THE THEORY OF IONIZATION BY COLLISION.

IV. CASES OF ELASTIC AND PARTIALLY ELASTIC IMPACT.

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Introduction.—In previous papers by one of the writers[1] a theory was developed by which the rate of ionization of molecules of a gas at pressure p by electrons moving in a field of intensity X could be calculated in two particular cases, viz., if the collisions of electrons with molecules are inelastic and if the collisions other than ionizing collisions result in no loss of energy. In the latter case, which was called "the case of elastic impact," it was shown that the average number of ionizing collisions α made by an electron while advancing one centimeter bears to the pressure p and the intensity X the relation

$$\frac{\alpha}{p} = \psi\left(\frac{X}{p}\right), \tag{1}$$

in which the form of the function ψ is determined by

$$\alpha = P\nu = pN\sqrt{P} \tag{2}$$

and

$$\epsilon^{P\nu x_0} = \frac{2}{1+P}\left\{\frac{1}{[\nu x_0(1+P)]} + \frac{2(\nu x_0-1)}{[\nu x_0(1+P)]^2} + \frac{3(\nu x_0-1)(\nu x_0-2)}{[\nu x_0(1+P)]^3} + \cdots + \frac{(\nu x_0-1)(\nu x_0-1)!}{[\nu x_0(1+P)]^{\nu x_0-1}} + \frac{\nu x_0!}{[\nu x_0(1+P)]^{\nu x_0}}\right\}. \tag{3}$$

In these equations P is the probability of ionization at a collision; ν is the average number of collisions made by an electron while advancing one centimeter toward the anode; pN is the average number of collisions made by an electron while moving one centimeter in its actual zig-zag path; N is this quantity calculated for 1 mm. pressure, and is the reciprocal of the mean free path at 1 mm. pressure; $x_0 = V_0/X$, where V_0 is the minimum ionizing potential. These equations were found to agree well with experimental determinations of α in helium[2] when the constants V_0 and N were given values differing very little from accepted experimental values. The small discrepancy between theory and experiment was attributed to impurities in the helium.

[1] PHYS. REV., 7, pp. 489, 501, 509, 1916.

[2] Phil. Mag., 23, p. 837, 1912.

Two lines of evidence, however, have recently indicated that the assumptions underlying equations (1), (2) and (3) must be modified if they are to be applied to helium and similar gases, and have suggested the nature of this modification. The first of these is the fact that, even though collisions in helium are perfectly elastic, yet sufficient energy is transferred from the electron to the molecule at impact to affect appreciably the rate of ionization of the gas. A detailed study of this loss of energy has recently been published by the writers.[1] The second line of evidence is based on the following study of Stoletow's constant.

Stoletow's Constant.—It has been shown by Townsend[2] that, if there is a functional relation of the type

$$\frac{\alpha}{p} = f\left(\frac{X}{p}\right),$$

it necessarily follows that the ratio of the intensity X to the pressure p_m at which α is a maximum is constant for all values of X. This ratio X/p_m, whose value is characteristic of the gas, is Stoletow's constant and has been measured and verified in the case of a number of gases of the inelastic type.

If there were a gas in which electrons lose no energy at impacts, except in the process of ionization, it is obvious that for such a gas p_m would be infinite and Stoletow's constant X/p_m would equal zero. The following experiments were made to test this point in the case of helium.

Carefully purified helium was introduced at various pressures into an ionization chamber containing two parallel electrodes. From one of these, electrons were liberated by ultra-violet light and moved under the influence of the applied field to the second electrode, which was connected to an electrometer, shunted with a high resistance. The details of the purification of the helium and the construction of the apparatus have been described in an earlier paper. The experimental procedure was to vary the pressure, keeping other conditions constant, until the pressure was discovered at which the current through the gas was maximum. A small correction of these results was necessary to take account of the regular decrease of photoelectric emission from the cathode as the pressure was increased. This correction was easily determined by a control experiment. Fig. 1 shows the result of a number of such tests with various values of the field X and the distance d between the electrodes.

It is very evident that X/p_m cannot be considered constant. That this

[1] Phys. Rev., 10, pp. 77, 80, 1917.

[2] Electricity in Gases, p. 300.

lack of constancy is not due to insufficient purity of the helium is proven by our previously reported measurements of the elasticity of impact in this same helium. It is necessary to conclude, therefore, that the functional relation of equation (1) is not true in the case of helium, which

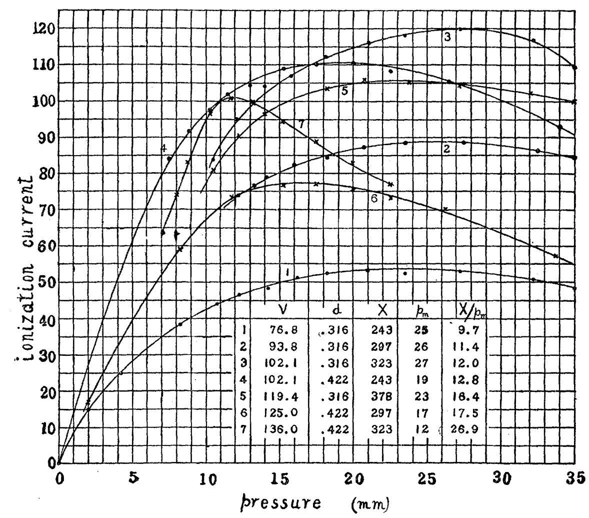

Fig. 1.

proves that the energy lost at non-ionizing collisions in helium cannot be neglected.

The following treatment of the theory takes account of small energy losses at collisions and should be applicable to all cases of elastic and nearly elastic impact.

Theory.—Let Δe, where e is the charge on an electron, represent the average amount of energy lost by an electron at a non-ionizing collision. Then, of the energy Xe acquired from the field while advancing 1 cm., an electron loses on the average an amount $\nu\Delta e$ by these collisions. Thus, if $X'e$ represents the *net* gain of energy per centimeter, we have

$$X'e = Xe - \nu\Delta e. \tag{4}$$

Obviously, if we insert X' in place of the actual intensity X in equations (1) and (3), we take account of losses of energy at non-ionizing collisions. Thus equation (1) in its general form should be written

$$\frac{\alpha}{p} = \psi\left(\frac{X'}{p}\right) = \psi\left(\frac{X - \nu\Delta}{p}\right). \tag{5}$$

In case there is no loss of energy except in ionization, $\Delta = 0$ and equation (5) reduces to equation (1). In case collisions are entirely inelastic, Δ is proportional to X and equation (5) reduces to Townsend's relation

$$\frac{\alpha}{p} = f\left(\frac{X}{p}\right),$$

in which form of the function f has been discussed in preceding papers. For cases in which collisions are nearly or entirely elastic, it is evident that Δ depends on the maximum energy V_0e acquired and not appreciably on the field X. We shall proceed to develop equation (5) into a form applicable to experimental measurements in gases of this latter type.

It was shown in an earlier paper[1] that

$$\nu = \frac{\frac{1}{2}mv^2p^2N^2}{Xe},$$

where v is the average velocity of an electron just before it ionizes. From equation (4),

$$\frac{1}{\nu} = \frac{X'e + \nu\Delta e}{\frac{1}{2}mv^2p^2N^2}.$$

But

$$\frac{X'e}{\frac{1}{2}mv^2} = \alpha = P\nu$$

is the average number of times an electron ionizes while advancing one centimeter, while

$$\frac{\Delta e}{\frac{1}{2}mv^2} = \delta,$$

where δ is the ratio of the average energy lost at a non-ionizing collision to that lost at an ionizing collision. Thus

$$\frac{p^2N^2}{\nu^2} = (P + \delta). \tag{6}$$

Eliminating P by the relation $P\nu = \alpha$, and solving for ν we obtain

$$\nu = \frac{2N^2p}{\frac{\alpha}{p} + \sqrt{\frac{\alpha^2}{p^2} + 4N^2\delta}}. \tag{7}$$

If we substitute this value of ν in equation (5) we obtain the relation

$$\frac{\alpha}{p} = \psi\left(\frac{X'}{p}\right) = \psi\left(\frac{X}{p} - \frac{2\Delta N^2}{\frac{\alpha}{p} + \sqrt{\frac{\alpha^2}{p^2} + 4N^2\delta}}\right), \tag{8}$$

[1] Phys. Rev., 7, p. 510, 1916.

which is in a form suitable for experimental test. The most convenient method of handling experimental data is to substitute the observed values of X/p in equation (8) and calculate X'/p. Equations (2) and (3) are then directly applicable if we put $x_0 = V_0/X'$.

Equation (3) has been solved for P corresponding to the values of νx_0 given in Table I. Corresponding values of pNV_0/X' are determined by use of equation (2). Intermediate values may be determined graphically.

TABLE I.

νx_0.	P.	$\frac{pNV_0}{X'}$.	νx_0.	P.	$\frac{pNV_0}{X'}$.
1	0.2490	0.499	20	0.0265	3.255
2	0.1610	0.802	30	0.0186	4.090
3	0.1213	1.045	40	0.01436	4.793
4	0.0984	1.255	60	0.00989	5.965
5	0.0831	1.441	80	0.00755	6.950
6	0.0723	1.613	100	0.00613	7.830
8	0.0574	1.917	150	0.004175	9.695
10	0.0478	2.186	200	0.003175	11.270
15	0.0340	2.768	275	0.002332	13.300

Comparison with Experiment. Helium.—The values of X/p and α/p in Table II. were determined experimentally by Gill and Pidduck,[1] and the values of X'/p were calculated by equation (8). To do this V_0 and N were chosen to give the best agreement between theory and experiment; Δ was taken to be the energy lost at an impact by an electron moving with half the ionizing energy, and is known with considerable accuracy as a result of our recent measurements of energy losses; $\delta = \Delta/V_0$.

TABLE II.

$V_0 = 21$ volts. $N = 8.7$.
$\Delta = 0.00282$ volts. $\delta = 0.000134$.

$\frac{X}{p}$.	$\frac{\alpha}{p}$.	$\frac{X'}{p}$.
5.0	0.127	3.83
10.0	0.275	9.31
10.0	0.285	9.33
20.0	0.560	19.63
20.0	0.597	19.65
38.1	1.035	37.90
40.0	1.080	39.80
80.0	1.835	79.90
120.0	2.100	120.0
200.0	2.370	200.0

[1] Phil. Mag., 23, p. 837, 1912.

The last two sets of observations are not plotted, since the experimental conditions under which they were taken have been shown in an earlier paper to be misleading.

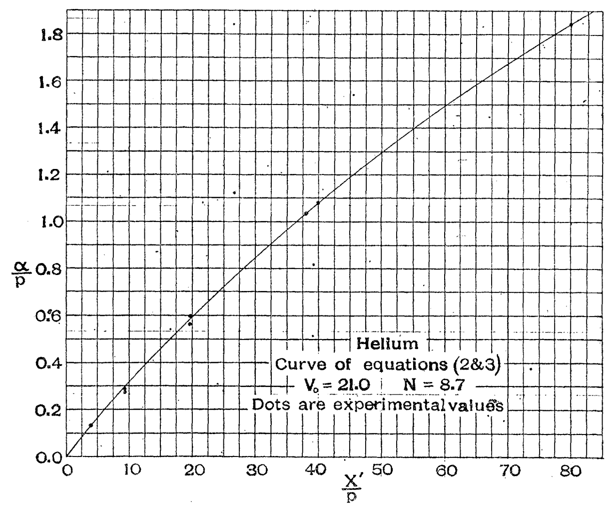

Fig. 2.

The remarkable agreement between theory and experiment is shown by Fig. 2, in which the solid curve represents equations (2) and (3) and the dots represent the observations in Table II. The discrepancies are certainly within the limits of experimental error.

Further support of the theory is afforded by the values of V_0 and N, which are the parameters of the equations. Probably the minimum ionizing potential is nearer 20 volts than 21 volts, but 21 volts is within the range of accepted direct measurements. It is not so easy to decide on the correct value of N, since we estimate N from considerations based on the kinetic theory of gases, and it is not certain that the effective molecular cross section which functions in collisions of molecules with each other is pertinent to the present problem. Assuming that it is, however, we find values ranging from $N = 8.3$ to $N = 13.5$, depending on the method of calculation. The smaller values result from taking the electronic free path to be $4\sqrt{2}$ times that of a gas molecule and the larger values from $N = \pi r^2 n$, where r is the molecular radius and n the number of molecules per unit volume. The former method of calculation has been more widely accepted, and there is no reason, therefore, for doubting the accuracy of the value $N = 8.7$.

The Case of Hydrogen.—It is supposed that impacts in hydrogen are more elastic than those in other gases, with the exception of the monatomic gases. This view is supported by rough measurements by Franck and Hertz[1] of the average energy lost at a collision and by attempts to apply equations for inelastic impact to the case of ionization in hydrogen. For instance, if an attempt is made to fit the equation for inelastic impact developed by one of the writers[2] to the experimental data published by Townsend[3] and Townsend and Hurst,[4] good agreement is obtained if the minimum ionizing potential is taken to be $V_0 = 9.56$ volts. In dealing with all other gases the equation leads to values of V_0 which are too large, while in this case it leads to a value which is distinctly too small. The most probable explanation of this discrepancy is that electrons retain some energy after non-ionizing impacts. The equations of this paper, however, are much less successful than those of inelastic impact. This supports the evidence, which we have advanced in our former papers, that losses of energy at impacts in hydrogen are due to processes similar to those which are effective in the so-called inelastic gases, and which are typically different from those which produce energy losses in gases like helium. The energy lost in inelastic gases, we believe, appears as energy of vibration of parts of the molecular complex.

Discussion.—The equations developed in this paper should be, and appear to be, more accurate than any that have been proposed for the case of elastic impact. The reason for this lies in the fact that all such equations must be based on some assumption regarding the probability that an electron, whose energy is greater than the minimum ionizing energy, will ionize at a collision. Until the mechanism of ionization is better understood, the expressions suggested for this probability must be entirely empirical and the best of them is probably only an approximation to the truth. Any error in the form of this expression, however, affects the accuracy of equations for elastic impact much less than those for inelastic impact. For if an electron, possessing at least the minimum ionizing energy, fails to ionize at an inelastic impact it loses its chance until it has gathered a new supply of energy; while if it fails to ionize at an elastic collision it retains its ability to ionize at the next collision. Since collisions are comparatively numerous in elastic gases, this means that an electron advances very little beyond the point at which it has accumulated the ionizing energy until it ionizes. There is reason, therefore, for confidence in equations (2), (3) and (8).

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[1] Verh. d. D. Phys. Ges., 15, p. 373, 1913.
[2] Loc. cit.
[3] Phil. Mag., 6, p. 598, 1903.
[4] Ibid., 8, p. 738, 1904.