* was on the straight line joining the focal spot and the axis of rotation of the crystal.

It was necessary to have a precise reference line marked on the photographic plate near the point where the undeviated portion of the X-ray beam would strike in order that a photograph might be taken with the crystal set to reflect toward one side of the apparatus and later one taken on another plate with the crystal turned to reflect to the other side of the center line. From these two plates the mean distance of any particular spectral line from this reference line could be found and having once determined the position of this reference line with respect to the true center it was possible to determine the true deviation of any wave-length from a photograph taken on one side of the instrument. To check against changes of position the instrument was frequently calibrated by taking photographs on both sides of the center. The reference line was made by allowing part of the portion of the X-ray beam which passed undeviated through the crystal to pass through the narrow slot between the two plane surfaced lead bars N and N' which were soldered to the brass bars M and M' for the purpose of strength and stiffness. These lead surfaces were separated by thin strips of paper between their upper and lower edges and the narrow beam of X-rays that passed through marked a very fine line on the plate.

While the photographs were being taken the crystal was slowly rotated by means of a fine wire which extended from the pulley R, Fig. 3, to a lever which was connected to a float in a tank of water. Water was siphoned into this tank from another tank in which the level was maintained constant and by regulating the rate of flow, the rate of rising of the float, and the rotation of the crystal could be regulated to any value desired.

While taking the photographs of the L radiation the current for the Coolidge tube was supplied by a transformer excited directly from the 110-volt alternating current mains. The transformer stepped the voltage up to a maximum potential of 58,000 volts and the tube rectified its own current, a well-known property of the Coolidge tube provided, as in this case, that the temperature does not become too high.

In order to avoid the necessity of remaining in the room during the long time required for taking the spectral photographs a motor-operated rheostat was placed in the heating circuit of the Coolidge tube and the motor controls were placed in another room. A wattmeter in this room indicated the power input to the transformer and it was possible by regulating the heating current of the tube to secure any power input desired. It was found that when the heating current was such that the power input of the transformer was 240 watts the target remained at a cherry red heat but did not get hot enough to cause damage to the tube. Of this power about 100 watts went to supply the losses in the transformer and the remaining 140 watts represented the power actually used in the tube.

For the K radiations the same method was followed except that the applied maximum potential was raised to 80,000 volts and the current through the heating circuit was set at such a value as to cause the tube to take 140 watts from the transformer as before.

It was found that the power input would remain constant for an hour to within five or ten watts, hence it was possible to work at other things during the long time of exposure required and thus the labor was very much reduced.

EXPERIMENTAL RESULTS FOR THE *L* RADIATIONS.

Some writers on this subject have used the first letters of the alphabet to designate the shorter wave-lengths and others have used these same letters to indicate the longer wave-lengths, while others have used Greek letters. Owing to these confusing methods of nomenclature it has been thought wise to submit the following means of identifying each particular wave-length. The first three significant figures denoting the wave-length in Ångström units are used as subscripts to the Greek letter λ which is usually used to denote a wave-length. If the knowledge of X-ray spectra shall increase to that point where three figures no longer distinguish two neighboring wave-lengths it will be possible to use four or more figures.

In the experimental work a number of photographs were taken using different distances from the crystal to the plate, always keeping the distance from the source to the crystal as nearly as possible equal to this distance. The method of procedure is shown by the following example. Plate No. 104 was placed so as to register the center line and the spectrum on the left side. Later Plate No. 105 was similarly placed on the right side, each being given an exposure of more than twenty-four hours. When measured on the dividing engine the distance of the most deviated side of the spectral line $\lambda_{1.27}$ from the central reference line was found to be 29.99^3 cm. to the left on Plate No. 104 and 30.03^9 cm. to the right on Plate No. 105. The reference line was therefore one half of the difference or 0.023 cm. to the left of the true center. This correction could then readily be applied to photographs taken later on only one side of the apparatus. The deviation of the outer edge of this spectral line was therefore 30.01^6 cm. and since the slit width was 0.032 cm. subtracting one half of this according to the previous theory gives the true deviation of the line to be 30.00^0 cm. The distance from the axis of rotation of the crystal to the plateholder was 61.10^0 cm., from which must be subtracted the thickness of the plate 0.260 cm., also the thickness of the paper envelope enclosing it, which was 0.013 cm., giving 60.82^7 cm. as the distance from the film side of the plate to the axis of rotation. The quotient of the distance from the center to the spectral line divided by the distance from crystal to film gives the tangent of twice the glancing angle of reflection and denoting this angle by θ we have

$$\text{Tan } 2\theta = \frac{30.00^0}{60.82^7}.$$

Whence $\theta = 13^0\ 7'\ 35''$, from which by the use of the formula $n\lambda = 2d \sin\theta$, in which n is unity and d has the value 2.814×10^{-8} cm. We find λ to be $1.278^1 \times 10^{-8}$ cm.

Table I.

Summary of Results for The L Radiations Wave-Lengths $\times 10^{-8}$ Cm.

Line.	Plates 104 and 105.	Plates 115 and 117.	Plate 121.	Plate 122.	Plate 123.	Average.
$\lambda_{1.48}$				1.4820	1.4836	1.482^8
$\lambda_{1.47}$			1.4719	1.4725	1.4723	1.472^2
$\lambda_{1.41}$					1.4163	1.416^3
$\lambda_{1.29}$	1.2979		1.2968	1.2976	1.2983	1.297^7
$\lambda_{1.28}$					1.2868	1.286^8
$\lambda_{1.27}$	1.2781	1.2781	1.2780	1.2784	1.2793	1.278^4
$\lambda_{1.25}$	1.2589	1.2580	1.2588	1.2593	1.2598	1.258^6
$\lambda_{1.24}$	1.2418	1.2412	1.2413	1.2414	1.2421	1.241^6
$\lambda_{1.22}$				1.2205	1.2199	1.220^2
$\lambda_{1.20}$				1.2102	1.2094	1.209^8
$\lambda_{1.17}$					1.1773	1.177^3
$\lambda_{1.12}$				1.1297	1.1286	1.129^2
$\lambda_{1.09}$	1.0948	1.0951	1.0948	1.0955	1.0963	1.095^3
$\lambda_{1.07}$					1.0705	1.070^5
$\lambda_{1.06}$	1.0645	1.0649	1.0643	1.0645	1.0656	1.064^8
$\lambda_{1.05}$	1.0587	1.0586	1.0581	1.0587	1.0593	1.058^7
$\lambda_{1.04}$					1.0427	1.042^7
$\lambda_{1.02}$	1.0250	1.0246	1.0258	1.0250	1.0262	1.025^3
[1]$\lambda_{.91}$	.9153	.9153	.9158	.9165	.9171	$.915^9$
$\lambda_{.70}$				.7058	.7079	$.706^8$
[2]$\lambda_{.48}$	.4835	.4838	.4838	.4828	.4838	$.483^3$

[1] Wave-lengths shorter than λ .91 are selectively absorbed by the bromine in the plate causing a dark band at the position of this wave-length.

[2] The silver of the plate selectively absorbs wave-lengths shorter than λ .48 thus causing dark band at the position of this wave-length.

In a similar way the angles of reflection and the wave-lengths were determined for the other characteristic L rays and the results of five separate tests are recorded in Table I. These results were computed from an average of eight separate measurements of each plate. The agreement between the different tests is a fair test of the accuracy of the work since the distances to be measured were different in each case. Table II. gives a summary of the results of different investigators each of whom had either used rock salt crystals directly or had compared the gtating constant of some other crystal with that of rock salt so that in every case the results are based on the value of 2.814×10^{-8} cm. for the

TABLE II.

Tungsten X-Ray Wave-Lengths in Ångström Units as Determined by Different Investigators.

Moseley, Phil. Mag., Vol. 27.	de Broglie,[1] Comptes Rendus, Jan., 1914.	Barnes, Phil. Mag., Sept., 1915.	Gorton, Phys. Rev., Feb., 1916.	Hull, Proc. Nat'l. Acad., May, 1916.	Compton, Phys. Rev., June, 1916.	Siegbahn and Friman, Phil. Mag., July, 1916.	Ledoux-Lebard and Dauvillier, Comptes Rendus, April 30, 1917.	Dershem.
			a_1, Faint, 1.477	a', 1.480	l, 1.5044	α_2, 1.481	9 1.504, Very weak	$\lambda_{1.48}$ 1.482^8, Weak
1.486	Strong ray, 1.471	1.477, Strong	a, Strong, 1.465	a, 1.468	k, 1.4993	α_1, 1.471	8 1.492, Intense	$\lambda_{1.47}$ 1.472^2, Strong
								$\lambda_{1.41}$ 1.416^3, Very faint
					i, 1.3543, Line uncertain			
		1.312, Weak	b_1, Faint, 1.290	b', 1.300	i, 1.3160	β_4, 1.296	7 1.309, Very intense	$\lambda_{1.29}$ 1.297^7, Medium
			b_2, Very faint, 1.283					$\lambda_{1.28}$ 1.286^8, Very faint
	Intense doublet							
	Second ray, 1.279	1.296, Strong	b, Strong, 1.277	b, 1.280	h, 1.2962, Strong	β_1, 1.278	6 1.290, Very intense	$\lambda_{1.27}$ 1.278^4, Strong
	Intermediate, 1.258	1.277, Weak	c, Faint, 1.255	c, 1.260	g, 1.2771	β_3, 1.258	5 1.269, Very intense	$\lambda_{1.25}$ 1.258^6, Medium
	First ray, 1.241	1.258, Strong	d, Strong, 1.238	d, 1.242	f, 1.2587, Strong	β_2, 1.241	4 1.250, Very intense	$\lambda_{1.24}$ 1.241^6, Strong
					e, 1.2349			$\lambda_{1.22}$ 1.220^2, Very faint
								$\lambda_{1.20}$ 1.209^8, Very faint
								$\lambda_{1.17}$ 1.177^3, Very faint
								$\lambda_{1.12}$ 1.129^2, Weak
	Strong ray, 1.096	1.113, Strong	g, Strong, 1.095	g, 1.100	d, 1.1107, Strong	γ_1, 1.095	3 1.101, Intense	$\lambda_{1.09}$ 1.095^3, Strong
								$\lambda_{1.07}$ 1.070^5, Faint
		1.083, Weak	h, Medium, 1.057	h $\begin{cases} 1.073 \\ 1.065 \end{cases}$	c_2, 1.0796 c_1, 1.0725 } Line very close double	γ_2, 1.064 γ_1, 1.058	2 1.067, 1 1.062 } Weak doublet	$\lambda_{1.06}$ 1.064^8, Medium $\lambda_{1.05}$ 1.058^7, Medium
			k, Medium, 1.024	k, 1.0387	b, 1.0539, Line somewhat doubtful			$\lambda_{1.04}$ 1.042^7, Very faint
								$\lambda_{1.02}$ 1.025^3, Medium
	Second band weak, .9192				a, 1.0387			$\lambda_{.91}$ $.915^9$, Bromine absorption line
	First band (strong,...... .4823							$\lambda_{.70}$ $.706^8$, Medium
								$\lambda_{.48}$ $.483^3$, Silver absorption line

Above table illustrates the various methods of nomenclature adopted by the different investigators as well as the values which they have obtained for the different wave-lengths. In classifying these it seemed necessary to put those wave-lengths into the same row which had been described as strong or weak, etc., by the different authors.

[1] Above results given as if by de Broglie were computed from values which he gave for angles of reflection from rock salt in article referred to at head of column.

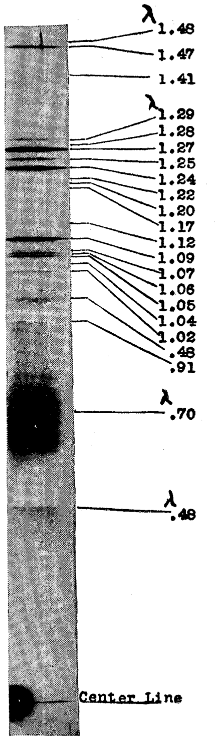

Fig. 5.

Showing the position of the 19 lines of the L group and also the boundaries of regions of greater blackening of the plate corresponding to wave-lengths of .9159 and .4833 Ångström units which are due to selective absorption by the bromine and silver of the plate of waves just shorter than their own K radiations. (de Broglie, Comptes Rendus, Vol. 158, p. 1493, and Vol. 163, p. 87; Wagner, Annalen der Physik, Vol. 46, p. 868.)

Lines λ .48 are *Ag* absorption lines, the upper one being first order, the lower one second order. Line λ .91 is Br absorption line.

Fig. 6.

ELMER DERSHEM.

distance between the atomic planes in halite. Fig. 5 is a photograph showing the position of the L lines of the tungsten spectrum.

Before doing the preceding work it was thought possible that the distance between planes of atoms in a crystal might not be identical for all crystals of the same substance but might vary with the conditions of growth of the crystal. To test this some preliminary measurements were made using crystals of halite obtained from different parts of the earth. The results showed that to within the limits of error of measurement there was no variation of the grating constant.

EXPERIMENTAL RESULTS FOR THE K RADIATIONS.

In securing the photographs of the K radiations the same methods were followed as in the case of the L radiations except that a higher potential was required. On account of the great penetrability of these rays the use of a thin crystal was much more imperative. Fig. 6 shows a photograph of the four K lines of tungsten. Owing to the use of a thin crystal these lines are all clearly separated in the first order. Other observers using a thick crystal have found difficulty in separating the two lines of shortest wave-length in the first order. Table III. gives the results of four tests for the wave-lengths of the K lines of tungsten and

TABLE III.

THE K RADIATIONS OF TUNGSTEN.

Wave-Lengths in Ångström Units.

	Plate 58.	Plate 109.	Plate 114.	Plate 119.	Weighted Average.
$\lambda_{.21}$........	.2121	.2126	.2118	.2126	.212^4
$\lambda_{.20}$........	.2075	.2075	.2069	.2078	.207^6
$\lambda_{.18}$........	.1833	.1818	.1831	.1837	.183^4
$\lambda_{.17}$........	.1784	.1786	.1778	.1785	.178^4

TABLE IV.

A COMPARISON OF THE RESULTS OBTAINED BY DIFFERENT INVESTIGATORS OF THE K RADIATIONS OF TUNGSTEN.

Wave-Lengths in Ångström Units.

de Broglie, Comptes Rendus, April, 1916.	Hull, G. E. Review, July, 1916.	Ledoux-Lebard and Dauvillier, Comptes Rendus, December, 1916.	Dershem.
α_K .2032	α .212	α_1 .2128	$\lambda_{.21}$ .212^4
	.208	α_2 .2053	$\lambda_{.20}$ .207^6
β_K .1768	β .185	β_1 .1826	$\lambda_{.18}$ .183^4
		β_2 .1768	$\lambda_{.17}$ .178^4

Table IV. gives a comparison with the results of other observers. In Table III. in finding the weighted average the last plate is assigned a weight of three and the others a weight of unity since they were not so perfect as the last. In these tests the distance from crystal to plate varied slightly for the different plates, but was always between 60 and 61 centimeters.

ACCURACY OF THE MEASUREMENTS.

Since the extreme variation from the mean value is not greater than 0.1 per cent. for any characteristic line of the *L* group the probable error is less than this amount. In the same way the probable error for the *K* lines is less than 0.8 per cent. On account of the smaller angles these cannot be so easily measured as the *L* lines.

There is very little possibility that the lines observed may in part be due to impurities in the tungsten target. I have no direct information in regard to the purity of the latter but understand that no impurities can be shown by chemical analysis.

These results agree well with such results as are reported by Siegbahn and Friman and also with those computed from the values of the reflection angles as given by de Broglie but disagree with most of the others. This is to be expected in some cases. Gorton used a film wrapped onto a cylindrical surface. It would seem possible that the film might either shrink or stretch in the process of development. Compton recorded the deflections of an electrometer photographically on a moving film. This gives a graphical representation of the relative intensities of the different lines but it would be difficult to get a precise measurement of wave-length in this way since the angular position of the crystal is not accurately known at the moment when the electrometer deflection is being recorded by the photographic film.

THEORETICAL CONSIDERATIONS.

Considerable work has already been done, notably the work of Moseley,[1] in correlating the X-ray spectra of the different elements but little progress has been made toward determining whether, or not, the lines of a single element might be grouped into series such as some of the spectral lines in ordinary light are grouped to form the well-known Balmer's series. The theoretical work of Bohr[2] shows that these series in the case of some of the lighter elements may be derived from a theory of atomic structure and it is the belief of many that X-rays are to the

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[1] Phil. Mag., Vol. 26, pp. 1024–34, and Vol. 27, p. 703.

[2] Phil. Mag., Vol. 26, pp. 1–25, pp. 476–505, and pp. 857–75.

heavier elements what light rays are to those of lesser atomic weight. If X-rays are produced by the change of motion of electrons near the central nucleus it might be possible to work back from an empirically derived series to the mechanism by which these rays are excited. So far such a series has not been found, but this may easily be due to the fact that so far only a comparatively small number of lines has been found. The failure to find them is more probably due to a lack of resolving power rather than to the existence of but few lines. In the case of the plate giving 19 lines in the L group the resolving power was less than 170 and we know that with such low resolving powers we would have learned but little of that which we now know of light spectra.

By the use of Bohr's theory Kossel[1] has attempted to explain the origin of the K and L radiations by assuming several stable orbits of different radii near the nucleus and that the hardest of the K lines is due to the falling of an electron from the outer to the inner orbit. These theories led to the conclusion that the difference in frequency of the two K lines (at the time he wrote the K lines were treated as only two but these are now known to be double lines) should be the frequency of the L line of longest wave-length. This has been said to hold true for a number of elements, but if we take the average wave-length of the K doublets as found in this work we should have

$$\frac{1}{.1809} - \frac{1}{.2100} = \frac{1}{\lambda L}.$$

Whence $\lambda L = 1.30$ instead of 1.48 Ångström units as it should if the theory were correct. This is a greater variation than is permissible, even granting the greatest possible errors in these measurements.

Summary.

1. This work shows that accurate wave-length measurements and the separation of close doublets can only be achieved by limiting the thickness of the crystal and the width of source and making the distance between crystal and photographic plate as great as is practicable with regard to the necessary intensity.

2. The L group of the tungsten X-ray lines by these means is shown to contain at least 19 lines and measurements correct to 0.1 per cent. are given of their wave-lengths. From considerations of the resolving power of the apparatus it seems possible that the true number may be as great as the number of lines in the light spectra of an element.

3. It is shown that the K lines of tungsten may be clearly separated

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[1] Ber. d. Physik. Gesel., Vol. 12, p. 953, 1914.

in the first order if the conditions required for the highest practicable resolving power are complied with.

In conclusion I wish to thank the staff of the physics department and especially Professor G. W. Stewart, who directed the work, for many helpful suggestions and encouragement in the carrying out of this task and also to Mr. A. M. McMahon, who gave much assistance in the performance of the work.

PHYSICS LABORATORY,
UNIVERSITY OF IOWA,
December, 1917.

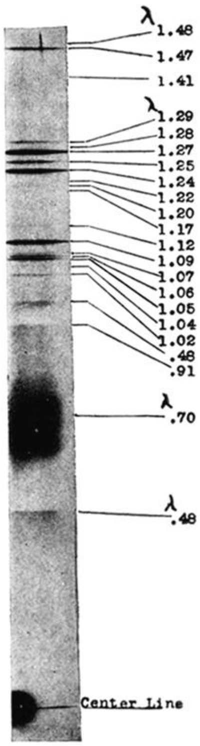

Fig. 5.

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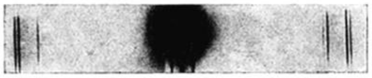

Fig. 6.