THE MOMENT OF MOMENTUM ACCOMPANYING MAGNETIC MOMENT IN IRON AND NICKEL.

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THE importance of ascertaining whether or not mass is associated with the electric current was recognized by Maxwell, who outlined the principles of three different experimental methods of attacking the problem. Phenomena with which Maxwell was unfamiliar have offered more suitable means of measuring the mass of electricity, and, on account of experimental difficulties, not until quite recently have his methods been successfully applied.

In Electricity and Magnetism, § 577, Maxwell suggests that acceleration of a conductor may generate a current; such currents have been found by R. C. Tolman and T. D. Stewart.[1]

The converse experiment is described in § 574—varying the current might set the conductor in motion. This effect will be discussed later in this article; it probably is too minute to be detected.

The general idea of § 575, namely, that a magnet (or a paramagnetic molecule) acts like a gyroscope, forms the basis of the work of S. J. Barnett, who showed that a rotating cylinder of iron becomes magnetized;[2] and of A. Einstein and W. J. de Haas, who showed that mechanical moment of momentum accompanies magnetic moment.[3]

This was first specifically pointed out by O. W. Richardson, who calculated the relation between the two moments according to the electron theory.[4] If magnetism is due to the motion of charged particles in circular orbits within the atom, a magnetized body must possess internal moment of momentum, the amount of which about any axis is proportional to the component of the magnetic moment along that axis. Working in this laboratory, Richardson made an attempt to verify experimentally his equation,

$$U' = 2\frac{m}{e}M'\frac{1 - \dfrac{e}{E}\dfrac{M}{m}\dfrac{A}{a}}{1 - \dfrac{A}{a}}. \qquad (1)$$

[1] Tolman and Stewart, PHYS. REV., VIII., p. 97, 1916.
[2] Barnett, PHYS. REV., VI., p. 239, 1915.
[3] Einstein and de Haas, Deut. Phys. Gesell., 17, p. 152, 1915.
[4] Richardson, PHYS. REV., XXVI., p. 248, 1908.

M' is the magnetic moment, and U' is the corresponding moment of momentum; the multiplying factor (which hereafter we shall call K) is a constant determined by the nature of the rotating sub-atomic corpuscles. These may be positive or negative; for the former M, E, and A, for the latter m, e, and a, respectively, denote the mass, charge in electromagnetic units, and average areal velocity, resolved in the plane perpendicular to the direction of the magnetic intensity. If, as is generally assumed, only the negative electrons are rotating, then

$$K = 2\frac{m}{e} = 1 \cdot 13 \times 10^{-7}. \tag{2}$$

Barnett's derivation of equation 2 follows: Suppose only negatively charged corpuscles are rotating, one in each orbit; then, if r represents the radius vector, ω the angular velocity, μ the magnetic moment, and u the angular momentum of each system, we have

$$\mu = ea, \qquad a = \tfrac{1}{2}r^2\omega, \qquad \text{and} \qquad u = mr^2\omega = 2ma.$$

Thus

$$\frac{u}{\mu} = 2\frac{m}{e}.$$

Since for any given electron orbit the vectors u and μ are in the same direction, summation through any volume gives

$$\frac{U}{M} = \frac{\Sigma u}{\Sigma \mu} = \frac{u}{\mu} = 2\frac{m}{e}.$$

Richardson thought that the operation of the principle of the conservation of angular momentum would give a means of experimentally detecting the existence of this internal momentum, and of measuring K. If the intensity of magnetization along any axis in a body be changed it follows from (1) that the internal moment of momentum about that axis will correspondingly vary, and from Newton's third law it seems probable that the whole body will tend to rotate in the perpendicular plane. The tendency to rotate will be greater as the moment of inertia of the body about the axis of magnetization is less; this suggests the use of a piece of soft iron wire suspended vertically by a fine quartz fiber within a vertical solenoid. Any rotation may be indicated by the movement of a beam of light reflected from a mirror attached to the wire. When the current through the solenoid is suddenly varied we may expect a temporary vibration of the suspended system.

From time to time, since 1908, unsuccessful efforts to observe this effect with such an apparatus have been made in this laboratory. The most recent attempt was begun in the spring of 1915 by Mr. Maurice Pate and the writer, under the direction of Prof. H. L. Cooke, and has been carried on by the writer alone. The difficulty was in eliminating the comparatively large disturbances due to the direct action of the field upon the magnetized wire. It was not until after this work was begun that we learned that Einstein and de Haas had succeeded in observing the effect predicted by Richardson, and had determined the value of K to be that which would be due to negative electrons. Barnett, on the other hand, had found a value only half so large. Since the method of Einstein and de Haas was somewhat different from ours (they got rid of disturbing influences by using for the suspension a comparatively tough glass fiber and building up the effect by resonance, with an alternating current), and since, moreover, the numerical data they published seemed inadequate, we thought it worth while to continue the experiment, using our more direct method.

Our results show that Barnett was right in 1915 in estimating the value of K to be only one half that given by (2). A detailed discussion of our work follows.

Description of the Apparatus.

The apparatus included the solenoid, the optical system, various compensating coils, and the wires to be tested.

Fig. 1 is a drawing of the solenoid, with the horizontal scale twice the vertical. The solenoid was built up of three sizes of brass tubing. The two smallest tubes carrying the parallel mirrors, M, M, fitted into one of intermediate diameter, which formed the framework; and over it fitted the larger tube on which was wound the solenoid proper. This consisted of 2126 turns of number 24 double-silk-insulated copper wire, wound in six layers, which varied in diameter from 3.52 to 4.52 cm.; its length was 22.2 cm. The winding was done very carefully: a thin strip of celluloid was wrapped around each layer on its completion, after the wire had been shellacked, in order that the layer next outside might be wound smoothly. The calculated field at the center due to a current of i amperes was 126 i gausses; the calculated self-inductance was 13 henrys; and the calculated resistance was about 25 ohms, agreeing well with the observed value. The temperature rise per minute in the copper wire, neglecting all heat losses, was, in Centigrade degrees, $10i^2$.

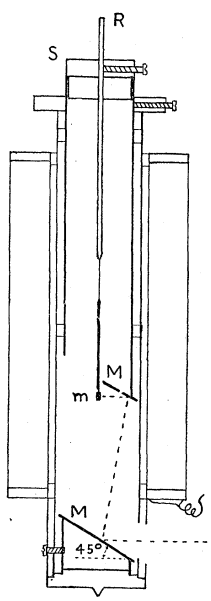

Fig. 1.

To admit of adjustment to the vertical, the solenoid was mounted as is shown in Fig. 2. The lower end rested on a brass strip which could be moved back and forth over the wooden block B, which in turn could be rotated into any desired direction. The upper end fitted into a hole in a brass disc fastened into the brass pipe D; the hole was half an inch off center. By this arrangement the horizontal component of the solenoid field could be brought into the direction of the block B, and then reduced to zero. A screw adjustment was used at the bottom, and one would have been very convenient at the top. In order to keep the whole solenoid from rotating, a projecting rod was held between the prongs of the fork F.

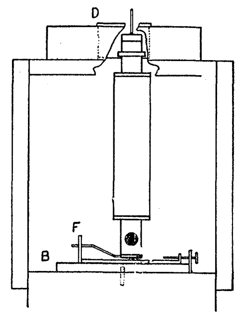

Fig. 2.

The solenoid was mounted in a solidly built box secured to a table with brass screws. One side of the box was left open, and faced north toward another table on which were placed the lamp and scale. Although these two tables were put together with iron bolts this iron was not very near the solenoid, and it caused no trouble.

The optical system was arranged as follows. A Nernst glower was the source of the light, which was reflected by the flat mirrors, M, M (Fig. 1), up to the middle of the solenoid, to the small mirror, m, on the end of the iron wire, and out again to the scale. As the scale distance never was more than sixty centimeters (allowing, of course, for the distance the light travelled down the solenoid), it was unnecessary to make the mirror m concave. A number of small flat mirrors, in size about 0.8 mm. by 3 mm., were cut out of thin microscope cover glass, silvered on one face. The band of light reflected on the scale from one of these mirrors was about 2 mm. broad, and sharply enough defined along its vertical edges. Its horizontal edges were not well defined; and in finding the magnetic moment of the wire, when it was necessary to measure vertical displacements of the spot of light, a convex lens had to be placed in front of the Nernst lamp.

The brass tubes to which the mirrors M, M were attached by soft wax were cut off at exactly 45 degrees; the plane of either mirror could be changed slightly by using extra wax. (Some sort of screw adjustment would have been advantageous.) When things were properly fixed the spot of light had a range of six or seven centimeters on the scale.

Six compensating coils were required to eliminate the earth's magnetic field. A cubical framework was constructed and wound with wire, and fastened to the table with the solenoid at the center. The two horizontal coils, each of 78 turns, were designed to neutralize the vertical component; they were connected in series to act together. Two other coils of 30 turns each were made to oppose the S. N. component, and two coils of 6 turns each took care of any stray E. W. field. All the coils were approximately square, 60 cm. on a side, and opposite members of a pair were almost that distance apart. The axes of all six met in a point at the center of the cube, where the iron wire was hung. A current of about 0.25 ampere in the proper direction through the vertical and S. N. coils neutralized the earth's field at the center. (78/30 is 2.60, which was the tangent of the angle of dip; in practice, however, each pair of coils was in a separate circuit.)

The field at a point on the axis of a square coil of sides $2a$ at distance y from its plane is

$$H = \frac{8\pi a^2 ni}{(a^2 + y^2)\sqrt{2a^2 + y^2}}, \tag{3}$$

for current i and number of turns n. Since $\partial^2 H/\partial y^2 = 0$ when $y = 0.545a$, a more uniform field at points not very close to the center of the cube would have been secured had the coils in each pair been 32.7 cm. apart.

Two exploring coils were needed for a variety of purposes, as will be explained. The coil C was 90 cm. square, and consisted of 180 turns of number 23 copper wire. It was fastened in a vertical plane on top of a heavy box, so that it could be moved about with the center of the coil at the same height above the floor as the center of the solenoid. A large cardboard scale of degrees was attached, for determining the direction of the normal to the coil with reference to a fixed line on the floor. Since it was of importance to know the number of ampere-turns, the coil was constructed in two divisions, one of 72, the other of 108 turns; the field strengths produced by these were compared, in order to make sure that there were no short circuits. Another exploring coil c was made, similar to C in size and mounting, but of only 10 turns.

The electrical connections were as follows: All the rheostats used were solenoidal, with sliding contacts. The rheostats were kept ten or twelve feet away from the solenoid, except the two employed in eliminating the horizontal component of the earth's field; these were placed within six feet. Each of the earth's field compensating coils was in a separate circuit. All lead wires were closely twisted in pairs. The solenoid was connected in series with a commutator, and could be thrown either into circuit 1 or circuit 2. The E.M.F. in circuit 1 could be varied by a shunt from 0 to 120 volts, and could be either direct or alternating (60 cycles). The E.M.F. in circuit 2 was about 20 volts, and the current was regulated by rheostats. This circuit could be closed through a switch, or, momentarily, through a mercury contact: a drop of mercury falling through a glass tube made, in passing, instantaneous electrical contact with the amalgamated ends of two copper wires. One or other of the coils C and c could be thrown in circuit with a commutator, rheostats, and a source of direct E.M.F. that could be varied from 0 to 120 volts.

This completes the discussion of the auxiliary parts of the apparatus, but the essential feature, the iron wire itself, remains to be described. The specimen of wire that was being tested was suspended near the center of the solenoid by a quartz fiber from the brass rod R, Fig. 1, which slid in the removable brass stopper S. A number of these brass pieces were constructed in order that several wires could be mounted at one time.

All the wires were pointed at each end, and rolled out straight, in order that the direction of the magnetic moment might lie along the central axis of the wire, and that the mirror, and especially the quartz fiber, might be attached exactly at that central axis. In spite of these precautions there always was a small component of the magnetic moment transverse to the axis of rotation; and this was the cause of the disturbing effects. The quartz fibers were attached to the top, the mirrors to the bottom ends of the wires; the fibers by shellac, burnt hard when an electrically heated, non-magnetic wire was brought near, the mirrors by minute pieces of soft wax. It was found very convenient in mounting the fiber and mirror to have the wire held vertically between the plane parallel ends of two brass rods which slid toward each other through opposite holes in a brass ring; this ring was fastened to a stand immediately below another brass piece that held the stopper S and rod R. The fiber until attached could be handled by a U-shaped piece of wire.

Before mounting the mirror the torsion constant of the fiber was determined; a small brass disc of known moment of inertia was attached centrally to the lower end of the wire, and the period of vibration was observed with the system free from magnetic control. At different times, and to check each other, two such discs were used. Each was about an eighth inch in diameter, and the calculated moments of inertia were 3.61 and 3.53 by 10^{-4}.

The choice of the size of the wire and fiber is of importance, but it can only be made after a consideration of equation 4, which will now be derived.

Size of the Effect.

This is given by equation 4. The equation of motion of the suspended iron wire is

$$I\frac{d^2\theta}{dt^2} + k\frac{d\theta}{dt} + c\theta = 0,$$

where c is the torsion constant of the quartz fiber, k the damping coefficient, I the moment of inertia of the suspended system, and θ the angle through which it is rotated at time t. In practice, $k^2 < 4cI$ and writing $m^2 = 4cI - k^2$, the solution is

$$\theta = \frac{2U}{m} \epsilon^{-\frac{k}{2I}t} \sin \frac{m}{2I} t,$$

for the special case in which alone we are interested, viz., when the rotation is due to an impulse U units of angular momentum which acted when t and θ were zero.

The period of this damped vibration is

$$T = \frac{4\pi I}{\sqrt{4cI - k^2}}.$$

The amplitude of the first swing is

$$\theta' = \frac{U}{\sqrt{cI}} \epsilon^{-\frac{\lambda}{\pi} \tan^{-1} \frac{\pi}{\lambda}},$$

where λ is the logarithmic decrement, usually small. The exponential term can be expanded in a series, and finally we have for δ, the linear deflection at scale distance L corresponding to θ',

$$\delta = \frac{2LKM}{\sqrt{cI}} (1 - 0.500\lambda + 0.227\lambda^2), \tag{4}$$

neglecting $(\lambda/\pi)^3$ and higher powers. KM is substituted for U; M is the change in magnetic moment giving rise, by (1), to the impulse U.

Two other equations we shall need are

$$c = \frac{4\pi^2 I}{T^2}\left(1 + \frac{\lambda^2}{\pi^2}\right), \tag{5}$$

and

$$I = \frac{cT^2}{4\pi^2}\left(1 - \frac{\lambda^2}{\pi^2}\right). \tag{6}$$

To return to the choice of the size of the wire and mirror—it is governed by four considerations.

First. The effect sought for must be large enough for easy observation.

Second. The magnitude of disturbing effects must be small.

Third. The wire must not be so tiny as to require excessive care in manipulation.

Fourth. The wire must not be longer than 8 or 9 cm., or the solenoid field may be non-uniform near the ends; but it should not be very short, or the demagnetizing factor, as well as the probable value of the transverse moment, will be high.

In (4) the factor $M/\sqrt{cI}$ depends upon the size of the suspended system, and should be as large as practicable. For unit intensity of magnetization this factor becomes $V/\sqrt{cI}$, where V is the volume of the wire. If l is the length and a the diameter, $V/\sqrt{cI}$ is proportional to $la^2/\sqrt{cla^2 \cdot a^2}$ or to $\sqrt{l/c}$, provided the mirror is quite small. The maximum weight the fiber can sustain varies directly as the square, and its torsion constant as the fourth power of its diameter. Calling the latter r, and supposing the fiber loaded to its maximum, or to its maximum divided by a factor of safety, c varies as r^4, and la^2 varies as r^2, or as $\sqrt{c}$; which gives $V/\sqrt{cI}$ proportional to $1/a^2\sqrt{l}$. It is, therefore, of great advantage to use wires of small diameter.

It is disadvantageous to use the smallest possible fibers. In (4) the factor $(1 - 0.500\lambda + 0.227\lambda^2)/\sqrt{c}$ depends upon the size of the fiber, and increases as c decreases; on the other hand, the disturbing direct action of the field, which is not necessarily impulsive, is much less influenced by the increase in damping, and rises rapidly in importance as c is lessened.

Of course, considerably larger values of δ can be obtained if the wire is suspended in a vacuum, but this was deemed unnecessary.

Disturbing Effects and Their Elimination.

The rotation which Richardson predicted does not depend upon the magnetic field produced by the solenoid, but upon the change in orientation of the magnetic molecules which that field causes. The solenoid field, however, as well as the earth's field, acts directly upon the magnetized wire, and the rotation produced by this direct action usually is of a much higher order of magnitude. Such rotation could not be produced if there were no transverse component of the magnetic moment (by symmetry), but that transverse component is never absent. It is possible to find by mathematical treatment the exact value of the rotation θ when the wire is magnetized uniformly at a known angle with its axis of rotation, and is hanging in a uniform field of given strength and direction. The uniform field can exert only a couple on the uniform magnet, and equilibrium is attained when this couple (magnetic) is balanced by the two other couples acting: one due to the twisted quartz fiber (torsional), and the other to the opposite pulls of the tension of the fiber and the weight of the wire (gravitational). Since there are no sidewise forces the fiber remains vertical, and its point of attachment to the wire remains fixed in position. Perhaps the assumption of uniform magnetization of the wire is not absolutely correct, but it seems likely that the departures from uniformity are not large enough seriously to invalidate this analysis.

Einstein and de Haas turned from the ballistic method of detecting the Richardson effect to the method of resonance because they believed the elimination of disturbing effects was impossible. It may well have been impossible in the case they had in mind, viz., when the magnetic moment of the wire is reversed by reversing a large, continuous current in the solenoid. Successful elimination of disturbing effects has been attained only when it was the *residual* magnetism of the wire that was varied. It was possible to work with the residual magnetism in this research, since for the wires used the ratio length to diameter was so large that the demagnetizing factors were unimportant, and a high value of M remained after the solenoid field was discontinued if the wire previously had been magnetized to saturation. To reverse such residual magnetization required a large, though only instantaneous, current in the solenoid; but to reduce it to zero a relatively small field (the coercive force) sufficed.

When M is thus varied the behavior of m, the small, accidental transverse moment, requires special comment. Suppose the wire is hanging in the solenoid and the spot of light is reflected on the scale. When the wire is magnetized with the north end up (hereafter, for convenience, simply "magnetized up") and the fiber is untwisted, suppose that m is represented by the vector *o1*, Fig. 3, making an angle j with the magnetic meridian. By means of the exploring coil C the position of m at any time can be determined; for when C is placed as indicated in Fig. 3, and only then, will a heavy current through C cause no deflection. When the coil c is placed at right angles to C the magnitude as well as the direction of the vector m can be determined. If now the value of M be reduced to zero and reversed, by a succession of increasing momentary demagnetizing currents through the solenoid, then the vector m will be found to rotate successively to the positions *o2*, *o3*, *o4*, and not until M has been completely reversed will m return to its original direction, *o5*.

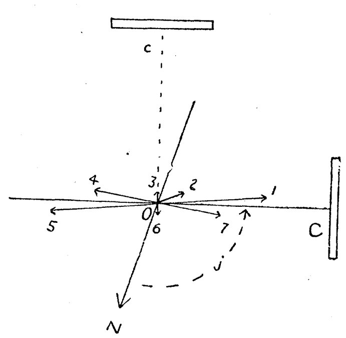

Fig. 3.

When m is at right angles to its original direction (*o3*, *o6*), M is zero as nearly as can be determined—its value then is certainly less than 5 per cent. of the saturated value. The ratio length *o3* to length *o1* (much exaggerated in the diagram) is of the same magnitude as the ratio diameter of the iron wire to its length.

All the wires tested showed this rotation of m, in some cases in the other direction. The cause of the phenomenon is plain enough; m is made up of two components, one of which is the horizontal component of the total moment of the wire, while the other, very much smaller, is due to an actual transverse magnetization of the wire, and remains unchanged, except under large fields.

M, then, is certainly zero when m has rotated through 90 degrees—that is, when a current through c produces no deflection.

The essential condition that must be satisfied before the Richardson effect can be observed is this: the suspended system must be free from magnetic control as regards changes in the value of θ. This result is attained if there is no horizontal field, for it is the horizontal field alone that exerts a couple on the unavoidable transverse component of magnetic moment. The rotation of this component as M changes makes necessary, and also possible, an accurate elimination of the horizontal field. Part of the field is that of the solenoid, and can be reversed or reduced to zero at will; the rest of the field is mainly that of the earth.

The currents through the various compensating coils required exactly to neutralize the earth's field are determined by a method of trial and error. The vertical component is eliminated most easily: an alternating current is sent through the solenoid (circuit 1), and gradually reduced to zero; if after this M is not zero the current through the compensating coil is changed, and the process repeated. This allows of a very delicate adjustment of the compensating current. The position on the scale occupied by the reflected band of light when the wire is exactly demagnetized is taken as the zero position. The period of vibration is determined; this is the period of the system when free from magnetic control. A current is passed for an instant through the solenoid sufficient to leave the wire magnetized quite strongly; now the spot of light, by adjusting the current in the S. N. and E. W. pairs of compensating coils, is brought back to its zero position. There the fiber is untwisted, therefore what horizontal field (say h) is remaining must be in the direction of the transverse component of magnetic moment. By varying together the compensating currents in the horizontally acting coils in such a fashion as to keep the band of light at zero the value of h can be changed. When the period of the suspended system becomes that which it had when the wire was demagnetized we can be sure that h is zero.

The elimination of any horizontal component of the solenoid field is carried out in the same manner, still working only with the residual magnetism of the wire. First the block B, Fig. 2, is rotated into the direction of the transverse magnet, OC, Fig. 3. Then the pipe D is revolved until a deflection no longer results on the application of a small solenoid field in the direction of M. Then the wire is demagnetized by the direct current (circuit 2), so that m is at right angles to its previous direction, and the brass plate on the block B is moved until there is no motion of the spot of light on reapplying the demagnetizing field.

The method of taking periods may not completely have got rid of the horizontal earth's field; in that case the deflection when the wire is approximately demagnetized will not be zero. It can be made zero by adjusting the resistances in the compensating circuits; and this is the final adjustment for the horizontal earth's field.

Even if the earth's field is accurately eliminated, and the solenoid field accurately vertical, there remains one disturbing effect: on applying a large solenoid field the wire tends to swing out of its normal position, for ordinarily the direction of its magnetic moment is not quite vertical. Except with a coarse fiber it is impossible to observe the Richardson effect on reversing the residual magnetism of the wire, for this requires too large a field, and the wire is greatly agitated. By far the best method is merely to reduce the residual magnetism to zero; a relatively small field (the coercive force), applied only for an instant, suffices for this.

Aside from the magnetic disturbing effects the only other trouble was caused by shifts in the zero position of the suspended system due to temperature variations. To guard against this a current was never allowed to flow in the solenoid for more than a few seconds at a time; and the zero was redetermined rather frequently. With most of the wires this effect was absent or negligible, with a few it was annoying, it was serious with none.

Observations Necessary.

Equation 4 is fundamental, but may be transformed into a more convenient working formula. Let T_1 be the period of the suspended system when a known moment of inertia I_1, large compared with that of the wire, has been added. From (5),

$$c = \frac{4\pi^2 I_1}{T_1^2},$$

for in practice the damping here is negligible. Substituting for c in (6) gives

$$I = I_1 \frac{T^2}{T_1^2}\left(1 - \frac{\lambda^2}{\pi^2}\right),$$

where T is the period of the wire and mirror. Here λ, the logarithmic decrement, cannot be neglected, since I is small. Substituting in (4) these values for c and I,

$$\delta = \frac{K}{\pi I_1} \frac{T_1^2}{T} (1 - 0.500\lambda + 0.278\lambda^2 \cdots) ML. \tag{7}$$

For the two inertia-discs used at different times the values of I_1 were, respectively, 3.61 and 3.53 by 10^{-4}. Substituting the value of K given in (2), and expressing L in meters and δ in millimeters, the magnitude of the constant factor $K/\pi I_1$ comes out 0.100 for inertia-disc 1, and 0.102 for inertia-disc 2. When I was not negligible in comparison with I_1 correction had to be made.

To calculate, then, what δ would be if the value of K were that for the negative electrons, it is necessary to observe T_1, T, λ, L, and M. M is found from an observation of the angle ψ in the vertical plane between the normal position of the wire and its position when a horizontal field H, due to coil C, acts along OC, Fig. 3. If the spot of light reflected on the scale moves vertically a distance p when H is set up, then

$$\psi = \frac{P}{2L} \cos j,$$

supposing the direction of the normal to the mirror is that of the magnetic meridian. (The angle j is measured from the magnetic meridian—see Fig. 3.) Equating the magnetic and gravitational couples,

$$MH = \tfrac{1}{2} Wgl\psi, \tag{8}$$

where Wg is the weight of the wire in dynes, and l is its length. Allowance must be made for the weight of the mirror also; in milligrams this was 0.40 times its area in square millimeters. If $\cos j$ is small M cannot be found, and the mirror must be readjusted.

Manipulation.

The wire to be tested was pointed, weighed, measured, and straightened, the inertia disc was attached by a little soft wax, and the fiber was mounted. The wire was placed inside the cubical framework of coils that compensated the earth's field; and the solenoid also was slipped over the wire, which was then demagnetized by gradually reducing to zero an alternating current through the solenoid. The solenoid was removed, and T_1 observed with the suspended system thus freed from magnetic control. The inertia disc was removed, the mirror was attached, and the wire was ready for the test. Usually several suspended systems were constructed at one time.

When the solenoid and compensating coils were back in position, the wire was lowered into the solenoid until the reflection of its mirror could be seen. It was demagnetized by the alternating current, and turned until the spot of light appeared in a central position on the scale. Then the wire was magnetized with the north end up by a momentary current of about an ampere (circuit 1), and the horizontal component of the earth's field was eliminated more or less completely. By the exploring coil *C* the position of the transverse component *m* was found, the block *B* was brought parallel to it, and the lower mirror was turned to bring the spot of light back to its original zero. By turning the pipe *D* the horizontal component of the solenoid field was brought into the direction of *m*. The coil *c* was placed at right angles to the coil *C*, and the solenoid, with the commutator reversed, was thrown into circuit 2. By trial the instantaneous current just sufficient to demagnetize was found. The criterion for demagnetization was that no deflection be produced by a current in coil *c*. Once this demagnetizing current—the coercive force—had been determined no further adjustment of the rheostats in circuit 2 was made. Before the coercive force could be found the vertical component of the earth's field had to be eliminated, but this was practically a permanent adjustment. The adjustment of the solenoid to the vertical was next completed, and the final compensation for the horizontal earth's field was effected.

The solenoid was returned to circuit 1, and the wire again strongly magnetized up. The Richardson effect, a sudden throw to the left, could be observed on again demagnetizing by circuit 2.

The horizontal earth's field and the current in the compensating coils would keep varying slightly; and before every observation the deflection had to be reduced to zero by slight changes in the adjustment of the rheostats. With most of the wires, however, the band of light remained nearly steady on the scale, in satisfactory fashion.

After δ had been determined, T, λ, and M were measured. In finding M several readings were made for two or more values of p, and the wire was once or twice remagnetized between times. M always was the same up as down.

The Experimental Results.

Twenty-four wires were tested—seventeen of iron, six of nickel, and one of silver. The effect sought for was shown by all but the silver wire. Of its reality there can be no question, for it was shown not only by every wire but also by every observation, and the observations agree quantitatively as well as qualitatively.

For nearly all the wires the Richardson effect—a sudden throw to the left when the wire had been magnetized up, or to the right when the wire had been magnetized down—appeared in company with another impulsive twist, the direction of which was independent of the sense of the magnetization. It can hardly be called a disturbing effect, for it always was of about the same magnitude as the Richardson effect, and obviously the two could easily be distinguished. It varied irregularly in direction and magnitude from wire to wire, and also with the same wire under different conditions; it may have been caused by magnetostriction.

It was impossible to cause variation in what was believed to be the Richardson effect, provided things were not thrown far out of adjustment, when observations could not be taken. Every reasonable test left it unaltered. The same sudden throw was obtained when the demagnetizing field was applied permanently, as when it was allowed to act only instantaneously; but it seemed safer to apply it only instantaneously, in order to eliminate all chance of inaccuracy from imperfect adjustment of the solenoid to the vertical. Changing the time-constant of circuit 2 in the ratio 20/1 left δ unchanged. When one of the wires was rotated through 180 degrees and the light reflected from the other side of the mirror, δ was the same as before.

A very certain disproof of the presence of any effect due to the solenoid field was this: with a rather coarse fiber it was possible to get the Richardson effect when the residual magnetism was reversed, instead of being merely brought to zero; and this admitted of varying the field without changing the flux through the wire. No change in δ was found when the momentary reversing field was increased a hundred per cent. and more. This proved that the effect reached a maximum when the magnet was saturated. That it decreased in ratio with M was also shown. Furthermore one of the wires was so well constructed that it was possible to get the effect on *magnetization*; it was of opposite sign to that obtained on demagnetization, but of exactly the same size.

Table I. shows the results for fifteen of the iron wires and six nickel wires. (Numerical results were not obtained for the first two iron wires tried.) In column 11 δ is the observed deflection reduced to scale-distance 50 cm.; in no case did the scale-distance differ from this by more than a few centimeters. The significance of the results is brought out in column 13, which gives the values of K calculated from the observed values of δ by equation 7. For convenience in interpretation these values of K are expressed as the ratios of the observed K to the value (1.13×10^{-7}) which K would have if only negative electrons were moving, and if all the reaction were effective in imparting angular momentum to the wire. The ratio observed K to calculated K is the same as the ratio observed δ to δ calculated by (7).

TABLE I.

Complete Table of the Experimental Results.

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.
No.	Diam. Mm.	Length Mm.	Weight Mg.	Intensity of Magn.	Coerc. Force Gausses.	$\frac{\text{Obs. } I}{\text{Calc. } I}$	Obs. $I \times 10^6$.	$10^5\, c$.	λ.	δ Mm.	M.	$\frac{K}{1\cdot 13 \times 10^{-7}}$
Fe:												
1	0.315	52	28.3	950	4.7	1.3	4.7	21.2	0.22	6.6	3.7	0.56
2	......	52	28.6	870	2.8	1.5	5.3	4.1	38	10.5	3.4	.47
3	......	52	26.5	590	3.9	1.4	5.1	2.5	67	8.2	2.3	.46
4	......	48	24.9	470	4.3	1.3	4.3	4.9	43	5.5	1.7	.51
5	......	35	17.9	700	6.0	1.3	2.7	4.8	37	9.0	1.8	.58
6	......	34	18.2	880	6.3	1.5	4.2	16.9	19	4.9	2.3	.55
7	0.48	69	84.6	320	3.8	1.1	25.2	4.8	46	5.7	4.0	.52
8	.. ..	75	105.9	340	2.8	1.2	36.6	32.7	19	1.8	4.6	.40
9	0.20	49	10.9	520	5.2	1.5	1.7	10.7	37	3.3	0.78	.58
10	......	52	12.9	540	5.0	1.7	1.5	5.9	46	7.2	0.87	.84
11	......	49	11.8	620	5.0	1.8	1.1	7.8	35	8.4	0.93	.88
12	0.13	40	3.6	730	8.8	2.0	1.2	8.0	12	2.1	0.37	.51
13	......	39	3.4	640	7.4	2.2	0.9	6.8	28	4.4	0.32	1.04
14	......	31	2.7	460	6.1	2.0	1.0	3.7	48	1.9	0.18	.68
15	......	52	4.2	670	6.4	2.2	0.9	2.4	51	4.3	0.45	.49
Ni:												
16	0.50	61	100.6	120	30	1.1	35.5	14.0	18	1.2	1.4	.60
17	......	49	77.6	100	30	1.1	27.2	11.2	26	0.8	1.0	.40
18	0.31	44	28.0	120	28	2.0	7.5	5.2	40	2.5	0.39	1.30
19	......	52	33.1	80	28	1.8	6.7	5.7	41	2.1	0.33	1.30
20	......	53	32.6	160	28	1.5	6.0	8.5	32	1.0	0.6	.40
21	0.25	25	10.8	110	38	1.5	2.4	6.2	32	0.5	0.13	.50
Ag:												
22	0.33	39	32.4	......	......	1.5	6.5	2.5	48	No effect		

In the remainder of this paper K will be expressed as this ratio; absolute values of K will not be employed.

In Table I. the value taken for δ is in each case an average of six or eight observations. Naturally it is the least accurately determined of all the observed quantities. In some instances the figure in the decimal place was almost guessed at; but the estimate was made before M had been determined and K calculated, and was never revised after K had been figured out.

A few examples are given below of the consistency of the individual observations. "Up" means the wire had been magnetized up; a throw of negative sign corresponds to a clockwise rotation, viewed from above, which was the direction of rotation of the negative electrons when the wire was magnetized up. The Richardson effect was distinguished from the accompanying irreversible effect in this manner: Suppose that the observed throw when the wire had been magnetized up was δ_1, and when the wire had been magnetized down suppose that the observed throw was δ_2. Then δ', the throw due to the irreversible effect, was $\frac{1}{2}(\delta_1 + \delta_2)$, while the magnitude of δ, the throw due to the Richardson effect, was $\frac{1}{2}(\delta_1 - \delta_2)$. The sign of δ, calculated by this formula, always came out negative—which means that the effect always was in the direction predicted for negative electrons. Examples of the observations of δ follow:

Wire 2, $L = 50$ cm. Up, + 5, 7, 5, 4, 7; Down, + 28, 27, 27, 26, 27 mm. (Average agreement.) Wire 9, $L = 49$. Up, − 5.5, − 5, − 5.3, − 4.7, − 4.5, − 5.7, − 5.0; Down, + 1.5, 1, 1.7, 1.5, 1, 1.5, 1.4. (Average agreement.) Wire 12, $L = 51$. $2\delta = 3.8$, 4, 2.5, 5.0, 5.5, 4.5, 3.2, 2.5, 4.5, 3.2, 4.4, 4.3, 5.5, 5.8, 5. (Worst of all the wires.) For nickel—Wire 20, $L = 50$. Up, − 5.2, − 5.0, − 5.2, − 4.8, − 4.8; Down, − 3.3, − 2.8, − 3.3, − 3.3, − 2.8.

No observation was recorded unless the steady deflection was zero before and after the wire was demagnetized. All the throws were sharp and distinct.

The numbers in column 7 of the table indicate how well each system was constructed. The observed value of I was in every case greater than the calculated value. (Values of I were calculated from the geometrical dimensions of the systems, taking into account the mirrors. For no mirror did the moment of inertia about its own central axis exceed 10^{-6}.) Those systems were best constructed for which the ratios in column 7 are nearest unity. Some of the smaller wires apparently were injured in the process of mounting, and a few of these gave wild values of K. All the wires, however, for which the ratio observed I to calculated I was less than 1.6 gave consistent values of K, and these only should be considered in taking the final averages.

The smallest wires were not intended to improve the mean value of K, but to prove that K does not vary with the diameter and hence that the internal angular momentum actually is proportional to the volume, as (1) demands. This constancy of K seems sufficiently established. The table of results also makes it very evident that the observed K was independent of such factors as the intensity of magnetization, the coercive force, etc.

The value of K seems to be about the same for nickel as for iron. The numerical accuracy of the results is less for nickel, because nickel is far less magnetizable; and because its coercive force is much larger, which makes it harder to eliminate disturbing effects. The mean value of $K/1.13 \times 10^{-7}$ for the nine iron wires for which the ratios of column 7 are less than 1.6 is 0.51 ± 0.04. For four nickel wires the corresponding mean is 0.47 ± 0.11.

The large departures from the means all are positive. The cause of this phenomenon is unknown (unless it be simply that there is more room for error on that side).

Einstein and de Haas obtained for K a value about twice that found by the writer.[1] They tried only two wires; the first gave $K = 0.75$, and they built a new apparatus. The second gave $K = 0.98$, but they published only seven numerical observations of the value of the "double throw," these all on the same resonance curve—and in taking the mean they discarded the three smallest ones. The ratios observed I to calculated I for their wires were 1.5 and 1.2, respectively. Afterwards another experiment was made by de Haas by a slightly different method; of this later work the writer has seen only the brief account published in Science Abstracts. "An electromagnet is hung from a unifilar suspension with its magnetic axis vertical and performs torsional oscillations. The current is reversed automatically, so that it can be observed whether the magnet has a moment of momentum depending upon and reversed with its magnetism. In one case the moment of momentum was detected and found to be 1.35×10^{-4}. By theory this must be 1.13×10^{-7} the magnetic moment, which gave 1,200 for the magnet instead of 1,400."[2] This would make $K = 0.86$. The resonance method is ingenious, but one cannot be sure that it really does eliminate all disturbing effects. Still another resonance method has been developed by Einstein, but this one is apparently only a lecture-table experiment. The residual magnetism of a suspended iron rod is reversed periodically by an instantaneous current.[3]

Maxwell's Second Effect.

When one of the iron or nickel wires was demagnetized the change in magnetic moment was accompanied by a change of flux and a momentary induced current. It is necessary to show that this current did not produce the sudden throws that were observed.

If the current in the wire moves the wire either this motion is caused by ordinary electromagnetic reaction between the current and the external field, or it is not. Proof has been given in a previous paragraph

[1] Einstein and de Haas, *loc. cit.*

[2] de Haas, Sci. Abs., XIX., p. 351, No. 938, Aug. 25, 1916. K. Akad. Amsterdam, Proc. 18, No. 8, pp. 1281–1299, 1916.

[3] Einstein, Chem. Abs., *11*, p. 1777, 1917.

that the throw δ was independent of the field. (Even the irreversible throw that usually accompanied δ was not caused by the induced current, for it was dependent upon the vertical, not the horizontal, component of the solenoid field.) If the current moves the wire, then, it must move it itself. Such a phenomenon would be of interest, but it does not exist. Since the current is momentary Maxwell's second suggestion does not apply; his second effect can appear only when the current is changed.

According to any electron theory of metallic conduction transference of electricity is by the convection of electrons in the direction opposite to the electric field, and per unit volume there is an exactly equal quantity of positive charge. So long as the current flows steadily a state of statical equilibrium exists, and there is no resultant force of the field upon the body as a whole. When the current is increasing, however, although the positive charge remains immobile, the state of motion of the negative electrons is being subjected to change; and to effect this change a certain amount of the negative field is being used. The result is an unbalanced force in the direction opposite to the negative current, which would give rise to Maxwell's second effect.

Assume the free electron theory. Let there be N free electrons per unit volume; if their average excess velocity over the free path in the direction of the negative current is v, then the current density, i, is Nev. The average momentum per electron is mv, or mi/Ne, and the momentum in volume V is

$$G = \frac{m}{e} Vi.$$

This is the fundamental equation for Maxwell's second effect, on the free electron theory.

Application of this equation to the case of our suspended iron wires shows that the Maxwell effect could not produce an impulse comparable to that caused by the Richardson effect, even if the induced current could be made to keep on flowing (as in a super-conductor), unless demagnetization took place in 10^{-7} second.

Nevertheless, in order to make perfectly certain that it was not some effect of the electrons concerned in conduction that was being observed, a silver wire was tested in the same manner as the magnetic wires. It showed a trace of magnetization, due probably to clinging particles of dust, or to the wax or mirror. The usual adjustments were made, and the steady deflection remained accurately zero when a solenoid field of a hundred gausses was suddenly applied. Shifting of the zero on account of temperature changes was annoying; but δ certainly was less than 0.2 mm., and it seemed to be zero. Of course there was no magnetic moment of the silver wire, but, in order to bring the calculation into the same form as for the other wires, it is convenient to assume that the flux, $B = \mu H$, was due to a magnetic moment $M = BV/4\pi$, instead of to the solenoid field H. $V = 3.32 \times 10^{-3}$ cm³., the volume of the wire, and μ is the permeability, which is unity for silver. When $H = 101$, $M = 0.0266$. Accordingly, if K in silver were five times as large as in iron (which is what one would expect from the ratio of the conductivities), the observed value of δ would have been 0.5 mm. Since δ certainly was less than 0.2 mm., we may suppose that the effect is absent in silver.

Einstein and de Haas reported that there is no effect in copper. There probably is no effect in copper, but they did not prove it. Although the conductivity of copper is a few times greater than that of iron the permeability is so much less that it would have required an alternating field of 2,000 gausses, instead of the 50 they used, to get a "double throw" of a millimeter with their apparatus—even if the effect did exist in copper.

Summary of the Experimental Results.

A momentum effect such as Richardson predicted for magnetizable substances exists in iron and nickel.

The direction of this momentum is that which would be due to the rotation of negative electrons within the atom; but the magnitude of the effect is only half that which Richardson supposed would result from such rotation of negative electrons.

No such effect exists in silver, whence the effect in iron and nickel cannot be attributed to the conducting electrons.

Conclusions: The Bearing Upon the Structure of the Atom of the Value Found for K.

The internal moment of momentum observed in iron and nickel must be due to the rotation of matter within the atom. It has usually been assumed that only negative electrons are moving, but this assumption leads to an internal momentum twice that observed. It is important to find a reason for the diminished effect.

There are two possible explanations:

1. Negative electrons alone are moving, but cannot react upon the suspended wire with the full effect predicted.

2. Positive as well as negative charges are rotating, in opposite directions.

1. To produce the twist of the suspended wire the rotating electrons must react upon the atom, and the atom, in turn, must react upon the wire as a whole. There are these two chances for loss of part of the

original momentum. Richardson suggested when he predicted the effect that the reaction might take place upon the electromagnetic field, or that the iron atoms might be loose and unable to transmit the momentum to the wire as a whole. The known facts of magnetism, however, render the latter supposition improbable; and if the reaction had taken place upon the electromagnetic system that produced the exciting field the observed effect would not have been independent of the intensity of magnetization.[1]

Barnett's first experiment on the production of magnetization by rotation—an effect the converse of the Richardson effect—agreed with this experiment in giving only half the full effect calculated for negative electrons. The coincidence between the writer's results and those of Barnett not only is evidence of the correctness of both experiments, but also seems to make untenable the loose-atom explanation of the diminished K.

Quite recently[2] Barnett has obtained somewhat larger values of K for steel, nickel, and cobalt. He finds that K has about 80 per cent. of the full predicted value; but the experimental errors are so large that, as Barnett himself states, his results can be considered as agreeing with those of Einstein and de Haas. They certainly agree equally well with those of the writer.

2. We are thus led to the important conclusion that the internal angular momentum in iron and nickel is only half what it would be if negative electrons alone were in motion.

By (1), if expressed as a fraction of $2m/e$,[3]

$$K = \frac{1 - \frac{e}{E}\frac{M}{m}\frac{A}{a}}{1 - \frac{A}{a}}.$$

It follows that

$$\frac{A}{a} = -\frac{1 - K}{K - \frac{e}{m}\frac{M}{E}}. \tag{9}$$

Experimentally, K has been proved constant with respect to changes in magnetic intensity. Therefore A/a is constant; we proceed to calculate its value.

Assume that the atom is composed of negative electrons and hydrogen nuclei (positive electrons), and that in the iron atom these electrons are

[1] Richardson, The Electron Theory of Matter (1914), p. 396.

[2] Barnett, Proc. Nat. Acad. Sci., *3*, p. 178, 1917.

[3] A mistake in sign made by Richardson is corrected here. He wrote $NE = ne$.

not packed so closely together that their mass is appreciably changed. Then, if $E = 1$ and $M = 1$, $e = -1$ and $m = 1/1850$. If the positive and negative electrons, respectively, are moving with angular velocities Ω and ω in circular orbits of radii R and r, it follows that $A = \Sigma\Omega R^2$ and $a = \Sigma\omega r^2$. Substitute these values in (9), and substitute for K the observed value, 0.51. Then

$$\frac{\Sigma\Omega R^2}{\Sigma\omega r^2} = -\frac{1}{3800}. \tag{10}$$

According to Sir Ernest Rutherford's theory of atomic structure, all the positive charges are concentrated in a very small "nucleus" at the center of the atom, while about half the negative electrons are rotating around this nucleus at distances very large compared with its diameter. Equation 10, if true, signifies that the central positive nucleus itself is rotating, but in the opposite direction. A rough calculation based on the assumptions of Rutherford and Bohr, shows that the ratio in (10) will be of the order of magnitude there indicated if the angular velocity of the rotating positive nucleus is about equal (but opposite in sign) to that of the inner ring of electrons.

H. S. Allen[1] has imagined an atom with a rotating positive core surrounded by a ring of revolving electrons, but he assumed $\Sigma\Omega R^2 = \Sigma\omega r^2$. Qualitatively, his assumption of the same sign for Ω as for ω is necessary for his explanation of the magneton.

Summary.

This paper is devoted chiefly to an account of an experiment which showed that iron and nickel, when magnetized, possess internal angular momentum, as was predicted by Richardson in 1908. The magnitude of this momentum can be accounted for if positive, as well as negative, charges are moving within the atom, but in opposite directions. The experimental results of Barnett and of Einstein and de Haas are in qualitative agreement with those described.

The writer is indebted to Professor H. L. Cooke for initial assistance and to Professor K. T. Compton for criticism. It should be possible, with the experimental method described in this paper, to observe the Richardson effect in cobalt and the Heusler alloys, and perhaps also in magnetite.

Palmer Physical Laboratory,
Princeton University,
July 14, 1917.

[1] Allen, Phil. Mag., XXIX., p. 714, 1915.