Decay of the Σ^+ Hyperon and its Antiparticle*

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Two decay modes of the Σ^+ hyperon and the corresponding ones for its antiparticle are investigated. The branching ratio may be different for both, if charge-conjugation invariance or time-reversal invariance does not hold.

THE purpose of this note is to investigate the decay modes of Σ^+ and $\bar{\Sigma}^+$.

$$\Sigma^+ \to p+\pi^0, \tag{1a}$$

$$\Sigma^+ \to n+\pi^+, \tag{1b}$$

$$\bar{\Sigma}^+ \to \bar{p}+\pi^0, \tag{2a}$$

$$\bar{\Sigma}^+ \to \bar{n}+\pi^-, \tag{2b}$$

where "barred" quantities mean antiparticles (e.g., $\bar{\Sigma}^+$ is the antiparticle of Σ^+). As a result of the TCP theorem,[1] the total lifetimes of Σ^+ and $\bar{\Sigma}^+$ are equal. However, this is generally not true for partial lifetimes, e.g., for (1a) and (2a), if charge conjugation or time-reversal invariance does not hold. Therefore, we may test the validity of these conditions by comparing the modes (1a) and (2a).

In the lowest order of the weak interaction H_W, which causes these decays, the matrix elements for the processes $i \to f$ and $\bar{i} \to \bar{f}$ are given by

$$M(i \to f) = [\Psi_{(-)}^*(f) H_W \Psi_{(+)}(i)], \tag{3a}$$

$$M(\bar{i} \to \bar{f}) = [\Psi_{(-)}^*(\bar{f}) H_W \Psi_{(+)}(\bar{i})]. \tag{3b}$$

By applying the TCP theorem[1,2] to (3b), we have

$$M(\bar{i} \to \bar{f}) = [\Psi_{(-)}^*(i) H_W \Psi_{(+)}(f)] = \sum_n M^*(i \to n) S(f \to n), \tag{4}$$

where we have replaced $\Psi_{(-)}(i)$ by $\Psi_{(+)}(i)$, which is correct to the lowest order in H_W for $i=\Sigma^+$, and where $S(i \to f)$ is the S-matrix element.

We may put

$$M_1 = M(\Sigma^+ \to p+\pi^0) = [(\tfrac{2}{3})^{\frac{1}{2}} A_3 - (\tfrac{1}{3})^{\frac{1}{2}} A_1] + [(\tfrac{2}{3})^{\frac{1}{2}} B_3 - (\tfrac{1}{3})^{\frac{1}{2}} B_1](\boldsymbol{\sigma}\cdot\mathbf{k})/k, \tag{5a}$$

$$M_2 = M(\Sigma^+ \to n+\pi^+) = [(\tfrac{1}{3})^{\frac{1}{2}} A_3 + (\tfrac{2}{3})^{\frac{1}{2}} A_1] + [(\tfrac{1}{3})^{\frac{1}{2}} B_3 + (\tfrac{2}{3})^{\frac{1}{2}} B_1](\boldsymbol{\sigma}\cdot\mathbf{k})/k, \tag{5b}$$

and

$$\bar{M}_1 = M(\bar{\Sigma}^+ \to \bar{p}+\pi^0) = [(\tfrac{2}{3})^{\frac{1}{2}} \bar{A}_3 - (\tfrac{1}{3})^{\frac{1}{2}} \bar{A}_1] + [(\tfrac{2}{3})^{\frac{1}{2}} \bar{B}_3 - (\tfrac{1}{3})^{\frac{1}{2}} \bar{B}_1](\boldsymbol{\sigma}\cdot\mathbf{k})/k, \tag{6a}$$

$$\bar{M}_2 = M(\bar{\Sigma}^+ \to \bar{n}+\pi^-) = [(\tfrac{1}{3})^{\frac{1}{2}} \bar{A}_3 + (\tfrac{2}{3})^{\frac{1}{2}} \bar{A}_1]/k + [(\tfrac{1}{3})^{\frac{1}{2}} \bar{B}_3 + (\tfrac{2}{3})^{\frac{1}{2}} \bar{B}_1](\boldsymbol{\sigma}\cdot\mathbf{k})/k, \tag{6b}$$

where $\mathbf{k}$ is the momentum of the pion in the rest system of the Σ hyperon, and we have assumed that Σ has spin $\frac{1}{2}$.

A_3, B_3, etc. (or A_1, B_1, etc.) represent the part of the matrix element belonging to the total isotopic spin $T=\frac{3}{2}$ (or $T=\frac{1}{2}$) in the final pion-nucleon system. If parity is conserved, either all the A's or all the B's are zero.

The application of (4) to the processes (1) and (2) gives the following equations.

$$\bar{A}_3 = A_3^* e^{2i\delta_3}, \quad \bar{A}_1 = A_1^* e^{2i\delta_1}, \quad \bar{B}_3 = -B_3^* e^{2i\delta_{13}}, \quad \bar{B}_1 = -B_1^* e^{2i\delta_{11}}, \tag{7}$$

where the δ's are the usual phase shifts for pion-nucleon scattering.

By using (7), we can easily verify that

$$\sum_{\text{spin}} \int d\Omega_{\mathbf{k}} (|M_1|^2 + |M_2|^2) = \sum_{\text{spin}} \int d\Omega_{\mathbf{k}} (|\bar{M}_1|^2 + |\bar{M}_2|^2), \tag{8}$$

which means that the total lifetime of Σ^+ and $\bar{\Sigma}^+$ are equal. However, in general,

$$\sum_{\text{spin}} \int d\Omega_{\mathbf{k}} |M_1|^2 \neq \sum_{\text{spin}} \int d\Omega_{\mathbf{k}} |\bar{M}_1|^2.$$

If charge conjugation holds, then in addition to (7), we have

$$A_3 = A_3^* e^{2i\delta_3}, \quad A_1 = A_1^* e^{2i\delta_1}, \quad B_3 = -B_3^* e^{2i\delta_{13}}, \quad B_1 = -B_1^* e^{2i\delta_{11}}; \tag{9}$$

if time reversal is valid, then

$$A_3 = -A_3^* e^{2i\delta_3}, \quad A_1 = -A_1^* e^{2i\delta_1}, \quad B_3 = -B_3^* e^{2i\delta_{13}}, \quad B_1 = -B_1^* e^{2i\delta_{11}}. \tag{10}$$

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[1] Lee, Oehme, and Yang, Phys. Rev. **106**, 340 (1957); G. Lüders and B. Zumino, Phys. Rev. **106**, 385 (1957), *Proceedings of the Seventh Annual Rochester Conference on High-Energy Nuclear Physics* (Interscience Publishers, Inc., New York, 1957), IV, p. 55; S. Okubo (to be published).

[2] W. Pauli, in *Niels Bohr and the Development of Physics*, edited by W. Pauli (Pergamon Press, London, 1955), p. 30.

In the derivation of (10), we have assumed that Σ^+ has the same parity as the nucleon. In the case of opposite parity, we only have to change all the negative signs in (10) into positive signs.

Returning to the general case, let us put

$$A_3=|A_3|e^{i(\delta_3+\Delta_3)},\quad A_1=|A_1|e^{i(\delta_1+\Delta_1)},\quad B_3=i|B_3|e^{i(\delta_{13}+\Delta_{13})},\quad B_1=i|B_1|e^{i(\delta_{11}+\Delta_{11})},\tag{11}$$

where the Δ's are unknown phase factors. Then, the identity (7) gives

$$\bar{A}_3=|A_3|e^{i(\delta_3-\Delta_3)},\quad \bar{A}_1=|A_1|e^{i(\delta_1-\Delta_1)},\quad \bar{B}_3=i|B_3|e^{i(\delta_{13}-\Delta_{13})},\quad \bar{B}_1=i|B_1|e^{i(\delta_{11}-\Delta_{11})}.\tag{12}$$

Specifically, if invariance under charge conjugation or time reversal holds, then by (9) or (10),

$$\Delta_3,\ \Delta_1,\ \Delta_{13},\ \Delta_{11}\equiv 0,\quad \text{or } \pm\pi\tag{13}$$

for charge-conjugation invariance, and

$$\Delta_3,\ \Delta_1=\pm\pi/2;\quad \Delta_{13},\ \Delta_{11}=0\quad \text{or } \pm\pi\tag{14}$$

for time-reversal invariance.

We define the relative frequency of the events (1a) and (2a) by

$$\rho_1=\sum_{\text{spin}}\int d\Omega_k|M_1|^2\Big/\sum_{\text{spin}}\int d\Omega_k(|M_1|^2+|M_2|^2),\tag{15a}$$

$$\bar{\rho}_1=\sum_{\text{spin}}\int d\Omega_k|\bar{M}_1|^2\Big/\sum_{\text{spin}}\int d\Omega_k(|\bar{M}_1|^2+|\bar{M}_2|^2),\tag{15b}$$

and their ratio by

$$\theta=\frac{\bar{\rho}_1}{\rho_1}=\sum_{\text{spin}}\int d\Omega_k|\bar{M}_1|^2\Big/\sum_{\text{spin}}\int d\Omega_k|M_1|^2,\tag{16}$$

where we have used the identity (8).

By (5a), (6a), (11), and (12), we get

$$\theta\equiv\frac{1-2x\sin[\delta_3-\delta_1-\frac{1}{2}(\Delta_3-\Delta_1)]\sin[\frac{1}{2}(\Delta_3-\Delta_1)]-2y\sin[\delta_{13}-\delta_{11}-\frac{1}{2}(\Delta_{13}-\Delta_{11})]\sin[\frac{1}{2}(\Delta_{13}-\Delta_{11})]}{1+2x\sin[\delta_3-\delta_1+\frac{1}{2}(\Delta_3-\Delta_1)]\sin[\frac{1}{2}(\Delta_3-\Delta_1)]+2y\sin[\delta_{13}-\delta_{11}+\frac{1}{2}(\Delta_{13}-\Delta_{11})]\sin[\frac{1}{2}(\Delta_{13}-\Delta_{11})]}.\tag{17}$$

where

$$x=\tfrac{2}{3}\sqrt{2}\frac{1}{D}|A_1|\cdot|A_3|,\quad y=\tfrac{2}{3}\sqrt{2}\frac{1}{D}|B_1|\cdot|B_3|,\tag{18}$$

$$D=\tfrac{2}{3}(|A_3|^2+|B_3|^2)+\tfrac{1}{3}(|A_1|^2+|B_1|^2)-\tfrac{2}{3}\sqrt{2}|A_1|\cdot|A_3|\cos(\delta_3-\delta_1)-\tfrac{2}{3}\sqrt{2}|B_1|\cdot|B_3|\cos(\delta_{13}-\delta_{11}).$$

If invariance under charge conjugation or time reversal is valid, so that according to (13) and (14)

$$(\Delta_3-\Delta_1),\ (\Delta_{13}-\Delta_{11})=0,\ \pm\pi,\quad\text{or } \pm 2\pi,\tag{19}$$

then (17) gives

$$\theta=1,\tag{20}$$

which means the equivalence of the partial lifetimes for the decays (1a) and (2a).

The pion-nucleon phase-shifts are given[3] by

$$\delta_3-\delta_1\simeq-23^\circ,\quad \delta_{13}-\delta_{11}\simeq 7^\circ\quad(\text{at}\sim 130\text{ Mev}).$$

Thus, if $x\neq 0$ and the deviation from time-reversal or charge-conjugation invariance is rather large, (17) will give a value different from unity and hence may be detected by experiment. Otherwise x will be of order unity.

In this derivation, we assumed that the spin of Σ is $\frac{1}{2}$. If it is $\frac{3}{2}$, the final pion must be in a P or D state. We may probably neglect the contribution from the D wave at this energy ($\sim$130 Mev). Then instead of (17), we have

$$\theta\equiv\frac{1-2y\sin[\delta_{33}-\delta_{31}-\frac{1}{2}(\Delta_{33}-\Delta_{31})]\sin[\frac{1}{2}(\Delta_{33}-\Delta_{31})]}{1+2y\sin[\delta_{33}-\delta_{31}+\frac{1}{2}(\Delta_{33}-\Delta_{31})]\sin[\frac{1}{2}(\Delta_{33}-\Delta_{31})]},\tag{21}$$

where

$$y=\frac{\frac{2}{3}\sqrt{2}|B_1|\cdot|B_3|}{\frac{2}{3}|B_3|^2+\frac{1}{3}|B_1|^2-\frac{2}{3}\sqrt{2}|B_1|\cdot|B_3|\cos(\delta_{33}-\delta_{31})}.\tag{22}$$

In this case[3]

$$\delta_{33}-\delta_{31}\simeq 33^\circ\quad(\sim 130\text{ Mev}),$$

therefore, the deviation of θ from unity will be greatly amplified.

Finally, we remark that if the initial Σ hyperon is polarized, we can obtain much information. For example, the validity of parity conservation may be tested by observing the angular distribution of the final pions.[4]

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[3] de Hoffmann, Metropolis, Alei, and Bethe, Phys. Rev. **95**, 1586 (1954).

[4] Lee, Steinberger, Feinberg, Kabir, and Yang, Phys. Rev. **106**, 1367 (1957).