Polarization in Proton-Proton Scattering at 3 and 4 Mev*

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A phase-shift analysis of the proton-proton scattering data of Worthington, McGruer, and Findley has been carried out in an attempt to find fits which also lead to polarization of non-negligible size. Such fits, having P-wave phase shifts separated in spin-orbit order with first Born approximation spin-orbit splittings, have been found, giving a polarization as large as 4%. Phase shifts for $L\leq 2$ have been included, with those for higher values of L, J put equal to zero.

I. INTRODUCTION

A PHASE-SHIFT analysis has been performed on the 3.036 and 4.203 Mev proton-proton scattering experiments of Worthington, McGruer, and Findley[1] in order to determine whether solutions exist which fit the scattering cross section and in addition give measurable values of polarization. These data have been analyzed previously by Hall and Powell[2] in terms of a ^1S phase shift, K_0, and a very small effective P-wave phase shift. In the present work all three ^3P phase shifts, δ_0, δ_1 and δ_2 and a ^1D phase shift, K_2, have been included. By proper choices of these phase shifts it has been found possible to obtain polarizations of up to 4 or 5% while still maintaining a good fit to the scattering cross section. This is possible because the angular distribution due to the D wave effectively cancels that due to the P waves, enabling large P-wave phase shifts (up to about 13°) to be used. Two types of solution were found, corresponding to P-wave splittings in direct and inverse spin-orbit order, respectively.

Notation

K_0, K_2 = singlet phase shifts for $L=0$, 2, respectively.

δ_0, δ_1, δ_2 = triplet P phase shifts for $J=0$, 1, 2, respectively.

α_c^S, α_c^T = singlet and triplet Coulomb amplitudes.

$\bar{\delta}=(\delta_0+3\delta_1+5\delta_2)/9$ = mean P phase shift.

$\langle\delta^2\rangle_{\text{Av}}=[(\delta_0)^2+3(\delta_1)^2+5(\delta_2)^2]/9$ = mean square P phase shift.

$\Delta=-2\delta_0-3\delta_1+5\delta_2$.

$$e_{L0}=\exp[2i(\sigma_L-\sigma_0)]=\exp[2i(\tan^{-1}(\eta/L)+\tan^{-1}[\eta/(L-1)]+\cdots\tan^{-1}(\eta/1))].$$

$$f=-2\ln\eta+q_0/\eta+(C_0^2/\eta)\cot K_0=f^{(0)}+Ef^{(1)}+E^2f^{(2)}+\cdots.$$

II. METHOD OF OBTAINING PHASE SHIFTS

The cross section and polarization for nucleon-nucleon scattering can be expressed conveniently in terms of the phase shifts by means of the scattering amplitudes. This has been done by Breit and Hull[3] and Breit, Ehrman, and Hull[4] (BEH).

In the present work only the effects of phase shifts K_0, δ_0, δ_1, δ_2, and K_2 have been included, all phase shifts with $L>2$ having been set equal to zero. It has been assumed that all phase shifts except the S wave are small; i.e., the approximation $\sin\delta\sim\delta$; $\cos\delta\sim 1$ has been used, except for K_0. Effects of the latter have been treated exactly in the nonrelativistic formulas. The cross section can be written as

$$k^2\sigma_{pp}=k^2\sigma_{pp}(\text{Coul})+k^2\sigma_{pp}(\text{nuc})+k^2\sigma_{pp}(\text{int}),\quad(1)$$

where the BEH formulas yield[5]

$$k^2\sigma_{pp}(\text{Coul})=|\alpha_c^S|^2+3|\alpha_c^T|^2,\quad(1.1)$$

$$\begin{aligned}k^2\sigma_{pp}(\text{nuc})={}&\sin^2K_0+9\langle\delta^2\rangle_{\text{Av}}+[5+(90/7)P_4(\cos\theta)]K_2^2\\&+P_2(\cos\theta)\{10K_2\sin K_0\cos(K_0-2\sigma_{20})+(9/5)\langle\delta^2\rangle_{\text{Av}}+(81/5)\bar{\delta}^2-\tfrac{1}{10}\Delta^2\\&+K_2^2[(50/7)+10\sin K_0\sin(K_0-2\sigma_{20})]\},\end{aligned}\quad(1.2)$$

$$\begin{aligned}k^2\sigma_{pp}(\text{int})={}&2\sin K_0[\cos K_0\,\text{Re}\alpha_c^S+\sin K_0\,\text{Im}\alpha_c^S]\\&+18P_1(\cos\theta)[\bar{\delta}\,\text{Re}(e_{10}\alpha_c^{T*})-\langle\delta^2\rangle_{\text{Av}}\,\text{Im}(e_{10}\alpha_c^{T*})]+10P_2(\cos\theta)\\&\times[K_2\,\text{Re}(e_{20}\alpha_c^{S*})-K_2^2\,\text{Im}(e_{20}\alpha_c^{S*})],\end{aligned}\quad(1.3)$$

while the polarization is given by

$$\begin{aligned}k^2(P\sigma)_{pp}={}&\sin\theta\cos\theta\{\delta_2[9\delta_1(\delta_1-\delta_2)+6\delta_0(\delta_0-\delta_2)]\}\\&+\sin\theta\{\langle\Delta^2\rangle_{\text{Av}}\,\text{Re}(e_{10}\alpha_c^{T*})+\Delta\,\text{Im}(e_{10}\alpha_c^{T*})\}.\end{aligned}\quad(2)$$

The abbreviations

$$\bar{\delta}=(\delta_0+3\delta_1+5\delta_2)/9,\quad(3.1)$$

$$\langle\delta^2\rangle_{\text{Av}}=[(\delta_0)^2+3(\delta_1)^2+5(\delta_2)^2]/9,\quad(3.2)$$

$$\Delta=-2\delta_0-3\delta_1+5\delta_2,\quad(3.3)$$

$$\langle\Delta^2\rangle_{\text{Av}}=-2(\delta_0)^2-3(\delta_1)^2+5(\delta_2)^2,\quad(3.4)$$

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[1] Worthington, McGruer, and Findley, Phys. Rev. **90**, 899 (1953).

[2] H. H. Hall and J. L. Powell, Phys. Rev. **90**, 912 (1953).

[3] G. Breit and M. H. Hull, Jr., Phys. Rev. **97**, 1047 (1955).

[4] Breit, Ehrman, and Hull, Phys. Rev. **97**, 1051 (1955).

[5] Breit, Kittel, and Thaxton, Phys. Rev. **57**, 255 (1940) have obtained the cross section in a form similar to that given below, with a similar grouping of angular dependent terms according to $P_1(\cos\theta)$ and $P_2(\cos\theta)$. This form showed that a combination of ^3P and $^1K_0$, $^1K_2$ can be found to yield an angular distribution differing little from pure S scattering.

have been used. It is seen that the P-wave phase shifts enter in the cross section only in the above combinations. Also, since the P-wave phase shifts are assumed small, large polarization will occur only when the term linear in the phase shifts in (2) is large; i.e., when Δ is large. For Δ large, appreciable polarization can be obtained at these energies in the interference region because the Coulomb amplitudes are large [$\mathrm{Im}(e_{10}\alpha_c{}^{T*})$ ~ 0.35 at 12° and 4.2 Mev]. Thus, the condition was applied to the phase shifts that Δ have a stationary value subject to the auxiliary conditions $\bar{\delta}=$constant, $\langle\delta^2\rangle_{\mathrm{Av}}=$constant. This leads to the results

$$\Delta=\pm 3[12\langle\delta^2\rangle_{\mathrm{Av}}-12\bar{\delta}^2]^{\frac{1}{2}}, \tag{4}$$

$$\delta_0=\bar{\delta}-\Delta/6, \tag{5.1}$$

$$\delta_1=\bar{\delta}-\Delta/12, \tag{5.2}$$

$$\delta_2=\bar{\delta}+\Delta/12. \tag{5.3}$$

It is to be noted that the splitting of the P-wave phase shifts is in the ratio 1:2, i.e., precisely the splitting given by a spin-orbit force in first Born approximation. Thus the assumption of large splittings as given by Eqs. (5) is equivalent to the assumption of strong spin-orbit forces.

It has been shown by Breit[6] that strong polarization effects are more easily accounted for by spin-orbit than by tensor forces. Conversely, it is expected that, since the conditions applied to the phase shifts in this work lead to splittings and ordering characteristic of spin-orbit forces, the nuclear polarization will be sizable. The calculations reported here show that this is indeed so; the nuclear polarization exceeds the interference polarization in several cases.

The final choice of phase shifts was made as follows. Equation (4) was used to eliminate Δ^2 from the cross section, leaving the four quantities K_0, K_2, $\bar{\delta}$, $\langle\delta^2\rangle_{\mathrm{Av}}$ to be determined. The pure Coulomb cross section (1.1) was programed for an IBM 650 computer and subtracted from the experimental cross section. This program also computed the other Coulomb quantities appearing in Eqs. (1.2), (1.3), and (2). A value of K_0 was then selected and the S-wave terms in (1.2) and (1.3) computed. The first terms in (1.2) and (1.3), which involve only K_0 and Coulomb amplitudes, were subtracted from $k^2\sigma_{pp}(\mathrm{expt})-k^2\sigma_{pp}(\mathrm{Coul})$. The residual cross section, which was a relatively smooth function of angle, was interpolated to find the value at the zero of $P_2(\cos\theta)$, yielding

$$k^2\sigma(\mathrm{resid})_{54.7°}=9\bar{\delta}\{2P_1[(\tfrac{1}{3})^{\frac{1}{2}}]\,\mathrm{Re}[e_{10}\alpha_c{}^{T*}]\}$$
$$+9\langle\delta^2\rangle_{\mathrm{Av}}\{1-2P_1[(\tfrac{1}{3})^{\frac{1}{2}}]\,\mathrm{Im}[e_{10}\alpha_c{}^{T*}]\}. \tag{6}$$

This equation was used to express $\langle\delta^2\rangle_{\mathrm{Av}}$ in terms of $\bar{\delta}$. Finally $\bar{\delta}$ and K_2 were obtained by fitting the residual cross section at 90° and 12°. This gives two simultaneous quadratic equations for $\bar{\delta}$ and K_2, which were solved by successive approximations. Only one solution gave physically reasonable phase shifts; $\langle\delta^2\rangle_{\mathrm{Av}}$ was then determined from (6) and two values of Δ from (4). Finally the three P-wave phase shifts were determined (for each sign of Δ) from the Eqs. (5). This procedure was repeated for several values of K_0 at each energy. If the value of K_0 is chosen to be greater than that for a fit with almost pure S wave[2] ($K_0=50.971°$ at 3.037 Mev and $K_0=53.808°$ at 4.203 Mev), then the value of $\langle\delta^2\rangle_{\mathrm{Av}}$ given by (6) is negative and no solution is possible. This result, of course, is to be expected, since in order to introduce P-wave phase shifts and still fit the total nuclear cross section, one must decrease K_0.

It was found in using the above procedure for a given energy that Δ^2, $\bar{\delta}^2$, and $\langle\delta^2\rangle_{\mathrm{Av}}$ were almost perfect linear functions of K_0. In order to reduce the computational labor, these quantities were calculated for a few values of K_0 and the linear dependence on K_0 used to obtain further values. In this way, trial sets of phase shifts were obtained which were expected to fit the experimental cross section and to give large values of polarization. All trial sets obtained were used as input data for an IBM 650 program which computed exactly the cross section and polarization at those angles for which there existed experimental values of the cross section. This program has the additional feature of allowing a search to be made on any specified phase shift to find a "best fit" to the cross section, the criterion of best fit being to minimize the quantity

$$E_\sigma{}^2=\frac{1}{n}\sum_{i=1}^{n}\left[\frac{\sigma_{\mathrm{expt}}(\theta_i)-\sigma_{\mathrm{calc}}(\theta_i)}{\Delta\sigma_{\mathrm{expt}}(\theta_i)}\right]^2, \tag{7}$$

where n is the number of experimental points. A value of $E_\sigma\lesssim 2$ will, for present purposes, be considered satisfactory. In a few cases, searches on K_2 were performed in an attempt to improve the fit.

III. RESULTS

The results of the calculations are given in Table I. In this table are given the phase shifts, the value of E_σ [Eq. (7)], and the maximum value of the polarization P in (a) the Coulomb interference region, and (b) the nuclear region, together with the experimental angles at which they occur. A letter A following a case number indicates the result of a search on K_2. In a few cases this search gave a significant improvement in the fit to the cross section [compare E_σ for cases (3–15) and (3–15A)].

Table I shows that it is possible to introduce considerable amounts of P-wave splitting and still maintain a good fit to the cross section (keep E_σ small). This shows that the method of obtaining trial phase shifts outlined in Sec. II is satisfactory. Polarizations up to 4 or 5% are obtained in the nuclear region and up to 2% in the interference region, while a reasonable fit to

[6] G. Breit, Phys. Rev. 106, 314 (1957), especially the appendix. Related considerations are discussed in footnote 17 of this reference.

TABLE I. Phase shifts and polarization resulting from the analysis.

Case[a] No.	K_0	K_2	Phase shifts in degrees δ_0	δ_1	δ_2	Effective P wave, $\bar{\delta}$ in degrees	E_σ	Maximum polarization in percent, Interference θ	Interference P	Nuclear θ	Nuclear P
1–1	50.97	0.026	− 0.573	−0.275	0.321	0.0230	1.84	24	−0.157	70	0.007
1–2	50.47	0.099	− 3.042	−1.530	1.495	−0.0175	1.88	24	−0.840	70	0.072
1–4	49.47	0.246	− 5.288	−2.693	2.498	−0.0975	1.86	24	−1.454	60	0.313
1–5	48.47	0.393	− 6.893	−3.535	3.180	−0.178	1.69	24	−1.852	60	0.702
1–7	47.47	0.540	− 8.205	−4.228	3.724	−0.252	1.56	24	−2.135	60	1.190
1–9	45.97	0.760	− 9.895	−5.139	4.372	−0.381	1.16	24	−2.432	60	2.055
1–10	44.97	0.908	−10.898	−5.684	4.744	−0.470	1.58	24	−2.565	60	2.716
2–1	50.97	0.026	0.619	0.321	−0.275	0.0230	1.84	24	0.150	70	−0.009
2–2	50.47	0.099	3.008	1.495	−1.530	−0.0175	1.85	24	0.664	60	−0.148
2–4	49.47	0.246	5.094	2.498	−2.693	−0.0975	1.75	24	0.931	60	−0.584
2–5	48.47	0.393	6.537	3.180	−3.535	−0.178	1.48	20	1.007	50	−1.223
2–8*A*[b]	46.97	0.60613	8.205	3.953	−4.549	−0.298	1.08	20	1.075	50	−2.448
2–9	45.97	0.760	9.150	4.372	−5.139	−0.381	1.37	20	1.062	50	−3.400
2–10	44.97	0.908	9.958	4.744	−5.684	−0.470	2.31	20	1.024	50	−4.417
3–1	53.8	0.0372	− 1.209	−0.614	0.575	−0.0196	0.66	25	−0.263	70	0.009
3–2	53.2	0.1272	− 3.560	−1.822	1.612	−0.107	0.62	25	−0.795	70	0.074
3–4	52.0	0.309	− 5.953	−3.071	2.693	−0.189	0.73	25	−1.287	60	0.386
3–6	51.0	0.457	− 7.403	−3.845	3.272	−0.286	0.79	25	−1.522	60	0.746
3–9	49.5	0.678	− 9.179	−4.801	3.976	−0.412	1.03	25	−1.708	60	1.436
3–12	48.0	0.900	−10.691	−5.626	4.503	−0.562	1.68	25	−1.784	60	2.224
3–15	45.0	1.344	−13.270	−7.053	5.380	−0.837	3.30	16	−2.000	50	4.287
3–15*A*	45.0	1.14212	−13.270	−7.053	5.380	−0.837	1.12	16	−1.994	50	4.303
4–1	53.8	0.0372	1.171	0.575	−0.614	−0.0196	0.66	25	0.236	70	−0.016
4–2	53.2	0.1272	3.500	1.612	−1.822	−0.107	0.60	25	0.555	60	−0.172
4–4	52.0	0.309	5.575	2.693	−3.071	−0.189	0.68	16	0.633	50	−0.677
4–6	51.0	0.457	6.830	3.272	−3.845	−0.286	0.77	16	0.727	50	−1.264
4–9	49.5	0.678	8.331	3.976	−4.801	−0.412	1.12	16	0.805	50	−2.333
4–12	48.0	0.900	9.568	4.503	−5.626	−0.562	2.01	16	0.838	50	−3.563
4–15*A*	45.0	1.09103	11.597	5.380	−7.053	−0.837	1.84	16	0.819	50	−6.492

[a] The first digit in the case number has the following significance: 1 means $E=3.036$ Mev, $\Delta>0$; 2 means $E=3.036$ Mev, $\Delta<0$; 3 means $E=4.203$ Mev, $\Delta>0$; and 4 means $E=4.203$ Mev, $\Delta<0$.
[b] The letter *A* signifies that this case is the result of a search on the phase shift K_2.

the cross section is still maintained. For these extreme cases the *P*-state phase shifts are large enough to cast some doubt on the linear approximation used in obtaining the trial solutions. This suggests the possibility of improving the fit by additional searching on the phase shifts. Little searching has been done because the present work was intended only to show that such fits were possible.

Figures 1 and 2 give angular distribution for typical cases yielding large polarization and reasonable fits to cross section *viz.*, cases (1–9), (2–8*A*), (3–15*A*), (4–15*A*) of Table I. In Fig. 1 the quantity $\mathcal{R}=(\sigma^S-\sigma)/\sigma^S$ is plotted as a function of angle. Here σ^S is the p-p cross section calculated with only the *S* wave nonzero. The experimental errors are obtained from those given in reference 1. When theoretical points do not appear, they are coincident with the experimental points on the scale of this graph. The large size of this ratio over part of the angular region indicates the considerable contribution of *P* and *D* waves to the cross section in these cases. Angular distributions for the polarization are given in Fig. 2. Case (3–15*A*) is perhaps of interest. The fit to the cross section is within 1.5 experimental errors at all but one point (25°) and the polarization is large (2% interference, 4.3% nuclear). Also the nuclear polarization with $\Delta>0$ is of the same sign as obtained experimentally at higher energies.

In the present work the data have been corrected for relativistic kinematic effects on center-of-mass angles and cross sections, and the relativistic value of η[7] has been employed in calculating the Coulomb amplitudes. The electrodynamic corrections[8] have not been considered in this investigation which has been thought of as exploratory. Any complete analysis of data in final form at these energies must include all the effects mentioned.

The introduction of *P* waves in these fits requires that K_0 be reduced from the values obtained by Hall and Powell,[2] and for the fits yielding large polarization effects the change is large. For cases (4–15*A*) or (3–15*A*) of Table I, for example, K_0 is 8.8° smaller than the Hall-Powell value. This implies an increase in the value of the f-function of Breit, Condon, and Present[9] to $\sim$14.34, compared to the value $\sim$11.65 implied by the Hall-Powell K_0. The latter value is in agreement with the analysis of Yovits, Smith, Hull, Bengston, and

[7] G. Breit, Phys. Rev. **99**, 1581 (1955); M. E. Ebel and M. H. Hull, Jr., Phys. Rev. **99**, 1596 (1955); A. Garren, Phys. Rev. **101**, 419 (1955). See also reference 6.
[8] L. L. Foldy and E. Eriksen, Phys. Rev. **98**, 775 (1955); L. Durand, thesis, Yale University (unpublished).
[9] Breit, Condon, and Present, Phys. Rev. **50**, 825 (1926).

Breit.[10] For 3 Mev, cases (1–9) and (2–9) of Table I imply a value for the f-function of 11.83 compared to 10.57 for Hall-Powell and the YSHBB analysis. If it is assumed that the P waves disappear at smaller energies so that the f-function analysis of YSHBB is valid there, it is difficult to justify increases of the size noted at 3 and 4 Mev. In the present analysis all values of K_0 between the Hall-Powell values and the extremes listed in Table I have been fitted, so that split P waves and polarization can occur with less drastic changes in the f function. However, in case (1–4) for example, with $f=10.80$, the maximum polarization is only -1.4% (in the interference region). Such a small effect might be difficult to detect.

If, on the other hand, one assumes for the sake of discussion that $f=11.83$ at 3 Mev and takes the YSHBB value of 7.78 at zero energy, a linear interpolation between these values implies an increase over the YSHBB value of about 0.3 in f at the intermediate energy of 0.7 Mev. This corresponds to a decrease of about 1 degree in K_0. It is not expected that very large

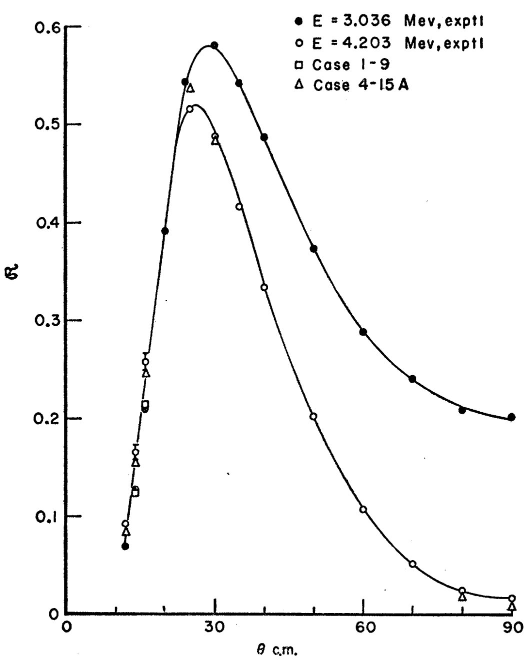

FIG. 1. Plot of the ratio $\mathcal{R}=(\sigma^S-\sigma)/\sigma^S$ as a function of the center-of-mass scattering angle, $\theta_{\text{c.m.}}$, for $E=3.037$ and 4.203 Mev. The quantity σ^S is the theoretical cross section with only the S wave non zero. The value of K_0 used is that for cases (3–15A) and (4–15A) of Table I for $E=4.203$ Mev, and that for case (1–9) for $E=3.036$ Mev. The quantity σ is the experimental cross section, or the theoretical cross section of cases (1–9), (2–8A), (3–15A), and (4–15A). For cases (2–8A) and (3–15A) the theoretical and experimental points are coincident on the scale of this graph, except that at $\theta_{\text{c.m.}}=25°$, $\mathcal{R}$ for (3–15A) is high, about the same as the plotted point for case (4–15A). A few of the points for case (1–9) are different enough from the experimental points to show up.

[10] Yovits, Smith, Hull, Bengston, and Breit, Phys. Rev. **85**, 540 (1952); referred to as YSHBB.

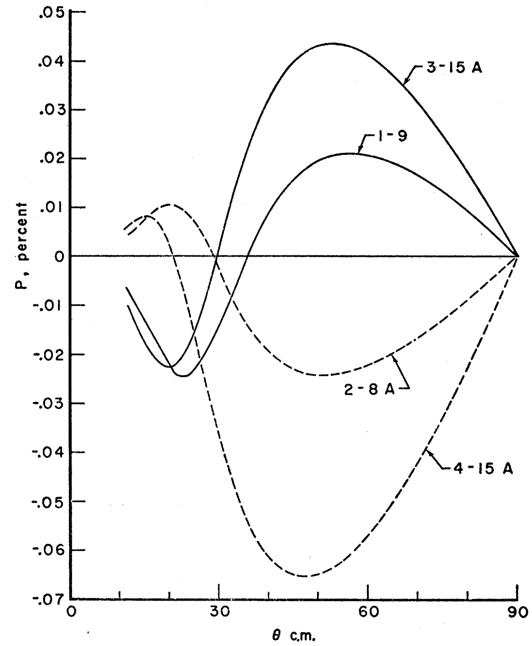

FIG. 2. Polarization, P, in percent plotted *vs* center-of-mass scattering angle, $\theta_{\text{c.m.}}$, as predicted for the phase shifts of cases (1–9), (2–8A), (3–15A), and (4–15A) of Table I.

P waves would be needed to make up the difference in cross section, so that consistency with no P waves at zero energy and sizable ones at 3–4 Mev might be realized. The implied increase in the slope, $f^{(1)}$, of the f,E curve implies an increase in the range[11] of a potential used to fit the curve. This is consistent with the early incidence of waves of higher angular momenta, but an increase of $\sim 0.16e^2/mc^2$ in the range of a Yukawa type potential, which occurs if the values mentioned are taken literally, is rather larger than is readily believed.

The reduction in the S wave is accompanied by large P and D waves, as is shown in Table I: some cases have $K_2 \sim 1°$, and P waves as large as 10°. Estimates based on potential fits to data yield smaller values than this. For example, a value of $K_2 \lesssim 0.06°$, and even smaller values, at 3 Mev might be expected.[12] Values in that range occur only for a few of the cases tried, when K_0 is close to the Hall-Powell value and polarization is small.

Thus the unusual fits obtained, while consistent as a representation of the available data at 3 and 4 Mev, appear to be somewhat improbable from a more general viewpoint.

Despite the cautions indicated against taking literally the more extreme of the results quoted, it seems that

[11] Hatcher, Arfken, and Breit, Phys. Rev. **75**, 1389 (1949), and Eq. (4) of YSHBB.
[12] G. Breit (private communication).

the possibility of having low-energy fits with split P waves and polarization is not ruled out. As this analysis shows, even the very precise cross-section measurements of the type carried out at the University of Wisconsin[1] cannot distinguish between theoretical fits using only S and effective P waves and those employing split P waves and D waves as well. Since the latter type of fit implies polarization while the former does not, an experimental decision as to the presence or absence of a double-scattering asymmetry at these energies would be welcome.

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Investigation of Excited States in Be^8 by Alpha-Particle Scattering from He†*

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Partial phase shifts have been deduced from the data on the scattering of α particles (12.3 to 22.9 Mev) from He, reported in an earlier publication. The interpretation of the phase shifts is as follows: The monotonic decrease of the S-wave phase shift throughout the energy range investigated indicates no broad S states in Be^8 for excitations between 0.5 and 11.45 Mev. The D-wave phase-shift energy dependence clearly indicates the 2.9-Mev 2^+ state in Be^8. The G-wave phase shift shows a broad 4^+ state in the neighborhood of 11 Mev with a reduced width of about 2 Mev. The I-wave phase shift is first observed at 20 Mev and is positive.

Moderate success has been achieved in explaining the scattering results in terms of Haefner's alpha-particle model of Be^8.

I. INTRODUCTION

IN either the alpha-particle[1,2] or central-force model,[3,4] Be^8 states with $l=0$, 2, and 4 at excitation energies of about 0, 3, and 10 Mev are predicted. Yet, the abundant experimental data[5] concerning Be^8 indicate the existence of levels at 2.2, 2.9, 3.4, 4.0–4.1, 4.62, 4.9, 5.3, 6.8, 7.2–7.5, and 10–12 Mev above the ground state. However, more recent experiments[6] of greater accuracy with particle reactions favor the first-mentioned level scheme.

The most direct method of studying the levels of Be^8 with even spin and parity is by the investigation of the angular distribution of alpha particles scattered from helium. This paper reports the nuclear-scattering phase-shift analysis of such an alpha-alpha particle scattering experiment.[7] Phase shifts from the low-energy scattering data of the Carnegie Institution, Department of Terrestrial Magnetism[8] and the Rice Institute[9] are also presented. In reviewing these phase shifts, which are now established over the range of bombarding α-particle energies 0.4–6 Mev, and 12–22.9 Mev, we propose, in addition to the well-known D state at 2.9 Mev, the existence of a G state at about 11 Mev. These states can be reasonably well interpreted as rotational levels of an alpha-particle model. One result of the S-wave phase-shift behavior is that the existence of a 0^+ 7.5-Mev state is excluded, in disagreement with the results of similar experiments of Steigert and Sampson at the University of Indiana.[10]

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