Electron-Electron Interaction and Heat Conduction in Gaseous Plasmas*

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The phenomenon of heat conduction in gaseous plasmas is observed in order to explore the problem of the mutual electron interaction. In addition to the technique of interaction of pulsed microwaves in decaying plasmas, the phenomenon of "afterglow quenching" is exploited in the experiments. The experimental values of the thermal conductivity, in low-gas-pressure neon and xenon plasmas of adequately high charge density, are determined by two different methods. They have been found to be of the order of $10^{-6} - 10^{-5}$ (joules/cm sec degree) for the electron density range 10¹¹-10¹³ (cm⁻³) at room temperature (~300°K). The most significant result of these experiments is that the thermal conductivity in the plasmas described is chiefly determined by heat flow in the electron gas of the plasma. Thus the mutual interaction of the electrons plays a predominant role in the phenomenon of heat conduction. The experimentally obtained values of the thermal conductivity are in agreement within less than one order of magnitude with those given by the theory of Spitzer and Härm.

1.

FOR the complete description of gaseous plasmas, the knowledge of the interaction processes that occur among all plasma constituents is necessary. In recent years, efforts have been directed toward the detailed studies of^{1-5} the interactions between the electrons and neutral molecules (excited and nonexcited) and electrons and ions in gaseous plasmas. This was made possible by the advent of new and improved experimental techniques. However, in view of the methods used in those investigations where the electrical conductivities were studied, the problem of the mutual interaction of the electrons in plasmas was not accessible to experimental studies. Indeed the mutual interaction of electrons on the electrical conductivity of a plasma is a second-order effect in the slightly ionized gases investigated. The purpose of this work is to remedy this situation. The paper reports the first results of an experimental study of thermal conductivity in gaseous plasmas where the mutual interaction of the electrons plays a predominant role.

The technique of interaction of pulsed microwaves in decaying plasmas previously described^{1,2,5,6} is employed for two purposes: (1) to heat selectively the electron gas of the plasma to increments above the temperature of the ions and molecules, initially at 300°K, and (2) to detect the change in electron temperature in the heated volume as well as at various known distant points along the plasma. The selective heating of the electron gas takes place in a very limited volume of the plasma, of

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small cross section but elongated. The rise in temperature away from the heated portion is studied in an equally small volume of plasma. In addition to the microwave technique, the phenomenon of "afterglow quenching"' due to electron temperature increase is also exploited to detect the change in the electron temperature both in the heated volume and at all points along the plasma.

The plasma is contained in a 5-mm diameter thinwall cylindrical glass tube which traverses, perpendicularly in the center, the broad walls of two identical rectangular wave guides as shown in Fig. 1. These wave guides are specially designed, both in height and width $(4 \times 33 \text{ mm})$, in order (1) to enhance heating of the plasma electrons for any given radio-frequency input



FIG. 1. Main part of experimental setup-glass tube and two special waveguides.

⁷ Goldstein, Anderson, and Clark, Phys. Rev. 90, 486 (1953).

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⁶ A. A. Dougal and L. Goldstein (to be published).
⁶ L. Goldstein, in Advances in Electronics and Electron Physics,

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power, (2) to permit relatively high resolution in position, and (3) to accommodate the gaseous discharge tube without undue disturbance of wave propagation, thus permitting a more precise determination of the appropriate plasma parameters which are to be measured (electron number density n_e and collision frequency ν). The heating of the electrons in a rather limited volume of the plasma-contained in the guide (0.075 cm³)-takes place in wave guide 1. The maximum power level available for this purpose is $\sim 5/10$ watt. Guide 2 is used for probing the electron temperature deviation at any distant point from the heated volume of plasma. The distance (d) between the two wave guides can be varied and fixed at any desired value by a screw mechanism. Both guides are connected to standard X-band wave-guide systems (independent or not) through tapered sections. A small aperture (diam < 1 mm) is provided in the narrow walls of each wave guide, enabling photomultiplier tubes to detect the visible light radiated from the particular limited volume of the decaying plasma.

The plasma is produced in low-pressure noble gases (Xe, to 5 mm Hg; Ne, to 12 mm Hg) by short-duration $(2-\mu sec)$ high-voltage dc pulses at a repetition frequency of 400 per second. At appropriate times in the afterglow, delayed with respect to the discharge pulse, the microwave heating signal (~8600 Mc/sec) is propagated in the lower guide (No. 1) in pulses of 2- to 30- μ sec duration. The probing 9000-Mc/sec frequency wave, whose amplitude is insufficient to effect any detectable heating in the plasma, is continuous.

3.

As mentioned above, we are concerned with decaying plasmas in low-pressure Xe and Ne gases. The measurements of thermal conductivity K in such plasmas are performed after the electrons have attained thermal equilibrium with the heavier plasma constitutents which are isothermal at 300°K. It has been previously shown^{1,5} that, in view of the long range of the Coulomb forces, the interaction of the electrons with ions is more important than with the neutral plasma constituents in plasmas with a degree of ionization of $\geq 10^{-5}$. The properties of the decaying plasma vary as a function of time since the degree of ionization decreases in time. However, such a plasma is suitable for the study of thermal properties as a function of electron density. The thermal properties of such a plasma are expected to be a function of the temperature of the electron gas, its temperature gradient, and its geometrical configuration, as well as of the degree of ionization.

The determination of the thermal conductivity in the plasmas as defined above, in which the electron gas is locally heated above the initial plasma temperature T_0 , involves the measurements—at appropriate times in the decay—of the electron density (n_o) , electron temperature (T_e) , and its gradient (∇T_e) . In the determination of the thermal conductivity (K) in such plasmas, it is necessary to know the "relaxation time"⁸ (τ) or the corresponding electron collision frequency for momentum transfer (ν) . The quantities $n_e(t)$ and $\nu(t)$ are determined, for the geometry of Fig. 1, from the measurements of the complex reflection coefficient of the heating microwave signal. It is also possible to measure ν or τ by two different and independent methods, described elsewhere.^{1,2,5} The knowledge of ν or τ enables us to estimate the electron temperature (T_e) .

The experimental value of the thermal conductivity K has been determined by two different methods. One is based upon the measurements of the "time element" involved in heat propagation, and the other is based upon the analysis of the "quasi"-steady-state temperature distribution along the plasma. These are referred to hereafter as the "transient method" and the "steady-state method," respectively.

The energy balance equation for the electron gas in a plasma, written on a per electron basis, is

$$\frac{\partial U_{e}}{\partial t} = \left[\frac{dU_{E}}{dt}\right]_{\text{source}} - \frac{1}{\tau}(U_{e} - U_{0}) - \left[-\frac{1}{n_{e}}\nabla \cdot (K\nabla T_{e})\right], \quad (1)$$

where $U_e = \frac{3}{2}kT_e$ (k being Boltzmann's constant) is the mean thermal energy of an electron and $U_0 = \frac{3}{2}kT_0$ is the mean thermal energy of a neutral molecule and/or an ion. In view of the low degree of ionization, even a high transient electron temperature rise cannot produce a noticeable temperature increase of the neutral and ionic plasma constituents. The mean thermal energy of these heavy plasma constituents can be considered as always equal to the kinetic energy corresponding to the initial temperature T_0 . The left-hand side of Eq. (1) represents the net time rate of energy increase of an electron. The first term of the right-hand side is the time rate of energy absorbed by an electron from the heating microwave, and is given by

$$\left[\frac{dU_{E}}{dt}\right]_{\text{source}} = \frac{\sigma E_{h}^{2}}{n_{e}} = \frac{e^{2}E_{h}^{2}}{m\omega^{2}}\nu \qquad (\text{if } \omega^{2} \gg \nu^{2})$$
$$= \frac{e^{2}E_{h}^{2}}{m\omega^{2}G} \cdot \frac{1}{\tau}, \qquad (2)$$

where $\omega = 2\pi f$ and E_h are the angular frequency and root-mean-square value of electric field of the heating microwave signal in the plasma, respectively; σ is the electrical conductivity of the plasma; e and m are the charge and mass of an electron, respectively; and τ is the "relaxation time" referred to above, which is related to the effective collision frequency (ν) through the expression $\nu = 1/G\tau$, where G = 2m/M (M being the

⁸ The relaxation time (τ) represents the characteristic time for equipartition of energy through electron collisions with molecules and ions.

mass of a molecule or an ion). The second term, $-(1/\tau)(U_e-U_0)$, is the time rate at which the excess mean electron energy is transferred to the neutral molecules and ions (through elastic collisions). In Eq. (1), $-\nabla \cdot (K\nabla T_e)$ represents the time rate at which the thermal energy flows outward because of heat conduction from the unit volume of plasma. The division of this quantity by the number density of the electrons (n_e) gives the time rate at which a single electron loses its thermal energy by heat conduction.

Strictly speaking, the thermal conductivity K, being a function of n_e , T_e , and ∇T_e , will vary along the plasma. However, for a small temperature deviation, for a slight temperature gradient and constant n_e , the quantity K can be regarded as a constant. Hence $\nabla \cdot (K\nabla T_e) = K\nabla^2 T_e$.

Equation (1) can be written in terms of T_e as

$$\frac{\partial T_{e}}{\partial t} = \left(\frac{\Delta T_{e0}}{\tau}\right) - \frac{1}{\tau} (T_{e} - T_{0}) + D\nabla^{2} T_{e}, \qquad (3)$$

where

$$D \equiv 2K/3kn_e, \tag{4}$$

$$\Delta T_{e0} \equiv \frac{2e^2 E_h^2}{3km\omega^2 G}.$$
(5)

D can be regarded as the "diffusivity" of the plasma. For $\partial T_e/\partial t=0$ and D=0, $T_e=T_0+\Delta T_{e0}$; hence ΔT_{e0} would be the electron temperature rise in the heated plasma volume under steady-state conditions in the absence of outward heat flow due to heat conduction.

Equation (3) includes neither the possibility of heat transfer by mass diffusion, which is proportional to $(\nabla n_e \cdot \nabla T_e)$, nor heat losses by radiation and absorption of quanta. As will be apparent later, both of these effects may be considered negligible under the conditions of our experiments.

The "transient" method is based upon the time element that is involved in heat propagation which takes place in the nonheated region of the plasma (x>0) at time t > 0, when the electron temperature at t = 0 is suddenly raised by ΔT_0 in the plane x=0. The smaller of the time constants involved in the energy transfer from the warm electrons to the heavy plasma constituents within the heated volume is about one order of magnitude larger than the measured time length of the electron temperature increase detected at distances of one centimeter or more away from the heated volume. For this reason it seems justifiable to neglect, in a first approximation, the term $-(1/\tau)(T_e-T_0)$. Heat loss to the wall of the tube, where the electron density is considerably reduced, occurs through direct electron transport by diffusion. However, on grounds of experimental observations, the time element involved in this process is also considerably longer than the time mentioned above. Consequently, such a term can also be neglected within this approximation. On the basis of these considerations the problem can be treated as a one-dimensional case. Thus Eq. (3) reduces to $\partial T_e/\partial t = D(\partial^2 T_e/\partial x^2)$.

The solution of this equation is given by

$$\Delta T_{e}(x,t) = (\Delta T_{0}) \left[1 - \Phi \left(\frac{x}{2(Dt)^{\frac{1}{2}}} \right) \right], \qquad (6)$$

where $\Delta T_e(x,t)$ is the electron temperature rise at the plane x at the time t, and Φ is the error function,

$$\Phi(z) = \frac{2}{\sqrt{\pi}} \int_0^z \exp(-\lambda^2) d\lambda.$$

Equation (6) gives the curve of ΔT_e versus time t for fixed value of x. The point of maximum slope ("inflection point") on the time axis, t_0 , is

$$t_0 = x^2/6D.$$
 (7)

It can be readily seen from Eq. (6) that the value of ΔT_e at $t=t_0$ is only about 8% of the final temperature deviation (ΔT_0). This particular value of the time $(t=t_0)$ will be referred to hereafter as the "delay time." Equation (7) provides a means to determine the experimental value of the thermal conductivity K for known values of n_e , through the diffusivity D, by direct measurement of the delay time (t_0) .

On the other hand, the "steady-state" method is based upon the temperature distribution along the plasma in the steady state of heating while only a limited volume of it is heated continuously. In our experiments, pulsed microwave heating occurs. The duration of the heating pulse is such that steady-state conditions are reached within the pulse duration. Therefore we observe a "quasi"-steady-state temperature distribution along the plasma. Upon again assuming one-dimensional heat flow in the elongated plasma, Eq. (3) for the nonheated region reduces to

$$D\frac{d^{2}T_{e}}{dx^{2}} - \frac{1}{\tau}(T_{e} - T_{0}) = 0, \qquad (8)$$

since $\partial T_e/\partial t \equiv 0$.

If it is assumed that the range of variation of electron temperature is rather limited so that τ as well as K can be regarded as constants, the solution⁹ of Eq. (8) has the functional form of $e^{-\alpha x}$; α is the electron temperature "attenuation" constant of the plasma and is given by $\alpha = 1/(D\tau)^{\frac{1}{2}}$. A "relaxation distance" ρ_0 or characteristic length can also be defined as

$$\rho_0 = 1/\alpha = (D\tau)^{\frac{1}{2}}.$$
 (9)

A direct measurement of the relaxation distance ρ_0 allows an independent determination of the thermal

⁹ The distribution of steady-state temperature in both the heated and nonheated regions of plasma will be presented in detail in a later paper.



FIG. 2. Heat propagation phenomenon: for xenon plasma at 2 mm Hg pressure. Heating microwave pulse (trace a, 20 µsec duration, applied 400 μ sec after the dc high-voltage discharge pulse) and the consequent "delayed" light "quenching" pulse (trace c) observed at various distances (d) from the heated volume of the plasma. Incident rf power level of the heating wave, $P_{in} = 215$ milliwatts (fixed). Electron number density $n_e = 9.9 \times 10^{11}$ (cm⁻³); electron temperature $T_e \sim 300^{\circ}$ K. Time scale = 2 μ sc per division. The base line of the light output traces coincides with the highest horizontal line of the scale.

conductivity K, provided that the electron density (n_e) and the effective relaxation time (τ) are known.

4.

It has been ascertained previously from the observation of the light "quenching" pulse in the heated volume (guide 1) that the increase of the electron temperature in the heated volume starts with no measurable delay with respect to the leading edge of the heating wave pulse. The phenomenon of heat propagation in the plasma is demonstrated by a typical series of photographs of the oscilloscope traces in Fig. 2. Each photograph in this figure shows the detected envelope of the heating wave pulse in waveguide 1 (trace a) and the corresponding "delayed" light "quenching" pulse (trace c) observed at various distances (d) from the heated volume of the plasma for a fixed incident power level (P_{in}) of the heating wave. Trace b is the base line of the detected microwave signal and trace d is the photocurrent output that is produced by the afterglow in the absence of any heating microwave 400 μ sec after the gas discharge pulse. Both the detected amplitude of the heating microwave and the magnitude of the light output are increasing with downward deflection of the oscilloscope traces. The experimental conditions are those indicated in the legend of Fig. 2. It is seen that the "delay time," corresponding to t_0 as defined by Eq. (7), varies with the distance (d).

A sequence of photographs, such as shown in Fig. 2, has been taken at various power levels of the heating wave as well as at various values of d, under different

experimental conditions both in Xe and in Ne. These photographs reveal that the "delay time" (t_0) is proportional to the square of the distance (d) for fixed values of P_{in} . This result is an experimental justification of the validity of Eq. (7). The slope of the curves of t_0 versus d^2 (straight lines) determines the value of the diffusivity D [Eq. (7)]. By knowing D we may find the experimental value of the thermal conductivity K, through Eq. (4), the electron number density n_e having being determined independently. The values of the thermal conductivity K obtained by the "transient" method are shown in Fig. 3 as a function of the power level of the heating wave (P_{in}) for both xenon and neon under the experimental conditions indicated in Table I. We shall now describe the results for K obtained by the "quasi"-steady-state analysis. The luminous intensity, I, of the afterglow, which results from recombination of electrons with positive ions within the decaying plasma, is proportional to the electron-ion recombination coefficient, the electron density (n_e) , and the ion density (N_i) . The recombination coefficient is dependent upon the electron temperature. If one assumes a temperature dependence of $T_{e^{-\frac{3}{2}}}$, as suggested by Bates,11 the luminous intensity becomes (for

¹⁰ Since the inflection point on the light-quenching pulse was not always clear, the time point at which the light-output trace begins to diverge from that in the absence of heating pulse has been taken here to determine the value of t_0 . This seems to be justified to some extent, since, as mentioned before, the electron temperature deviation at the inflection point is theoretically only 8% of the final (quasi-)steady-state temperature deviation. ¹¹ D. R. Bates, Phys. Rev. 78, 492 (1950).

$$n_e = N_i$$

$$I \propto n_e^2 T_e^{-\frac{1}{2}}.$$
 (10)

The analysis of the light output from the afterglow showed that the electron-ion recombination process is the predominant disintegrating mechanism of the plasma in the time range in the afterglow where our experiments have been carried out. Except for a very high power level ($P_{\rm in}$) of the heating wave, no appreciable variation of n_e has been observed either during the application of the heating or in its wake. Therefore, the decrease in light output at various points along the plasma in the "quasi"-steady state (from I_0 to I_1 in Fig. 2) may be attributed to the increase of the electron temperature due to heat conduction. Consequently Eq. (10) yields an estimated electron temperature deviation ΔT_e

$$\Delta T_{e} = T_{e} [(I_{0}/I_{1})^{\frac{3}{2}} - 1], \qquad (11)$$

where T_e is the initial electron temperature.

If the temperature distribution along the plasma can be expressed by a single exponential $e^{-\alpha x}$ as described before, the curve of $\ln(\Delta T_e)$ versus the distance (d) for fixed values or P_{in} must be a straight line. The slope of this curve determines α or the relaxation distance ρ_0 . Examination of the photographically recorded data, obtained under varied experimental conditions, shows that the above statement is verified by the experiments for larger values of d and lower levels of heating power (P_{in}) . This is no longer the case for smaller values of d and higher levels of \bar{P}_{in} . This is understandable because in the latter cases the temperature gradient may be so large that the temperature decay along the plasma cannot be expressed by a single exponential. Other reasons are that due to drastic electron heating (up to inelastic collision levels) "excitation" of the heated plasma and enhanced diffusion of the electrons occur. The effect of "excitation" has been observed on the light output for the higher range of P_{in} . Therefore, the determination of α (i.e., ρ_0) and consequently K from the slope of such straight lines is applicable only for the case of relatively small temperature gradients, which corresponds in these experiments to the lower values of P_{in} (Fig. 3).

The values of the thermal conductivity K, deter-

TABLE I. Experimental conditions of Fig. 3.

No. of curve	Gas	Pressure (mm Hg)	Time position in afterglow (µsec)	Measured ^{<i>n_s</i>} (10 ¹² cm ⁻³)	Measured (10 ¹⁰ sec ⁻¹)
1	Xe	2.0	200	2.8	3.4
2	Xe	2.0	300	1.7	2.0
3	Xe	2.0	400	0.99	1.8
4	Xe	1.1	300	0.96	2.9
5	Xe	2.0	500	0.59	1.5
6	Ne	11.2	300	0.50	0.39
7	Xe	3.2	300	0.39	2.0
8	Xe	2.0	600	0.37	1.8
9	Xe	4.0	300	0.32	2.2

EXPERIMENTAL RESULTS EXPERIMENTAL BY "TRANSIENT" METHOD RESULTS BY "STEADY STATE" METHOD (1) THERMAL CONDUCTIVITY, K (JUULE 0 0 (2) (1) (3)14 (2) (5) (6) (8) (3) £(9) (6) 10-6 (ē) N THEORY OF SPITZER AND HARM 10 20 40 70 100 200 400 600 10 HEATING MICROWAVE, Pin (MW)

FIG. 3. Experimental results of thermal conductivity (K) (by "steady-state method" and "transient method").

mined from the analysis of "quasi"-steady-state data, are also shown in Fig. 3 in horizontal, short, solid lines. Each of these lines corresponds to the curve of K, with the same number, that has been obtained by the "transient" method.

Inspection of Fig. 3 shows that the experimental values of the thermal conductivity K in the plasma described, which are determined by the two different methods, are in fair agreement. This agreement is closest for the lowest heating power level $(P_{\rm in})$, the only case where the "quasi"-steady-state treatment is applicable.

5.

The most significant result of these experiments is that the thermal conductivity in plasma of low gas pressure but adequately high charge density (degree of ionization $10^{-6}-10^{-5}$) is determined by the heat flow chiefly in the electron gas of the plasma. Indeed, the rate at which thermal energy is transferred from a small volume of warm electron gas to equal volumes of cool electrons within the same plasma is considerably faster (at least one order of magnitude faster) than the rate of heat transfer to any of the two other heavy plasma constituents (ions and neutral atoms).

This conclusion is also borne out by the experimental fact that at the same electron density n_e in one gas (here Xe) no gas pressure dependence on the thermal conductivity is observed (compare curve No. 3 with

No. 4, or No. 8 with No. 9 in Fig. 3 and Table I). A further support for the above conclusion is provided by the fact that, again at the same charge density, the mass of the gas (here xenon and neon) has little or no influence on the thermal conductivity in these plasmas (compare the curves No. 5, 6, and 7). A rather strong dependence of this thermal conductivity on the electron density, however, is apparent.

The most appropriate comparison of the experimentally obtained coefficient of thermal conduction in the above-mentioned plasmas can be made with the recent detailed calculations of Spitzer and Härm.12 These calculations are relative to plasmas in fully ionized gases. The comparison of our experimental results with this theory is appropriate because in the plasmas described in our work the interaction of electrons with the charged constituents (electrons and ions) predominates over that with the neutral gas constituents of the plasma owing to the long range of the Coulomb force. This comparison shows that the experimentally obtained values of thermal conductivity are in agreement, within less than one order of magnitude, with those given by the theory of Spitzer and Härm. A closer comparison with theory at this stage of the experiments does not appear to be justified in view of the present uncertainty of the initial values of the electron temperature (T_e) in the experiments performed with xenon. The initial values of the electron temperatures in the decaying neon plasmas investigated are known, however, with more certainty. In this case the experimental and theoretical values of the thermal conductivity of the plasma are in rather excellent agreement. (See Fig. 3.)

These experiments are being extended to other gases and it is expected that their continuation will yield more precise measurements of the plasma parameters. A subsequent paper will contain a more detailed description of the experiments and their results as well as a more complete comparison with theory.

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Shape of the High-Energy End of the Electron-Bremsstrahlung Spectrum*

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The elastic scattering of photons by the 15.11-Mev level in C¹² has been used to study with good energy resolution the number of photons at the high-energy end of a bremsstrahlung spectrum. The bremsstrahlung was produced by electrons accelerated in a betatron, the energy of which was varied in 35-kev increments. Targets were: a 0.025-inch-diameter tungsten wire and the following foils, 0.001- and 0.010-inch tungsten, 0.002-inch thorium and 0.010-inch nickel. The foils were used to study the dependence of the spectrum shape upon target thickness and atomic number. When compared with Bethe-Heitler spectra corrected for target thicknesses, the data indicate an excess number of photons in the tip of the spectrum. The experimental number depends on the atomic number of the target and cannot vary more rapidly than Z¹.

I. INTRODUCTION

HERE have been a number of experimental measurements of the bremsstrahlung spectra generated by electrons having energies in the range from 1 to 20 Mev. A review of these experiments and the comparison of the experiments with the available theories is given by Starfelt and Koch.¹ The data indicate that for high energies the general shape of the bremsstrahlung spectrum is well described by the results obtained from a calculation made in Born approximation when the effect of screening by the atomic electrons is taken into account.2 The absolute magnitudes of the bremsstrahlung cross section for electrons with energies in the 10- to 20-Mev range have been found to agree with the Born approximation result to within about 10%.^{1,3}

Up to the present time there have been no detailed measurements or satisfactory theoretical calculations of the shape of the high-energy tip of the bremsstrahlung spectrum (energies within mc^2 of the incident electron kinetic energy). The shape of the spectrum in this energy range is of utmost importance in interpreting the breaks that have been found in activation curves.⁴ Poor instrumental resolution has been the principal reason for the lack of experimental information about this portion of the bremsstrahlung spectrum. The failure to obtain a satisfactory theoretical prediction for the shape of the tip of the spectrum can be laid to

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FIG. 2. Heat propagation phenomenon: for xenon plasma at 2 mm Hg pressure. Heating microwave pulse (trace a, 20 μ sec duration, applied 400 μ sec after the dc high-voltage discharge pulse) and the consequent "delayed" light "quenching" pulse (trace c) observed at various distances (d) from the heated volume of the plasma. Incident rf power level of the heating wave, $P_{in}=215$ milliwatts (fixed). Electron number density $n_e=9.9\times10^{11}$ (cm⁻³); electron temperature $T^1_e\sim300^{\circ}$ K. Time scale = 2 μ sec per division. The base line of the light output traces coincides with the highest horizontal line of the scale.