ELASTICITY OF IMPACT OF ELECTRONS WITH GAS MOLECULES.

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Introduction.—In the theory of ionization by collision, as originally developed by Townsend, it was assumed that an electron lost practically all of its kinetic energy at each impact with a gas molecule. Subsequent discoveries have shown that this view is substantially correct in the case of most gases.

On the other hand, it was pointed out by Franck and Hertz[1] that the strong ionization in helium, whose molecules are difficult to ionize, could only be explained by assuming that in this gas the electrons retained a considerable portion of their energy at encounters, so that their energy at any instant has been accumulated during the entire path since the preceding ionizing collision. They proved the existence of this type of collision in helium, and also in the other monatomic gases neon and mercury vapor, by showing that in these gases the ionization current increases abruptly whenever the applied difference of potential between the electrodes is increased to an exact multiple of the minimum ionizing potential.[2] These experiments have been amply verified by Goucher,[3] Bazzoni,[4] Todd and others. We are therefore justified in distinguishing two general types of impact, inelastic and elastic.

The question immediately suggests itself: "Are these really two distinct types of impact, or may there be all degrees of elasticity between the two extremes of perfect elasticity and complete inelasticity?" At first sight, the case of hydrogen appears to support the latter alternative. Impacts in hydrogen are known to be less elastic than those in monatomic gases, but more elastic than in other multiatomic gases.

As far as we know, the only attempts to measure directly the amount of energy lost by an electron at a collision were made by Franck and Hertz.[5] They projected electrons with a known maximum velocity

[1] Verh. d. D. Phys. Ges., 15, p. 34, 1913.

[2] Verh. d. D. Phys. Ges., 16, p. 457, 1914. Professor Bergen Davis and Mr. F. S. Goucher have shown, in the case of mercury vapor, that these successive discontinuities occur also at multiples of the "minimum radiating potentials."

[3] PHYS. REV., 8, p. 561, 1916.

[4] Phil. Mag., 32, p. 566, 1916.

[5] Verh. d. D. Phys. Ges., 15, p. 373, 1913.

through a gauze into a long chamber filled with gas at low pressure. If any electrons returned to an electrode in the plane of the gauze, these must have been reflected from gas molecules. By measuring the retarding field against which these reflected electrons could reach the electrode, the energy retained after a collision was found and thus the energy lost at a collision was determined. It is to be noted that, in these experiments, electrons approaching the detecting electrode obliquely were treated as if approaching directly, with the result that the apparent average energy loss was much greater than the actual loss. Realizing this, Franck and Hertz can only conclude that the average loss of energy at a collision, expressed in equivalent volts, is less than 0.3 volt in helium, 1.6 volts in hydrogen, and that in the common gases practically all the energy is lost. These considerations, of course, apply only to impacts with velocities less than the minimum ionizing velocity. When ionization occurs, the electron must lose at least the amount of energy necessary to ionize the molecule.

Recently the writers[1] have suggested a theory of the loss of energy by an electron while passing through a gas, according to which the electron should lose very little energy in a monatomic gas, whereas in multiatomic gases, the loss of energy should be least in light gases of simple molecular structure and greatest in heavy complex gases. Qualitatively, at least, this is in accordance with the facts. The vital point in the theory, however, is that *the loss of energy in the two types of gases is due to distinctly different processes*, so that we should *not* expect to find all degrees of elasticity of impact between the most and the least elastic gases.

In the present investigation we have developed a method for measuring the loss of energy at an impact which has enabled us to measure accurately losses of the order of magnitude of a thousandth of a volt. Measurements of the loss in helium indicate that impacts of electrons with helium atoms are perfectly elastic in their nature, *i. e.*, that the only energy lost by the electron is due to the motion imparted to the atom during impact. The method is so sensitive to changes in the elasticity of impact that the experimental measurements prove the coefficient of restitution at impacts in helium to be unity with a possible experimental error of not more than 0.01 per cent. In other words, the coefficient of restitution, if not unity, is at least greater than 0.9999. In hydrogen and oxygen the loss of energy is much greater and is shown to be of a more complicated type than in helium. Attempts to measure the loss in argon have failed, up to the present, owing to the failure to obtain gas of sufficient purity for these experiments. These points will

[1] PHYS. REV., 8, p. 449, 1916.

be discussed more fully after the experimental evidence has been presented.

Calculation of the Average Energy lost by an Electron at a Collision with a Gas Molecule.—Let us consider, for the moment, the case of an electron of mass m moving with velocity v and colliding in "head on" fashion with a stationary molecule of mass M. After impact the velocities of electron and molecule are v_1 and V_1 respectively. The electron loses a fraction k' of its original kinetic energy, which we may easily calculate from the relations

$$mv = MV_1 - mv_1,$$
$$ev = V_1 + v_1,$$

where e is the coefficient of restitution. We find

$$k' = \frac{v^2 - v_1^2}{v^2} = \frac{M^2(1 - e^2) + 2Mm(1 + e)}{(M + m)^2}.$$

Since we may take $M = M + m$ without appreciable error, this expression may be written

$$k' = (1 - e^2) + 2(1 + e)\frac{m}{M}.$$

In the actual case of electrons traveling through a gas, not all collisions are of the "head on" type, in which the velocities are in the direction of the line of centers at impact. Many electrons strike more or less "glancing" blows, and we have to average the effect of all. To make calculation possible we shall assume the molecule to be spherical. We may then multiply the energy lost by an electron which strikes the surface of the molecule at a given angle by the probability of striking at that angle, and integrate over all possible angles (0 to $\pi/2$), thus determining the average loss of energy at a collision. Even this calculation is difficult except in the particular case of interest to us, when e is very near to unity. In this case we find approximately

$$k = \frac{(1 - e^2)}{2} + \frac{(1 + e)m}{M} \qquad (1)$$

for the average fraction of its energy lost by an electron at a collision. This approximation becomes more accurate as e approaches unity, and if collisions are perfectly elastic the relation is exact, taking the form

$$k = \frac{2m}{M}, \qquad (2)$$

which is just half the value of k' for "head on" collisions alone.

We have assumed, thus far, that the molecule is at rest when struck. The question therefore arises: What is the effect of the thermal motion of the molecules on the decrease in the kinetic energy of the electron at impact? We may take this into account by averaging the effects of two types of collisions: between electrons and molecules moving in opposite directions and between those moving in the same direction before impact. Assuming perfect elasticity and denoting the average molecular velocity before impact by V, we find that, out of N collisions, there are

$$\tfrac{1}{2}N\frac{v+V}{v}$$

collisions of the first type, resulting in an average energy loss equal to

$$M\left\{\left(\frac{m}{M}\right)^2 v^2 + \frac{m}{M}vV\right\}$$

and

$$\tfrac{1}{2}N\frac{v-V}{v}$$

collisions of the second type with an average energy loss equal to

$$M\left\{\left(\frac{m}{M}\right)^2 v^2 - \frac{m}{M}vV\right\}.$$

We can therefore obtain the total loss of energy by the electrons in all N collisions and thence find the average loss per collision. When this is divided by the average energy before collision, we obtain

$$k = 2\left(\frac{m}{M} - \frac{V^2}{v^2}\right). \tag{3}$$

This expression illustrates the equipartition theorem, for it shows that, in the absence of external forces, the two types of particles will exchange energy until their average kinetic energies are equal, when the proportion k determining the average loss of energy at a collision becomes zero.

In the present case, however, the velocities v with which we have to deal so far exceed the thermal velocity V that the second term is entirely negligible in comparison with the first. We are therefore justified in taking equation (1) to represent the fraction of energy lost by an electron at a collision, if impacts are very elastic.

In the case of helium, substitution of the relative masses of an electron and a helium atom leads to the value

$$k = 0.0002685 \tag{4}$$

if the collisions are perfectly elastic.

We shall proceed to a description of the experimental method of determining k. If the experimental value of k should differ somewhat from 0.0002685, the appropriate value of the coefficient of restitution could be calculated from equation (1).

Method.—Since the energy lost at a single collision between an electron and a monatomic molecule is known (or assumed for the present) to be small, in order to measure this loss it is necessary to deal with the aggregate effect of a large number of successive collisions. This has been done by liberating electrons at a negative plate and driving them through the gas to a second electrode parallel to the first and positively charged. The number of collisions made by an electron is a function of the gas pressure p and the distance d between the plates.

Curves representing the increase in the electronic current with increasing potential difference indicate by an upward inflection, or "break," the potential at which ionization begins; and this occurs as soon as an appreciable number of electrons have a quantity of energy, in the case of helium, corresponding to a fall through 20 volts, the well-known ionization potential. The difference between the applied potential and 20 volts represents the energy lost by collisions with molecules, and can be made as large as we please by increasing the product pd.

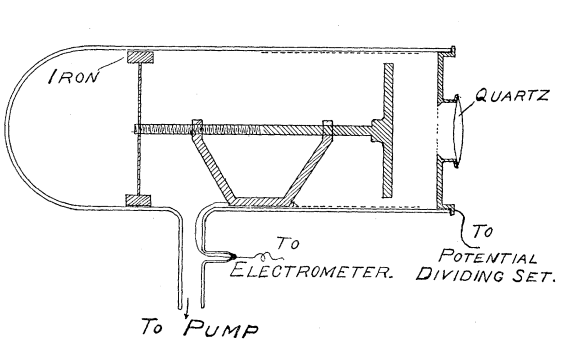

Fig. 1.

The earlier curves were plotted from data obtained by the use of an ionization chamber similar to that used by Partzsch in his work on Stoletow's constant,[1] but later a simpler and more satisfactory one was substituted. The latter is shown in Fig. 1. It consists of a glass tube of about 5 cm. diameter and 14 cm. length, with other parts as shown in proportion. The brass cap on the end, with a fine wire gauze flush with its inner surface serves as one of the electrodes. Behind this electrode

[1] Ann. d. Phys., 40, p. 157, 1913.

and parallel to it is a brass disk, the second electrode, which is mounted on a horizontal threaded shaft held in a nut and journal coaxial with the glass container. On the rear end of the shaft is a cross bar with two iron lugs which, with the aid of an external electromagnet, serve to adjust the distance between the electrodes. A wire gauze closely fitting the inner surface of the chamber surrounds the adjustable electrode and is electrically connected to it. This prevents the accumulation of a charge on the surface of the glass when the distance d is large. The surfaces of both electrodes were heavily coated with platinum, by sputtering, to insure constancy of photoelectric effect and to avoid contact difference of potential.

Ultra-violet light from a quartz mercury vapor lamp enters the chamber through a quartz window and the gauze and liberates electrons from the movable electrode, which is connected to a sensitive quadrant electrometer shunted with a resistance of about 100 megohms. The electrometer gave a deflection of about 2,000 mm. per volt, so that the arrangement is equivalent to a galvanometer with a sensitivity of about $5(10)^{-12}$ amperes per division. (The shunt resistance was very satisfactory and consists of a thin film of platinum deposited on hard rubber or glass, with globules of mercury for contacts.) The fixed electrode is connected to a conveniently adjustable potential source, and voltmeter.

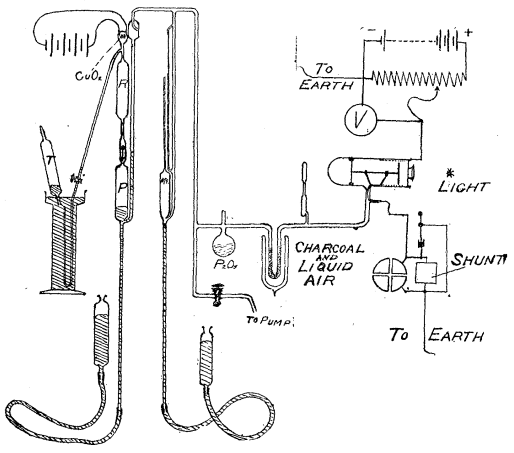

Fig. 2.

The ionization chamber is connected to a hand mercury pump, gas reservoir and McLeod gauge, as shown in Fig. 2. Before introducing gas for investigation, the apparatus was exhausted by a Gaede pump, allowed to stand for some time and again pumped down to the lowest

attainable pressure to get rid of adsorbed gases. As far as possible, the glass parts were heated during part of this process.

The helium was introduced as follows: A U tube, with one arm drawn out to form a capillary tube with the end open, was immersed in mercury as shown at the left of the figure. The stopcock above the U tube was opened while the apparatus was being exhausted and mercury allowed to rise in the tube to a point a little above the stopcock, so that the U tube was entirely filled with mercury and the open end was beneath the surface of the mercury in the cylinder. The tube containing the gas to be introduced was scratched and the end broken off under mercury. The end of the capillary was then introduced into this gas container, which was pressed down allowing the gas to be forced into the apparatus when the stopcock was opened. In this way not more than a cubic millimeter of gas was lost in the transfer. The first bit of gas transferred was pumped out again, in order to carry out traces of other gases remaining in the apparatus. Finally, the introducing tube was sealed off. By means of the hand pump the gas could be pumped from the ionization chamber into the reservoir, so as to get any desired pressure in the chamber. The mercury sealed valve between the pump and the reservoir carried an iron weight on the stem so that the valve could be held open by an electromagnet when it was desired to let gas flow back into the ionization chamber.

A spectrum tube connected with the ionization chamber was used with a direct reading Hilger spectrometer to indicate the presence of impurities in the gas. When working with helium, a U tube filled with cocoanut charcoal and surrounded by liquid air was used to remove impurities. This was very effective except in the removal of hydrogen. It was found that the hydrogen spectrum was much reduced when an electrodeless discharge tube was substituted for the original one, which had aluminium electrodes. This indicates that much of the hydrogen came from the electrodes, as had been proved by Winchester.[1] In order to remove the remainder of the hydrogen, the following method was found the most satisfactory of several methods tried. A small bulb containing a platinum coil which was heavily copper-plated and well oxidized was attached as shown in Fig. 2. After keeping the copper oxide at a bright red heat for several days the hydrogen spectrum was so much reduced as to be almost invisible at low pressure discharges, though it was quite evident at the higher pressures. In this connection it should be remembered that the presence of helium in a discharge tube has the effect of greatly enhancing the spectra of any other gases which

[1] PHYS. REV., 3, p. 287, 1914.

may happen to be present in the tube. It is possible, also, that most of the hydrogen observed was liberated by the discharge in the spectrum tube, and may not have been present in the ionization chamber during the tests. At any rate we feel quite safe in assuming that our helium could be considered pure, for a simple calculation shows that the results of our experiments would have been impossible had there been present in the gas as much as one part of hydrogen in one hundred thousand parts of helium. Several mercury lines were also faintly visible, but with the liquid air trap the amount of mercury vapor present could not have been serious, and even this small amount would not be likely to affect the results because it is fairly well established that collisions in mercury are elastic.

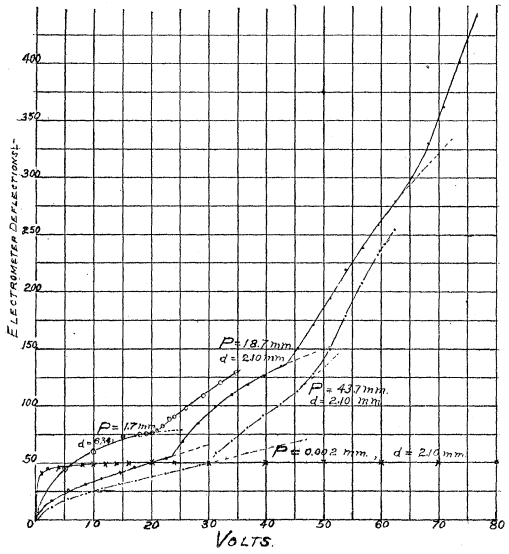

Fig. 3.

Figs. 3 and 4 give typical examples of a large number of curves obtained by plotting the electronic current in helium as a function of the applied potential V for various gas pressures p and distances d between the plates. At extremely low pressures there is no evidence of ionization of the gas, the currents quickly reaching saturation as the potential drop is increased. When the product of the pressure and distance pd is larger, so that an appreciable number of collisions occur, ionization sets in when the applied potential is 20 volts, as indicated by the "break" in the curve. For larger values of pd this "break" is shifted toward larger values of the applied potential, proving that energy is being lost

by the electrons at collisions. Some of the curves are extended to show two or three "breaks," indicating potentials at which the electrons liberated by the preceding ionization are themselves ionizing the gas.

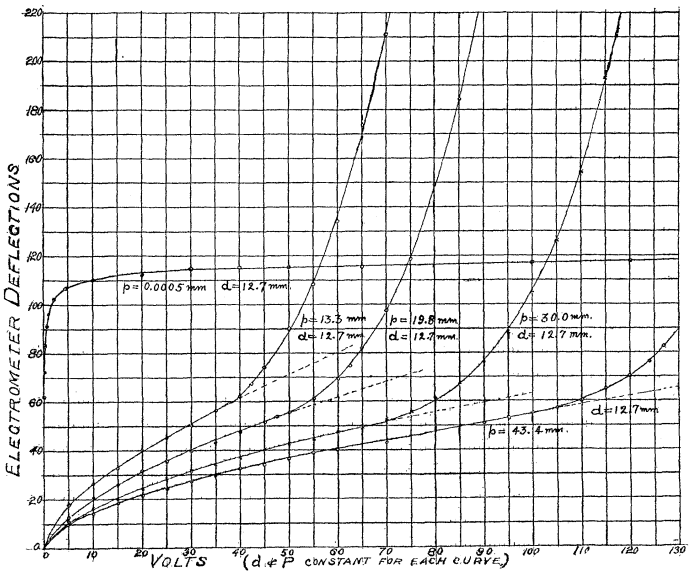

Fig. 4.

It is interesting to note that the second and third "breaks" do not come at exactly two and three times the potential of the first, except when this is at 20 volts. This is due to the fact that the average number of collisions made by an electron while acquiring sufficient energy to ionize

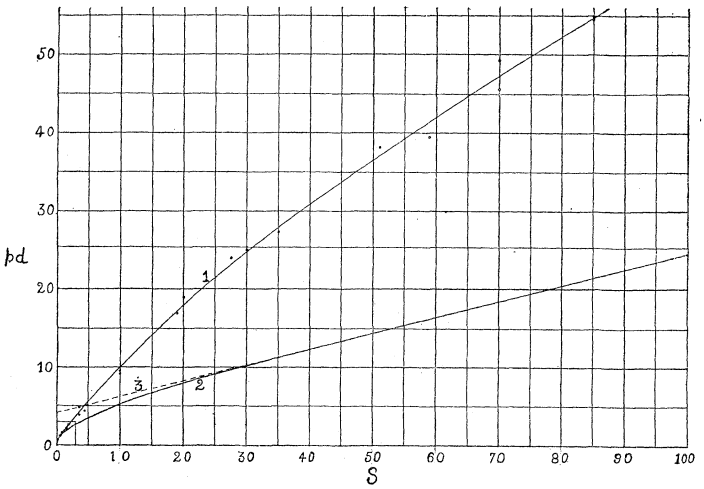

Fig. 5.

is less in the second and third cases than in the first because of the greater potential gradient in these cases.

The conclusions regarding the energy lost at impact are drawn from the variation of the shift of the "break" point (observed ionizing potential) with the product of the pressure and distance pd. The data from observations made after the apparatus was working satisfactorily are given in Table I. and are shown graphically by Curve 1, Fig. 5.

TABLE I.

p (Mm.).	d (Cm.).	pd.	Shift s.
11.70	0.425	4.97	3.7
11.70	0.2125	2.48	1.8
5.55	0.2125	1.18	0.5
4.35	0.900	3.91	3.7
4.35	1.010	4.39	4.5
4.34	0.476	2.06	1.6
4.34	0.370	1.60	0.8
4.34	0.265	1.15	0.5
46.60	0.846	39.40	59.0
46.60	1.060	49.30	70.0
43.00	0.636	27.30	35.0
43.00	0.848	36.50	60.0
43.00	1.060	45.50	70.0
43.00	1.270	54.60	85.0
43.40	1.270	55.00	85.0
30.00	1.270	38.10	51.0
19.80	1.270	25.06	30.0
13.30	1.270	16.90	19.0
43.70	0.210	9.20	10.0
18.70	0.210	3.93	3.7
10.55	1.800	19.00	20.0
13.30	1.800	23.95	27.5
5.40	1.800	9.72	10.0

In order to use these experimental results to determine the degree of elasticity of impact, it is necessary to picture to ourselves the phenomena accompanying the passage of an electron between the electrodes in the gas, and to express the energy of the electron at any point in its path in terms of the gain from the field and the loss from collisions.

Change of Kinetic Energy of an Electron passing through a Gas.—The photoelectric relation between the nature of the emitting cathode and the effective wave-length of the ultra-violet light is such that we may neglect the initial velocities of the electrons. We have to deal, therefore, with a group of electrons which start from rest at the cathode and move toward the anode, bounding and rebounding from the molecules with

which they collide. During each free path the motion of an electron is determined by the electric field and the velocity retained after its preceding impact. At each collision, however, a fraction k of its energy is lost. Our problem is to determine the average energy Ue of an electron after it has moved a distance d through the gas at pressure p under the action of a uniform field of intensity X, and thus to calculate the difference of potential through which the electron must move in order to acquire the energy necessary to ionize a molecule.

Let us express the average energy of an electron at any point in the gas by Ue, where U is the energy in equivalent volts. The average rate at which the electron is acquiring energy at this point of its path is $e(dU/dx)$. However the electron gains energy from the field at the rate eX per centimeter. Thus $e[X - (dU/dx)]$ represents the average amount of energy lost at collisions per centimeter advance toward the anode.

If N is the average number of collisions made by an electron in a centimeter path through the gas at one millimeter pressure, then pN is the average number of collisions per centimeter path at the pressure p. The average number of collisions made while *advancing* one centimeter toward the anode we shall denote by ν, which is much greater than pN because of the zig-zag character of the path. The relation between ν and pN is found as follows:

During a free path l the electron experiences an acceleration $X(e/m)$ in the direction of the field for a time equal to (l/v), where v is the average speed. Therefore

$$s = \tfrac{1}{2}al^2 = \frac{1}{2}\frac{Xel^2}{mv^2}$$

gives the average distance moved in the direction of the electric field during one free path. The reciprocal of s is ν and of l is pN, whence

$$\nu = \frac{2mv^2p^2N^2}{Xe}.$$

We could put $mv^2 = 2Ue$, were it not for the fact, discussed later, that the electrons quickly acquire Maxwell's distribution of speeds about the mean speed of advance, so that we must distinguish between the square of the mean speed v^2 and the mean square speed $\overline{v^2}$. Let the ratio $v^2/\overline{v^2}$ equal r^2, whence $mv^2 = 2r^2Ue$. Then

$$\nu = \frac{4r^2Up^2N^2}{X}. \qquad (5)$$

We may therefore write the average amount of energy lost per collision by an electron in the region of the gas specified by x in the form

$$\frac{e}{\nu}\left(X - \frac{dU}{dx}\right), \quad \text{or} \quad \frac{Xe\left(X - \frac{dU}{dx}\right)}{4r^2Up^2N^2}.$$

This expression must equal kUe, where k is the fraction of energy lost at a collision. Thus we obtain

$$\frac{dU}{dx} = X - \frac{4r^2p^2N^2kU^2}{X} \tag{6}$$

for the average net rate of gain of energy by an electron whose energy is U, expressed in equivalent volts.

If the anode is at a distance d from the cathode, the average energy of the electrons reaching the anode is given by

$$\int_0^U \frac{dU}{X^2 - 4r^2p^2N^2kU^2} = \int_0^d \frac{dx}{X},$$

whence

$$U = \frac{V}{2rN\sqrt{k}pd}\,\frac{\epsilon^{4rN\sqrt{k}pd} - 1}{\epsilon^{4rN\sqrt{k}pd} + 1}, \tag{7}$$

where V has been written for Xd, the total difference of potential between the anode and the cathode.

In order to adapt this relation to our experimental results in Table I., we note that we were able to ascertain the value of U as soon as it became equal to the minimum ionizing potential V_0, whence we shall consider equation (7) when U has the value V_0. Now $V - V_0 = s$ is the "shift" whose experimental values are given as a function of pd in Table I. and Fig. 5. Solving equation (7) for this quantity, we find

$$s = V_0\left[\frac{2rN\sqrt{k}pd(\epsilon^{4rN\sqrt{k}pd} + 1)}{\epsilon^{4rN\sqrt{k}pd} - 1} - 1\right]. \tag{8}$$

In order to understand the application of this equation to the experimental results, attention should be called to the fact that the equation applies to *mean* values of the kinetic energy of the electrons, while in our experiments we detect ionization and thus determine the values of s when an appreciable number of the *fastest* electrons attain the minimum ionizing energy. The following considerations enable us to take account of the difference between these points of view.

When the product pd is small, so that relatively few collisions are made by each electron, there is small probability that the speed of any electron will differ appreciably from the mean speed. Consequently equation (8) may be safely used for small values of pd. When pd is increased, however, the relation between s and pd approaches a linear form, which indicates that energy is being lost by collisions at almost the same rate that it is acquired from the field. This state of equilibrium is most easily expressed by placing $(dU/dx) = 0$ in equation (6), whence

$$U = \frac{X}{2rpN\sqrt{k}} \tag{9}$$

gives the mean energy of electrons in a steady state of drift in the field X. Strictly speaking, this state would not be reached until the electrons had moved an infinite distance through the gas, but it was reached within the limits of experimental accuracy in a number of our measurements with large values of pd. In other words, we were able to increase pd indefinitely, keeping X constant, without appreciably increasing the mean energy U.

Under these conditions, Langevin[1] and Boltzmann[2] have shown that the velocities of the electrons are distributed according to Maxwell's law about the mean velocity of drift. That this really applies to the case under discussion may be shown by an argument based on two equations derived by Pidduck[3] in a paper on "The Abnormal Kinetic Energy of an Electron in a Gas."

He considers electrons of mass m and charge e moving with a steady mean rate of drift u_0 in a field X through a gas consisting of perfectly elastics pherical molecules of mass M, each set having velocities distributed according to Maxwell's law. His equations, with certain symbols changed to avoid ambiguity with the present paper, are

$$u_0 = \frac{3eX}{16N'\sigma^2}\left(\frac{6}{\pi m\lambda MV^2}\right)^{1/2},$$

$$\frac{u_0^2}{V^2} = \lambda - 1,$$

where N' is the number of molecules per unit volume, σ is the molecular radius, V is the square root of the mean square velocity of molecular agitation and λ is the ratio of the mean kinetic energy of an electron to that of a gas molecule.

[1] Ann. Chim. Phys., 105, 5, p. 245, 1905.
[2] Boltzmann, Gastheorie, Vol. 1, p. 114.
[3] Roy. Soc. Proc., 88, p. 296, 1913.

If we substitute $\pi N'\sigma^2 = pN$ in the first equation and use the second equation to eliminate u_0, we obtain

$$\lambda \cdot \tfrac{1}{2} M V^2 = \frac{3eX}{32pN}\sqrt{\frac{6\pi M}{m}},$$

neglecting the factor $(\lambda - 1)/\lambda$ which does not differ appreciably from unity. The first member is, by definition of λ, equal to the mean kinetic energy of an electron, which we have expressed in the form eU. Thus

$$U = \frac{3X}{32pN}\sqrt{\frac{6\pi M}{m}}. \tag{10}$$

By equation (2) we may replace $\sqrt{M/m}$ by $\sqrt{2/k}$. Equation (10) thus becomes

$$U = \frac{3\sqrt{12\pi}}{3^2}\frac{X}{pN\sqrt{k}} = 1.151\,\frac{X}{2pN\sqrt{k}}.$$

This is seen to be identical with equation (9) of the present paper, since the numerical term 1.151 is identical with $1/r$, which is the ratio of the square root of the mean square speed to the average speed in a Maxwellian distribution.

The point of this discussion is that we may apply equation (8) directly to our experimental results only when dealing with such small values of pd that the maximum speed of the electrons at any point of the gas does not differ appreciably from the mean speed. Under these conditions the ratio r equals unity. As the value of pd increases, the ratio r diminishes, approaching the value 1.151^{-1} as a limit. For very large values of pd, equation (8) becomes

$$s = V_0(2rN\sqrt{k}pd - 1). \tag{11}$$

If we know, from the characteristics of our apparatus, the least proportion of the electrons whose ionization can be detected, we may apply equation (11) to our experimental results if we give to the average energy, not the value V_0, but such a smaller value as will give, according to Maxwell's distribution, the necessary proportion of electrons with energies equal to or greater than the minimum ionizing energy V_0. In this case the constant r in equation (11) should be given the value 1.151^{-1}.

We have, therefore, two methods of using the experimental results to determine the value of k. Of these methods, the one utilizing very small values of pd is the more direct and accurate.

Calculation of Elasticity of Impact in Helium.—The experimental determinations of the relation between s and pd are shown plotted along

Curve 1, Fig. 5. For very small values of pd the points are plotted on a larger scale in Fig. 6, which includes the region marked off by the small rectangle near the origin in Fig. 5.

In equation (8) V_0 has the value 20 volts, and N will be taken to be 8.5. This value is calculated from values of the mean free path of helium atoms at 1 mm. pressure by taking the mean free path of an electron to be $4\sqrt{2}$ times that of an atom, in accordance with Maxwell's conclusions regarding a small particle moving with relatively high speed among larger particles. Different methods of estimating the mean free path of a helium atom give somewhat different results, so that a weighted mean value of these results was used to determine the value $N = 8.5$. As a matter of fact, N enters into the equation in such a way that the conclusions arrived at would not be appreciably affected if any of the individual values rather than their mean had been used. With these values equations (8) and (11) become

$$s = 20\left[\frac{17r\sqrt{kpd}(\epsilon^{34r\sqrt{kpd}} + 1)}{(\epsilon^{34r\sqrt{kpd}} - 1)} - 1\right] \tag{12}$$

and

$$s = 20(17r\sqrt{kpd} - 1). \tag{13}$$

The ratio $r = 1$ when $pd = 0$ in equation (12) and decreases gradually as pd increases, approaching the value $r = 1.151^{-1}$ for equation (13).

In Fig. 5, Curve 2 represents equation (12) on the assumption that impacts are perfectly elastic, so that $k = 0.0002685$ by equation (4). It is seen to coincide with the experimental Curve 1 when pd is very

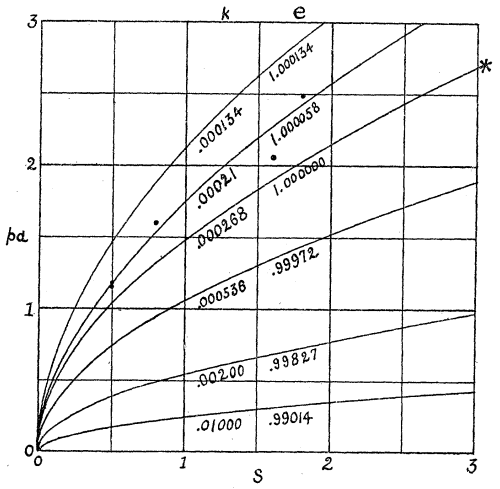

Fig. 6.

small, and at large values of pd to approach the straight dotted Curve 3, which represents equation (3).

(*a*) *Use of Small Values of pd to Determine k.*—In Fig. 6 are shown graphs of equation (12) for various arbitrarily chosen values of k. The correct value of k is the smallest value for which the curve of equation (12) lies entirely below the experimental results, approaching coincidence with the experimental curve at the origin. The reason for this choice is obvious from the discussion in the preceding section. For instance, k is less than 0.002, since a curve with a smaller value of k can obviously be drawn without passing above and intersecting the experimental curve. Similarly k is greater than 0.000134, since this curve, near the origin, lies above the experimental curve. An examination of the relation of the curves of Fig. 6 to the plotted experimental values shows that k cannot be smaller than about 0.00024 and cannot be larger than about 0.00035. Mechanical considerations show that k cannot be less than 0.0002685, which represents perfect elasticity. Thus the value of k is fixed with considerable certainty between 0.0002685 and 0.00035. These values of k, by equation (1), show that the coefficient of restitution cannot differ from unity by more than 0.01 per cent.

This degree of accuracy in the determination of e seems, at first sight, impossible. It is possible because of the very small proportion of energy lost per impact, whence a very slight decrease in the degree of elasticity would greatly increase the proportion k of energy lost.

(*b*) *Use of Large Values of pd to Determine k.*—A consideration of the sensitiveness and constancy characteristics of our apparatus leads us to the conclusion that a consistent increase of 5 per cent. in the electronic current is about the least increase which we could detect and take as definitely indicating a "break" in the experimental curves of Figs. 3 and 4. We will therefore take 5 per cent. to be approximately the proportion of the electrons present which must have energies equal to or greater than 20 volts in order that ionization may be detected. In a Maxwellian distribution it is found that 5 per cent. of the particles have kinetic energies equal to or greater than 2.6 times the mean energy. Thus, in the present case, 20 volts represents 2.6 times the mean energy U, whence the mean energy at the "break points" must have been close to $U = 7.7$ volts.

The slope of the theoretical $pd - s$ curve for large values of pd is shown by equation (11) to be $(2V_0rN\sqrt{k})^{-1}$, if the average energy were represented by V_0, or 20 volts for helium. We have just seen, however, that the average energy appropriate to our experiments must be taken to be about 7.7 volts. Substituting this value in place of V_0 and taking

$r = 1.151^{-1}$ and $N = 8.5$, we should obtain the slope of the experimental Curve 1, Fig. 5, which is very near 0.5. Thus

$$\frac{1.151}{2(7.7)8.5\sqrt{k}} = 0.5,$$

whence

$$k = 0.00031.$$

Obviously there is much greater uncertainty with regard to calculations by this method than with regard to results determined by the method previously discussed. However the order of magnitude cannot be in error, whence this method affords a confirmatory check of the results of the first method.

An Attempt to Apply the Method to Hydrogen and Oxygen.—Extensive series of measurements similar to those made with helium were made with carefully purified hydrogen and oxygen in the apparatus. In dealing with either of these gases it was found very difficult to determine definitely the point at which ionization begins, the "break points" in the experimental curves being much less sharply defined than in the case of helium. This was particularly true when working at small values of pressure times distance *pd*. We never found any indication of a second "break" in a curve. Furthermore, the upward inflections in the $pd - s$ curves, shown in Fig. 7, cannot be explained on the assumptions underlying equation (8). For this reason it is not deemed important to present here the original data or curves, although certain conclusions of a qualitative nature may be drawn from the results.

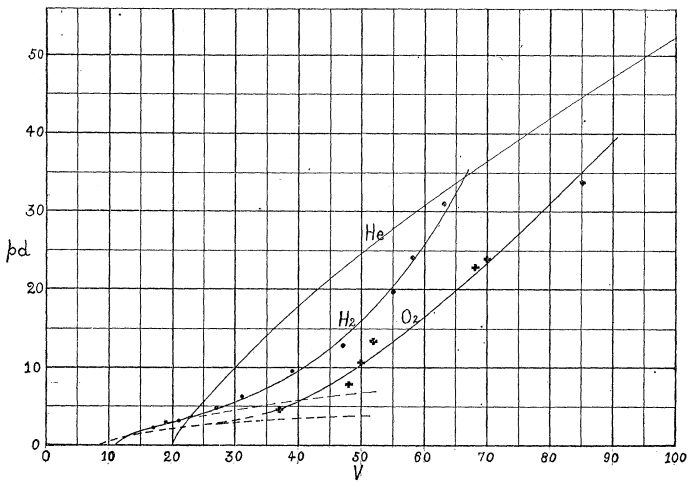

Fig 7.

In Fig. 7 are shown the potentials which must be applied to produce ionization for various values of *pd* for oxygen and hydrogen. Curve 1 of Fig. 5 for helium is also reproduced for comparison. The feet of the curves correspond to the minimum ionizing potentials 8.5, 11.0 and 20.0 volts respectively.

If ionization is due only to the impacts of electrons, and if the average energy lost by an electron at a collision may be represented by a constant fraction of its energy for all values of energy below that necessary for ionization, then there is no reason for an upward inflection like that in the oxygen and hydrogen curves. This inflection probably indicates either ionization by positive ions or by radiation from the molecules excited by the impacts, both of which phenomena would be expected to be more effective at the larger values of *pd*. If these, or other superimposed effects, account for the upward inflection, it appears that the course of the curves, had these effects been absent, would have been somewhat as shown by the dotted lines. At any rate, the trend of the curves for the smaller values of *pd* indicates that less energy is lost at impacts in hydrogen than in oxygen, but that both of these gases are much less elastic than helium.

The difficulty in obtaining sharp "break points" in the curves for small values of *pd* and the failure to find a *series* of "break points" indicates that the group of electrons emitted from the cathode loses its homogeneity more quickly as it travels through oxygen or hydrogen than if moving through helium. This again implies that energy is lost in relatively large amounts at individual collisions, and possibly that the amount lost may depend on the angle at which the molecule is struck.

SUMMARY.

1. A method is developed for measuring the average fraction of its energy lost by an electron at a collision with a gas molecule for impact velocities less than the minimum ionizing velocity. This method can only be applied to a study of those gases in which the amounts of energy lost are relatively small and in which no appreciable amount of ionization is produced, within the range of pressures, distances and applied potentials used, by any agency except the impacts of the electrons.

2. Collisions of electrons with helium atoms appear to be perfectly elastic for velocities less than the velocity corresponding to 20 volts. If any energy is lost by an electron in addition to that transferred to kinetic energy of translation of the atom, such a loss is certainly less than 0.02 per cent. of the energy before impact. From this it seems safe to conclude that:

(*a*) The constituents of a helium atom are held so firmly together that they are not appreciably displaced, relatively to each other, when the atom is struck by an electron whose velocity is less than the ionizing velocity. Or, if such displacement does occur, the natural frequency of the displaced parts must be so high that there is no appreciable lag between their motion and that of the approaching and receding electron.

(*b*) There is no "minimum radiating potential" below the ionizing potential 20 volts. The only appreciable effect of the passage of the electrons through the gas is to slightly increase the mean kinetic energy of the atoms and thus slightly increase the ordinary heat radiation. The same effect on the radiation from the gas could be produced by warming it.

3. Collisions of electrons with molecules of hydrogen and oxygen are much less elastic than in the case of helium and the loss of energy is of a more complicated type, to which the method of this paper cannot be applied.

It should be of great interest to apply this method to a study of the other inert gases and to mercury vapor.

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