RESULTS FOR $F^{19}+n$

Measured values of the total cross section of fluorine are shown in Fig. 8. The discrepancy between the high- and low-resolution measurements above 4 Mev was attributed to the presence of lower energy neutrons in the beam arising from improper beam collimation in the high-resolution measurements.

The rise in cross section at 800 kev appears to be due to a group of unresolved levels. Those at 1.1 and 2.25 Mev, as well as the structure above 3 Mev, have not been previously reported.[16–18]

Angular distributions of the scattered neutrons were measured at energies between the resonances in an attempt to obtain information concerning the magnitude of the potential-scattering phase shifts. Those results are shown in Fig. 9. Interference between neighboring levels was undoubtedly present, but the phase shifts extracted from the data should be more suitable for describing the scattering than those calculated from nuclear models. These phase shifts are listed in Table V.

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Fierz Interference of the Fermi Interactions in Beta Decay*

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The coupling constant combinations $(|C_S|^2+|C_S'|^2+|C_V|^2+|C_V'|^2)$ and $\mathrm{Re}(C_SC_V^*+C_S'C_V'^*)$ are evaluated for the beta-decay interaction by an analysis of data for 0→0 (no) transitions.

I. INTRODUCTION

The theoretical expression for the electron energy spectrum for allowed beta transitions is

$$N(W)dW = (2\pi^3)^{-1}pW(W_0-W)^2F(Z,W)\xi(1+b/W)dW, \quad (1)$$

where, in the case of pure Fermi transitions (0→0, no),

$$\xi=\left|\int 1\right|^2[k^{-2}(|C_S|^2+|C_S'|^2)+(|C_V|^2+|C_V'|^2)], \quad (2)$$

$$\xi b=\pm 2\gamma\left|\int 1\right|^2 \mathrm{Re}[k^{-1}(C_SC_V^*+C_S'C_V'^*)], \quad (3)$$

$$k=\int 1\Big/\int\beta. \quad (4)$$

The notation is that of Rose[1] with the modifications introduced by Lee and Yang[2] which allow for non-invariance under the parity transformation, charge conjugation, and time reversal. The parameter k, which does not appear in references 1 and 2, is introduced here because the scalar matrix element $\int\beta$ and the vector matrix element $\int 1$ are equal only in the approximation of nonrelativistic nucleon motion.[3] In Eq. (3) the upper sign applies for electron emission, the lower for positron emission. The term containing b is the Fierz interference term.

Upon integrating Eq. (1) over the range of electron energies, the ft value is obtained:

$$2\pi^3(ft)^{-1}\ln 2=\xi+\xi b\langle W^{-1}\rangle, \quad (5)$$

where

$$f=\int_1^{W_0} F(Z,W)pW(W_0-W)^2dW, \quad (6)$$

$$\langle W^{-1}\rangle=f^{-1}\int_1^{W_0} F(Z,W)p(W_0-W)^2dW. \quad (7)$$

$\langle W^{-1}\rangle$ is the average of W^{-1} over the allowed spectrum.

If the Fierz constant b is not zero, the electron spectrum will deviate from the allowed shape; also, the ft values for allowed transitions will depend on end-point energy. A similar Fierz interference effect occurs for the K capture to positron ratio.[4] The general linearity of Kurie plots has long been known and indicates that $\xi b\ll\xi$. Accurate determinations of the coupling constant combinations contained in b from spectral shape studies, however, have been restricted by the stringent instrumental requirements necessary if the weak W dependence of the Kurie plot is to be detected unambiguously. Such determinations have been made recently for the interference of Gamow-Teller interactions by several groups,[5] but no similar

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[1] M. E. Rose, in *Beta- and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (Interscience Publishers, Inc., New York, 1955), pp. 273–291.

[2] T. D. Lee and C. N. Yang, Phys. Rev. **104**, 254 (1956).

[3] S. R. DeGroot and H. A. Tolhoek, Physica **16**, 456 (1950). The notation introduced here is the same as that of Porter *et al.* (reference 5) except for a change of sign.

[4] This effect is discussed by R. Sherr and R. H. Miller, Phys. Rev. **93**, 1076 (1954). These authors find $b_{\mathrm{GT}}=-0.01\pm0.02$.

[5] Pohm, Waddell, and Jensen, Phys. Rev. **101**, 1315 (1956); Schwarzchild, Rustad, and Wu, Bull. Am. Phys. Soc. Ser. II, **1**, 336 (1956); Porter, Wagner, and Freedman, Phys. Rev. **107**, 135 (1957). These authors place somewhat different limits on the magnitude of b_{GT}. Their various results fall in the range $-0.15\leqslant b_{\mathrm{GT}}\leqslant 0.093$.

TABLE I. Data for 0→0 (no) transitions.

Decay	Half-life sec		End-point kinetic energy Mev		ft sec	$\langle W^{-1}\rangle$
$C^{10}\rightarrow B^{10**}$	1160±150	a b	1.08 ±0.10	b	5900±2700	0.573±0.025
			0.827 ±0.050	j, k	2020±570	0.640±0.013
$O^{14}\rightarrow N^{14*}$	72.5±0.5	c d	1.8097±0.0078	l	3103±62	0.441±0.001
$Al^{26*}\rightarrow Mg^{26}$	6.60 ±0.06	e	3.202 ±0.010	m	3092±52	0.306±0.001
$Cl^{34}\rightarrow S^{34}$	1.53 ±0.02	f	4.50 ±0.03	n	3110±103	0.236±0.002
$K^{38}\rightarrow A^{38}$	0.935±0.025	f	5.06 ±0.11	o p	3140±400	0.215±0.006
$Sc^{42}\rightarrow Ca^{42}$	0.62 ±0.05	g				
$V^{46}\rightarrow Ti^{46}$	0.44 ±0.01	e	6.1 ±0.3	e	2800±600	0.183±0.010
$Mn^{50}\rightarrow Cr^{50}$	0.28	h	>7	h	>2400	<0.162
$Co^{54}\rightarrow Fe^{54}$	0.18	h	>7.4	h	>3100	<0.154
$Cu^{58}\rightarrow Ni^{58}$	3.30 ±0.10	e	8.2 ±0.3	e	8×10⁴	0.142±0.006
$Ga^{66}\rightarrow Zn^{66}$	3.40 ×10⁴	i	4.144 ±0.041	i	6.3×10⁷	0.251±0.002

[a] Sherr, Muether, and White, Phys. Rev. **75**, 282 (1949). $t=19.1\pm0.8$ sec.

[b] R. Sherr and J. B. Gerhart, Phys. Rev. **91**, 909 (1953). Branching 1.65 ±0.20%.

[c] J. B. Gerhart, Phys. Rev. **95**, 288 (1954). $t=72.1\pm0.4$ sec.

[d] R. Sherr *et al.*, Phys. Rev. **100**, 945 (1955). Branching =99.40±0.10%.

[e] J. B. Gerhart (unpublished). Scintillation spectrometer measurements.

[f] R. M. Kline and P. J. Zaffarano, Phys. Rev. **96**, 1620 (1954).

[g] H. Morinaga, Phys. Rev. **100**, 431 (1955).

[h] W. M. Martin and S. W. Breckon, Can. J. Phys. **30**, 643 (1952).

[i] L. M. Langer and R. D. Moffat, Phys. Rev. **80**, 651 (1950).

[j] F. Ajzenberg and T. Lauritsen, Revs. Modern Phys. **27**, 77 (1955).

[k] Cook, Marion, and Bonner (unpublished, see reference j).

[l] D. A. Bromley *et al.* (to be published). $C^{12}(He^3,n)O^{14}$ threshold.

[m] J. D. Kington *et al.*, Phys. Rev. **99**, 1393 (1955).

[n] D. Green and J. R. Richardson, Phys. Rev. **101**, 776 (1956).

[o] Hunt, Kline, and Zaffarano (unpublished, see reference p).

[p] R. W. King, Revs. Modern Phys. **26**, 327 (1954).

determination has been possible for the Fermi interactions.[6] The most accurate determination for Gamow-Teller interactions is that of Sherr and Miller[4] from K/β^+ for Na^{22}, but, again, this approach has not been applicable for the Fermi interactions.

The remaining methods for determining Fierz interference are based on analyzing selected ft values. The approach of this paper is that suggested by Gerhart and Sherr[7] in which Eq. (5) is applied to data for 0→0 (no) transitions. Recently Kofoed-Hansen and Winther[8] have analyzed the ft values for superallowed transitions using semiempirical nuclear matrix elements and including an allowance for Fierz interference of Fermi interactions.

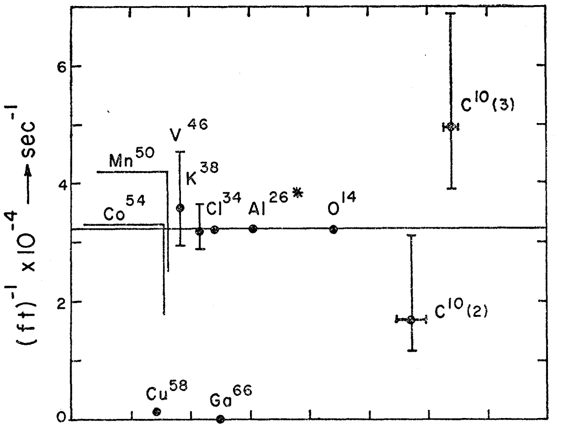

FIG. 1. Plot of $(ft)^{-1}$ *vs* $\langle W^{-1}\rangle$ for 0→0 (no) transitions. The solid line is drawn through the points for O^{14}, Al^{26*}, and Cl^{34}.

Until recently[2] it was customary to take $C_S'=C_V'=0$ and C_S and C_V real in Eq. (3), thus making b proportional to C_SC_V. It was then deduced from the smallness of b that one of the two interactions, S or V, was either absent or very weak. The experiment of Wu, Ambler, *et al.*[9] has shown this interpretation to be unwarranted. As a consequence, it is no longer possible to conclude from data on Fierz interference that either interaction is small. (For example, $C_S=C_S'=C_V=-C_V'$ would make $b=0$.[10]) Though the conclusions to be drawn from evaluations of b are less sweeping than formerly thought possible, the evaluations retain their importance as a necessary step in the full determination of the beta-decay interaction. With the data now avail-

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[6] A number of early attempts to determine the Fierz interference of Fermi interactions from shape studies were made using allowed transitions involving mixtures of Fermi and Gamow-Teller interactions. These analyses were hampered by the necessity of evaluating accurately the Gamow-Teller matrix elements, and by instrumental uncertainties in the older data. A discussion of these attempts is found in C. S. Wu's article in *Beta- and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (Interscience Publishers, Inc., New York, 1955), pp. 314–356.

[7] J. B. Gerhart and R. Sherr, Bull. Am. Phys. Soc. Ser. II, **1**, 195 (1956).

[8] O. Kofoed-Hansen and A. Winther, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **30**, No. 20 (1956). This analysis places heavy weight on the O^{14} ft value which subsequently has been corrected. This change may modify their results somewhat.

[9] Wu, Ambler, Hayward, Hoppes, and Hudson, Phys. Rev. **105**, 1413 (1957).

[10] The assumption that S or V and that A or T are absent or weak in the beta decay interaction, based on the smallness of the Fierz interference, has been used in all analyses of the interaction up to the present; e.g., see L. Michel, Revs. Modern Phys. **29**, 223 (1957). With the introduction of parity-nonconserving interactions these analyses require modification. Of the older experiments, the only ones which throw light on the question of which of the five interaction types are important contributors to the over-all interaction are the beta-neutrino angular correlations. The results of these experiments would allow significant amounts of *all five interaction types*.

able, the coupling constant combinations so determined are among those most precisely known.[11]

II. EXPERIMENTAL DATA

Table I gives the half-lives, end-point kinetic energies, *ft* values, and $\langle W^{-1}\rangle$ for the 11 positron transitions believed to be of the type $0\rightarrow 0$ (no). The identification of the first five rests on secure evidence detailed in the references given with the table. The others, excepting Ga^{66}, were first identified by Moszkowski and Peaslee[12] on the basis of an analysis of odd-odd, $N=Z$ nuclei. Aside from Cu^{58}, the available data on these transitions are in good agreement with both the decay energies predicted from the Coulomb energy differences of mirror nuclei and isotopic spin triplets, and the half-lives predicted from the O^{14} *ft* value. The observed decay energy of Cu^{58} is also in agreement, but the observed half-life is longer than expected by a factor of about 30. This could be the result either of an anomolously small nuclear matrix element for the transition, or undetected isomerism in the decay scheme. Ga^{66} is identified as a $0\rightarrow 0$ decay on the basis of the recent spin determination of Hubbs *et al.*[13] Their spin zero assignment is not completely certain since the experimental result might alternatively be interpreted as indicating an anomalously small magnetic dipole moment for Ga^{66}.

The values of $\langle W^{-1}\rangle$ in Table I are approximate values calculated by omitting the Fermi function $F(Z,W)$ in Eq. (7). The resulting expression is

$$\langle W^{-1}\rangle\cong\left(\frac{5}{2W_0}\right)\times\frac{2W_0^3p_0+13W_0p_0-3(4W_0^2+1)\ln(W_0+p_0)}{2W_0^3p_0-9W_0p_0-8(p_0/W_0)+15\ln(W_0+p_0)}. \tag{8}$$

Figure 1 is a plot of the data in Table I indicating the agreement of the data. The plotted points are expected to fall on a straight line provided $k=1$ and the various transitions have the same nuclear matrix element. This is seen to be the case, within experimental uncertainty, except for Cu^{58} and Ga^{66} which were discussed above, and C^{10} where the experimental values for the decay energy are very uncertain. However, because of the more certain identification and more accurate data, all further analysis will be limited to the data for O^{14}, Al^{26*}, and Cl^{34}.

TABLE II. Calcualted quantities.

Parent nucleus	O^{14}	Al^{26*}	Cl^{34}
$\gamma=(1-\alpha^2Z^2)^{\frac{1}{2}}$	0.9987	0.9961	0.9932
f	42.80±0.80	468.5±6.6	2033±62
$\langle W^{-1}\rangle$	0.4382	0.3010	0.2254
$2\pi^3(ft\lvert\int 1\rvert^2)^{-1}\ln 2\times 10^3$ (sec^{-1})	6.92±0.14	6.95±0.12	6.91±0.23
$2\gamma\lvert\int 1\rvert^2\langle W^{-1}\rangle$	1.750	1.199	0.8954

III. ANALYSIS AND DISCUSSION

Given in Table II are those quantities appearing in Eq. (5) which are computed from the experimental data. The f values are from the tables of Moszkowski and Jantzen.[14] The values of $\langle W^{-1}\rangle$ were calculated by numerical integration using the tables of Fano.[15] A comparison of these values with the corresponding approximate values in Table I indicates the accuracy of Eq. (8). Figure 2 is a plot similar to Fig. 1 except that only the data used in the analysis are shown. For this plot the data should lie on a straight line whose intercept at $2\gamma\lvert\int 1\rvert^2\langle W^{-1}\rangle=0$ is $(\lvert C_S\rvert^2+\lvert C_S'\rvert^2+\lvert C_V\rvert^2+\lvert C_V'\rvert^2)$, and whose slope is $\mathrm{Re}(C_SC_V^*+C_S'C_V'^*)$. The solid line is a least-squares fit to the data. The dashed lines were used to set limits of uncertainty on the evaluations of the two-coupling constant combinations.

This interpretation rests on two assumptions: (a) $k=1$; and (b) $\lvert\int 1\rvert^2=2$. The first is equivalent to the nonrelativistic approximation for calculating the nuclear matrix elements. This assumption has been made almost uniformly in discussions of beta decay and only

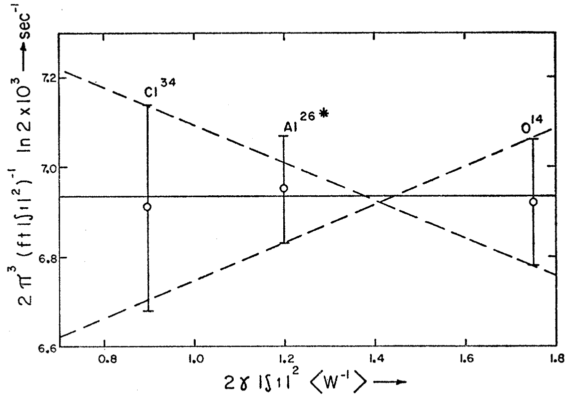

FIG. 2. Plot of data used in evaluating the coupling constant combinations.

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[11] The combinations $(\lvert C_S\rvert^2+\lvert C_S'\rvert^2+\lvert C_V\rvert^2+\lvert C_V'\rvert^2)$, $(\lvert C_T\rvert^2+\lvert C_T'\rvert^2+\lvert C_A\rvert^2+\lvert C_A'\rvert^2)$, $\mathrm{Re}(C_SC_V^*+C_S'C_V'^*)$, and $\mathrm{Re}(C_TC_A^*+C_T'C_A'^*)$ are determined with good accuracy from the *ft* values of O^{14} and the neutron, and determinations of b for the Gamow-Teller and Fermi interactions. From the beta-neutrino angular correlations the combinations $(\lvert C_S\rvert^2+\lvert C_S'\rvert^2)-(\lvert C_V\rvert^2+\lvert C_V'\rvert^2)$ and $(\lvert C_T\rvert^2+\lvert C_T'\rvert^2)-(\lvert C_A\rvert^2+\lvert C_A'\rvert^2)$ are determined, though with considerably less precision. Various other combinations can be determined from experiments involving polarized sources or electrons. See Jackson, Treiman, and Wyld, Phys. Rev. **106**, 517 (1957).

[12] S. A. Moszkowski and D. C. Peaslee, Phys. Rev. **93**, 455 (1954).

[13] J. C. Hubbs *et al.*, Phys. Rev. **105**, 1928 (1957).

[14] S. A. Moszkowski and K. M. Jantzen, University of California at Los Angeles Technical Report No. 10–26–55, 1956 (unpublished).

[15] U. Fano, *Tables for the Analysis of Beta Spectra*, National Bureau of Standards Applied Mathematics Series No. 13 (U. S. Government Printing Office, Washington, D. C., 1952).

recently has there been any investigation of its validity. Stech[16] has made an unpublished calculation for O^{14} which indicates that the exact value of $\int\beta$ may differ from $\int 1$ by as much as 4%. Further investigation of this problem is needed.

The value 2 for the Fermi matrix element $|\int 1|^2$ is calculated solely on the assumption of charge independence of nuclear forces and is independent of nuclear models. MacDonald[17] has estimated the effect of isotopic spin impurity on $|\int 1|^2$ and concluded that it differs from 2 by less than a few percent for any of the nuclei considered here, and by much less for those of lowest A. The empirically fitted wave functions of Sherr *et al.*[18] and of Visscher and Ferrell[19] for the $A=14$ triplet give $|\int 1|^2$ differing from 2 by less than 0.01%. Thus it appears that assumption (b) is reasonably well founded, and that assumption (a) is the chief source of uncertainty in the theoretical analysis.

The analysis of the data for O^{14}, Al^{26*}, and Cl^{34} leads to the results

$$(|C_S|^2+|C_S'|^2+|C_V|^2+|C_V'|^2) = (6.93\pm0.58)10^{-3}\ \text{sec}^{-1},$$

$$\text{Re}(C_SC_V^*+C_S'C_V'^*) = (0.00\pm0.83)10^{-3}\ \text{sec}^{-1},$$

$$b_F=\frac{\text{Re}(C_SC_V^*+C_S'C_V'^*)}{|C_S|^2+|C_S'|^2+|C_V|^2+|C_V'|^2}=0.00\pm0.12.$$

These results permit a re-evaluation of the beta decay constants[20] A and R.

From the above,

$$A=2\pi^3(|C_S|^2+|C_S'|^2+|C_V|^2+|C_V'|^2)\ln 2 = 6200\pm120\ \text{sec}.$$

Combining this with $ft=1220\pm90$ sec for the neutron decay[21]

$$R=\frac{|C_T|^2+|C_T'|^2+|C_A|^2+|C_A'|^2}{|C_S|^2+|C_S'|^2+|C_V|^2+|C_V'|^2}=1.36\pm0.14.$$

In evaluating R it was assumed that

$$\text{Re}(C_TC_A^*+C_T'C_A'^*)=0.$$

Note added in proof.—W. M. MacDonald has re-evaluated the O^{14} and Cl^{34} matrix elements taking into account both relativistic and Coulomb effects (reported at the New York meeting of the American Physical Society, January, 1958). He finds that Coulomb effects decrease $|\int 1|^2$ for O^{14} by about 0.1% and increase $|\int 1|^2$ for Cl^{34} by 1% or less. Relativistic effects result in decreases of $|\int\beta|^2$ of 4.1% for O^{14} and 4.2% for Cl^{34} while the corresponding vector matrix elements are unaffected.

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[16] B. Stech (unpublished); see Michel, reference 10.

[17] W. M. MacDonald, thesis, Princeton, 1954 (unpublished).

[18] Sherr, Gerhart, Horie, and Hornyak, Phys. Rev. **100**, 945 (1955).

[19] W. M. Visscher and R. A. Ferrell, University of Maryland, Physics Department Technical Report No. 19 (1955) (unpublished).

[20] J. B. Gerhart, Phys. Rev. **95**, 288 (1954).

[21] Using the half-life 12.0±1.5 min as determined by Spivac *et al.*, *Proceedings of the International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1955* (United Nations, New York, 1956), UN-650.