Paramagnetic Effect in Superconductors. V. Resistance Transition of Tin Wires

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The resistance of tin wires between 0.15 and 3 mm in diameter has been measured as a function of the current in the transition region to superconductivity. The samples used were, with one exception, single crystals and had residual-resistance ratios $r_0 = R_{0°K}/R_{273°K}$ of a few times 10^{-5}. Most of the transition curves showed a steep rise and a break at the critical current. The critical resistance, defined as the resistance at the break of the curves, is for single crystals independent of the temperature between 1.9°K and the critical temperature, if proper corrections for the temperature dependence and the field dependence of the resistivity are made. Extensive measurements of these dependences have been made for this purpose.

Measurements on one polycrystalline sample and comparison with measurements by Rinderer showed a strong influence of the electronic mean free path on the ratio of the critical resistance to the normal resistance, this ratio being smaller for the purer samples with the longer mean free paths. The single crystals which were used here were probably still not quite good enough to represent samples free from all imperfections.

The heat transfer from horizontal wires to the liquid helium was measured to assure that the temperature difference between sample and bath is not significant. It was found that the bubbles rising from a heater at the bottom of the Dewar simulate a forced convection which greatly increases the heat transfer coefficient and makes it independent of the temperature difference between sample and bath. It was observed that this temperature difference fluctuates at least by 10% of its value, and probably by a considerably larger amount.

Measurements on a tin-coated manganin wire showed that a heating of the wire displaces the critical curve, but does not give rise to a hysteresis. Below 3.4°K hysteresis occurred which was not connected to the heating and which can probably be explained in terms of Ginsburg's phenomenological theory of superconductivity.

A device for maintaining the temperature of the helium bath constant within a millidegree is described.

I. INTRODUCTION

A THEORY was developed in Part I,[1] which describes the state of a wire in the transition region to superconductivity when the wire is simultaneously exposed to a current and a longitudinal magnetic field. An analysis which was carried out in Part II[2] showed that no difficulties arise in the limit of vanishing current or vanishing field. Experiments[3–9] always show small but marked differences from the theoretical results. The longitudinal[1,3,4] as well as the circular flux[9] differ slightly from the predicted values. Neither case offers an easy possibility to amend the theory, because the experiments deal with integrated rather than local quantities. In the case of the longitudinal flux it is possible to apply empirical corrections to the theory[1] and bring it into agreement with the experiments. These corrections involve constants[1,10] which seem to vary in a peculiar way from superconductor to superconductor.

Also the resistance values which were experimentally found[5–8] differ slightly from the theoretical expectations. Calculations of the resistance are much easier,[2,4,8,11] so that one can hope to amend the theory from such measurements. So far, however, the experimental data on the resistance in the transition region are rather scant and, as we shall see later, some of them should certainly be corrected for secondary influences such as magnetoresistance.

The work here was started by using extruded polycrystalline tin wires which, close to the normal transition temperature, had wide transition curves similar to the ones observed by Rinderer.[8] Since it was known from Scott's experiments[7] that it is possible even in this region to obtain sharp transitions, single crystals of tin were grown. These had residual resistances between 1×10^{-5} and 8×10^{-5} of the ice-point resistance. This very low value has the following implications:

(1) Resistance measurements have to be made even if the potential differences across the sample are smaller than the total thermal emf in the measuring circuit. (The total thermal emf was usually about 3 to 7×10^{-7} volts.) With few exceptions the resistances were therefore measured by reversing the current and the recorded values were taken as the average of 3 to 24 readings.

(2) The resistivity is quite strongly dependent on the temperature and the magnetic field. In order to apply the necessary corrections, extensive measurements of the temperature-dependent resistance and the magnetoresistance were made.

(3) The effect of the heating of the wire is less serious than in experiments with higher residual resistances. Nevertheless, since the author of this article was unable to agree with previous discussions of the heat-transfer

[1] Hans Meissner, Phys. Rev. **97**, 1627 (1955), referred to as Part I.
[2] Hans Meissner, Phys. Rev. **101**, 31 (1956), referred to as Part II.
[3] James C. Thompson, Phys. Rev. **102**, 1004 (1956).
[4] James C. Thompson, J. Phys. Chem. Solids **1**, 61 (1956).
[5] L. W. Shubnikov and N. E. Alexeevsky, Nature **138**, 804 (1936).
[6] N. E. Alexeevsky, J. Exptl. Theoret. Phys. U.S.S.R. **8**, 342 (1938).
[7] R. B. Scott, J. Research Natl. Bur. Standards **41**, 581 (1948).
[8] L. Rinderer, Helv. Phys. Acta **19**, 339 (1956).
[9] Hans Meissner, Phys. Rev. **101**, 1660 (1956), referred to as Part III.
[10] A. H. Fitch and H. Meissner, Phys. Rev. **106**, 733 (1957), referred to as Part IV.
[11] F. London, *Superfluids* (John Wiley and Sons, Inc., New York, 1950), Vol. I, p. 120.

problem, extensive measurements of the heating of horizontal wires in liquid helium have been made in order to verify certain assumptions made in these discussions.

II. EXPERIMENTAL ARRANGEMENT

(a) Cryostat, High Current Connections, and Automatic Temperature Control

Cryostat and high current connections were the same as described in Part III.[9] The temperature was held constant by regulating the power input to the heater at the bottom of the Dewar with a carbon thermometer and a circuit as shown in Fig. 1. The ac Wheatstone bridge is operated slightly off balance, the outgoing signal amplified and fed into the phase detector. The phase detector gives an ac signal to the power amplifier only if the phase is correct. The heat input into the heater increases, therefore, with the square of the temperature difference if this temperature difference has the correct sign and is zero otherwise. Although the thermometer was only 1 cm above the heater, the system had a tendency to "hunt" so that the full gain could not be used. The average temperature was (around 3.6°K) so constant that no change in pressure could be detected over periods of about 10 minutes, while over periods of an hour the pressure rose slowly by an amount corresponding to the decrease in hydrostatic pressure of the helium. The meter in the heater circuit, however, indicated rapid fluctuations of the temperature, which were at times as big as 2 millidegrees (when the current reading was reconverted to temperature differences causing the unbalance of the bridge). No complete balance of the bridge was possible below the lambda point of the liquid helium. This was probably due to transmission of a 60-cycle signal from heater to thermometer by second sound. Rectifying of the output of the power amplifier would certainly cure this effect.

(b) Measuring Equipment

The temperature was obtained from the vapor pressure of the liquid helium using the 1955_E scale.[12] It is to be noted that the normal transition point of tin, which occurs at a vapor pressure of 461.5 mm Hg, falls on the 1955_E scale at $T=3.722$°K, while on the 1948 scale it was at $T=3.730$°K. No corrections were made for the hydrostatic pressure head.

The earth's magnetic field was compensated to a value of less than 3.5×10^{-3} amp/cm by a pair of Helmholtz coils.

The resistance of the samples was measured by reversing a known current through the sample and observing the deflection of a galvanometer connected to the potential leads of the samples.

The galvanometer (Leeds and Northrup type HS No. 2284-b) was used with a telescope and a scale at 5 m distance and had a sensitivity of 3.7×10^8 mm/v which could be shunted down by factors of 10^{-1}, 10^{-2}, and 10^{-3}. A Leeds and Northrup K-2 potentiometer was kept in the measuring circuit all the time and served for calibration purposes, for compensation of thermal emf's and for the measurement of larger potential differences, where the galvanometer was then used as a null instrument.

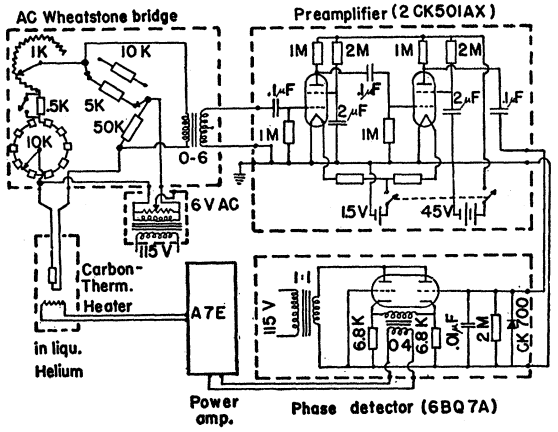

FIG. 1. Circuit of the automatic temperature control: Carbon thermometer: R_0°$_C=200$ ohms, $R_{3.7}$°$_K=3900$ ohms, $(dR/dT)_{3.7}$°$_K=3600$ ohms degree^{-1}. O-6, O-4: Ouncer transformers. A7E: Heathkit audio-amplifier used with input No. II. Heater: $R=20$ ohms, heater current indicated by meter (not shown).

A second pair of Helmholtz coils was used to provide uniform horizontal magnetic fields up to 10 amp/cm in the direction of the axis of the samples.

A cylindrical coil was placed around the helium Dewar in the nitrogen bath which produced vertical (perpendicular to the samples) fields up to 260 amp/cm.

(c) Samples

All samples were prepared from Vulcan Detinning Company "high-purity" tin of 99.998% purity. The manufacturer lists the following analysis: lead 0.0005%, iron 0.0002%, antimony 0.0005%, others 0.0008%. As mentioned above, most of the samples used were single crystals. The larger-diameter samples ($d=3.4$ mm) were grown in high vacuum in a furnace by slowly reducing the heater current, so that the temperature gradient moves across the sample.

The smaller samples were grown in thick-walled Pyrex capillaries which were sealed to a filler tube. The tin was first melted down under vacuum in the filler tube, and only after everything was well degassed was it allowed to run into the capillary. Air was then admitted to the filler tube and the sample was allowed to solidify in vertical position. The ends were then cracked away and the tin was pulled out of the capillary, being cooled if necessary with liquid nitrogen. Despite the use of silicon oil as releasing agent, only one out of five samples could be removed without damage. But

[12] Clement, Logan, and Gaffney, Phys. Rev. **100**, 743 (1955). See their "note added in proof."

TABLE I. Data on the samples.

Sample No.	Diameter mm	Length mm	Ice-point resistance ohm	Resistivity at 0°C ohm cm	Angle ϕ between crystal axis and wire axis	Residual-resistance ratio r_0
Sn IX[a]	3.390	61.5	0.983×10^{-3}	14.41×10^{-6}	0°	0.70×10^{-5}
Sn X[a]	1.02	60.5	7.62×10^{-3}	10.3×10^{-6}	55°	4.80×10^{-5}
Sn XV[a]	3.322	37.0	4.08×10^{-3}	9.5×10^{-6}	70°	1.27×10^{-5}
Sn XVI[a]	2.215	53.0	1.23×10^{-3}	9.0×10^{-6}	90°	2.21×10^{-5}
Sn XVII	1.00	40.0	4.62×10^{-3}	9.06×10^{-6}	90°	3.98×10^{-5}
Sn XVIII	0.357	41.5	38.13×10^{-3}	9.24×10^{-6}	79°	5.80×10^{-5}
Sn XIX[b]	1.05	54.5	5.77×10^{-3}	9.16×10^{-6}	82°	4.31×10^{-5}
Sn XXI	2.192	48.0	1.28×10^{-3}	10.1×10^{-6}	60°	2.24×10^{-5}
Sn XXIII	0.156	3.58	17.56×10^{-3}	9.39×10^{-6}	74°	$\begin{cases}7.88\times10^{-5}[c]\\9.76\times10^{-5}[d]\end{cases}$
Sn XXIV	0.325	59.5	9.16×10^{-2}	12.75×10^{-6}	polycrystalline	12.32×10^{-5}

[a] The ice-point resistivity was not very accurately determined and the angles are somewhat uncertain.
[b] This sample was in a glass capillary.
[c] If regions around potential leads are superconducting.
[d] If regions around potential leads are normal conducting.

since the growing was done easily enough, this was not a serious handicap. All samples were etched and proved to be single crystals. They were then electrolytically polished with a solution of 20 parts perchloric acid and 70 parts acetic acid. The thinnest samples, of about 0.15 mm diameter, were prepared from thicker ones by continued electropolishing. Pieces of 3–4 mm length were found to be of uniform diameter within 2%.

Sample Sn XIX was left in the glass capillary (the ends of which were removed) to see what influence a change in the heat-transfer coefficient would have.

Sample Sn XXIV was an extruded wire. It was used to study both the influence of higher residual resistance and of higher heat input.

The samples were mounted horizontally in order to eliminate temperature differences along their axis due to hydrostatic pressure differences. Their position relative to heater and thermometer and the position of current and potential leads are shown in Fig. 2. Concentric return tubes for the current were omitted, because they would curtail the cooling of the liquid helium.

The potential leads from commercial copper wire were soldered at a distance of a few millimeters from the current leads. The current leads were from extruded lead (Pb) wire. All solder joints were made with a very small amount of lead-tin solder. This way of attaching caused little trouble except for the thinnest sample, Sn XXIII.

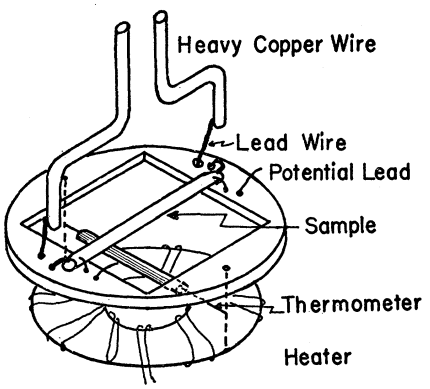

FIG. 2. Mount for samples with heater, thermometer, current and potential leads.

The orientation of the crystal axis with respect to the wire axis was obtained from the ice-point resistivity by use of the equation[13]

$$\cos^2(\rho_{\mathrm{III}},z)=(\rho_z-\rho_{\mathrm{I}})/(\rho_{\mathrm{III}}-\rho_{\mathrm{I}}), \qquad (1)$$

where ρ_{III} and $\rho_{\mathrm{I}}=\rho_{\mathrm{II}}$ are the principal resistivities for which Bridgman[14] found the values

$$\rho_{\mathrm{I}}=\rho_{\mathrm{II}}=9.088\times10^{-6}\ \text{ohm cm},$$

$$\rho_{\mathrm{III}}=13.08\times10^{-6}\ \text{ohm cm}.$$

The diameter of the samples was obtained with a traveling microscope for the thicker samples, and with an eyepiece micrometer for the thinner samples.

Table I lists the dimensions of the samples, their ice-point resistances, their resistivities, the angle ϕ between crystal axis and wire axis, and their residual-resistance ratios $r_0\equiv R_{0°\mathrm{K}}/R_{273°\mathrm{K}}$. This ratio is, without further correction, not an indication of the quality of the sample because at least in the thinner samples boundary scattering gives a sizable contribution to it.

Dingle[15] has calculated this contribution. In the range in which we are interested, his results can be approximated by the equation

$$\rho_0=\rho_{00}(1+al/d), \qquad (2)$$

where ρ_{00} is the impurity resistivity, l the mean free path of the electrons, d the diameter of the sample, and a a constant which has the values $a=1.045$ for completely-inelastic scattering, and $a=0.388$ for 50% inelastic scattering. By division of Eq. (2) by the ice-point resistivity, one obtains a similar equation for the resistance ratios.

[13] W. Meissner, *Handbuch der Experimentalphysik* (Verlag-Julius Springer, Berlin, 1935), Vol. 11, Part 2, p. 21, Eq. (47).
[14] P. W. Bridgman, Proc. Am. Acad. Arts Sci. **68**, 95 (1933).
[15] R. B. Dingle, Proc. Roy. Soc. (London) **A201**, 545 (1950).

In Fig. 3 the residual-resistance ratio is plotted as function of the reciprocal diameter. One can see that the samples Sn X, XVII, XVIII, XIX, and XXIII have about the same impurity resistance ratio of $r_{00}\approx 4.2\times10^{-5}$, while the thicker samples IX, XV, XVI, and XXI are actually purer. The slope of the line in Fig. 3 gives the product $al=0.0126$ cm. Assuming an intermediate value for a of $a=0.5$, one finds $l=0.0252$ cm. With an ice-point resistivity of $\rho=9.09\times10^{-6}$ ohm cm and an impurity resistance ratio $r_{00}=4.2\times10^{-5}$, one finds for the ratio of conductivity to mean free path:

$$\sigma/l=1/\rho l=10.4\times10^{10}\ \text{ohm}^{-1}\ \text{cm}^{-2}.$$

This value compares well with the findings of Andrews[16] and Chambers.[17]

III. MEASUREMENTS OF RESISTANCE AS A FUNCTION OF THE CURRENT

Figure 4 shows plots of the resistance ratio $r=R(T,I,H)/R(0°\text{C},0,0)$ as a function of the current for various temperatures above and below the normal transition temperature of tin for the single-crystal samples Sn XVII ($d=1.00$ mm), Sn XVIII ($d=0.357$ mm), and Sn XXIII ($d=0.156$ mm). All three diagrams are plotted so that they have a common $H_{\phi0}$ scale, where $H_{\phi0}$ is the value of the circular component of the magnetic field at the surface of the sample: $H_{\phi0}=I/\pi d$.

Samples Sn XVII and XVIII exhibit an increase of resistance with current even in the normal-conducting state $T\geqslant 3.722$°K due to the influence of the magnetic field set up by the current.

Sample XXIII shows—even above 3.722°K—an additional increase which arises from regions around the potential leads becoming normal conducting. These regions amount to only a few percent of the total length of the sample but they contain—owing to high concentrations of impurities—about 20% of the total resistance when normal conducting. As we shall see later, consistent results are always obtained when low-current measurements are referred to measurements where these regions remain superconducting and high-current measurements are referred to ones where they are normal-conducting.

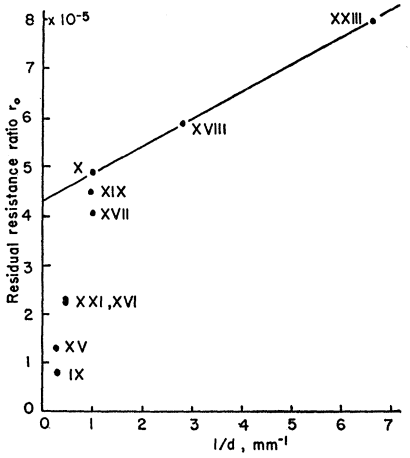

FIG. 3. Residual-resistance ratio $r_0=R_0°\text{K}/R_{273}°\text{K}$ as function of the reciprocal diameter.

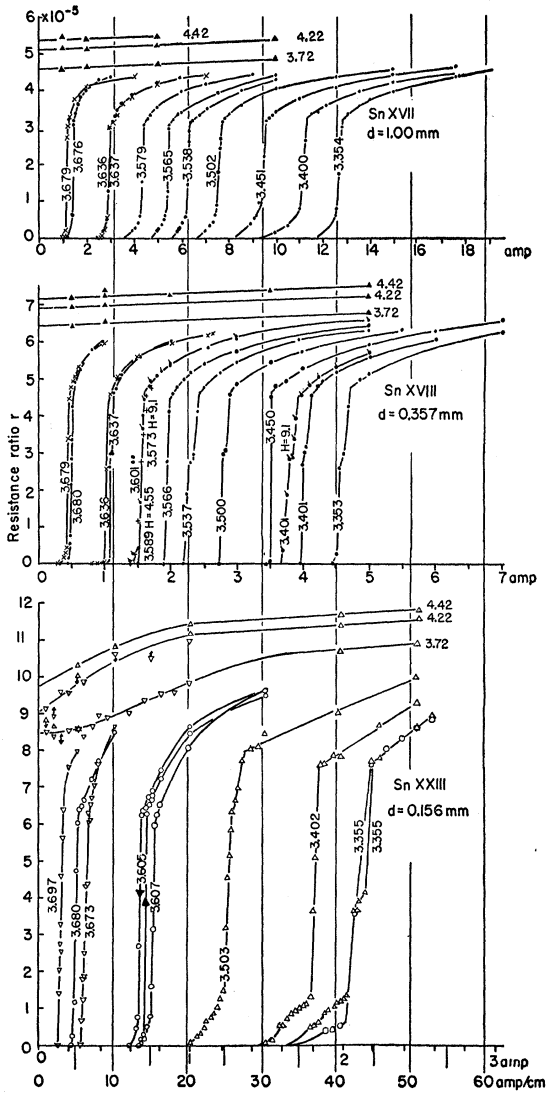

FIG. 4. Resistance ratio r as function of current plotted on a uniform $H_{\phi0}$ scale for various temperatures and (for sample Sn XVIII) values of the longitudinal field H. The numbers on the curves refer to the temperature. The different symbols refer to different runs. Note that some measurements are with superimposed magnetic field H_{z0}.

It can be seen further that the temperature-dependent part of the resistance is a sizable part of the total resistance, amounting to about 10 to 20% of the latter.

Below the critical temperature one obtains quite sharp transitions, where at the critical current, $I=I_c$, the resistance ratio jumps to a value $r=r_c$ and rises more slowly as the current is increased further. It is

[16] E. W. Andrews, Proc. Phys. Soc. (London) **26**, 77 (1949).
[17] R. G. Chambers, Proc. Roy. Soc. (London) **A215**, 481 (1952).

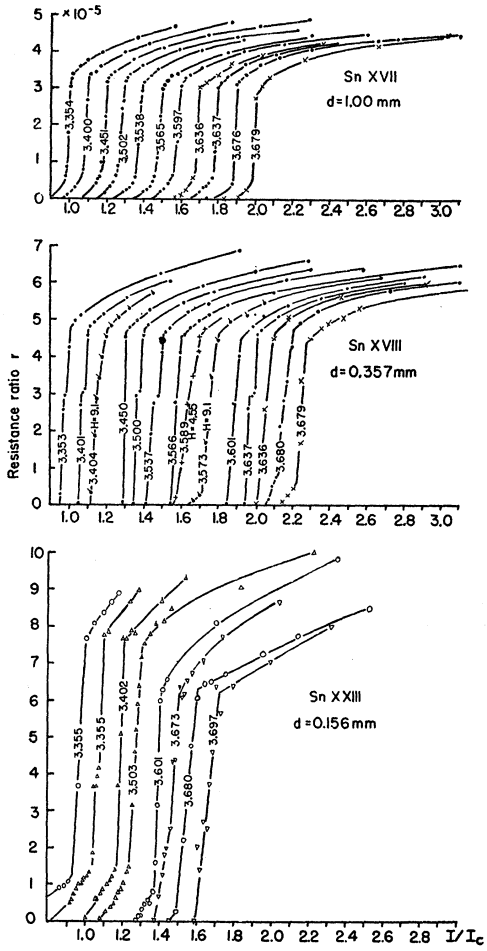

FIG. 5. Resistance ratio r as function of I/I_c, where I_c is the critical current. Note: The curves are displaced against each other by amounts of $I/I_c=0.1$. The numbers on the curves refer to the temperature. The symbols are the same as in Fig. 4.

the value of r_c or r_c/r_n with which we are mainly concerned. r_n is the resistance ratio which the sample would have if it had the resistivity of the normal conducting regions.[2,4,8,11]

Two curves (at 3.605°K) of sample Sn XXIII were measured without reversing the current. They give a slightly higher value for r_c. This is probably due to a shift in the thermal emf's while they were measured. At this temperature and $I=I_c$, the potential drop across the sample was 7.78×10^{-7} v as compared to a total thermal emf of at least 3×10^{-7} v.

A few curves on sample Sn XVIII were measured with a superimposed longitudinal field. They are quite similar to the other curves and yield the same values of r_c.

The value of r_c can, especially for curves near the normal transition point, be much better determined if the curves are plotted as function of I/I_c as in Fig. 5. In order to omit confusion the curves have been displaced by amounts of $I/I_c=0.1$. The values of r_c appear to be somewhat higher at the lower temperatures. As we shall see later, r_c/r_n remains constant, but the increase of r_n by magnetoresistance overcompensates the decrease of r_n by temperature resistance, resulting in an increase of r_c at the lower temperatures. The curves with superimposed field (with sample Sn XVIII) are also, in this plot, quite similar to the other ones. Their somewhat larger spread is probably due to an imperfect alignment of the sample with the magnetic field.

Sample Sn XVIII shows quite a number of curves with a step in the rising part at a value of $r\approx\frac{1}{2}r_c$. These steps were first observed for tin by Steiner,[18] and later on, by the author *et al.*[19]; and for indium by Meissner and Doll.[20] All of these older observations were made on recordings where one suspects time-dependent phenomena as the cause of the steps. In order for the steps to appear in the type of measurement used here, they have to be stationary for at least 20 seconds, the time to read the galvanometer, an interval which is enormously longer than any length of time for which they have been observed before.

Figures 6 and 7 show results on the polycrystalline sample Sn XXIV ($d=0.325$ mm). The resistance of this sample was sufficiently high so that it was possible to measure most of the curves also without reversing the current, that is, by increasing or decreasing it stepwise and observing the deflection of the galvanometer. It can be seen that this leads to a substantial amount of hysteresis at lower temperatures. Furthermore, it was found that there is no difference in r_c above and below the lambda point of the liquid helium ($T_\lambda=2.18$°K), indicating that heating of the wires is not sufficient to influence r_c. The only curve which was measured close to the normal transition point of tin is very wide—as one can see easily on the r *vs* I/I_c plot. This is in

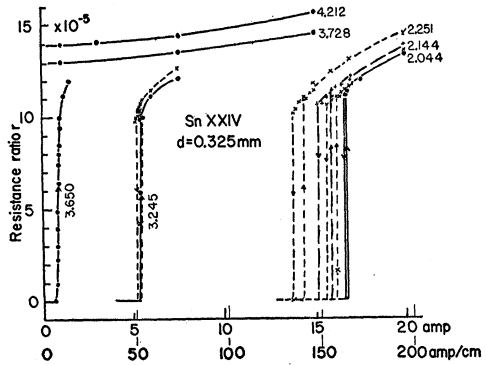

FIG. 6. Resistance ratio r as function of the current for the polycrystalline sample Sn XXIV. ● —— ●, measurements with the current reversed; × – – – ×, measurements with the current monotonically increased or decreased.

[18] K. Steiner (private communication).

[19] Meissner, Schmeissner, and Meissner, Z. Physik **130**, 521 (1951).

[20] W. Meissner and R. Doll, Z. Physik **140**, 340 (1955), Figs. 2 and 3.

agreement with findings of Rinderer[8] on polycrystalline samples.

IV. CRITICAL-FIELD CURVE

Figure 8 shows the critical-field curve for samples Sn XVIII, XXIII, and XXIV. All samples which are not shown give results which agree with the curve for Sn XVIII. The critical temperature is $T_c=3.722$°K, and the slope at T_c is $(dH_c/dT)_{T=T_c}=-115$ amp/cm°K $=-144.5$ oe/°K. This has to be compared with -151.5 oe/°K found by Lock *et al.*[21] and -147.4 oe/°K found by Muench.[22] Some of the points of sample Sn XVIII are measured with a superimposed field H_{z0}. In this case H_c is taken as

$$H_c=[(I_c/\pi d)^2+H_{z0}^2]^{\frac{1}{2}}. \quad (3)$$

It can be seen that they fall, within the accuracy of measurement, on the same curve as the other points. Sample Sn XXIII gave the same critical temperature, but a larger slope of the H_c-T curve. This could have been caused by an incorrect measurement of the diameter. The diameter was therefore measured again and was found—if at all different from the previous measurement—rather smaller than larger, which would increase the slope further.

Sample Sn XXIV did show hysteresis, when the current was monotonically increased or decreased. One can see that the points taken with increasing current ("up") and the points taken with reversal of the current agree with each other and fall on the H_c-T curve obtained by Muench,[22] which is drawn as a solid line. The points taken with decreasing current ("down") fall on a distinctly lower curve. There is some scatter around the lambda point of the liquid helium, which may be due to the limits of accuracy of the simple mercury manometer and to lack of correction for the hydrostatic pressure head. The latter would shift the points at 2.251°K toward higher temperatures, but would leave the points at 2.044 and 2.114°K unchanged. On the other hand, the scatter could be caused by a temperature difference between sample and bath, the correction for which would again shift the points at 2.251°K towards higher temperatures but leave the ones below 2.18°K unchanged.

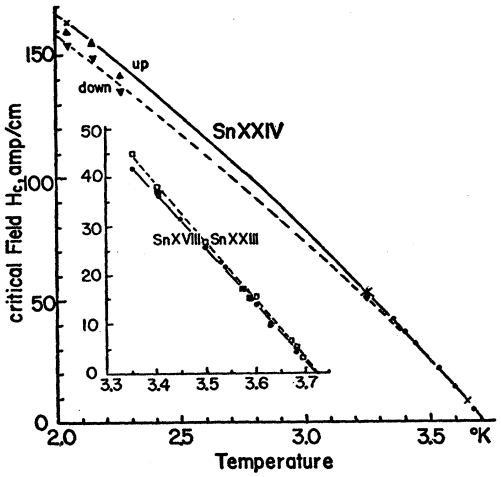

FIG. 8. Critical field curve for samples Sn, XVIII, XXIII (both in the insert) and XXIV. Practically all other samples gave curves which agreed with that for Sn XVIII. Symbols: Sn XXIX: × current reversed, ▲ current increased, ▼ current decreased. Sn XXIII: □ current reversed; Sn XVIII: ● current reversed $H_{z0}=0$; ■ current reversed $H_{z0}>0$.

All samples show that even the points very close to the normal transition temperature do not deviate from a straight line in the manner observed by Baird[23] for tin of lower purity, and by Haley and Andrews[24] for NbN.

V. TEMPERATURE DEPENDENCE OF THE RESISTANCE

Table II lists the values of r in the limit of vanishing current. All are taken at temperatures above the critical point. It would have been desirable to obtain values below this temperature by application of a magnetic field and extrapolation to $H=0$. This was not possible, as will be seen from the discussion in Sec. VI, because the magnetoresistance showed an irregular behavior. Instead two r values at 20°K have been used from measurements of Meissner and Voigt.[25] While the corresponding values at lower temperatures are somewhat uncertain owing to the high residual resistance of their sample, the points at 20°K are certainly quite accurate.

According to Mathiessen's rule,

$$\rho=\rho_0+\rho_T \quad \text{or} \quad r=r_0+r_T, \quad (4)$$

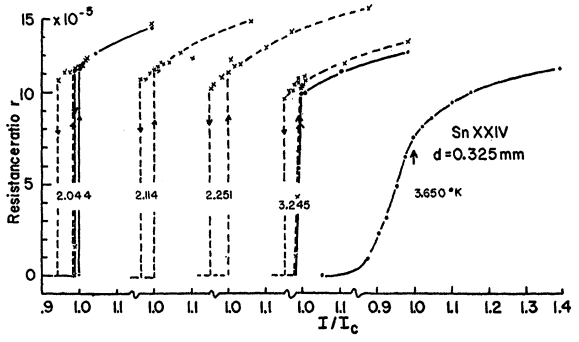

FIG. 7. Resistance ratio r as a function of I/I_c for the polycrystalline sample Sn XXIV. In case of hysteresis I_c has been chosen as the critical current for increasing current. ●—— ●, measurements with the current reversed; × – – – × measurements with the current monotonically increased or decreased. Note: The curves have been displaced by various amounts of I/I_c against each other.

[21] Lock, Pippard, and Shoenberg, Proc. Cambridge Phil. Soc. 47, 811 (1951).
[22] Nils L. Muench, Phys. Rev. 99, 1814 (1955).
[23] D. C. Baird, Can. J. Phys. 34, 725 (1956).
[24] F. C. Haley and D. H. Andrews, Phys. Rev. 89, 821 (1953).
[25] W. Meissner and B. Voigt, Ann. Physik 7, 761, 892 (1930).

TABLE II. Temperature dependence of the resistance ratio.

Sample	ϕ	T°K	$10^5 \lim_{I\to 0} r$	$10^5 r_0$	$10^5 r_T$	$1.938 \times 10^{-3} T^{4.40}$
Sn XVII	90°	4.428	5.35	3.98	1.37	1.36
Sn XVII	90°	4.223	5.10	3.98	1.12	1.10
Sn XVII	90°	3.722	4.60	3.98	0.62	0.63
Sn XVIII	79°	4.428	7.15	5.80	1.35	1.36
Sn XVIII	79°	4.223	6.90	5.80	1.10	1.10
Sn XVIII	79°	3.722	6.42	5.80	0.63	0.63
W. Meissner and B. Voigt	poly-crys-talline	20.37	1140	75	1065	1114
		20.41	1197	75	1122	1124

where ρ_0 and r_0 are the residual resistivity and residual-resistance ratio and ρ_T and r_T the corresponding temperature-dependent quantities. While ρ_0 and r_0 vary from sample to sample, ρ_T and r_T are universal functions for tin which, however, still depend on the crystal orientation.[26] Most of the samples used here have the crystal axis oriented very closely at an angle of $\phi=90°$ to the wire axis. We took as representative samples Sn XVII and XVIII with $\phi=90°$ and 79°, respectively. We found that a good fit to the measured points for the temperature region in question could be obtained (see Table II) by

$$r_T=aT^n, \quad \text{with} \quad a=1.938\times10^{-8} \quad \text{and} \quad n=4.40. \tag{5}$$

The measurements here could be represented as well by power laws of $n=4$ or $n=5$, but only $n=4.40$ gives a good match to the 20°K values. However, it is of course not possible to conclude from these data either that the resistance curve follows a single power law up to temperatures of 20°K or that the points for polycrystalline samples and single-crystals fall exactly on the same curve.

In all events, however, the temperature-dependent resistivity is well enough known that one can correct the critical-resistance ratio.

VI. DEPENDENCE OF THE RESISTANCE ON THE MAGNETIC FIELD

As mentioned in Sec. III, the resistance of the wire increases with increasing current due to the magnetoresistance created by the magnetic field of the current. In the normal-conducting state as well as in the intermediate state, the current and magnetic field are perpendicular to each other if no external longitudinal field is superimposed. One is therefore mainly interested in the magnetoresistance in transverse fields. Furthermore, for single-crystals the magnetoresistance usually depends on the direction between crystal axis and magnetic field.

Fortunately, the magnetoresistance due to the current does not have to be known with great accuracy, and measurements of the magnetoresistance in transverse fields were taken only for one, undetermined, direction between crystal axis and field. Measurements on samples Sn XVII, XVIII, and XXIII are shown in Fig. 9. Already in fields of less than 200 amp/cm the resistance doubles. This is to be expected for small r, since according to Kohler's theory[27] the magnetoresistance $\Delta r/r$ depends on H/r. It appears furthermore, in Fig. 9, that curves at different temperatures cross over, the resistance at certain values of the magnetic field being lower at the higher temperatures. This has been found repeatedly in different runs. The reason for this behavior is not quite clear. There are three possible explanations: (1) it is the de Haas–van Alphen effect; (2) it follows from band theory as shown by Sondheimer and Wilson[28]; (3) it is of the type predicted for thin films in transverse fields[29] and found in thin sodium wires.[30] The last of the three possibilities seems to be by far the most likely one.

In sample Sn XXIII the regions around the potential leads are normal-conducting at fields above 80 amp/cm. The dashed lines show the shape which the curves would have if these regions stayed normal-conducting down to the lowest fields.

All curves depend slightly on the measuring current. It would have been desirable to make measurements at several currents and then extrapolate to zero current. It was felt, however, that other uncertainties are bigger than this and that this time-consuming procedure would not be necessary. Nevertheless, the surface fields $H_{\phi 0}$ due to the measuring current are noted in Fig. 9.

Figure 10 is a Kohler diagram, in which $\Delta r/r_{H=0}$ is plotted as function of $H/r_{H=0}$ on a double-logarithmic scale. The oersted has been chosen as the unit for H in order to facilitate comparison with measurements by other authors. Owing to the irregularities of the curves in Fig. 9, one obtains of course a certain scattering of the points. The best-fitting straight line through the points was found to be represented by

$$\frac{\Delta r}{r_{H=0}}=8.45\times10^{-3}\times\left[\frac{H}{r_{H=0}}\times10^{-5}\right]^{1.30} \tag{6}$$

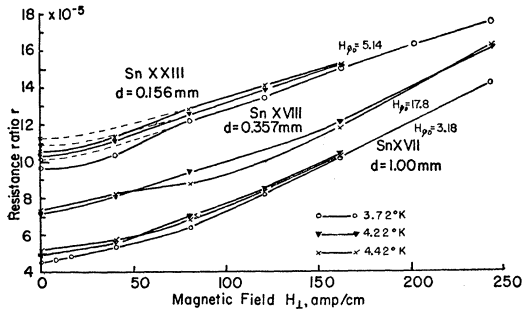

FIG. 9. Increase of resistance ratio in a transverse magnetic field for single-crystal samples Sn XVII, XVIII, and XXIII. The crystal axis is in all cases roughly perpendicular to the wire. The angle between the crystal axis and the magnetic field is not known.

[26] P. W. Bridgman, Proc. Am. Acad. Arts Sci. **68**, 104 (1933).

[27] M. Kohler, Ann. Physik **32**, 211 (1938).

[28] E. H. Sondheimer and A. H. Wilson, Proc. Roy. Soc. (London) **A190**, 435 (1947).

[29] E. H. Sondheimer, Phys. Rev. **80**, 401 (1950).

[30] J. Babiskin and P. G. Siebenmann, Bull. Am. Phys. Soc. Ser. II, **2**, 140 (1956).

if H is measured in oe, and by

$$\frac{\Delta r}{r_{H=0}}=6.29\times10^{-3}\times\left[\frac{H}{r_{H=0}}\times10^{-5}\right]^{1.30} \quad (6a)$$

if H is measured in amp/cm.

It was felt that Eq. (6) is accurate enough to allow all necessary corrections for magnetoresistance, especially since it was found that the polycrystalline sample Sn XXIV gave almost the same constants as in Eq. (6).

VII. CORRECTION OF THE r_c/r_n VALUES FOR CHANGE IN RESISTIVITY WITH TEMPERATURE AND MAGNETIC FIELD

Calculations of resistance in the transition region are usually expressed in terms of $R_c/R_n=r_c/r_n$. For the samples used here, R_n or r_n is no longer constant, since the resistivity changes with temperature and magnetic field:

$$\rho_{H,T}=\rho_0+\rho_T+\Delta\rho(H), \quad (7)$$

or

$$r_{H,T}=r_0+r_T+\Delta r(H). \quad (8)$$

We are here interested only in making corrections for $r=r_c$ (i.e., at the break). At this point the field at the surface and wherever the material is normal-conducting is equal to H_c. Unfortunately there is very probably always some paramagnetic effect set up, as was shown in Part II.* As a result, the magnetic field in the center is longitudinal and parallel to the current.

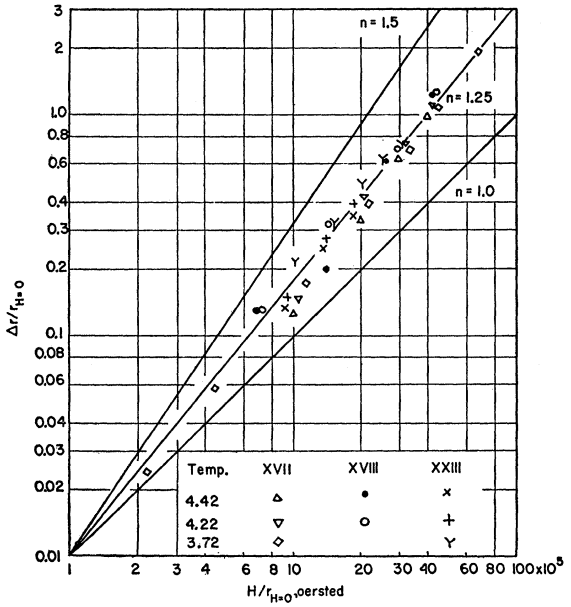

FIG. 10. Kohler diagram for the magnetoresistance in transverse fields for tin. The best fitting straight line through the measured points gives $\Delta r/r=8.45\times10^{-3}\times[(H/r)\times10^{-5}]^{1.30}$ if H is measured in oe, and $\Delta r/r=6.29\times10^{-3}\times[(H/r)\times10^{-5}]^{1.30}$ if H is measured in amp/cm.

* *Note added in proof.*—New measurements have shown that this is not the case.

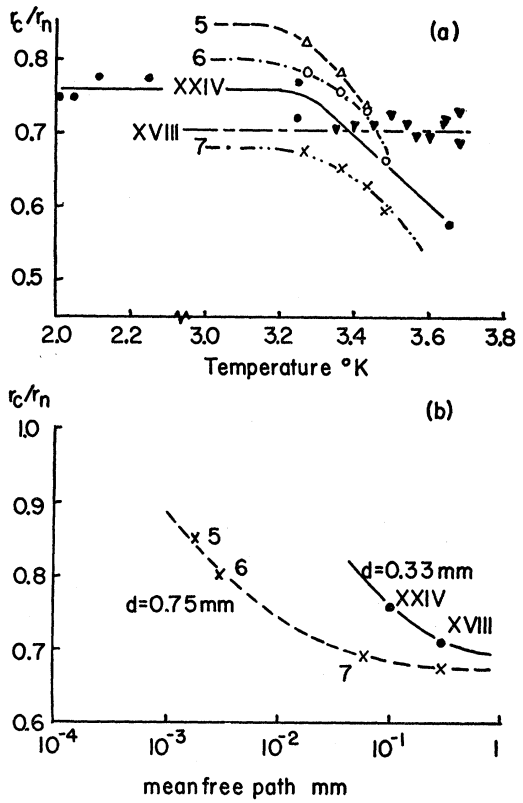

FIG. 11. (a) Dependence of r_c/r_n on temperature for polycrystalline samples. Curves number 5, 6, and 7 are from measurements by Rinderer. Curve marked XXIV is for sample Sn XXIV. For comparison the points for the single-crystal sample Sn XVIII are also plotted. Note the break in the temperature scale. (b) Temperature dependence of the low-temperature value of r_c/r_n on electronic mean free path. Points 5, 6, and 7 are derived from measurements of Rinderer.

This region, however, might be very small, and the total resistance is probably more determined by the outer regions, where the field is perpendicular to the current. It is felt, therefore, that one can correct r_n as:

$$r_n=(r_0+r_T)\left[1+b\left(\frac{H_c}{r_0+r_T}\right)^n\right], \quad (9)$$

with b and n given as in Eq. (6a) and r_T as in Eq. (5).

Table III lists the values of r_c and r_0 as measured and the values of r_n and r_c/r_n as derived from them.

VIII. INFLUENCE OF THE MEAN FREE PATH ON THE VALUE OF r_c/r_n

Rinderer[8] studied the effect of residual resistance or —since they are related—mean free path on the value of r_c/r_n. His measurements can be compared with our measurements on the polycrystalline sample Sn XXIV. Figure 11(a) shows the values of r_c/r_n for Rinderer's samples 5, 6, and 7 and for our sample Sn XXIV plotted as a function of the temperature. The r_c/r_n values drop at temperatures close to the critical temper-

TABLE III. Corrected values of r_c/r_n.

Sample	d mm	$10^5 r_0$	T°K	$10^5 r_c$	$10^5 r_n$	r_c/r_n	Remarks
Sn IX	3.39	0.70	3.681	0.95±0.10	1.35	0.70 ±0.10	a
			3.640	0.83±0.10	1.39	0.60 ±0.10	a
Sn XV	3.32	1.27	3.679	1.10±0.10	1.91	0.58 ±0.05	a
			3.636	1.10±0.10	1.95	0.58 ±0.05	a
Sn XVI	2.21	2.21	3.679	1.70±0.15	2.84	0.60 ±0.05	a
			3.636	1.60±0.15	2.88	0.56 ±0.05	
Sn XXI	2.19	2.24	3.681	1.75±0.10	2.87	0.610±0.04	
			3.603	1.70±0.10	2.94	0.578±0.04	
			3.355	2.10±0.10	3.33	0.630±0.04	
Sn XVII	1.00	3.98	3.679	2.85±0.15	4.61	0.618±0.030	
			3.676	3.05±0.05	4.60	0.663±0.010	
			3.637	3.00±0.10	4.63	0.649±0.020	
			3.636	3.00±0.10	4.63	0.649±0.020	
			3.597	3.05±0.10	4.66	0.654±0.020	
			3.565	2.95±0.06	4.69	0.629±0.015	
			3.538	3.13±0.10	4.71	0.665±0.020	
			3.502	3.15±0.05	4.77	0.660±0.010	
			3.451	3.15±0.06	4.82	0.654±0.015	
			3.400	3.25±0.10	4.88	0.666±0.020	
			3.354	3.25±0.10	4.95	0.657±0.020	
Sn XIX	1.05	4.31	3.681	3.10±0.15	4.94	0.628±0.030	a
			3.679	2.80±0.15	4.94	0.567±0.030	a
			3.636	2.95±0.15	4.96	0.593±0.030	a
			3.603	3.35±0.10	4.99	0.671±0.020	
Sn X	1.02	4.80	3.681	3.70±0.15	5.43	0.681±0.030	
			3.640	3.70±0.15	5.44	0.680±0.030	
Sn XVIII	0.357	5.80	3.679	4.40±0.15	6.43	0.686±0.030	
			3.680	4.70±0.20	6.43	0.731±0.040	
			3.636	4.65±0.15	6.44	0.723±0.030	
			3.637	4.60±0.05	6.44	0.715±0.010	
			3.601	4.50±0.15	6.48	0.694±0.030	
			3.566	4.50±0.15	6.49	0.694±0.030	
			3.537	4.65±0.10	6.52	0.713±0.02	
			3.500	4.75±0.10	6.57	0.723±0.020	
			3.450	4.70±0.10	6.61	0.711±0.020	
			3.401	4.75±0.05	6.67	0.712±0.010	
			3.353	4.80±0.10	6.83	0.703±0.020	
			3.573	4.60±0.15	6.48	0.710±0.030	$H_{z0}=9.09$
			3.589	4.55±0.10	6.49	0.701±0.020	$H_{z0}=4.55$
			3.404	4.70±0.15	6.66	0.706±0.030	$H_{z0}=9.09$
Sn XXIV	0.325	12.32	3.650	7.46±1.00	12.95	0.576±0.080	a, b
			3.245	9.80±0.20	13.24	0.740±0.016	b
			3.245	10.18±0.20	13.24	0.770±0.016	b, c
			2.251	10.98±0.20	14.04	0.783±0.016	b, c
			2.114	11.22±0.20	14.24	0.788±0.015	b, c
			2.044	11.06±0.20	14.54	0.762±0.015	b, c
			2.044	11.17±0.20	14.59	0.766±0.015	b
Sn XXIII	0.156	7.88	3.697	6.20±0.20	8.51	0.729±0.015	d
			3.680	6.30±0.15	8.51	0.740±0.010	d
			3.673	6.20±0.20	8.51	0.729±0.015	d
			3.601	6.05±0.15	8.55	0.708±0.010	d
Sn XXIII	0.156	9.76	3.503	7.55±0.15	10.50	0.719±0.015	e
			3.402	7.60±0.15	10.58	0.719±0.015	e
			3.355	7.70±0.20	10.66	0.722±0.020	e
			3.355	7.70±0.20	10.66	0.722±0.020	e

[a] Wide transition; r_c not well defined.
[b] Polycrystalline sample.
[c] Current monotonically increased.
[d] Low-current measurements; regions around potential leads superconducting.
[e] High-current measurements; regions around potential leads normal-conducting.

ature from a plateau at lower temperatures. (Note the break in the temperature scale.) Rinderer has unfortunately not measured to temperatures low enough to show the plateau. The extrapolation of his data represents therefore only a fair estimate. The points for sample Sn XVIII, which have been plotted for comparison, show that the plateau for pure enough samples extends to temperatures very close to T_c. Thus, apparently the plateau value of r_c/r_n is characteristic of the sample.

In Fig. 11(b) the plateau values of r_c/r_n have been plotted as a function of the electronic mean free path. The mean free path has been calculated from the residual impurity resistance, correction being made for boundary scattering. Since there were no measurements with single crystals of $d=0.75$ mm, an appropriate intermediate value between samples Sn XVII and XVIII has been used. The small difference in diameter between samples Sn XXIV and XVIII has been neglected. It can be seen from Fig. 11(b) that the values of r_c/r_n decrease with increasing mean free path. For samples with smaller diameter, the curves are shifted toward larger values of the electronic mean free path.

One might argue that since sample Sn XVIII shows no decrease in r_c/r_n near T_c, it should represent the "ideally pure" case and that the curves in Fig. 11(b) should be drawn with a break. Measurements which are currently undertaken on indium indicate, however, that even samples which do not show a decrease in r_c/r_n near T_c still show a strong dependence of r_c/r_n on the residual resistance.

IX. DEPENDENCE OF r_c/r_n ON SAMPLE DIAMETER

Table IV lists the values of r_c/r_n averaged over all temperatures, where proper weight has been given according to the errors of the individual measurements. In the assignments of the error limits of the averages it has been taken into account that any error in r_n influences all individual values of r_c/r_n in the same way for the sample in question. In cases where measurements on more than one sample of nearly the same diameter exist, it can be seen that although the error limits overlap, there is a definite tendency for the r_c/r_n values to increase with increasing residual resistance r_0.

TABLE IV. Dependence of r_c/r_n on diameter.

Sample	Diameter mm	$10^5 r_0$	Av (r_c/r_n)	Remarks
Sn IX	3.39	0.70	0.65 ±0.10	
Sn XV	3.32	1.27	0.58 ±0.05	
Sn XVI	2.21	2.21	0.58 ±0.05	
Sn XXI	2.19	2.24	0.606±0.030	
Sn XVII	1.00	3.98	0.653±0.010	
Sn XIX	1.05	4.31	0.670±0.020	a
Sn X	1.02	4.80	0.680±0.030	
Sn XVIII	0.357	5.80	0.711±0.010	
Sn XVIII	0.357	5.80	0.705±0.020	b
Sn XXIII	0.156	7.88	0.726±0.010	c
Sn XXIII	0.156	9.76	0.720±0.015	d
Sn XXIII	0.156	...	0.724±0.010	e

[a] Sample in glass tube; the measurements with wide transitions have been neglected.
[b] Measurements with superimposed magnetic field H_{z0}.
[c] Low-current measurements; regions around potential leads superconducting.
[d] High-current measurements; regions around potential leads normal-conducting.
[e] Average of low-current and high-current measurements.

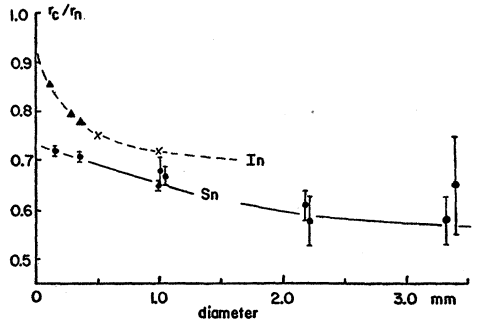

FIG. 12. Dependence of r_c/r_n on the diameter of the sample. Lower curve for tin of residual-resistance ratio due to impurities of $r_{00}\approx 4.2\times10^{-5}$. Upper curve for indium with a residual-resistance ratio of about 1×10^{-3}. ▲ from Scott's measurements; × preliminary measurements in this laboratory.

Figure 12 shows a graph of the values of r_c/r_n as function of the sample diameter. It appears that it is not possible to draw a smooth curve through the measured points which ends for $d=0$ at $r_c/r_n=1$, as was proposed by Scott[7] for indium. Scott's results, together with two preliminary points which were measured in this laboratory on indium samples of similar purity, are shown for comparison in the same diagram. It is, however, quite certain that purer indium samples would give a considerably lower curve. Also the points at the smallest diameters for tin would be, according to the discussion in Sec. VIII, somewhat lower for ideally pure tin.

On the whole, the result of these measurements is that for ideally-pure tin, r_c/r_n depends only slightly on the diameter between $d=0.1$ mm and $d=3$ mm and has values of $0.57\leqslant r_c/r_n\leqslant 0.73$.

X. HEAT TRANSFER FROM HORIZONTAL WIRES TO THE LIQUID HELIUM

Previous investigators[7,8] have argued as follows: As long as the sample stays superconducting, it has the temperature of the bath T_B. At $I_c=d\pi H_c(T_B)$ the resistance appears, and the sample heats up and goes over into a state for which the critical current is $I_c'=d\pi H_c(T_B+\Delta T)$. Since $H_c(T_B+\Delta T)<H_c(T_B)$, we have $I_c'<I_c$. The current, however, is still kept at $I=I_c$ and the sample is in a state where $I/I_c'>1$, that is, partially normal-conducting. The present theory[2,4,8,10] predicts for this state a value $r=r_c'>r_c$. In other words: it is not possible on the rising curve to obtain the true value of r_c. On the falling curve, however, the sample remains in the intermediate state down to currents I_c' and one would actually observe the true value of r_c or very nearly so.

All measurements (see references 7 and 8 and our

TABLE V. Data on heat-transfer samples.

	Ht I	Ht II
o.d. of tube (mm)	3.18	1.21
Wall thickness of tube (mm)	0.20	0.10
Length between potential taps (mm)	39.5	34.8
Resistance of tube at 4.2°K (ohms)	8.16×10^{-3}	3.307×10^{-2}
Thermometer resistance at 273°K (ohms)	116	799
Thermometer resistance at 4.2°K (ohms)	2 334	25 550
Thermometer sensitivity		
dR/dT at 4.2°K (ohms/°K)	1.10×10^{3}	14.9×10^{3}
dR/dT at 3.6°K (ohms/°K)	2.44×10^{3}	
dR/dT at 3.35°K (ohms/°K)	3.70×10^{3}	

sample Sn XXIV) where the current is monotonically decreased or increased show indeed approximately this type of hysteresis.

The argument is then continued in the following way: Since the heating increases as I_c^2, the temperature difference ΔT should also increase as I_c^2 and the difference between rising and falling curves should increase as I_c^2. Moreover r_c/r_n should also increase with I_c, though not as I_c^2. Since this is not observed, the heating is negligible.

The assumption behind this argument is that the heat-transfer coefficient h is a constant. The coefficient h is defined by $q=hA\Delta T$, where q is the heat input, A the surface area, and ΔT the temperature difference. The heat transfer from these samples should, however, be a case of free convection; that is, the temperature difference ΔT is the driving force which causes the convection in the liquid. This means that h is not a constant, but increases with ΔT. The free convection is commonly[31] described by an empirical relation between the Nusselt number N_{Nu}, the Grasshof number N_{Gr}, and the Prandtl number N_{Pr}, of the type $N_{\mathrm{Nu}}=\phi(N_{\mathrm{Gr}}N_{\mathrm{Pr}})$. This relation is shown in reference 31, p. 524, Fig. 25–3, and p. 641, Fig. 29–21.

If r_c/r_n is found to be independent of I_c after all other corrections have been made, it might still be that one should make a correction for heating which leads to a reduction in the values of r_c/r_n for large I_c if the heat-transfer coefficient were actually as small as one would expect for free convection.

It was felt that one should not use the free-convection equation untested for liquid helium, although it has been tested for a great number of other substances.

TABLE VI. Heat transfer data.

Sample	T°K	Heater at bottom of Dewar watts	Range of ΔT millidegrees	h w/cm² °K	N_{Nu}	Effective flow velocity v_{eff} cm/sec
Ht I	4.208	0	1.6–132	1.53×10^{-2}	18.8	1.1
	4.207	0.2	0.4–67.5	2.77×10^{-2}	34.0	3.8
	4.206	0.8	0.7–73.2	2.89×10^{-2}	35.5	4.2
	3.590	0.4	0.8–81.5	2.38×10^{-2}	31.7	3.3
	3.358	0.4	0.3–70.6	2.75×10^{-2}	38.0	4.9
Ht II	4.212	0.2	6–224	3.81×10^{-2}	17.8	2.7

[31] Max Jakob, *Heat Transfer* (John Wiley and Sons, Inc., New York, 1949), Vol. I, pp. 443 ff and pp. 483 ff.

Two heat-transfer samples (see Table V) were constructed as follows: A carbon thermometer was placed inside a thin-walled supernickel tube. The leads to the thermometer were made from manganin to reduce thermal conduction through them and brought out through Kovar seals at the ends of the tube. The rest of the inside space was filled with Silicon oil. Current and potential leads were attached to the tube. The samples were mounted in exactly the same position as the tin samples, and the temperature difference between the cases of current and no current through the tube was measured with the carbon thermometer.

It was found (see Fig. 13 and Table VI) that the heat-transfer coefficient is independent of the temperature difference over a certain range, and that the Nusselt numbers are much larger than one would expect for free convection according to reference 31, Fig. 25–3. The measurements were done in the range of $800\leqslant N_{\mathrm{Gr}}N_{\mathrm{Pr}}\leqslant100\,000$, where one expects to find Nusselt numbers $0.5\leqslant N_{\mathrm{Nu}}\leqslant5$, whereas we have found

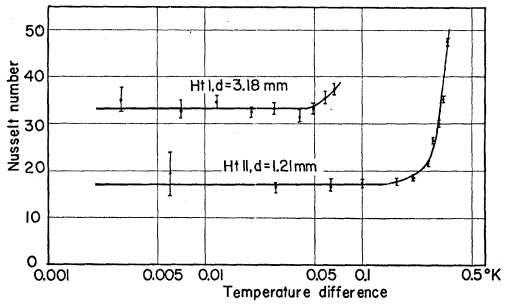

FIG. 13. Nusselt number $N_{\mathrm{Nu}}=hd/k$ as function of temperature difference for heat transfer in liquid helium. $T=4.21$°K, heater at bottom of Dewar vessel dissipates about 0.4 w.

Nusselt numbers of $15\leqslant N_{\mathrm{Nu}}\leqslant40$, that is, 10 to 20 times larger than for free convection.

Furthermore, although the temperature differences were measured with a Wheatstone bridge and a very slow ($\tau=20$ sec) galvanometer, *they always fluctuated by at least 10%*. A fast-reading instrument would probably have revealed that the fluctuations were still much larger. They were certainly much larger than any fluctuations in the bath temperature.

It was further found that the heat-transfer coefficient was strongly dependent on the current through the heater at the bottom of the vessel (see Table VI).

It is apparent from these observations that one is not dealing with free convection, but with simulated forced convection, caused by the rising bubbles from the heater. Forced convection is commonly described by a relationship between the Nusselt number and the Reynolds number, N_{Re}. The relation is usually given as $N_{\mathrm{Nu}}=\psi(N_{\mathrm{Re}})$, where the function ψ is tabulated in reference 31, p. 560. The tabulated function is valid for gases. If it is applied to other fluids, corrections for the change in the Prandtl number should be made. Fortu-

nately, liquid helium has almost the same Prandtl number as gases, and the same function can be used in estimating the effective flow velocities v_{eff} which are shown in Table VI. These effective flow velocities are of a very reasonable order of magnitude. They decrease with sample diameter and would probably be very much smaller if the sample diameter is comparable to the bubble diameter or about 0.2 mm. It is obviously not possible to make a heat transfer sample of the same construction as Ht I and Ht II of so small a diameter.

Instead, a manganin wire was coated with tin in the hope that the temperature at which the coating becomes superconducting might shift with the current in a manner which allows one to calculate the temperature difference between wire and bath. The outcome of this experiment was quite unexpected and will be discussed in the next section. Unfortunately it does not allow one to obtain reliable data on the heat transfer.

To complete the discussion on the heat transfer, it should be mentioned that at temperature differences of 0.2 to 0.5°K nucleate boiling occurs. (Compare Fig. 13 with Fig. 29–21 of reference 31, p. 641.) This causes a steep rise in the heat-transfer coefficient with temperature difference. Astonishingly enough it was found that the fluctuations in the temperature difference for nucleate boiling, as observed by the rather slow galvanometer, are very small, about as small as the fluctuations in bath temperature.

The measurements on Sn XXIV above and below the lambda point substantiate the conclusion that the heat-transfer coefficient is large enough to allow one to neglect the heating of the samples.†

XI. MEASUREMENTS ON A TIN-COATED MANGANIN WIRE

The wire, which is designated as heat-transfer sample Ht IV, had a diameter of 0.204 mm and a length of 8.3 mm between potential taps. It was coated with pure tin by using a clean soldering iron. The following resistance values were found: $R_{293°K}=8.99\times10^{-2}$ ohm, $R_{77.3°K}=5.82\times10^{-2}$ ohm, and $R_{4.2°K}=0.719\times10^{-2}$ ohm. Treating the manganin and the tin coat as a parallel combination of resistors and assuming that the ratio $(R_{77.3°K}/R_{293°K})_{\text{tin}}=0.20$, while the corresponding ratio for manganin equals unity, one finds for the resistance of the tin coating $(R_{293°K})_{\text{tin}}=0.66$ ohm, and $(R_{4.2°K})_{\text{tin}}=0.77\times10^{-2}$ ohm, yielding for the residual-resistance ratio of the tin the value $r_{4.2°K}=1.2\times10^{-2}$. With the dimensions of the sample and the room-temperature resistivity of tin, one obtains a mean thickness of the tin coating of $t=1.82\times10^{-4}$ cm.

Since the original plan was to observe the heating of the wire above the bath temperature, transition curves were measured with rising and falling temperature for different constant currents (see Fig. 14). It was found that close to T_c rather wide transitions and no hysteresis occurred, while at lower temperatures, sharp transitions and large hysteresis were found.

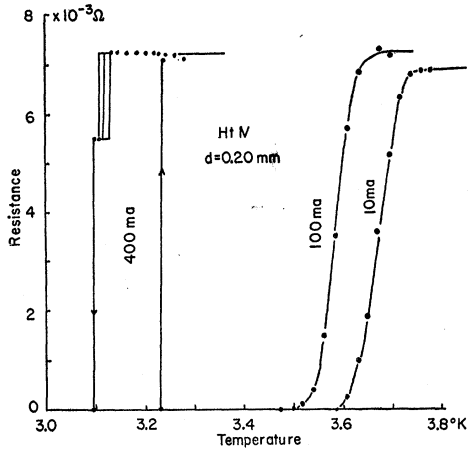

FIG. 14. Transition of tin-coated manganin wire at constant current. Thickness of tin coat about 1.8×10^{-4} cm.

At first sight, this looked as if the hysteresis was due to heating and was suppressed close to the critical temperature only because the temperature differences, ΔT, between wire and bath were smaller than the transition width.

The results were then plotted on a $H_{cI}-T$ diagram (see Fig. 15) and it was found that the reduction of the critical field was considerably larger than one would expect from the thinness ($t=1.82\,\mu$) of the film, according to the equation[32]

$$H_{cI}=H_{c\,\text{bulk}}\tanh(t/\delta), \tag{10}$$

where H_{cI} is the magnetic field created by the critical current, $H_{c\,\text{bulk}}$ the critical field for bulk superconductors, t the thickness of the tin coating, and δ the penetration depth. In the derivation of Eq. (10) the curvature of the thin film is neglected, but the fact that the magnetic field on one side of the film is zero is taken into account. (The current through the manganin core has been neglected.)

Furthermore the plot of H_c *vs* T looks very much like the graphs obtained by Lutes[33] for the transition of tin whiskers in a longitudinal external magnetic field. Lutes explains his results in terms of Ginsburg's[34] theory, which gives second-order transitions close to the critical point, and first-order transitions at lower temperatures for specimens which are of the same dimension as the penetration depth and which are subject *mainly* to an external field. In the region of second-order transition, hysteresis is not possible, while

† *Note added in proof.*—R. Hilsch has recently published an article [Z. Physik **149**, 1 (1957)] in which he comes to a different conclusion.

[32] M. von Laue, *Theorie der Supraleitung* (Springer-Verlag, Berlin, 1949), second edition, p. 95.

[33] O. S. Lutes, Phys. Rev. **105**, 1451 (1957).

[34] V. L. Ginsburg and L. D. Landau, J. Exptl. Theoret. Phys. (U.S.S.R.) **20**, 1064 (1950).

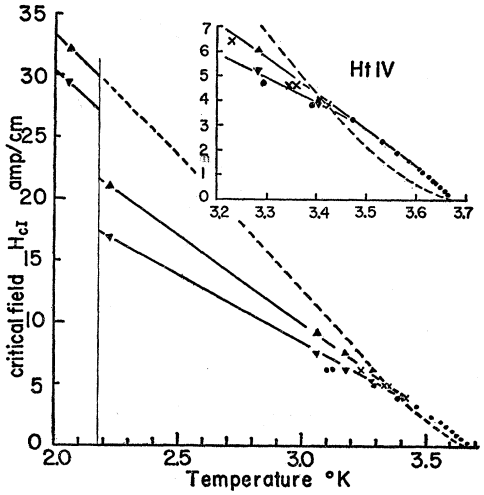

FIG. 15. Critical field H_{cI} for transitions of a tin-coated manganin wire through which a current flows. The following symbols are used. For constant current: × increasing temperature, ● decreasing temperature. For constant temperature: ▲ increasing current, ▼ decreasing current. Note the break at the lambda point of liquid helium. The dashed line is calculated according to Eq. (14) and matched to the measured point at 2.066°K.

in the first-order region, hysteresis is possible. Ginsburg's theory, however, does not state how much hysteresis will be observed.

In order to test further whether the hysteresis was due to heating or due to effects of the type considered in Lutes' experiments, the measurements were extended to temperatures below the lambda point and were now carried out at constant temperature with monotonically increasing or decreasing current. The results are also plotted in Fig. 15. The critical fields are much larger below the lambda point than above, indicating a shift above the lambda point due to heating. The hysteresis, however, continues nearly unchanged below the lambda point and apparently has nothing to do with heating.

An attempt was made to determine whether Eq. (10) would give a reasonable H_c-T curve. If $t\ll\delta$ (which actually is true only near T_c), one has

$$H_{cI}=H_{c\ \mathrm{bulk}}(t/\delta), \tag{11}$$

and with the temperature dependences:

$$H_{c\ \mathrm{bulk}}=H_{c0}[1-(T/T_c)^2], \tag{12}$$

$$\delta=\delta_0[1-(T/T_c)^4]^{-\frac{1}{2}}, \tag{13}$$

one obtains

$$\frac{H_{cI}(T_1)}{H_{cI}(T_2)}=\frac{[1-(T_1/T_c)^2][1-(T_2/T_c)^4]^{\frac{1}{2}}}{[1-(T_2/T_c)^2][1-(T_1/T_c)^4]^{\frac{1}{2}}}. \tag{14}$$

If one takes the measurement for rising current below the lambda point as the true critical field, the critical fields at the higher temperatures can thus be calculated according to Eq. (14). The result is the dashed line drawn in Fig. 15. The line departs significantly from what one would reasonably expect close to the critical point. The reason seems to be not so much a failure of the approximation $t\ll\delta$, which actually becomes better just in this region, but the failure of Eq. (10).

The most likely explanation seems to be that in the transition region superconducting filaments are formed, each of which is subject not only to the field due to its own current, but to the fields due to the currents in the others. It has not been checked, however, whether one can find a combination of current and field which gives in Ginsburg's theory[32] a rather strong reduction of the critical field but still leads to a Curie point, above which transitions are of the second order.

XII. CONCLUSIONS

The values of r_c/r_n are, for pure samples, independent of critical current or temperature and depend only slightly on the diameter. Even for the largest diameters they are clearly larger than the theoretical value 0.5. The rise is certainly not caused by heating of the wire above the bath temperature.

The values of r_c/r_n increase markedly with decreasing electronic mean free path. This increase is larger for samples of smaller diameter.

It appears to us that the differences between theory and experiment can be explained by assuming that the superconducting domains have the shape of very long and thin filaments. The diameter of the filaments should be of the order of the correlation length[35] of the superconducting wave functions. The arguments on which this opinion is based are rather lengthy, and will be presented in a later paper. The connection of the findings of this paper with the constants I_g, which appear in the empirical correction of the paramagnetic effect, is still completely dark. From the observations above one would expect that the values of I_g should depend on the electronic mean free path, at least in samples of smaller diameter. Precision measurements of the values of I_g therefore seem to be necessary.

XIII. ACKNOWLEDGMENTS

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[35] A. B. Pippard, Proc. Roy. Soc. (London) **A216**, 547 (1953).