the possibility of having low-energy fits with split P waves and polarization is not ruled out. As this analysis shows, even the very precise cross-section measurements of the type carried out at the University of Wisconsin[1] cannot distinguish between theoretical fits using only S and effective P waves and those employing split P waves and D waves as well. Since the latter type of fit implies polarization while the former does not, an experimental decision as to the presence or absence of a double-scattering asymmetry at these energies would be welcome.

ACKNOWLEDGMENTS

The authors wish to express their appreciation to Professor G. Breit for suggesting the calculation in view of its possible bearing on an experimental test of the presence of polarization, and for discussions of the results. K. D. Pyatt and R. Fischer carried out much of the programming for the computer, and performed many of the calculations reported. The courtesy of the International Business Machines Corporation at Poughkeepsie in allowing the use of their IBM 650 computer is gratefully acknowledged.

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Investigation of Excited States in Be^8 by Alpha-Particle Scattering from He†*

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Partial phase shifts have been deduced from the data on the scattering of α particles (12.3 to 22.9 Mev) from He, reported in an earlier publication. The interpretation of the phase shifts is as follows: The monotonic decrease of the S-wave phase shift throughout the energy range investigated indicates no broad S states in Be^8 for excitations between 0.5 and 11.45 Mev. The D-wave phase-shift energy dependence clearly indicates the 2.9-Mev 2^+ state in Be^8. The G-wave phase shift shows a broad 4^+ state in the neighborhood of 11 Mev with a reduced width of about 2 Mev. The I-wave phase shift is first observed at 20 Mev and is positive.

Moderate success has been achieved in explaining the scattering results in terms of Haefner's alpha-particle model of Be^8.

I. INTRODUCTION

IN either the alpha-particle[1,2] or central-force model,[3,4] Be^8 states with $l=0$, 2, and 4 at excitation energies of about 0, 3, and 10 Mev are predicted. Yet, the abundant experimental data[5] concerning Be^8 indicate the existence of levels at 2.2, 2.9, 3.4, 4.0–4.1, 4.62, 4.9, 5.3, 6.8, 7.2–7.5, and 10–12 Mev above the ground state. However, more recent experiments[6] of greater accuracy with particle reactions favor the first-mentioned level scheme.

The most direct method of studying the levels of Be^8 with even spin and parity is by the investigation of the angular distribution of alpha particles scattered from helium. This paper reports the nuclear-scattering phase-shift analysis of such an alpha-alpha particle scattering experiment.[7] Phase shifts from the low-energy scattering data of the Carnegie Institution, Department of Terrestrial Magnetism[8] and the Rice Institute[9] are also presented. In reviewing these phase shifts, which are now established over the range of bombarding α-particle energies 0.4–6 Mev, and 12–22.9 Mev, we propose, in addition to the well-known D state at 2.9 Mev, the existence of a G state at about 11 Mev. These states can be reasonably well interpreted as rotational levels of an alpha-particle model. One result of the S-wave phase-shift behavior is that the existence of a 0^+ 7.5-Mev state is excluded, in disagreement with the results of similar experiments of Steigert and Sampson at the University of Indiana.[10]

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[1] R. R. Haefner, Revs. Modern Phys. **23**, 228 (1951).
[2] C. H. Humphrey (private communication), and Bull. Am. Phys. Soc. Ser. II, **2**, 72 (1957).
[3] D. R. Inglis, Revs. Modern Phys. **25**, 390 (1953).
[4] D. Kurath, Phys. Rev. **101**, 216 (1956).
[5] For excellent summaries of work up to 1954 see R. Malm and D. R. Inglis, Phys. Rev. **92**, 1326 (1953), and E. W. Titterton, *ibid.* **94**, 206 (1954). More recent work is discussed in Part V of this paper.
[6] See Part V of this paper.
[7] Nilson, Kerman, Briggs, and Jentschke, Phys. Rev. **104**, 1673 (1956).
[8] Cowie, Heydenburg, Temmer, and Little, Phys. Rev. **86**, 593(A) (1952); G. M. Temmer and N. P. Heydenburg, *ibid.* **90**, 340(A) (1953); N. P. Heydenburg and G. M. Temmer, *ibid.* **104**, 123 (1956).
[9] Russell, Phillips, and Reich, Phys. Rev. **104**, 135 (1956).
[10] F. E. Steigert and M. B. Sampson, Phys. Rev. **92**, 660 (1953).

II. PHASE-SHIFT ANALYSIS

The expression for the center-of-mass differential scattering cross section is

$$\sigma_{\text{c.m.}}(\theta)=\frac{Z^4e^4}{M^2V^4}\Bigg|\csc^2\theta\exp(-i\alpha\log_e\sin^2\theta)+\sec^2\theta\exp(-i\alpha\log_e\cos^2\theta)+\frac{2i}{\alpha}\sum_{\substack{L=0\\(L\text{even})}}^{L_{\max}}(2L+1)P_L(\cos\theta)\times\exp[2i(\zeta_L-\zeta_0)]\exp[(2i\delta_L)-1]\Bigg|^2, \quad (1)$$

where θ is the laboratory angle, e is the unit electron charge in esu, Z is the atomic number of helium, M is the alpha-particle mass, V is the incident alpha-particle velocity in the laboratory system in cm/sec, P_L is the Legendre polynomial of order L, and α is a velocity-dependent parameter defined as

$$\alpha=Z^2e^2/\hbar V. \quad (2)$$

The phase terms ζ_L represent the phase shifts of the Lth-order partial waves from the Coulomb interaction and are given by

$$\zeta_L=\zeta_0+\sum_{s=1}^{L}\tan^{-1}(\alpha/s). \quad (3)$$

The phase terms δ_L represent the phase shifts of the Lth-order partial waves from nuclear forces other than the Coulomb force, and are the parameters which are determined from the cross sections. The mixing of the scattered and recoil waves is such that the odd L terms vanish inside the sum—a direct consequence of the symmetry of the wave functions which are needed to describe a system of two Bose-Einstein particles. The maximum L in the summation can be determined roughly by considering whether the *impact parameter* of the Lth-order wave lies within the range of the nuclear forces. For incident alpha particles up to about 12 Mev, only S and D waves are scattered by non-Coulomb forces. Nuclear scattering of G waves begins at 12 Mev, and of I waves probably around 20 Mev.

A. Graphical Method of Determining the δ_L

A graphical method of determining the nuclear phase shifts from the experimental cross sections has been employed by Wheeler[11] for the early scattering experiments of Mohr and Pringle[12] and Devons.[13] The procedure is to rewrite (1) as

$$\left[\frac{\sigma_{\text{c.m.}}(\theta)M^2V^4}{Z^4e^4}\right]^{\frac{1}{2}}=\csc^2\theta\exp(-i\alpha\log_e\sin^2\theta)+\sec^2\theta\exp(-i\alpha\log_e\cos^2\theta)+\frac{2i}{\alpha}\sum\cdots, \quad (4)$$

and then solve for the δ_L by treating each term as a vector in the complex plane. More than one set of δ_L will usually satisfy Eq. (4).

21.8- and 22.9-Mev Data

The graphical method was used in analyzing *only* the two proportional-counter scattering experiments at 21.8 and 22.9 Mev.[14] Just two phase-shift solutions at 22.9 Mev and one at 21.8 Mev were found which satisfy the requirements that the theoretical cross sections computed with the determined phase shifts in expression (1) lie within the experimental errors of the measured cross sections. These solutions have been denoted as Solutions I and II; Solution I is always that one which agrees better with experiment. The two solutions are tabulated in Table I and the theoretical cross sections are plotted in Fig. 1 along with the experimental values. The errors shown for the phase shifts were estimated from uncertainties in the cross sections and incident-particle energies.

B. Computer Method of Determining the δ_L

The graphical method is very time-consuming and tedious, especially when many partial waves must be included. Therefore, the phase-shift analyses problem has been programed for ILLIAC, the University of Illinois digital computer.[15] The program is able to handle phase shifts up to δ_{10} and the solution is determined by considering simultaneously the experimental cross sections at a maximum of 32 angles between 10.5° and 45° (lab).

By varying the phase shifts δ_L, the computer minimizes the following function:

$$f(\delta_0,\delta_2,\cdots\delta L_{\max})=\sum_{i=1}^{n}\frac{[\sigma_{\exp}{}^*(\theta_i)-\sigma_{\text{th}}{}^*(\theta_i;\delta_0,\cdots\delta L_{\max})]^2}{\Delta_i^2}. \quad (5)$$

TABLE I. Graphically-determined phase shifts for alpha-alpha particle scattering experiments at 21.8 and 22.9 Mev.

E_{lab} (Mev)	δ_0 (deg)	δ_2 (deg)	δ_4 (deg)	δ_6 (deg)
Solution I				
21.8	-8.2 ± 0.5	95 ± 0.4	46.7 ± 0.4	0.5 ± 0.2
22.9	-10.5 ± 0.5	94.1 ± 0.4	59.1 ± 0.4	0.95 ± 0.2
Solution II				
21.8	No second solution was found graphically			
22.9	86.5	119	-51	1.8

[11] J. A. Wheeler, Phys. Rev. **59**, 16 (1941).
[12] C. B. O. Mohr and G. B. Pringle, Proc. Roy. Soc. (London) **A160**, 193 (1937).
[13] S. Devons, Proc. Roy. Soc. (London) **A172**, 564 (1939).
[14] Of the ten scattering experiments discussed previously by the authors [R. Nilson *et al.*, Phys. Rev. **104**, 1673 (1956)], these two have the best statistics.
[15] J. N. Snyder, Phys. Rev. **96**, 1333 (1954).

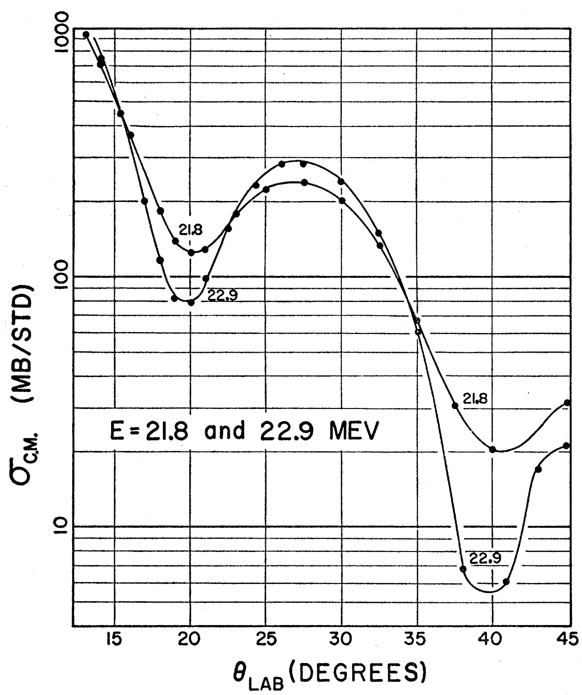

FIG. 1. Computed and observed cross sections at $E=21.8$ and 22.9 Mev. (See reference 7.) ● Experimental results (proportional counter), solid curve computed with either graphically-determined or ILLIAC-determined phase shifts of Solution I.

The $\sigma_{exp}^*(\theta_i)$'s are the experimental cross sections, at laboratory angles θ_i, scaled down to suit the range of ILLIAC. Likewise the $\sigma_{th}^*(\theta_i)$'s are the theoretical cross sections from expression (1) also scaled down. The Δ_i's represent the experimental root-mean-square errors associated with the cross sections at the angles θ_i.

TABLE II. Computer (ILLIAC)-determined phase shifts for alpha-alpha particle scattering experiments at 12.3 to 22.9 Mev. ϵ = rms error for the least-squares fit (see Part II, Sec. B).

E_{lab} (Mev)	δ_0 (deg)	δ_2 (deg)	δ_4 (deg)	δ_6 (deg)	ϵ
Solution I					
12.3	29 ±4	103 ±8	3.0±1.5	···	1.60
15.2	11 ±4	100 ±8	5.2±2	···	1.66
17.8	7 ±2	104 ±4	16.2±2	···	0.86
19.1	3 ±2	101 ±4	24.1±2	0	1.09
20.4	− 1.6±2	97.5±4	27.7±2	0.54	1.58
21.65	− 8.8±2	94.7±2	41.8±2	0.13	0.13
21.8	− 6.9±2	94.8±2	47.0±2	1.03	1.5
22.25	−10.2±2	93.3±2	48.1±2	0.09	0.95
22.81	− 9.4±2	91.7±2	56.4±2	1.07	0.89
22.9	−10.7±2	94.0±2	59.2±2	1.09	1.62
Solution II					
12.3	40.2	102	0	···	2.20
15.2	60.8	104.5	0.2	···	1.76
17.8	68.6	111	− 8.5	···	1.38
19.1	68.9	112	−16	2.7	2.38
20.4	79.3	114	−16	3.0	2.72
21.65	81.7	146	−23.7	−8.5	0.64
21.8	84.2	116.7	−36.3	3.3	4.27
22.25	89.7	128.9	−50.3	−3.7	8.4
22.81	87.8	119.8	−50.0	1.5	1.01
22.9	86.4	118.2	−51.3	1.6	2.1

The following criteria were established in order to determine which solution to a particular phase-shift analysis was the best one. First, acceptable solutions were required to give agreement with the experiment (all but two or three solutions were rejected on this basis). Second, it was necessary that acceptable phase shifts have a consistent and smooth variation with energy (none of the solutions rejected for poor least-square fits satisfied this criterion). These two criteria always reduce the number of possible solutions to at most two. Then, the best solution was taken to be the one whose energy variation was interpretable in terms of a Be^8 level structure that is consistent experimentally and theoretically. When the experimental data were sufficiently precise, it was always found that the solution finally chosen had the best least-squares fit.

A measure of the fit was obtained from the rms error for the least-squares fit. This quantity is called ϵ and is given by

$$\epsilon=\left(\frac{f_{\min}}{(n-m)k}\right)^{\frac{1}{2}},$$

where n is the number of observations, m is the number of parameters used in the least-squares fit (i.e., phase shifts), k a normalizing constant, and $f_{\min}$ is the minimum value of the function f. Since the rms error

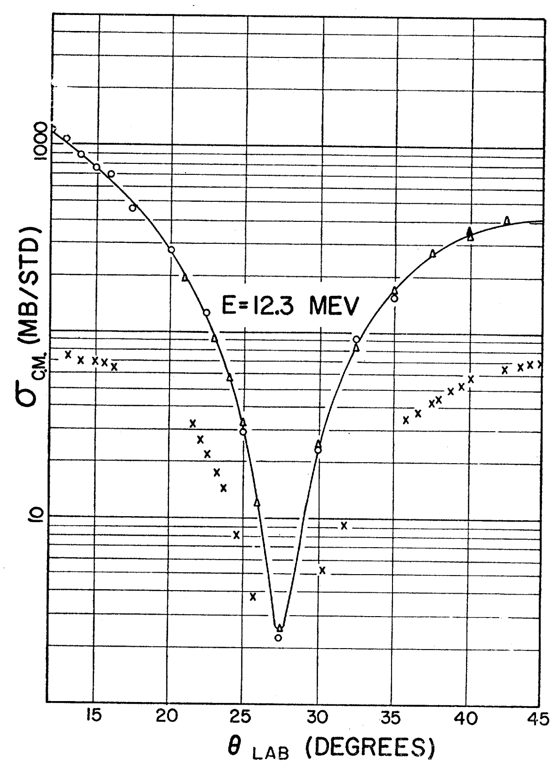

FIG. 2. Computed and observed cross sections at $E=12.3$ Mev (see reference 7). o Low-angle slit results, △ high-angle slit results, ▲ high-angle slit results $(\frac{1}{2}\pi-\theta)$, solid curve computed with ILLIAC-determined phase shifts of Solution I. Shown also are results of Steigert and Sampson[10] at 12.88 Mev (×).

associated with the numerical fit σ_{th}^* is given by $\epsilon\Delta$, a fit commensurate with the accuracy of the experimental data requires that ϵ be of the order one.

12.3- to 22.9-Mev Data

The fact that the graphical analysis previously discussed limited the possible solutions to at most two for the 21.8- and 22.9-Mev data provided a valuable aid in the computer analyses at lower energies. By starting the computer with initial sets of phase shifts equal to those at 22.9 Mev, the computer found two acceptable solutions for the 20.4-, 21.65-, 22.25-, and 22.81-Mev experiments. Then, by using the 20.4-Mev phase shifts as initial values, the phase shifts at 19.1 Mev were determined, etc.

The search for suitable solutions was not limited to the procedure just described. Many initial values of phase shifts were tried so that probably no suitable solutions were missed. The two solutions found at each energy are tabulated in Table II. The theoretical cross sections computed with the phase shifts of Solution I in Eq. (1) are plotted in Figs. 2 through 7 along with the experimental values. The phase shifts for the 21.8- and 22.9-Mev experiments determined by the computer check very well with those determined graphically (compare Tables I and II).

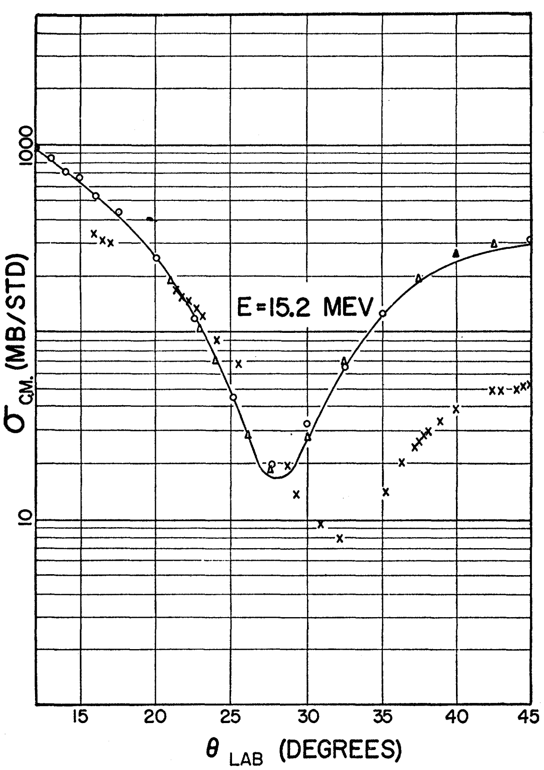

FIG. 3. Computed and observed cross sections at $E=15.2$ Mev. (See reference 7.) ○ Low-angle slit results, △ high-angle slit results, ▲ high-angle slit results $(\frac{1}{2}\pi-\theta)$, solid curve computed with ILLIAC-determined phase shifts of Solution I. Shown also are results of Steigert and Sampson[10] at 14.86 Mev (×).

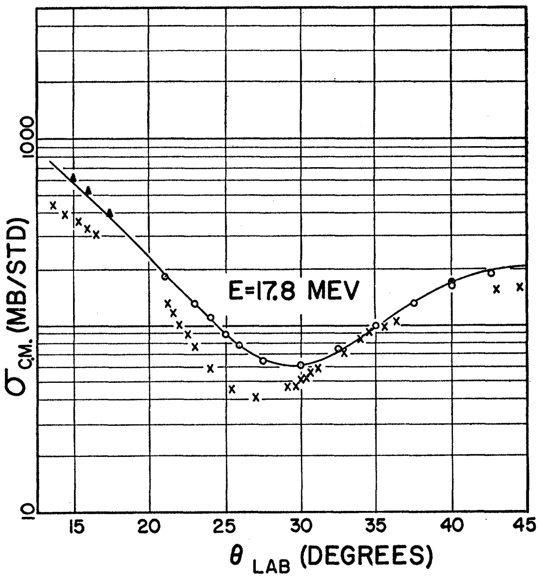

FIG. 4. Computed and observed cross sections at $E=17.8$ Mev. (See reference 7.) ○ High-angle slit results, ● high-angle slit results $(\frac{1}{2}\pi-\theta)$, ▲ extrapolated points from $d\sigma/dE$ curves taken from present data, solid curve computed with ILLIAC-determined phase shifts of Solution I. Shown also are results of Steigert and Sampson[10] at 18.32 Mev (×).

The errors assigned to the phase shifts of Solution I were determined by ascertaining how great a variation could be made in the phase shifts before the theoretical cross sections lay outside the errors in the experimental cross sections. For example, for the 15.2-Mev data, it was found that the D-wave phase shift could be varied as much as eight degrees (with suitable adjustment of δ_0 and δ_4) before too serious a disagreement with the experimental cross sections arose. At energies above 20 Mev, where energy resolution and statistics are better,[7] the assigned errors are smaller.

An interesting symmetry exists between the δ_0, δ_2, and δ_4 values of each solution. No theoretical explanation of this symmetry was tried. The symmetry is shown in Fig. 8.

20- and 20.4-Mev Data

Two scattering experiments have been performed at Washington University by Mather at 20 Mev,[16] and by Braden *et al.* at 20.4 Mev.[17] The experimental points are shown in Fig. 6. The absolute values show reasonable agreement with our 20.4-Mev results, but insufficient data prevented the computer from finding a suitable phase-shift solution.

30-Mev Data

The experimental cross sections obtained by Graves[18] at 30 Mev were also analyzed. Attempts with partial

[16] K. B. Mather, Phys. Rev. 82, 126 (1951).
[17] Braden, Carter, and Ford, Phys. Rev. 84, 837 (1951).
[18] E. Graves, Phys. Rev. 84, 1250 (1951).

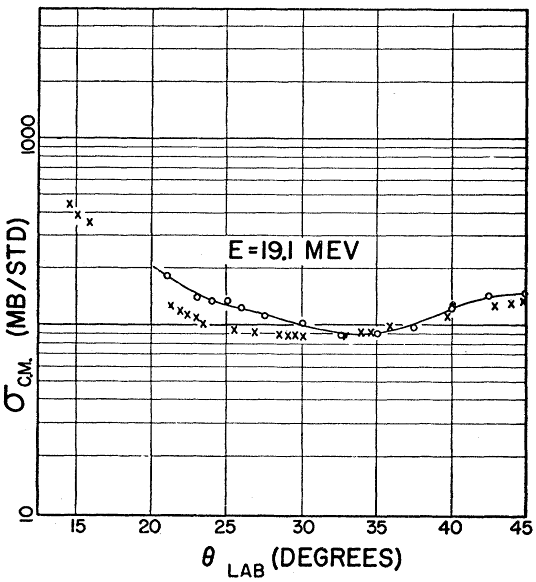

FIG. 5. Computed and observed cross sections at $E=19.1$ Mev. (See reference 7.) o High-angle slit results, ● high-angle slit results ($\frac{1}{2}\pi-\theta$), solid curve computed with ILLIAC-determined phase shifts of Solution I. Shown also are results of Steigert and Sampson[10] at 19.47 Mev (×).

waves up to δ_8 were made, but no solutions were found which gave satisfactory least-square fits. The solution which provided the best agreement between the computed and experimental cross sections is tabulated in Table III. The phase shifts reported by Graves[18] for his data are incorrect since the theoretical expression used in the analysis is not symmetric about laboratory angle 45 degrees.

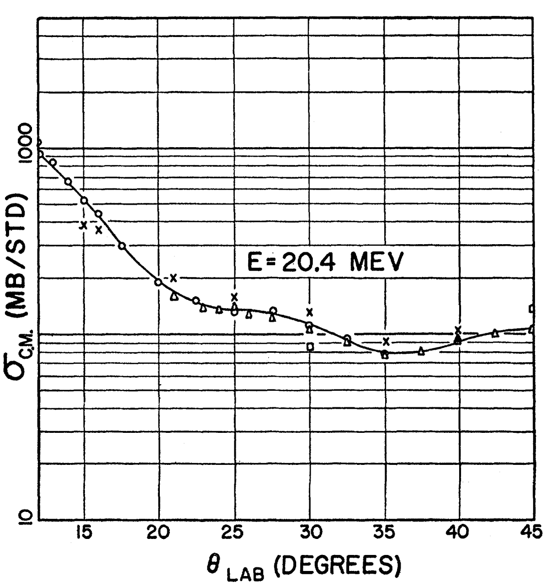

FIG. 6. Computed and observed cross sections at $E=20.4$ Mev. (See reference 7.) o Low-angle slit results, △ high-angle slit results, ▲ high-angle slit results ($\frac{1}{2}\pi-\theta$), solid curve computed with ILLIAC-determined phase shifts of Solution I. Shown also are results at 20 (×)[16] and 20.4 Mev (□).[17]

12.88- to 21.62-Mev Data

Phase-shift analyses of the alpha-alpha particle scattering cross sections measured by Steigert and Sampson[10] were carried out with the digital computer. The results agree qualitatively with the phase shifts published[10] except for the *G*-wave phase shift—the rise with increasing energy being much more gradual, attaining a value of 40 degrees at 21.6 Mev instead of the tentative value of ~115 degrees given by Steigert.[10] A complete discussion of the ILLIAC phase-shift analyses of the data of Steigert and Sampson[10] is given by Nilson.[19] The variance between our data and those of Steigert and Sampson[10] in the energy region from 12 to 15 Mev does not lie in the phase-shift analysis but arises from the hitherto unexplained differences in the experimental cross sections in the 12–15 Mev range.

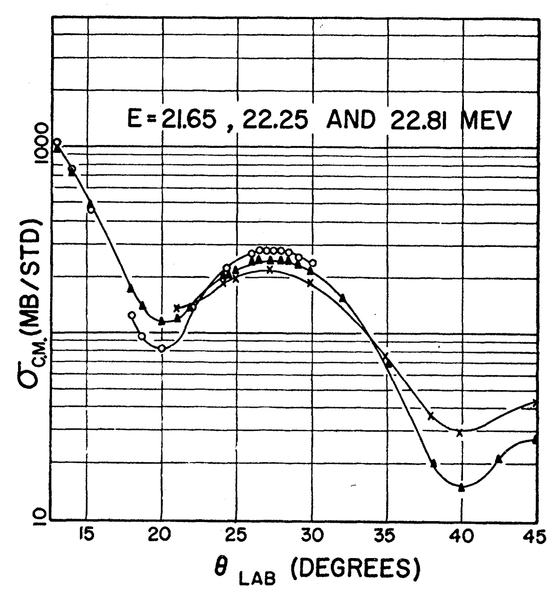

FIG. 7. Computed and observed cross sections at $E=21.65$ (×), 22.25 (▲), and 22.81 (o) Mev. (See reference 7.) Solid curves computed with ILLIAC-determined phase shifts of Solution I.

III. INTERPRETATION OF PHASE SHIFTS

Our experimental phase shifts (Solutions I) are shown in Fig. 9. Included also are the phase shifts from the data of the Carnegie Institution[8] and the Rice Institute.[9] The energy dependence of all phase shifts is very smooth and consistent for the three groups of experiments.

Application of the single-level approximation of the Wigner-Eisenbud dispersion theory[20] has had considerable success in the identification of virtual nuclear

[19] R. Nilson, Ph.D. thesis, University of Illinois, 1956 (unpublished.

[20] E. P. Wigner and L. Eisenbud, Phys. Rev. 72, 29 (1947); T. Teichmann and E. P. Wigner, Phys. Rev. 87, 123 (1952).

levels in many light element reactions.[21] It seemed worthwhile, therefore, to analyze the scattering phase shifts in terms of the dispersion theory. On the other hand, such a procedure may not be completely warranted since the idea of a compound nucleus being formed may not be applicable in a collision of two tightly bound alpha particles.

The single-level approximation of the dispersion theory is applied by assuming that only one state affects the resonance part of the phase shift over the energy range under consideration. This assumption is valid if the spacing of Be^8 levels of the same L is large compared to the level widths.

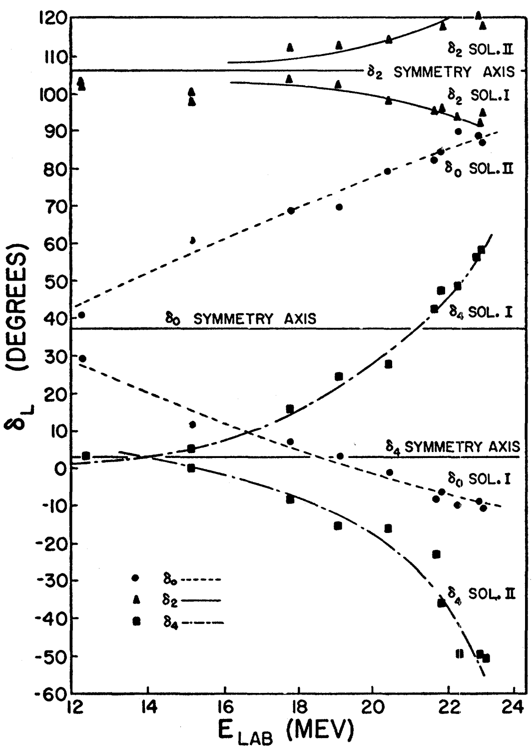

FIG. 8. Observed mirror symmetry of Solutions I and II phase shifts for data above $E=12.3$ Mev.

We express the phase shift δ_L as a sum of two terms

$$\delta_L=\phi_L+\eta_L. \qquad (6)$$

The first term in (6) is the potential phase shift and is given by the following relation:

$$\phi_L=-\tan^{-1}\left[\frac{F_L(kr,\alpha)}{G_L(kr,\alpha)}\right]_{r=a}. \qquad (7)$$

[21] See for example the following: p+He—C. L. Critchfield and D. C. Dodder, Phys. Rev. **76**, 602 (1949); R. K. Adair, *ibid.* **86**, 155 (1952); D. C. Dodder and J. L. Gammel, *ibid.* **88**, 520 (1952). $p+C^{12}$—H. L. Jackson and A. I. Galonsky, *ibid.* **89**, 370 (1953). $\alpha+C^{12}$—R. W. Hill, *ibid.* **90**, 845 (1953); J. Bittner and R. D. Moffat, *ibid.* **96**, 374 (1954). $\alpha+O^{16}$—J. R. Cameron, *ibid.* **90**, 839 (1953). α+Ne—E. Goldberg *et al.*, *ibid.* **93**, 799 (1954). d+He—A. I. Galonsky and M. T. McEllistrem, *ibid.* **98**, 590 (1955).

TABLE III. Computer (ILLIAC)-determined phase shifts for alpha-alpha particle scattering at 30 Mev.[a] ϵ=rms error for the least-squares fit (see Part II, Sec. B).

δ_0 (deg)	δ_2 (deg)	δ_4 (deg)	δ_6 (deg)	ϵ
90	30.5	25.3	3.9	3.8

[a] It is uncertain that this represents the physically correct solution.

F_L and G_L are the regular and irregular radial Coulomb functions[22]; k is the wave number of the center-of-mass motion of the two alpha particles, r is the interparticle separation, and α is the parameter previously defined. The quantity a is the channel or interaction radius.

The second term in (6) is the resonance phase shift η_L and is given by

$$\eta_L=\tan^{-1}\left[\frac{R_LP_L}{1-R_LS_L}\right]_{r=a}. \qquad (8)$$

R_L is given by

$$R_L=(\gamma_{\lambda,L}^2)/(E_{\lambda,L}-E). \qquad (9)$$

where $\gamma_{\lambda,L}^2$ and $E_{\lambda,L}$ are the reduced width and characteristic energy (center-of-mass) of the single level. E is the channel energy (one-half the kinetic energy of the incident alpha particle in our case). P_L in Eq. (8) is the penetration factor[23] and is given by

$$P_L=\left[\frac{kr}{G_L^2+F_L^2}\right]_{r=a}. \qquad (10)$$

S_L is the level-shift factor defined as

$$S_L=\left[\frac{kr(F_LF_L'+G_LG_L')}{F_L^2+G_L^2}\right]_{r=a}. \qquad (11)$$

The primes in (11) indicate differentiation with respect

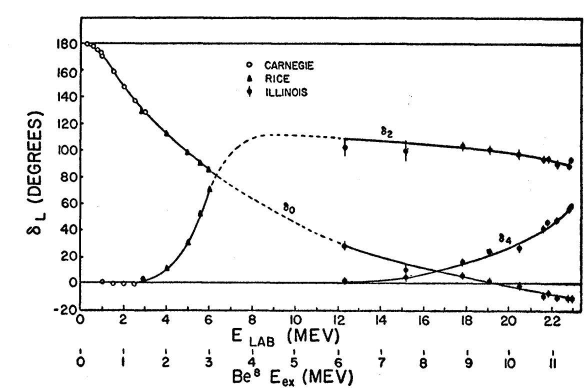

FIG. 9. Experimental S-, D-, and G-wave phase shifts for alpha-alpha particle scattering. 0.4 to 3 Mev, Carnegie Institution[8]; 3 to 6 Mev, Rice Institute[9]; above 12.3 Mev, Illinois Solution I.

[22] Bloch, Hull, Bouricius, Freeman, and Breit, Revs. Modern Phys. **23**, 147 (1951).

[23] This is not to be confused with P_L defined earlier as a Legendre polynomial.

to kr. E_{res} is defined as the channel energy at which $\eta_L = 90$ degrees,

$$E_{res} = E_{\lambda,L} - \gamma_{\lambda,L}^2 S_L. \tag{12}$$

For a compound nucleus state, the reduced width gives the probability of finding the decay particles at the nuclear surface; the partial width $\Gamma_{\lambda,L'}$, on the other hand, determines the probability for the decay particles escaping after arrival at the nuclear surface and depends on the nature of the potential barrier to be penetrated. The relationship usually given for $\Gamma_{\lambda,L}$ in terms of the reduced width is[24]

$$\Gamma_{\lambda,L} = 2P_L\gamma_{\lambda,L}^2. \tag{13}$$

The reduced width also is a measure of the overlap of the Be^8 compound-nucleus wave function with a two-alpha-particle wave function at the nuclear surface. If $\gamma_{\lambda,L}^2$ is near its maximum value, which is given by the first sum rule of Wigner and Teichmann[20] as $3\hbar^2/2\mu a^2$ (where μ is the reduced mass of two alpha particles), the level has a high degree of single-particle purity, the single particles being alpha particles.

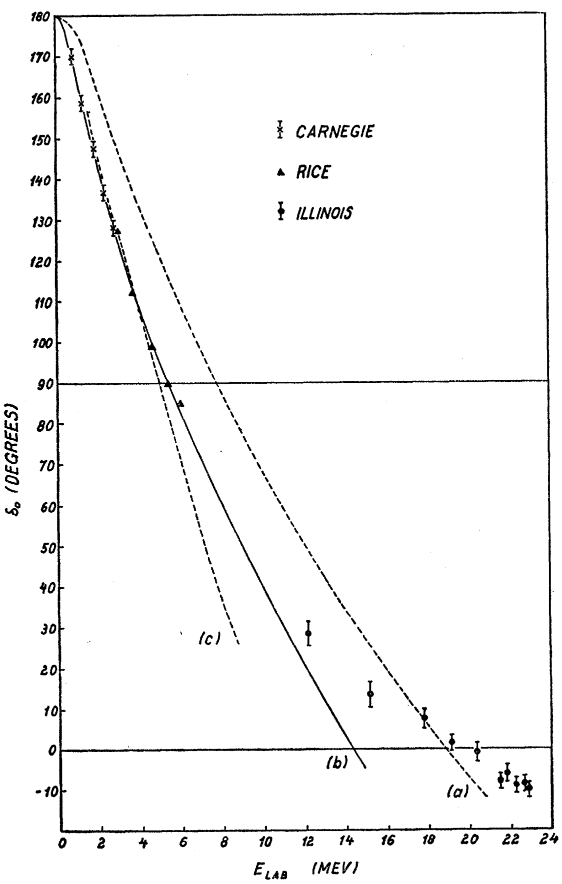

FIG. 10. Comparison of experimental and single-level dispersion theory S-wave phase shifts. Hard-sphere scattering: (a) $a=4.12\times10^{-13}$ cm ($r_0=1.3\times10^{-13}$ cm); (c) $a=5.71\times10^{-13}$ cm ($r_0=1.8\times10^{-13}$ cm). Hard-sphere scattering plus resonance scattering: (b) $a=4.4\times10^{-13}$ cm ($r_0=1.4\times10^{-13}$ cm); $\gamma_{\lambda,0}^2=0.624$ Mev-cm (0.75 Wigner limit); $E_{res}=0.096$ Mev (c.m.).

S Wave

The S-wave phase shift (Fig. 9) decreases monotonically from 180° below 1 Mev (lab) to −10° at 22.9 Mev. It is logical to begin the S-wave phase shift at 180° since in the (bombarding) energy region of 0 to about 200 kev, δ_0 is required to have risen sharply from 0° to 180° because of the $L=0$ ground state at 96 kev. There is no indication of any S states (assuming the extrapolation between 6 and 12.3 Mev can be made) for Be^8 excitation energies between 0.5 and 11.5 Mev.

The ground state width has been determined by Russell, Phillips, and Reich[9] to be 8.5±3 ev (c.m.). However, the reduced width is comparable with the Wigner limit, so the behavior of the S-wave phase shift at energies above the resonance is certainly influenced by this resonance level. Evidence of this is shown in Fig. 10 where theoretical curves based on dispersion theory are compared to the experimental phase shifts. Very good agreement can be obtained at energies less than 6 Mev (lab) with a combination of potential and resonance phase shifts based on the following parameters[25]: $a=4.44\times10^{-13}$ cm, $E_{res}=0.096$ Mev (c.m.), and $\gamma_{\lambda,0}^2=0.624$ Mev (c.m.) ($\frac{3}{4}$ Wigner limit).

To achieve agreement with potential scattering alone at the low energies requires an a value of 5.71×10^{-13} cm which is certainly too large. At the higher energies the experimental values indicate that a smaller interaction radius should be used.

D Wave

In the energy range 2 to 6 Mev (lab), the behavior of δ_2 is clearly one of resonance scattering. The phase-shift rise can be reproduced theoretically with the following single-level dispersion theory parameters[9]: $E_{res}=3$ Mev (c.m.) (based on the well-known 2.9-Mev state, i.e., $E_{res}=2.9+0.096=2.996$), $a=5.0\times10^{-13}$ cm ($r_0=1.6\times10^{-13}$ cm), and $\gamma_{\lambda,2}^2=0.9$ Mev (c.m.) (0.7 Wigner limit). As Fig. 11 shows, agreement between experiment and theory, however, does not continue beyond 6 Mev (lab). Assuming a narrower reduced width or a smaller interaction radius or both raises the theoretical values at the higher energies as shown in the figure, but no one set of one-level parameters will reproduce the experimental values over the entire energy range of 0–22 Mev.

G Wave

The G-wave phase shift begins differing from 0° around $E_{lab}=12$ Mev and also behaves like resonance

[24] This relation provides only an estimate of the reduced width if $\Gamma_{\lambda,L}$ is known. See R. G. Sachs, *Nuclear Theory* (Addison-Wesley Press, Cambridge, 1953), p. 308.

[25] Values of a considered were those calculated from the customary formula, $a=(4^{\frac{1}{3}}+4^{\frac{1}{3}})r_0$. The constant r_0 ranged between 1.2 and 1.8×10^{-13} cm.

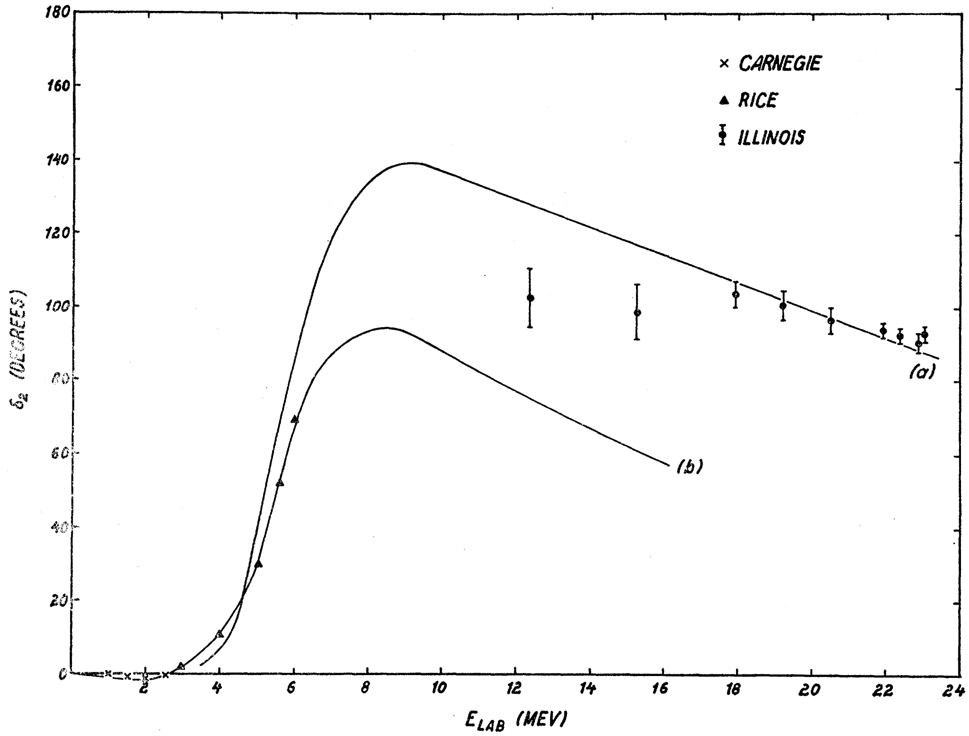

FIG. 11. Comparison of experimental D-wave phase shifts to that obtained from single-level dispersion theory with $E_{res}=3.0$ Mev. Data points are from Carnegie, Rice, and present experiments. (a) $E_{res}=3.0$ Mev (c.m.); $\gamma_{\lambda,2}{}^2=0.9$ Mev (c.m.) (0.4 Wigner limit); $a=3.5\times10^{-13}$ cm ($r_0=1.2\times10^{-13}$ cm). (b) $E_{res}=3.0$ Mev (c.m.); $\gamma_{\lambda,2}{}^2=0.9$ Mev (0.7 Wigner limit); $a=5.0\times10^{-13}$ cm ($r_0=1.6\times10^{-13}$ cm).

scattering. Depending on the choice of r_0, the center-of-mass energy at which the resonance part of the phase shift attains 90° is as follows:

r_0 (cm)	E_{res} (Mev)
1.6×10^{-13}	11
1.4×10^{-13}	11.8
1.2×10^{-13}	12.0

The G-wave phase shift can be reproduced with the single-level dispersion theory, the parameters being as follows: $r_0=1.4\times10^{-13}$ cm ($a=4.44\times10^{-13}$ cm), $\gamma_{\lambda,4}{}^2=2$ Mev (c.m.), and $E_{\lambda,4}=10.6$ Mev (c.m.) ($E_{res}=11.8$ Mev). The sum rule gives 1.59 Mev for the single-particle reduced width, and thus this state must be wholly a single-particle (alpha-particle) state. The experimental and theoretical phase shifts for $L=4$ are compared in Fig. 12.

I Wave

A small I-wave interaction improved the agreement between the theoretical and experimental cross sections above 20 Mev. It is perhaps significant that the required I-wave phase shift, though small, is positive (see Table II). The $L=6$ potential phase shift (which is negative) should begin to differ from zero around $E_{lab}=20$ Mev, and an experimentally observed δ_6 indicates that an I state may exist at a higher energy.

Although the experimental phase shifts at any given energy can be reproduced theoretically, the same value of a cannot be used throughout the whole energy range of 0 to 22 Mev. In addition to the requirement that the single-level theory be applicable, the Coulomb functions F_L and G_L plus their derivatives should not vary greatly over the energy range for the single-level approximation to be valid. This requirement is not met; for example, the value of F_2 for $r_0=1.4\times10^{-13}$ increases from $\sim$0.01 at 1 Mev (lab) to $\sim$1 at 26 Mev.

IV. ALPHA-PARTICLE MODEL

The high degree of single-particle purity indicated by the dispersion-theory analyses for the Be⁸ states below 12 Mev leads to the interpretation that these states can be described by means of an alpha-particle model. A very early discussion of allowed alpha-particle

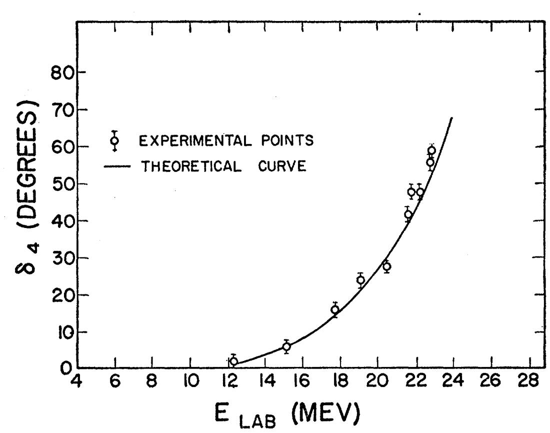

FIG. 12. Comparison of experimental G-wave phase shift to that obtained from single-level dispersion theory. $E_{res}=11.8$ Mev, $E_{\lambda,4}=10.6$ Mev, $\gamma_{\lambda,4}{}^2=2.0$ Mev, $r_0=1.4\times10^{-13}$ cm.

rotational states in Be^8 was given by Wheeler.[26] The rotational energy is

$$E_R = (\hbar^2/2A)L(L+1), \tag{14}$$

where A is the moment of inertia of the nucleus and L is the angular momentum. Only even L are allowed. Wheeler stated that only the lowest vibrational state would be expected to have a long enough life to be observable and that rotational levels with $L>4$ may be too much widened by dissociation to be of interest. For simplicity we consider only rotational states.

The energies of the first three rotational states given by (14) are in the ratio 0:3:10 for $L=0$, 2, and 4. The level energies of the Be^8 states discussed in the preceding section are in the ratio 0:2.9:11.7 for $L=0$, 2, 4,—a striking correspondence to that predicted on the basis of a rotational model.

Haefner[1] has proposed an alpha-particle interaction potential for Be^8 as follows:

$$V(r)=\begin{cases} 4e^2/r, & r>a \\ -D+(q^2\hbar^2)/(2\mu r^2), & r<a \end{cases}, \tag{15}$$

where r is the separation of the alpha-particle centers, e is the electronic charge, D is the well-depth parameter, μ is the reduced mass of two alpha particles, and q^2 is a parameter which allows convenient use of tabulated values of wave functions. The potential is proposed for investigation of the Be^8 compound nucleus for low excitations and its shape represents a strongly repulsive potential for very small r, an attractive potential for intermediate ranges and the Coulomb potential beyond the range of nuclear forces.

If the wave equation describing the relative motion of the two alpha particles is solved with the interaction potential given in Eq. (15), and the internal and external wave functions and their first derivatives (for each L) matched at the boundary $r=a$, two parameters, δ_L and A_L/B_L are determined as functions of energy. The parameter δ_L is identical to the nuclear phase shift determined in alpha-alpha particle scattering and A_L/B_L is the ratio of the internal $(r<a)$ wave-function amplitude to the external $(r>a)$ wave-function amplitude. The criterion for a virtual level of angular momentum L is given by the maximum of the ratio A_L/B_L.

The well-depth parameter D can be determined from the known position of the ground state at $E=96$ kev and turns out to be between 19 Mev and 50 Mev depending on the choice of a.

For values of a between 4.0 and 5.0×10^{-13} cm, Haefner found that the maximum in A_2/B_2 lies between $E=2.7$ to 3.8 Mev which brackets the known D state at 2.9 Mev. Extending Haefner's model to include $L=4$, we found that the energy range in which A_4/B_4 attains its maximum value is 9 to 12 Mev for values of a between 4.13 and 5.08×10^{-13} cm. This includes the G state at $E_{res}=11.8$ Mev which was determined from the dispersion-theory analysis.

A more quantitative test of the model consists of seeing how well the experimental phase shifts can be reproduced. Several curves of δ_0, δ_2, and δ_4 have been calculated for different values of a. As Fig. 13 indicates, the best agreement with the low-energy S-wave experimental phase-shift values requires a small value of a $(3.49\times10^{-13}$ cm).

None of the D-wave phase shifts calculated from Haefner's theory rise high enough to reproduce the experimental $L=2$ phase shifts beyond 12 Mev (lab) (Fig. 14).

A good fit to the experimental G-wave phase shifts is obtained with $a=4.44\times10^{-13}$ cm (Fig. 15). A more critical test of the particular potential would be a comparison between the experimental and theoretical G-wave phase shifts beyond 22.9 Mev. However, no experimental phase-shift data exist in this region. Recent alpha-alpha scattering data at 38.5 Mev (lab) by Burcham, Gibson, Prowse, and Rotblat[27] have not yet been analyzed.

Although the experimental S-, D-, and G-wave phase shifts are not exactly reproducible by Haefner's model,

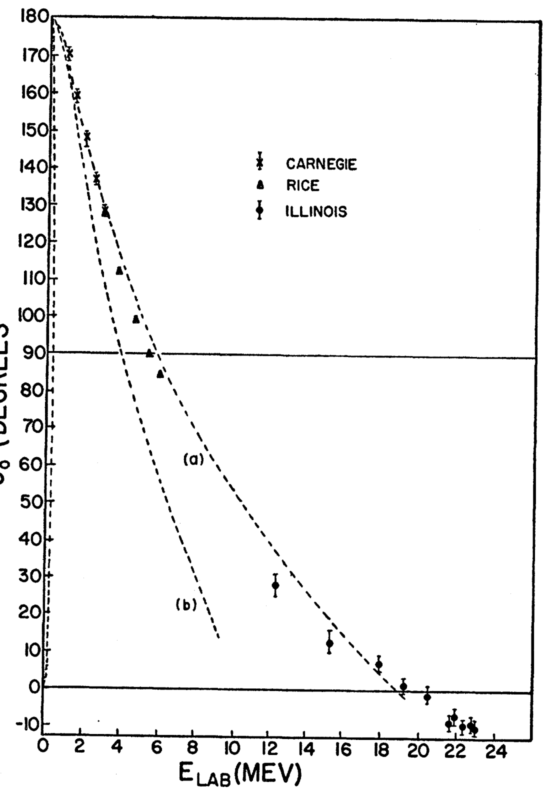

FIG. 13. Comparison of experimental S-wave phase shift with that calculated from alpha-particle model of Haefner. (a) $a=3.49\times10^{-13}$ cm, (b) $a=5.08\times10^{-13}$ cm.

[26] J. A. Wheeler, Phys. Rev. 52, 1083 (1937).

[27] Burcham, Gibson, Prowse, and Rotblat (private communication).

the similarities between the experimental and the rotational-model phase shifts are a strong indication that these three states can be described in terms of a two-body interaction. It is not surprising that deviations from this simple model occur at higher excitation energies. There will be more mixing of individual nucleons of each α particle in the compound nucleus instead of each alpha particle retaining on the average its own identity. More recently, Humphrey[28] has been able to reproduce the experimental alpha-alpha particle phase shifts for the entire energy range of 0 to 22 Mev with a modified Haefner model. He finds best agreement with $a=3.75\times10^{-13}$ cm and an L-dependent well depth D_L ($D_0=21$ Mev, $D_2=25$ Mev, and $D_4=32$ Mev). Naturally, it is questionable if the introduction of an L-dependent potential has any physical significance. He also has obtained good agreement with a Margenau potential which is infinite for very small r, a negative square well for intermediate ranges, and a Coulomb potential beyond the range of nuclear forces.

V. DISCUSSION OF RESULTS

The scattering phase-shift data indicate that only three states exist in Be^8 below 12-Mev excitation with even spin and parity.

A study of the Be^8 levels by particle reactions yields the result that only the same three states are evidenced as by alpha-alpha scattering if one relies on the more recent and most accurate experiments.

The investigations of the $Li^7(d,n)Be^8$ reaction by Trumpy, Grotdal, and Graue,[29] and by Trail and Johnson[30] for Be^8 excitations up to 9–10 Mev show no evidence of a 7.5-Mev state reported earlier.[10,31–34] Trail and Johnson's[30] experiments possess better statistics than the work of Gibson and Green,[31] and Ihsan.[32] LaVier, Hanna, and Gelinas[35] have recently studied the alpha spectrum from $Li^7(p,\gamma)Be^8\rightarrow2\alpha$ and find no structure other than that corresponding to the 2.9-Mev Be^8 state and a broad state in the region of $E_{ex}=10$ Mev. These authors state that the minimum observable intensity of a 7.5-Mev state which they should have been able to detect is 0.5% of the total alpha intensity. This compares to an intensity of one percent observed by Inall and Boyle[33] for the 7.5-Mev state in the same reaction. The work of Cuer *et al.*[34] is in disagreement with the work of Holland *et al.*[36] who, studying the same reaction [$B^{10}(d,\alpha)Be^8$], find no alpha groups other than that corresponding to the 2.9-Mev state. The far better statistics favor the results of Holland *et al.*

Thus there are contradictory results arising from the same reactions, concerning the existence of the 7.5-Mev state. The weight of good statistics favors the absence of the 7.5-Mev state, and our results concur with this conclusion.

Several other particle reactions also indicate a simple Be^8 level structure. Notably, the recent work of Kunz, Moak, and Good[37] on $Li^6(He^3,p)Be^8$, Malm and Inglis[38] and Holland *et al.*[36] on $B^{11}(p,\alpha)Be^8$, and Frost and Hanna[39] and Gilbert[40] on $Li^8\rightarrow\beta^-+Be^8\rightarrow2\alpha$ all give no

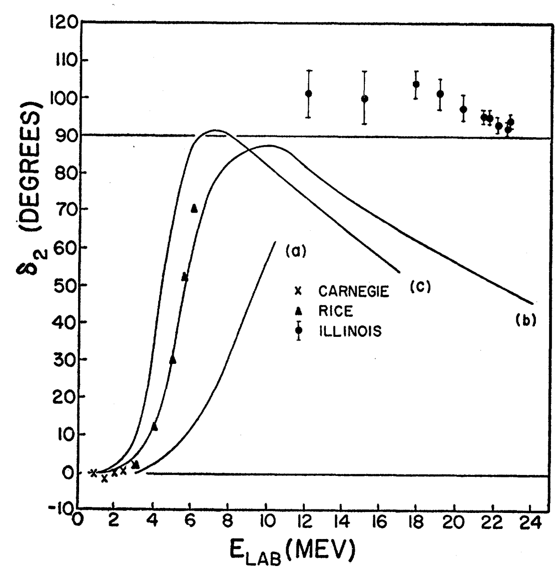

FIG. 14. Comparison of experimental D-wave phase shift with that calculated from alpha-particle model of Haefner. (a) $a=3.49\times10^{-13}$ cm, (b) $a=4.44\times10^{-13}$ cm, (c) $a=5.08\times10^{-13}$ cm.

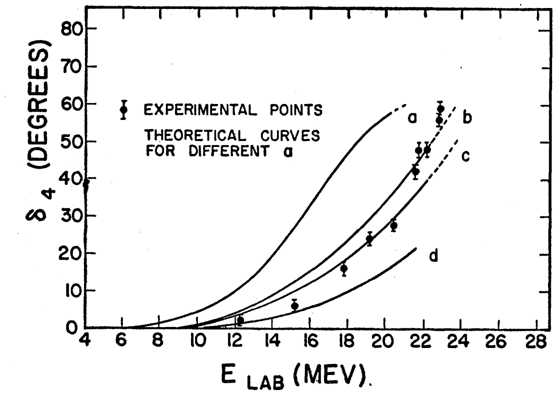

FIG. 15. Comparison of experimental G-wave phase shift with that calculated from alpha-particle model of Haefner. (a) $a=5.08\times10^{-13}$ cm, (b) $a=4.60\times10^{-13}$ cm, (c) $a=4.44\times10^{-13}$ cm, (d) $a=4.13\times10^{-13}$ cm.

[28] We are indebted to C. H. Humphrey for granting us permission to quote his general results.

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evidence for the existence of any states in Be^8 except the 2.9-Mev level up to excitations of 8 Mev. The recent results of Armstrong and Frye[41] on $B^{11}(n,\alpha)Li^8$-$(\beta^-)Be^{8*}(2\alpha)$ can, within the limits of their statistics, be described adequately by only the 2.9-Mev level up to an excitation of 10 Mev. In agreement with these results, Cameron[42] observed with the reaction $Be^9(p,d)$-Be^8 only the ground state and broad 2.9-Mev level in the region from 0- to 6.5-Mev excitation. With the $Be^9(d,t)Be^8$ reaction, no Be^8 levels were observed in the region of 7.1- to 15-Mev excitation.[42] All these recent experiments have in common good statistics.

The existence of a broad state in the vicinity of 10–12 Mev is now on secure footing. In addition to the previously mentioned $Li^7(p,\gamma)Be^8$ study of LaVier *et al.*,[35] which indicated a Be^8 state at $E_{ex}=10$ Mev, more recent work by Moak and Wisseman[43] on $Li^6(He^3,p)Be^8$ in which they have extended the Be^8 excitation energy to 14 Mev indicates a broad state at 12.3 Mev. Some older measurements purporting to a level at 11 Mev are summarized by Ajzenberg and Lauritsen.[44]

Inglis[3] and more recently Kurath[4] have shown that a central-force model with intermediate spin-orbit coupling is also consistent with Be^8 states with $L=0$, 2, and 4, and excitation energies of about 0, 3, and 10 Mev, respectively. There are no calculated states corresponding to observed levels at 4.2, 5.4, and 7.5 Mev.[4]

In summary, the findings of our scattering experiments taken together with those of Phillips *et al.*[9] and Heydenburg and Temmer[8] are consistent with the statement that there are no even states in Be^8 (broader than $\sim$100 kev) other than the S ground state, the 2.9-Mev D state, and a G state in the vicinity of 11–12 Mev. These results are in good agreement with those from most nuclear reactions.

VI. ACKNOWLEDGMENTS

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