No. 4, or No. 8 with No. 9 in Fig. 3 and Table I). A further support for the above conclusion is provided by the fact that, again at the same charge density, the mass of the gas (here xenon and neon) has little or no influence on the thermal conductivity in these plasmas (compare the curves No. 5, 6, and 7). A rather strong dependence of this thermal conductivity on the electron density, however, is apparent.

The most appropriate comparison of the experimentally obtained coefficient of thermal conduction in the above-mentioned plasmas can be made with the recent detailed calculations of Spitzer and Härm.[12] These calculations are relative to plasmas in fully ionized gases. The comparison of our experimental results with this theory is appropriate because in the plasmas described in our work the interaction of electrons with the charged constituents (electrons and ions) predominates over that with the neutral gas constituents of the plasma owing to the long range of the Coulomb force. This comparison shows that the experimentally obtained values of thermal conductivity are in agreement, within less than one order of magnitude, with those given by the theory of Spitzer and Härm. A closer comparison with theory at this stage of the experiments does not appear to be justified in view of the present uncertainty of the initial values of the electron temperature (T_e) in the experiments performed with xenon. The initial values of the electron temperatures in the decaying neon plasmas investigated are known, however, with more certainty. In this case the experimental and theoretical values of the thermal conductivity of the plasma are in rather excellent agreement. (See Fig. 3.)

These experiments are being extended to other gases and it is expected that their continuation will yield more precise measurements of the plasma parameters. A subsequent paper will contain a more detailed description of the experiments and their results as well as a more complete comparison with theory.

[12] L. Spitzer, Jr., and R. Härm, Phys. Rev. 89, 977 (1953).

ACKNOWLEDGMENTS

We wish to acknowledge the assistance of Mr. H. M. Lauby and Mr. R. Herndon in the experiments and computations. Thanks are also due to Dr. A. A. Dougal and Dr. R. C. Hwa for discussion of these problems.

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Shape of the High-Energy End of the Electron-Bremsstrahlung Spectrum*

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The elastic scattering of photons by the 15.11-Mev level in C^{12} has been used to study with good energy resolution the number of photons at the high-energy end of a bremsstrahlung spectrum. The bremsstrahlung was produced by electrons accelerated in a betatron, the energy of which was varied in 35-kev increments. Targets were: a 0.025-inch-diameter tungsten wire and the following foils, 0.001- and 0.010-inch tungsten, 0.002-inch thorium and 0.010-inch nickel. The foils were used to study the dependence of the spectrum shape upon target thickness and atomic number. When compared with Bethe-Heitler spectra corrected for target thicknesses, the data indicate an excess number of photons in the tip of the spectrum. The experimental number depends on the atomic number of the target and cannot vary more rapidly than $Z^{\frac{1}{2}}$.

I. INTRODUCTION

THERE have been a number of experimental measurements of the bremsstrahlung spectra generated by electrons having energies in the range from 1 to 20 Mev. A review of these experiments and the comparison of the experiments with the available theories is given by Starfelt and Koch.[1] The data indicate that for high energies the general shape of the bremsstrahlung spectrum is well described by the results obtained from a calculation made in Born approximation when the effect of screening by the atomic electrons is taken into account.[2] The absolute magnitudes of the bremsstrahlung cross section for electrons with energies in the 10- to 20-Mev range have been found to agree with the Born approximation result to within about 10%.[1,3]

Up to the present time there have been no detailed measurements or satisfactory theoretical calculations of the shape of the high-energy tip of the bremsstrahlung spectrum (energies within mc^2 of the incident electron kinetic energy). The shape of the spectrum in this energy range is of utmost importance in interpreting the breaks that have been found in activation curves.[4] Poor instrumental resolution has been the principal reason for the lack of experimental information about this portion of the bremsstrahlung spectrum. The failure to obtain a satisfactory theoretical prediction for the shape of the tip of the spectrum can be laid to

* This work was partially supported by the U. S. Air Force, through the Office of Scientific Research of the Air Research and Development Command.

[1] N. Starfelt and H. W. Koch, Phys. Rev. 102, 1598 (1956).
[2] H. A. Bethe and W. Heitler, Proc. Roy. Soc. (London) A146, 83 (1934).
[3] E. V. Weinstock and J. Halpern, Phys. Rev. 100, 1293 (1955).
[4] A. S. Penfold and B. M. Spicer, Phys. Rev. 100, 1377 (1955).

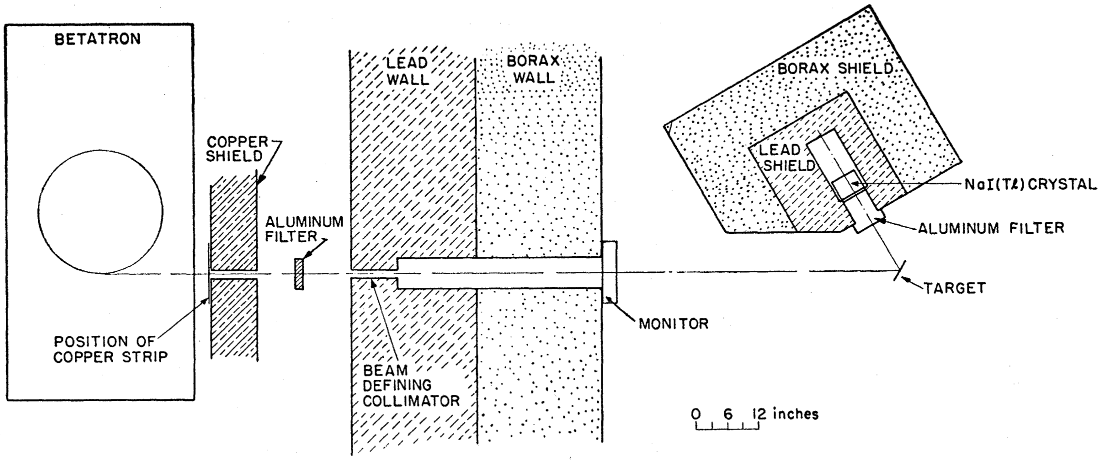

FIG. 1. The experimental arrangement.

the extreme difficulties encountered when calculations must be made using the exact wave functions to describe the incoming and outgoing electron.

The technique of studying the shape of the bremsstrahlung spectrum by use of the photoexcitation of nuclear energy levels has been shown to be possible at low energies[5] and in the 10- to 20-Mev range.[4,6] This paper will describe an experiment in which this technique was employed. In addition to the spectrum from a 0.025-inch tungsten wire target, which is normally used in the National Bureau of Standards betatron, spectra were also measured from 0.010-inch nickel, 0.001-inch tungsten, 0.010-inch tungsten, and 0.002-inch thorium foil targets placed inside the accelerating tube. The results for the shape of the high-energy tip of the spectrum are compared in this paper with spectra calculated from the Born approximation result given by Bethe and Heitler.[7]

II. EXPERIMENTAL METHOD

In this experiment the shape of the bremsstrahlung spectrum was studied by observing the scattering by the 15.11-Mev level in C^{12} as a function of the maximum energy of the betatron. In the laboratory system the threshold for scattering by this level is 15.12 Mev. The width of this level has been shown to be 79 ± 16 ev.[8] The number of photons scattered is then a measure of the number of photons in a very narrow energy band of the bremsstrahlung spectrum. For this work the yield curve is defined as the curve giving the relative number of 15.12-Mev photons per Mev in the total bremsstrahlung spectrum as a function of the peak energy of the spectrum. The object of the experiment was to study the shape of the yield curve for various types of bremsstrahlung targets placed inside the betatron donut.

The experimental arrangement used is shown in Fig. 1. The counting rate for the photons scattered by the 15.11-Mev level was optimized by placing as much aluminum absorber as space would allow in the scattered beam. This procedure had the effect of decreasing the counting rate resulting from low-energy photons and electrons relative to that resulting from the high-energy photons. The absorber in the primary beam was then adjusted until it was possible to run with full output from the betatron at 16 Mev without having appreciable pile-up in the peak of the pulse-height distribution produced by the photons scattered by the 15.11-Mev level. The pulse-height distribution produced by the photons scattered from a 2-g/cm² graphite target is shown in Fig. 2. The pulses between A and B on this distribution were used as a measure of the

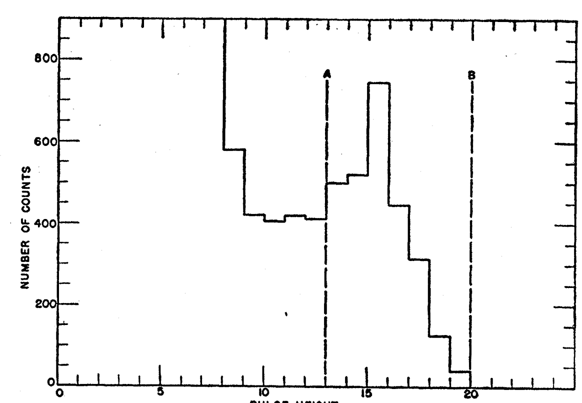

FIG. 2. The pulse-height distribution obtained when the carbon target was irradiated by 16- to 17-Mev bremsstrahlung beams. The number of counts between A and B were taken as a measure of the number of 15.12-Mev photons in the bremsstrahlung beam. This distribution contains relatively many more small pulses than the one shown in reference 8 because of the degradation of the scattered photons in the large aluminum filter used in the present experiment.

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[5] W. C. Miller and B. Waldman, Phys. Rev. **75**, 425 (1949).
[6] Fuller, Hayward, and Svantesson, Bull. Am. Phys. Soc. Ser. II, **1**, 10 (1956); Sargent, Bertozzi, and Demos, Bull. Am. Phys. Soc. Ser. II, **1**, 343 (1956).
[7] W. Heitler, *Quantum Theory of Radiation* (Oxford University Press, New York, 1954), third edition, Eq. (16), p. 245.
[8] E. Hayward and E. G. Fuller, Phys. Rev. **106**, 991 (1957).

number of 15.12-Mev photons in the incident spectrum. At 19 Mev the counting rate was approximately 200 counts per hour in this region of the pulse-height distribution.

When data were obtained for bremsstrahlung spectra with operating energies greater than 20 Mev, there was some evidence that counts in the region from A to B were produced by one or all of the following: (1) feeding of the 15.11-Mev level by higher levels in C^{12}, (2) the incomplete absorption by the NaI(Tl) crystal of higher energy photons scattered by the target, and (3) background produced by neutrons generated in the target. This background was determined by making a self-absorption measurement[8] with a 2-g/cm^2 carbon absorber in the incident beam when the betatron was operated at energies above and below 20 Mev. At 40 Mev the background amounted to a 10% correction.

Each run was monitored in terms of the charge collected from the transmission ionization chamber. This chamber was calibrated by comparing its reading with that of a standard chamber placed in the same beam. The standard chamber had been calibrated by the total spectrum method of Koch, Leiss, and Pruitt.[9]

The properties of the various targets used are listed in Table I. The target normally used in the betatron is a tungsten wire with a diameter of 0.025 inch. This target is located at a smaller radius than the equilibrium orbit in the betatron. The other targets were mounted, four at a time, in a special holder that allowed any one of the four to be rotated into position within the donut without breaking the vacuum or changing the donut position. These targets were all positioned at a radius larger than the equilibrium orbit but inside the radius of the injector.

In the case of the normal betatron target, x-rays were obtained by applying a rather slow pulse to an electron-orbit shift coil whose radius was smaller than that of the donut. The x-ray pulse from the betatron under these conditions was roughly triangular in shape with a base length of about 4 microseconds. For the data taken near the threshold for scattering by the 15.11-Mev line, the time width of the x-ray pulse and the rate of change of the betatron magnetic field resulted in an energy spread of about 80 kev. For the targets mounted at a radius larger than the equilibrium orbit, yield was obtained by applying a large pulse to a pair of coils located above and below the equilibrium orbit. This procedure resulted in an x-ray pulse whose width was less than 0.1 microsecond at the base. In this case the spread of the beam was less than 2 kev.

TABLE I. Target properties.

Target	Energy loss[a] kev	Beam width at one-half maximum Mev-radians	Observed energy loss[b] kev
0.025 in. W wire	1100	2.3	130± 40
0.001 in. W foil	44	2.0	117± 23
0.010 in. W foil	440	2.7	370± 70
0.010 in. Ni foil	240	2.4	350± 84
0.002 in. Th foil	49	2.3	112± 34

[a] Based on collision-energy loss expression as given by Goldwasser, Mills, and Hanson, Phys. Rev. **88**, 1137 (1952).
[b] Based on Muirhead *et al.* (reference 10) and on measured angular distribution of the bremsstrahlung beam.

During all spectrum measurements, the x-ray intensity from the betatron was kept below a limit fixed by the counting rate of an integral discriminator biased at about 1 Mev. This limit was fixed at about one-half the counting rate at which pile-up was observed in the region A to B of the pulse-height distribution given in Fig. 2, when the yield of counts in this region was measured as a function of the counting rate of the integral discriminator. The measurements were made for both the "slow-contract" and the "fast-expand" targets.

The energy scale of the betatron was based on the thresholds for the (γ,n) reactions in Be^9, Au^{197}, and Cu^{65} and on the threshold for scattering by the 15.11-Mev level in C^{12}. The latter threshold is by far the sharpest observed in this laboratory. A change of 20 kev on the energy scale produced a fivefold increase in counting rate. The threshold for this scattering was used to monitor any changes in the betatron energy scale. As would be expected, the energy calibration was different for the "expand" target and for the "contract" target. For any one type of machine operation, however, the energy calibration was constant to within 20 kev for the three-month period during which measurements were made. It was found that this stability could be obtained by warming up the betatron for about one hour before starting to take data.

For each of the targets used an attempt was made to determine the effective target thickness by measuring the angular distribution of the bremsstrahlung. These measurements were made by exposing a copper strip on the betatron side of the collimator. The strip was located on a diameter of the beam (see Fig. 1). After exposure of the strip to either 19- or 21-Mev bremsstrahlung (the energy differed for different targets), the 10-minute activity induced in the strip was counted as a function of position across the strip. After corrections were made for the decay of the induced radioactivity, the data gave the angular distribution for those photons in the high-energy part of the bremsstrahlung spectrum. Strips were exposed both vertically and horizontally. The angular distributions were not different within the errors of measurement. The measured angular distributions were used to determine the effective target thicknesses by comparing them with the angular distribution given by Muirhead *et al.*[10] For the 0.001-inch tungsten and 0.002-inch thorium targets there seems to be some evidence for multiple traversals of the target, since the target thicknesses in kev as inferred from

[9] Koch, Leiss, and Pruitt, Bull. Am. Phys. Soc. Ser. II, 1, 199 (1956).

[10] Muirhead, Spicer, and Lichtblau, Proc. Phys. Soc. (London) **A65**, 59 (1952).

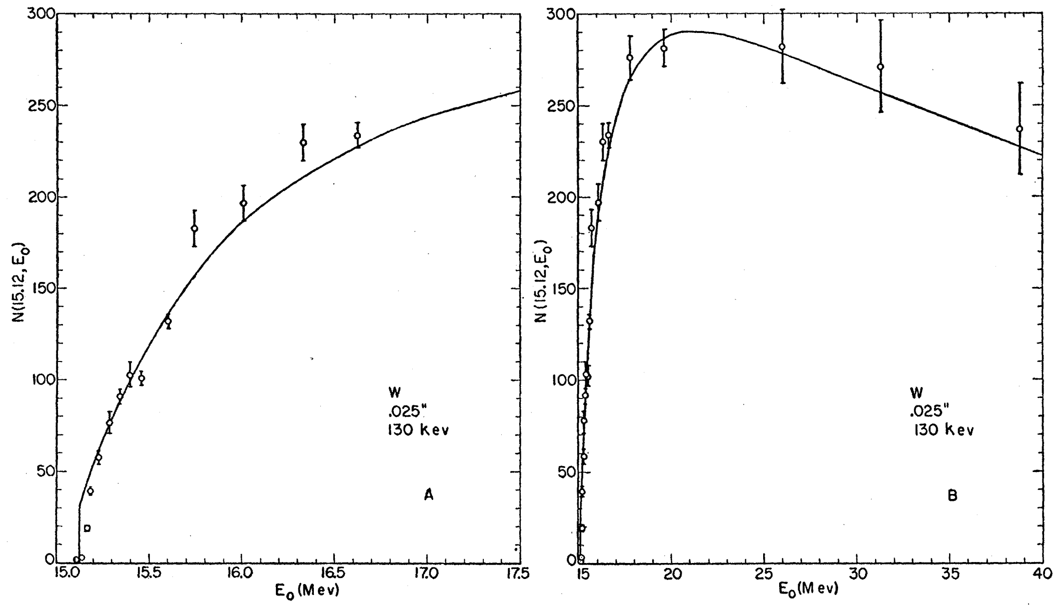

FIG. 3. The yield of 15.12-Mev photons obtained when electrons of energy, E_0, were slowly contracted into the 0.025-inch wire target normally used in the betatron. The solid curve is the Schiff spectrum normalized to the experimental curve near 20 Mev. The target thickness, as determined by a measurement of the angular distribution of the bremsstrahlung, is 130 kev.

reference 10 were larger than the collision-energy loss expected for these foils. The data for the 0.025-inch tungsten wire target clearly show the effect of a slowly contracted beam just grazing the edge of a target.

III. RESULTS

Figure 3 shows the yield curves obtained using the 0.025-inch tungsten wire target into which the betatron electron beam was slowly contracted. The ordinates of Fig. 3 are in units of the relative number of 15.12-Mev photons per Mev in the beam. Also plotted is the yield curve obtained from the Schiff bremsstrahlung spectrum as tabulated by Penfold and Leiss.[11] These tables are frequently used in the analysis of activation curves. The theoretical curve has been normalized to the experimental data near 20 Mev. The approximations made by Schiff[12] in evaluating the differential bremsstrahlung cross section of Bethe and Heitler were that the energies of both the incoming and outgoing electron were large compared to the rest energy of the electron. He also neglected certain screening terms in integrating over the outgoing electron angles. In integrating over the outgoing photon angle the approximation was made that the angles involved are small. These approximations produced the finite threshold to the yield curve shown in Fig. 3(a). If the theoretical curve shown in Fig. 3(a) is modified, by a method to be described later, to take into account the finite thickness of the target and the energy spread of the primary electrons, a considerably better fit can be obtained to the experimental data for energies near the threshold. The agreement of the experimental curve with the modified theoretical curve near threshold is only fortuitous, since the approximations made by Schiff are not valid in this energy region.

The yield curve for the tungsten-wire target was measured for electron energies up to 40 Mev, and is given in Fig. 3(b). The coincidence of the measured and calculated yield curves demonstrates, within the rather large experimental errors, the validity of the energy dependence of the monitor calibration used for these interpretations as well as the general validity of the Schiff calculations. The data are not accurate enough to distinguish between the Schiff integrated spectrum and the corrected forward spectrum.[13]

Figure 4 shows the measured yield curves obtained by using the "thin" targets into which the electron beam was expanded.[14] The solid curves are plots of a 15.12-Mev yield curve calculated from the Bethe-Heitler expression[7] normalized to the experimental data at 16.75 Mev.[15] This formula represents the brems-

[11] A. S. Penfold and J. E. Leiss, Phys. Rev. 95, 637 (1954); and private communication.

[12] L. I. Schiff, Phys. Rev. 83, 252 (1951).

[13] A. Sirlin, Phys. Rev. 106, 637 (1957); E. Hisdal, Phys. Rev. 105, 1821 (1957).

[14] Data were also taken by using a 0.020-inch aluminum target. These were not considered reliable because there was evidence that the x-rays came from two sources in the betatron. The measured angular distribution was very broad and the spectrum and magnitude of the yield obtained were consistent with the assumption that the x-rays came from both the aluminum target and the tungsten support for the assembly. The aluminum target was the only one for which this effect was expected.

[15] The normalization factor was a function of Z, varying by 11% from nickel to thorium. This difference is of the right magnitude and sign to be attributed to a screening effect.

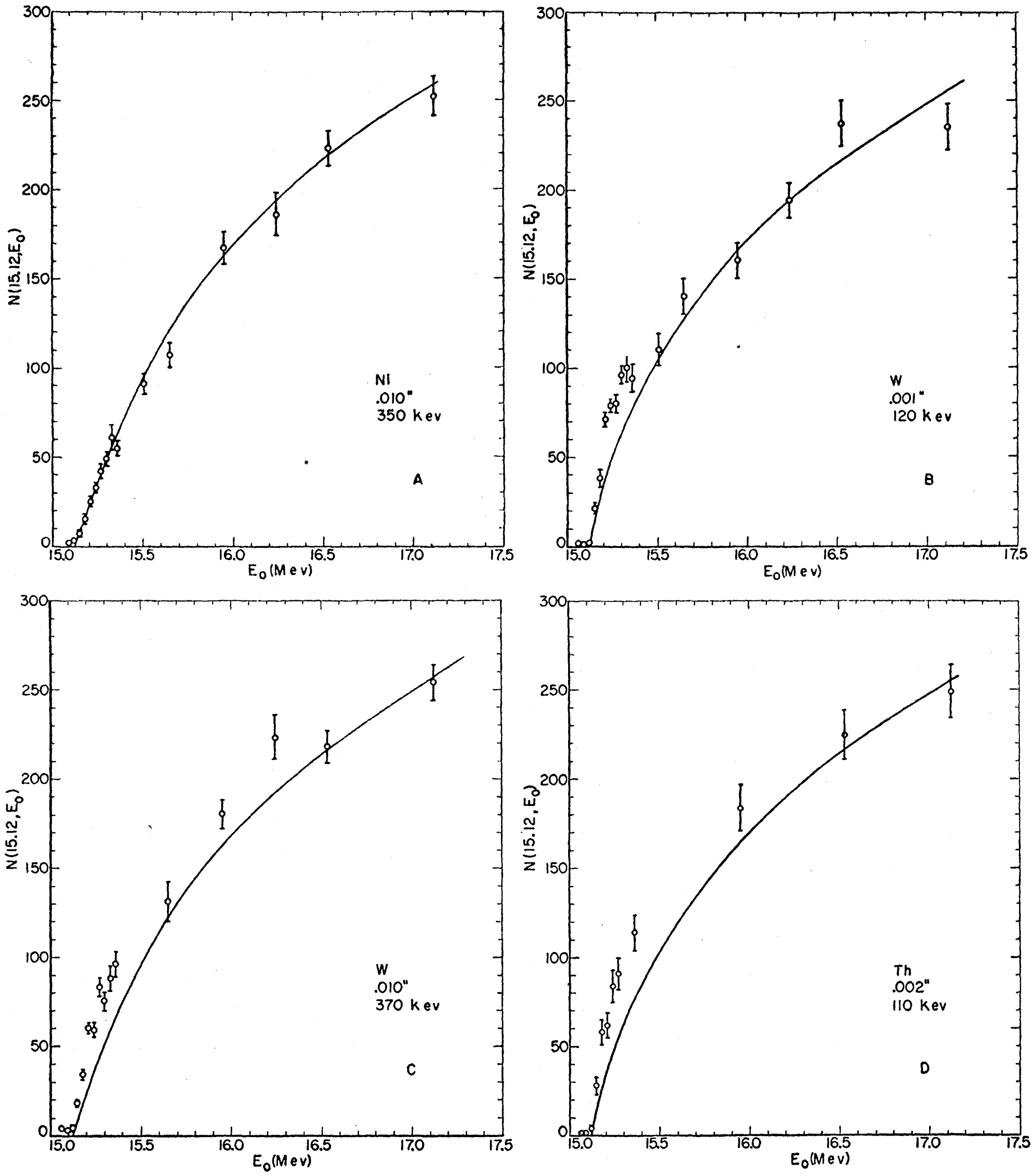

FIG. 4. The yield curves obtained when the electrons were rapidly expanded into targets of 0.010-inch nickel, 0.001-inch tungsten, 0.010-inch tungsten, and 0.002-inch thorium foils. The solid curves have been calculated from the Bethe-Heitler formula modified to take into account target thickness effects. Target thickness used was that determined from a measurement of the angular distribution of the bremsstrahlung from each target. This thickness is given in kev.

strahlung cross section integrated over the outgoing electron and photon angles. It should provide a more legitimate test of the validity of the Born approximation than would Schiff's calculation. The value of zero for the cross section at the threshold of the yield curve is a consequence of the Born approximation. It is evident that the experimental yield curves rise more rapidly from threshold for the heavier nuclei than do the theoretical curves.

The theoretical curves shown in Fig. 4 have all been modified to take into account the finite thickness of the various targets. The target thickness used for each target is given in kev on the graphs. The effective target thicknesses were determined from the angular distribution as described above. The distortions produced by multiple scattering and energy loss in the target may be taken into account by integrating the contributions to the bremsstrahlung generated in elements of target thickness, dx.[16] If $N(15.12, E_0)$ is the elementary yield

[16] Motz, Miller, and Wyckoff, Phys. Rev. **89**, 968 (1953).

curve, then the modified curve will be:

$$N'(15.12,E_0)=\int_0^T dxS(x)N\left(15.12,E_0+\frac{dE}{dx}dx\right),$$

where $S(x)$ is the probability that a photon generated in dx is detected. $S(x)$ therefore depends on the target properties and the acceptance angle of the detector. The functions, $S(x)$, have been calculated using a procedure outlined by Penfold[17] for the targets and the acceptance angle of 0.01 radian used in this experiment. In this approximation energy loss and scattering were treated as independent processes and radiation straggling was completely neglected. Penfold has shown that these simplifications do not alter the results significantly, but do make hand computation feasible.

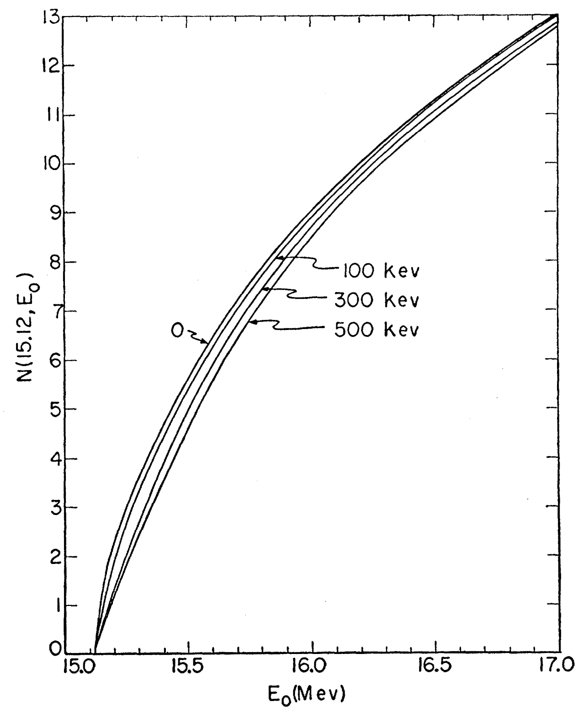

FIG. 5. A comparison of the calculated thin-target yield curve with those modified for tungsten targets 100, 300, and 500 kev thick.

The effects of the finite target thickness on the shape of a yield curve are shown in Fig. 5. The upper curve is a plot of the undistorted 15.12-Mev yield curve calculated from the Bethe-Heitler formula. The three lower curves resulted from folding the thick target weighting functions, $S(x)$, into the latter to take into account the effects of energy loss and multiple scattering in tungsten targets 100, 300, and 500 kev thick. The differences are not very great.

TABLE II. Ratio of observed counts/Mev to the modified Bethe-Heitler distribution.

Energy	Ni 0.010 in.	W 0.001 in.	W 0.010 in.	Th 0.002 in.
15.51	0.96±0.06	1.05±0.10		
15.36	0.87±0.06	1.22±0.10	1.48±0.11	1.48±0.13
15.33	1.09±0.13	1.43±0.11	1.50±0.12	
15.30	1.02±0.08	1.52±0.08	1.50±0.10	
15.27	1.05±0.10	1.45±0.10	1.98±0.12	1.63±0.16
15.24	1.03±0.10	1.68±0.08	1.74±0.12	1.78±0.19
15.21	1.0 ±0.12	1.92±0.11	2.4 ±0.12	1.68±0.17

A cursory inspection of Fig. 4 reveals that the yield curve measured using the nickel target agrees well with that given by the Bethe-Heitler formula corrected for target thickness. Within 300 kev of the tip of the spectrum the tungsten and thorium data lie above the calculated distributions. The ratio of the experimental points of Fig. 4 to the modified Bethe-Heitler distribution in the energy range 15.12 to 15.51 Mev are given in Table II. The ratios of the order of two near the tip for high atomic number targets are similar in magnitude to those found by Starfelt and Koch[1] for very thin gold targets and similar outgoing electron energies. These data indicate that for the heavy target nuclei there is an excess in the number of photons in the high-energy tip of the bremsstrahlung spectrum over that given by the Born-approximation calculation. The data may also indicate that there is a finite value for the end of the bremsstrahlung spectrum which depends somewhat upon the nuclear charge. To be consistent with the present data, this end point cannot vary faster than $Z^{\frac{1}{2}}$. An exact calculation is expected to result in a finite end point to the spectrum.[18] None of the various attempts to approximate a correction to the Bethe-Heitler cross section yields a result consistent with the data given in Table II.

ACKNOWLEDGMENT

The authors wish to thank Irwin Oppenheim for his interest and Ida Reingold for her help in the evaluations of the Bethe-Heitler cross section.

[17] A. S. Penfold (private communication).

[18] F. G. Nagasaka, Ph.D. thesis, University of Notre Dame, 1955 (unpublished); E. Guth, Phys. Rev. **59**, 325 (1941); G. Elwert, Ann. Physik **34**, 178 (1939).