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Energy Exchange between Electron and Ion Gases through Coulomb Collisions in Plasmas*

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The general aspects of energy exchange among the electron, ion, and molecule constituent gases, and their energy transfer to the boundary, are examined for a partially ionized gas. Coulomb collisions between electrons and ions are identified to contribute significantly to thermal energy transfer from the electron gas, even in very weakly ionized gases.

Experiments were performed to determine the time required for electron temperature deviations from equilibrium to return to equilibrium with the ion and molecule temperature. The method of guided microwave interaction within the plasma was employed for selectively heating the electron gas to increments above that of the ion and gas temperatures, and for sensing the subsequent electron temperature deviations through the electrical conductivity of the plasma. In addition, phototube detection of the plasma luminous intensity is employed to detect electron temperature deviations.

The experimental results show that energy exchange between electron and ion gases is predominant in the experimental plasmas, under appropriate conditions, where the ionization is less than 0.001%. In a plasma produced in neon gas at 2.25 mm Hg, 300°K, the characteristic time for equipartition of excess mean electron energy with the ion gas varies from 11 to 5 μsec as the ion concentration increases from 1.6 to 5.8×10^{11} per cm^3. These times are much shorter than the 160 μsec which would be required for the electron temperature to return to equilibrium in the absence of Coulomb collisions. The expected behavior with respect to ion concentration and ion mass is confirmed, but an anomaly with respect to electron temperature is found.

INTRODUCTION

WHEN the electron and ion gases in a gaseous plasma are not in thermal equilibrium, there is a net exchange of thermal energy through Coulomb collisions between electrons and ions. The time rate at which this energy is exchanged is of fundamental interest in the physical properties and behavior of plasmas. This paper reports an experimental study of that energy exchange.

We restrict our attention here to energy exchanged solely through *elastic* Coulomb collisions of electrons with ions. Energy exchange processes identified with the stopping power of the plasma to an incident charged particle, and with departures of the electron or ion velocity distributions from Maxwellian distributions are not considered. Rather, we consider the details whereby an electron gas, which has a Maxwellian velocity distribution with a mean energy different from that of the ion gas, exchanges excess mean energy with the ion gas. The time required for equipartition of energy, which establishes the net time rate of energy exchange between the electron and ion gases, is the principal physical quantity of interest.

Several theoretical studies of energy exchange between electron and ion gases have been made which pertain to this work. Landau[1] developed a kinetic gas theory for the case of Coulomb collisions between gases of electrically charged particles. Landau calculated the time rate at which electrons exchange energy in elastic collisions with ions where each gas has a Maxwellian velocity distribution corresponding to its respective temperature. In their review article on the plasma state of gases, Rompe and Steenbeck[2] discuss the subject of energy exchange between the constituents of the plasma through Coulomb collisions. They indicate that Steen-

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[1] L. Landau, Physik. Z. Sowjetunion **10**, 154 (1936).

[2] R. Rompe and M. Steenbeck, Ergeb. exak. Naturw. **18**, 257 (1939).

beck[3] has speculated that an appreciable fraction of the electron energy in a plasma, perhaps due to the far-reaching Coulomb forces, first is transmitted to the ions as kinetic energy and subsequently exchanged to the neutral gas through collisions of ions with neutral molecules.

An analogous problem in the field of stellar dynamics was studied independently by Spitzer[4] and Chandrasekhar.[5] The gravitational forces between stellar masses have a similar inverse square (of separation distance) law as do the Coulomb forces between electrical charges. Spitzer and Chandrasekhar calculated the time rate of approach of two star systems with Maxwellian velocity distributions toward an energy equilibrium state. Through changes of variables from gravitational force to Coulomb force, and by employing the Debye-Hückel[6] shielding radius in place of the mean distance between stars as the maximum collision impact parameter which is used to evaluate an otherwise divergent integral, their results pertain to Coulomb collisions between gases of electrons and ions. Spitzer[7] has employed the appropriate changes of variables and determined from his earlier result for stellar collisions the time rate at which energy is exchanged between two gases of electrically charged particles.

Allis[8] has discussed Coulomb collisions of electrons with ions and has derived the transport coefficients through use of the Fokker-Planck equation.

In an experimental study of ionization through strong shock waves in gases, Petschek and Byron[9] interpret energy transfer from ions to electrons as a predominant energy source for ionization. However, experimental substantiation of the theoretical predictions apparently has not appeared in the literature. This fact, combined with the advent of microwave experimental techniques for plasma research and the recognition that Coulomb collisions may play a dominant role even in very feebly ionized gases, encouraged the experimental research reported here. Ginsburg[10] calculated the high-frequency electrical conductivity of the plasma, showing that even for very weakly ionized gases the Coulomb collisions of electrons with ions may predominantly determine the conductivity. Anderson and Goldstein[11] subsequently measured the electron-ion collision contribution to the conductivity. Their experimental results agreed closely with the theory of Ginsburg and verified that Coulomb collisions may play a dominant role in plasma behavior, even in very feebly ionized gases of less than 0.001% ionization.

A suitable experimental method for investigating the energy exchange between electron and ion gases is that of guided microwave interaction of two electromagnetic waves propagated through the plasma, a method which was described by Anderson and Goldstein[11] and which was employed in this study. One microwave source served to selectively heat the electron gas so that an initial nonequilibrium thermal state with respect to the ion gas is established. A second microwave source provided electron temperature sensing through the high-frequency conductivity which is electron-temperature dependent. Also, photoelectric detection of the plasma luminous intensity afforded additional observation of electron temperature deviations. This latter technique is based on the observations of Kenty[12] and of Goldstein, Anderson, and Clark[13] who have identified the luminous intensity from electron-ion recombination within plasmas as being highly sensitive to electron temperature deviations.

The subsequent text reports our additional theoretical considerations, the theory and details of the experiment, and the experimental results.

ENERGY FLOW FOR A BOUNDED PLASMA

Plasmas produced in the laboratory generally occur in partially ionized gases. They may be nonuniform, and are subject to boundary effects due to their finite dimensions. Therefore, it is necessary to examine the general aspects of energy exchange among the electron, ion, and molecule gases which constitute the plasma, and their energy transfer to the plasma boundary. Energy exchange between the electron and ion gases is but one process within a complex system, rather than a unique process in an ideal system, and it must be interpreted within its relation to the other processes.

Let the electron gas be subjected to a selective energy source, such as a microwave electric field, which supplies thermal energy solely to the electron gas constituent of the plasma. The resultant energy flow from the electron gas to the ion and molecule gases, and to the boundary, is shown in Fig. 1. U_e, U_i, U_m, and U_r represent, respectively, the mean thermal energy per particle of the electrons, ions, molecules, and the reservoir (boundary). The net time rate at which electrons exchange mean thermal energy with ions is denoted by dU_{ei}/dt, with a similar notation for energy exchange between the other plasma constituents.

Based on the principles of conservation and equipartition of thermal energy, the following system of three simultaneous equations[14] describes analytically the time rates of energy change of the plasma

[3] M. Steenbeck, Physik. Z. 33, 809 (1932).
[4] L. Spitzer, Jr., Monthly Notices Roy. Astron. Soc. **100**, 396 (1940).
[5] S. Chandrasekhar, *Principles of Stellar Dynamics* (University of Chicago Press, Chicago, 1942), p. 48.
[6] P. Debye and E. Hückel, Physik. Z. 24, 185 (1923).
[7] L. Spitzer, Jr., *Physics of Fully Ionized Gases* (Interscience Publishers, Inc., New York, 1956), Chap. 5.
[8] W. P. Allis, in *Handbuch der Physik* (Springer-Verlag, Berlin, 1956), Vol. 21, p. 383.
[9] H. Petschek and S. Byron, Ann. Phys. **1**, 270 (1957).
[10] V. L. Ginsburg, J. Phys. (U.S.S.R.) 8, 253 (1944).
[11] J. M. Anderson and L. Goldstein, Phys. Rev. **100**, 1037 (1955).
[12] C. Kenty, Phys. Rev. **32**, 624 (1928).
[13] Goldstein, Anderson, and Clark, Phys. Rev. **90**, 486 (1953).
[14] Rationalized mks units are used in all equations. Quantities may be given in other units which are expressly stated.

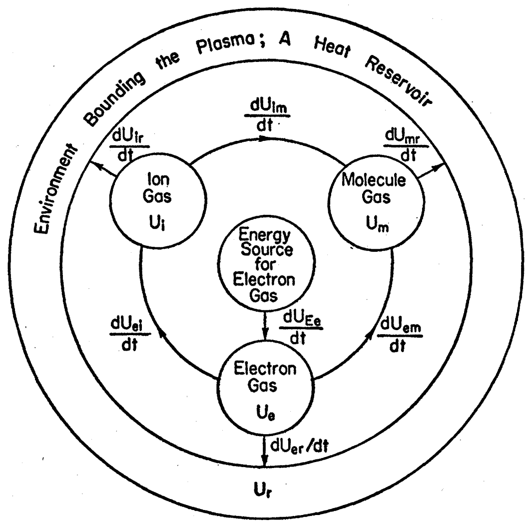

FIG. 1. Energy flow diagram for a bounded plasma in the presence of a selective energy source.

constituents:

$$\frac{dU_e}{dt}=\frac{dU_{Ee}}{dt}-\frac{(U_e-U_m)}{\tau_{em}}-\frac{(U_e-U_i)}{\tau_{ei}}-\frac{(U_e-U_r)}{\tau_{er}}, \quad (1a)$$

$$\frac{dU_i}{dt}=\frac{(U_e-U_i)}{\tau_{ei}}-\frac{(U_i-U_m)}{\tau_{im}}-\frac{(U_i-U_r)}{\tau_{ir}}, \quad (1b)$$

$$\frac{dU_m}{dt}=\frac{(U_e-U_m)}{\tau_{em}}+\frac{(U_i-U_m)}{\tau_{im}}-\frac{(U_m-U_r)}{\tau_{mr}}. \quad (1c)$$

In Eq. (1a), dU_{Ee}/dt represents the time rate at which thermal energy is supplied to an electron by the selective energy source, a microwave electric field in our experiment. The quantities denoted by τ, with appropriate subscripts, are defined as the characteristic times for equipartition of energy and have the dimensions of time. The time rate at which excess mean electron energy is transferred to the ion gas is given by the term $(U_e-U_i)/\tau_{ei}$ in Eq. (1a). τ_{ei}, the principal subject of this study, is the characteristic time for equipartition of energy through electron collisions with ions. When dU_{Ee}/dt is zero, the thermal equilibrium state, i.e., $U_e=U_i=U_m=U_r$, is clearly a solution to the set of equations.

Equations (1a, b, c) may be simplified for plasmas in weakly ionized gases where the mean electron energy is not deviated appreciably from thermal equilibrium. In our experimental plasmas the ratio of ion concentration (and electron concentration) to the neutral molecule concentration is less than 10^{-5}.

The thermal energy per unit volume in the electron gas is $3knT_e/2$, and in the molecule gas is $3kN_mT_m/2$, where k is Boltzmann's constant, n and N_m are the electron and molecule concentrations, and T_e and T_m are the electron and molecule temperatures, respectively. With $(n/N_m)\leq 10^{-5}$, electron energy transferred to the molecules, either directly or through the intermediary of the ions, cannot increase significantly the molecule temperature for only small electron energy deviations from thermal equilibrium. In addition, there exists strong thermal contact between the ion and molecule gases due to the comparable masses through which the average fraction of excess energy exchanged in a single encounter is the order of unity. This results in τ_{im} being much smaller than τ_{ei} in Eq. (1b), which precludes the ion temperature from increasing significantly above the molecule temperature. Therefore, throughout small deviations of the electron energy from thermal equilibrium, the mean thermal energies of the ions, molecules, and obviously that of the reservoir remain very nearly equal to the equilibrium value $U_i=U_m=U_r$. With this simplification for plasmas in gases of weak ionization, we need to consider only Eq. (1a) which reduces to

$$\frac{dU_e}{dt}=\frac{dU_{Ee}}{dt}-\left(\frac{1}{\tau_{em}}+\frac{1}{\tau_{ei}}+\frac{1}{\tau_{er}}\right)(U_e-U_{i,m,r}). \quad (2)$$

The characteristic time for equipartition of energy through elastic electron collisions with molecules, τ_{em} in Eq. (2), was calculated by Cravath[15] and, for comparable electron and molecule temperatures, is

$$\tau_{em}=\left(\frac{M}{2m}\right)\frac{3}{4qN_m}\left(\frac{\pi m}{8kT_e}\right)^{\frac{1}{2}}, \quad (3)$$

where m and M are the electron and molecule masses, respectively, N_m is the concentration of molecules per unit volume, and q is the effective cross section per molecule for electron collisions. Cravath's result can be expected to be accurate for electron collisions with monatomic gases in electron energy ranges where q is relatively constant.

The characteristic time for equipartition of energy through elastic electron collisions with singly charged ions, τ_{ei} of Eq. (2), for comparable electron and ion temperatures, is

$$\tau_{ei}=\left(\frac{M_i}{2m}\right)\frac{\alpha_1(m)^{\frac{1}{2}}\epsilon_0^2(\pi kT_e)^{\frac{3}{2}}}{N_ie^4\ln(\alpha_2,\Lambda)}, \quad (4a)$$

where

$$\Lambda=\frac{\pi(kT_e\epsilon_0)^{\frac{3}{2}}}{e^3(N_i)^{\frac{1}{2}}}; \quad (4b)$$

Landau[1] has $\alpha_1=6\sqrt{2}$ and $\alpha_2=8\sqrt{\pi}$, Spitzer[4,7] has $\alpha_1=6\sqrt{2}$ and $\alpha_2=12$, and Chandrasekhar[5] has $\alpha_1=4.50\sqrt{2}$ and $\alpha_2=12$. In Eq. (4), M_i is the ion mass, N_i is the ion concentration, ϵ_0 is the permittivity of free space, and e is the electron charge.

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[15] A. M. Cravath, Phys. Rev. 36, 248 (1930).

During this study we have derived[16] an alternative expression for τ_{ei} for a limiting model of the plasma. It is

$$\tau_{ei}=\left(\frac{M_i}{2m}\right)\times\frac{\alpha_1(m)^{\frac{1}{2}}\epsilon_0^2(\pi kT_e)^{\frac{3}{2}}}{N_ie^4\{[\mathrm{Ci}(1/\alpha_2\Lambda)]^2+[(\pi/2)-\mathrm{Si}(1/\alpha_2\Lambda)]^2\}}, \quad (5)$$

where $\alpha_1=8\sqrt{2}$, $\alpha_2=4\sqrt{2}$, and Ci(x) and Si(x) are the cosine and sine integrals[17] of x, respectively.

Whereas energy exchange from the electron gas to the ion and molecule gases occurs within the plasma volume, independent of the plasma boundary, energy transport through the electron gas from within the plasma to the boundary is embodied in the determination of τ_{er}. The principal process through which thermal electron energy is transported to the boundary is thermal conduction in the electron gas. Since we are concerned primarily with equipartition of energy among the plasma constituents and the boundary, electron energy loss associated with diffusion of electrons to the boundary need not be considered here.

τ_{er} can be calculated by straightforward solution of the thermal diffusion equation, employing the appropriate coefficient of thermal conductivity for the electron gas, for plasmas with a near uniform electron concentration and known boundary conditions. However, τ_{er} is not so simply susceptible to computation for energy transport to the walls of discharge tubes. The complications arise from the increasing thermal impedance identified with the relatively low electron concentrations in the near vicinity of the walls. Effectively, a thermal barrier exists there which can be expected to greatly decrease electron energy transport to the walls. A quantitative calculation of τ_{er} requires the prior determination of the small, but finite, electron concentration in the near vicinity of the walls. Schirmer[18] concludes that electron thermal conduction near the tube walls cannot be effective, and that only the molecule thermal conduction is effective in transporting energy to the walls. Therefore, in view of this discussion, we take $(1/\tau_{er})=0$ in Eq. (2) as an approximation, recognizing that some residual electron energy loss to the boundary may occur in the experiment. With this approximation, and relating the mean thermal energies to the temperatures, $U_e=\frac{3}{2}kT_e$, etc., Eq. (2) becomes

$$\frac{dT_e}{dt}=\left(\frac{2}{3k}\right)\frac{dU_{Ee}}{dt}-\left(\frac{1}{\tau_{em}}+\frac{1}{\tau_{ei}}\right)(T_e-T_{i,m,r}). \quad (6)$$

The predominant energy exchange occurs between the electron and ion gases, rather than between the electron and molecule gases, when $\tau_{ei}\leq\tau_{em}$. The ratio of τ_{ei} to τ_{em}, disregarding the logarithm term in Eq. (4), has the proportionality of $(\tau_{ei}/\tau_{em})\propto qT_e^2/(N_i/N_m)$. Therefore, the most favorable experimental conditions for studying energy exchange between the electron and ion gases prevail in ionized gases with low electron temperatures, small electron-molecule collision cross sections, and suitable degree of ionization. Electron temperatures as low as the molecule temperature exist in plasmas in the afterglow of pulsed electrical discharges in gases.

It is of interest to evaluate τ_{em} and τ_{ei}, Eqs. (3) and (4), for such a plasma in neon gas where, for example, ion concentrations of $N_i=5\times10^{11}$ per cm^3 can be achieved at a pressure of 1 mm Hg which corresponds to a molecule concentration of $N_m=3.22\times10^{16}$ per cm^3 at $T_m=300$°K. Evaluating Eq. (3) for $(M/2m)=1.85\times10^4$, $q=1.12\times10^{-16}$ cm^2, and $T_e=300$°K, then $\tau_{em}=363$ μsec. Evaluating Eq. (4) for α_1 and α_2 from Landau, $(M_i/2m)=1.85\times10^4$, $T_e=300$°K, $N_i=5\times10^{11}$ per cm^3, then $\tau_{ei}=11.7$ μsec. With $\tau_{ei}=11.7$ μsec and $\tau_{em}=363$ μsec, the calculation predicts that energy exchange between the electron and ion gases will predominate even though the ionization is only 0.001%.

EXPERIMENTAL APPARATUS

The experimental apparatus, Fig. 2, consists of (1) a gaseous discharge tube in which the plasma is periodi-

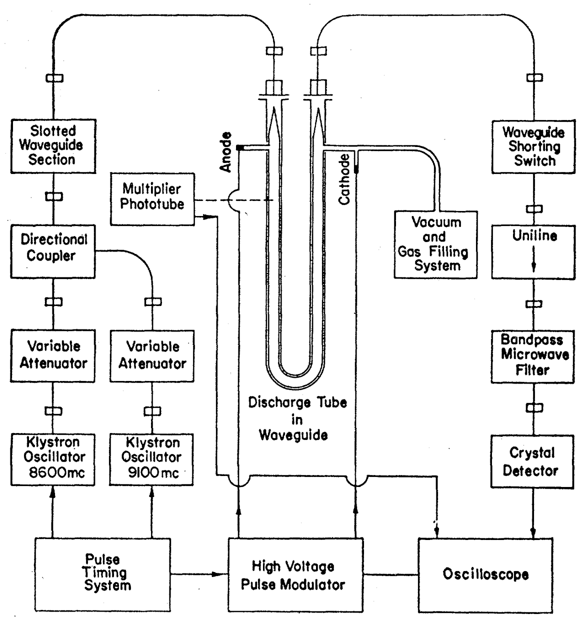

FIG. 2. Diagram of experimental apparatus.

[16] On a microscopic scale of a binary collision, the fraction of initial electron energy transferred to the ion in a single encounter is evaluated. This is averaged over the impact parameter and the azimuthal angle. The time rate at which binary encounters occur in the dense system is calculated, from which the time rate of electron energy transfer to the ions is obtained. Only direct electron-ion encounters are considered, inverse encounters being neglected. Averaging this electron energy transfer over the Maxwellian distribution of electron speeds determines the τ_{ei} of Eq. (5).

[17] E. Jahnke and F. Emde, *Function Tables* (Dover Publications, New York, 1945), fourth edition.

[18] H. Schirmer, Appl. Sci. Research **B5**, 196 (1955).

cally produced by a recurrent electrical discharge through the gas, (2) an X-band microwave circuit with two microwave sources to provide both selective heating of the electron gas, and measurement of the high-frequency electrical conductivity of the plasma, (3) a multiplier phototube for detecting the luminous intensity of the plasma, (4) a pulse-timing and video display system, and (5) a vacuum and gas-filling system for evacuating and admitting gas to the tube.

The discharge tube was fabricated from round Pyrex glass tubing of 20-mm o.d. and 1-mm wall thickness. This tube and the wave-guide assembly in which it is enclosed, were made with a 180° bend at the midpoint. The assembly was contained within a large Dewar flask which permitted cooling the gas to temperatures below room temperature (300°K) with coolant baths of liquid nitrogen (77.3°K) or solid carbon dioxide (195°K). The effective length of the plasma in the discharge tube was 72 cm.

The cross section of the wave guide which accommodates the discharge tube is square with inner dimensions of 2.07 cm×2.07 cm. The TE_{10} mode is utilized in the propagation of the electromagnetic waves. Two reflex klystron oscillators generate microwave power at separate frequencies of 8600 and 9100 Mc/sec. The 8600 Mc/sec klystron delivers up to 400 milliwatts power incident to the discharge tube, and serves as the selective heating source for the electron gas. We designate this wave as the "heating" wave. A directional coupler couples the power from the second klystron, 9100 Mc/sec, to the wave-guide circuit at a much lower level (a fraction of a milliwatt). This 9100 Mc/sec wave is of insufficient amplitude to heat the electron gas significantly; its propagation through the plasma is dependent on the high-frequency conductivity of the plasma. This latter wave is designated as the "sensing" wave.

Both waves are guided through the discharge tube, traverse the Uniline (a unilateral ferrite microwave transmission device), and are incident on the band-pass cavity filter resonating at 9100 Mc/sec. The heating wave, 8600 Mc/sec, is rejected by this filter and subsequently absorbed in the Uniline. Therefore, only the sensing wave is detected at the crystal detector. Microwave interaction between the two waves occurs due to alteration of the conductivity, which is effected through heating of the electron gas by the heating wave, influencing the propagation of the sensing wave through the plasma.

For measurement of the shift in phase of the sensing wave due to the plasma conductivity, the wave-guide shorting switch is employed to reflect the sensing wave in the reverse direction through the discharge tube. The resultant standing wave, in the absence of the heating wave, is detected at the slotted wave-guide section. Measurement of the phase shift permits determination of the electron (and ion) concentration in the plasma.

Electrical breakdown of the gas and establishment of the plasma occur during the application of a high-voltage pulse to the anode and cathode of the discharge tube. The pulse duration is from 2–10 μsec at a repetition frequency of 250 cps. Throughout the 4000-μsec period between pulses, the plasma disintegrates. The pulse timing system supplies modulation pulses to both klystrons which transmit the heating and sensing waves through the disintegrating plasma at appropriate time intervals.

A multiplier phototube, RCA type 931A, detects the luminous intensity of the plasma. The light passes through a 1.6-mm diameter aperture in the wave-guide wall, located along the axis of the discharge tube, 14.5 cm from the tip.

A vacuum system capable of evacuating the discharge tube to 10^{-7} mm Hg was employed. Laboratory grade gases (Linde, mass spectrometer controlled) were admitted to the discharge tube through an activated coconut charcoal filter (cooled) which isolates the tube from the system. Neon, helium, and neon 99.9% plus argon 0.1% gases were used in these experiments.

THEORY OF THE EXPERIMENT

The experiments are performed in the disintegrating plasma. In this plasma, the electron temperature returns to thermal equilibrium with the ion, and molecule, temperature. Selective heating of the electron gas is effected by the pulsed heating wave, resulting in deviation of the electron temperature from equilibrium. This electron temperature deviation affects the conductivity, and the rate of electron-ion recombination. We determine quantitatively here the resulting absorption of the sensing wave, and the change in luminous intensity of the afterglow.

The rms current density $\bar{J}_h$ and electric field intensity $\bar{E}_h$ of the heating wave are related through the complex conductivity σ_h by $\bar{J}_h=\sigma_h\bar{E}_h$, where $\sigma_h=\sigma_{rh}+j\sigma_{ih}$. The quantities σ_{rh} and σ_{ih} are the real and imaginary parts of the conductivity, respectively. The differential heating wave power, $dP_{\alpha h}$, absorbed in the differential plasma volume dV is $dP_{\alpha h}=\mathrm{Re}(\mathbf{J}_h\cdot\mathbf{E}_h^*)dV$. Therefore, $dP_{\alpha h}=E_h^2\sigma_{rh}dV$. Since there are ndV electrons in dV, the mean time rate at which an electron receives energy (dU_{Ee}/dt) from the heating wave is

$$\frac{dU_{Ee}}{dt}=\frac{dP_{\alpha h}}{ndV}=\frac{E_h^2\sigma_{rh}}{n}. \tag{7}$$

The real part of the conductivity for the heating wave of angular frequency ω_h is $\sigma_{rh}=ne^2(\nu_i+\nu_m)/m\omega_h^2$, where $(\nu_i+\nu_m)^2\ll\omega_h^2$; ν_i and ν_m are the effective collision frequencies for momentum transfer of the electrons with ions and molecules, respectively. Ginsburg[10] gives ν_i and ν_m as

$$\nu_i=\frac{N_ie^4\ln(4\sqrt{2}\Lambda)}{6(2m)^{\frac{1}{2}}\epsilon_0^2(\pi kT_e)^{\frac{3}{2}}},\quad \nu_m=\tfrac{4}{3}qN_m\left(\frac{8kT_e}{\pi m}\right)^{\frac{1}{2}}. \tag{8}$$

Comparison of ν_i and ν_m with τ_{ei} [Eq. (4a), Landau] and τ_{em} [Eq. (3)] yields $\nu_i=(M_i/2m)(1/\tau_{ei})$ and $\nu_m=(M/2m)(1/\tau_{em})$, except for a negligible difference due to slightly different factors within the logarithm terms. For ions of the parent gas, $M=M_i$. Therefore, $\nu_i+\nu_m=(M_i/2m)[(1/\tau_{ei})+(1/\tau_{em})]$.

The electron temperature balance equation, Eq. (6), upon substitution of Eq. (7) and the above relation for $(\nu_i+\nu_m)$ becomes

$$\frac{dT_e}{dt}=\left(\frac{1}{\tau_{ei}}+\frac{1}{\tau_{em}}\right)[\Delta T_e-(T_e-T_{i,m})],$$

where

$$\Delta T_e=\frac{M_ie^2E_h^2}{3km^2\omega_h^2}. \quad (9)$$

Time-dependent solutions to Eq. (9) are to be determined for the pulsed heating wave field which has the time variation of $E_h(t)=0$ for $t<t_1$, $E_h(t)=E_h$ for $t_1\leq t\leq t_2$, and $E_h(t)=0$ for $t>t_2$. Under the restriction that the electron temperature deviation is sufficiently small so that τ_{ei} and τ_{em} may be considered as constants, and with the initial condition of $T_e(t_1)=T_{i,m}$, the solutions for $T_e(t)$ are, for $t_1\leq t\leq t_2$,

$$T_e(t)=T_{i,m}+\Delta T_e\left[1-\exp\left(-\frac{t-t_1}{\tau}\right)\right], \quad (10)$$

and for $t_2<t$,

$$T_e(t)=T_{i,m}+\Delta T_e\left[1-\exp\left(-\frac{t_2-t_1}{\tau}\right)\right]\exp\left(-\frac{t-t_2}{\tau}\right). \quad (11)$$

The quantity τ is defined through the relation,

$$\frac{1}{\tau}=\frac{1}{\tau_{ei}}+\frac{1}{\tau_{em}}. \quad (12)$$

The electron temperature responses to the pulsed heating wave field are exponential with the time constant τ. Upon termination of E_h at t_2, T_e returns exponentially to $T_{i,m}$.

If the heating wave has a sufficiently long pulse duration such that $t_2-t_1\gg\tau$, then as $t\to t_2$, the electron temperature reaches a quasi-equilibrium value equal to $T_{i,m}+(M_ie^2E_h^2/3km^2\omega_h^2)$. This quasi-equilibrium temperature is independent of the electron concentration and effective collision frequencies, but is dependent on the ion (and molecule) mass. In particular, a given heating wave field E_h will effect larger electron temperature deviations in the heavier ion and molecule gases.

Transient behavior is effected in the absorption of the sensing wave propagating through the plasma by deviations in T_e. The differential sensing wave power absorbed $dP_{\alpha s}$ in the differential plasma volume dV is $dP_{\alpha s}=E_s^2\sigma_{rs}dV$, where $\sigma_{rs}=ne^2(\nu_i+\nu_m)/m\omega_s^2$. The effective collision frequencies have an electron temperature dependence of $\nu_i+\nu_m=AT_e^{-\frac{3}{2}}+BT_e^{\frac{1}{2}}$, assuming q is relatively independent of T_e. For small deviations in T_e, the time-dependent $T_e(t)$ can be expanded in series to obtain $T_e^{\frac{1}{2}}(t)$ and $T_e^{-\frac{3}{2}}(t)$; an example, for $t_1\leq t\leq t_2$, is

$$T_e^{-\frac{3}{2}}(t)\simeq T_{i,m}^{-\frac{3}{2}}\left\{1-\frac{3\Delta T_e}{2T_{i,m}}\left[1-\exp\left(-\frac{t-t_1}{\tau}\right)\right]\right\}. \quad (13)$$

Therefore, the conductivity $\sigma_{rs}(t)$ has the time dependence for $t_1\leq t\leq t_2$,

$$\sigma_{rs}(t)=\frac{ne^2}{m\omega_s^2}\left\{\nu_i+\nu_m+(\nu_m-3\nu_i)\left(\frac{\Delta T_e}{2T_{i,m}}\right)\times\left[1-\exp\left(-\frac{t-t_1}{\tau}\right)\right]\right\}, \quad (14)$$

and for $t_2<t$,

$$\sigma_{rs}(t)=\frac{ne^2}{m\omega_s^2}\left\{\nu_i+\nu_m+(\nu_m-3\nu_i)\left(\frac{\Delta T_e}{2T_{i,m}}\right)\times\left[1-\exp\left(-\frac{t_2-t_1}{\tau}\right)\right]\exp\left(-\frac{t-t_2}{\tau}\right)\right\}, \quad (15)$$

where ν_i and ν_m are the initial values at $t=t_1$. Since σ_{rs} decreases during the heating wave pulse if $3\nu_i>\nu_m$, the plasma is more transparent to the sensing wave; since it increases if $\nu_m>3\nu_i$, the plasma is more absorbing.

Integration of $dP_{\alpha s}(t)=E_s^2\sigma_{rs}(t)dV$ over the plasma-filled wave-guide volume V determines the total sensing wave power absorbed within the plasma. The observable in our experiment is the transmitted sensing wave power $P_{ts}(t)$ which emerges from the plasma. It is the difference between the incident and absorbed power, any reflected power being negligibly small. Assuming that τ is not spatially dependent within the plasma, as for a uniform plasma, the integration yields the time dependence of the transmitted sensing wave power $P_{ts}(t)$, for $t_1\leq t\leq t_2$ and $(t_2-t_1)\gg\tau$, as

$$P_{ts}(t)=P_{ts}(t_1)+[P_{ts}(t_2)-P_{ts}(t_1)]\left[1-\exp\left(-\frac{t-t_1}{\tau}\right)\right], \quad (16)$$

and for $t>t_2$,

$$P_{ts}(t)=P_{ts}(t_1)+[P_{ts}(t_2)-P_{ts}(t_1)]\exp\left(-\frac{t-t_2}{\tau}\right). \quad (17)$$

The time constant of the exponential response of $P_{ts}(t)$ is τ, where $(1/\tau)=(1/\tau_{ei})+(1/\tau_{em})$, which affords direct measurement of the characteristic time for equipartition of electron energy with the ion and molecule gases.

The luminous intensity I in the late afterglow is proportional to the electron-ion recombination coefficient α as $I=K_1\alpha nN_i$, where K_1 is a proportionality constant. The recombination coefficient may have the electron temperature dependence of $T_e^{-\frac{3}{2}}$ as suggested by Bates,[19] and implied by the electron-ion collision frequency ν_i of Eq. (8). Therefore, the luminous intensity becomes $I=K_2 n N_i T_e^{-\frac{3}{2}}$, where I decreases with increasing T_e. However, for small electron temperature deviations, the exact dependence of α on T_e is immaterial since the exponential time dependence of the electron temperature will effect an exponential time variation of the luminous intensity. The transient response of $I(t)$ resulting from the electron temperature variation in Eq. (13) is, for $t_1 \leq t \leq t_2$ and $(t_2-t_1) \gg \tau$,

$$I(t)=I(t_1)-[I(t_1)-I(t_2)]\left[1-\exp\left(-\frac{t-t_1}{\tau}\right)\right], \quad (18)$$

and for $t_2 \leq t$,

$$I(t)=I(t_1)-[I(t_1)-I(t_2)]\exp\left(-\frac{t-t_2}{\tau}\right). \quad (19)$$

The luminous intensity affords an additional, and independent, measurement of the characteristic time for equipartition of electron energy with the ion and molecule gases.

Correlation of the experimentally determined τ_{ei} with the ion concentration N_i requires experimental measurement of N_i. In a plasma containing predominantly singly charged ions, space-charge neutrality requires that $n=N_i$. Therefore measurement of n determines N_i. A solution for microwave propagation through an inhomogeneously filled wave guide containing plasma with complex conductivity was given by Anderson and Goldstein.[11] Evaluating their integrals for a plasma of uniform electron concentration, which is characteristic of plasma disintegration through volume recombination at the relatively high electron and ion concentrations of interest here, n is determined as a function of the shift in phase of the sensing wave. It is

$$n=N_i=1.76\times10^{18}\Delta_r, \text{ per m}^3, \quad (20)$$

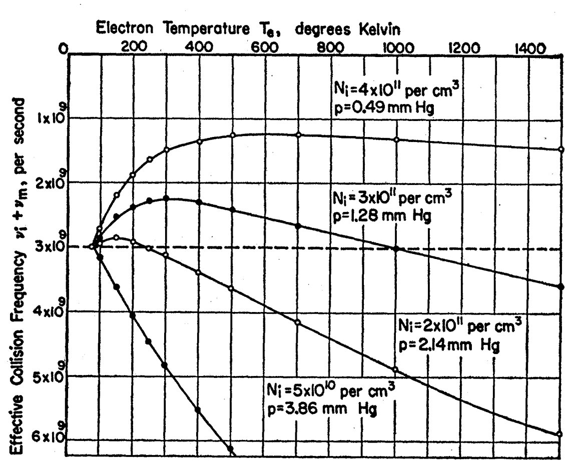

FIG. 3. Effective collision frequency of electrons with ions and molecules in ionized helium gas at 77°K.

[19] D. R. Bates, Phys. Rev. 78, 492 (1950).

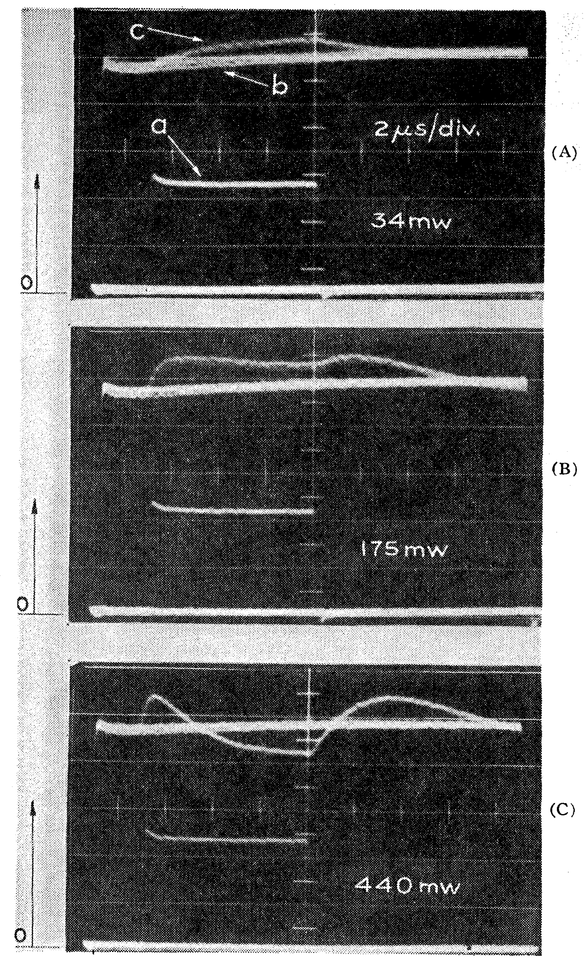

FIG. 4. Microwave interaction in ionized helium gas at 77°K.

where Δ_r, in meters, is the total shift in the minimum of the standing wave at the slotted wave-guide section, due to the plasma.

EXPERIMENTAL OBSERVATIONS AND RESULTS

Comparison of the expected role of Coulomb collisions between electrons and ions in the plasma with experimental observations of microwave interaction is shown in Figs. 3 and 4. Absorption of the sensing wave was determined through the conductivity σ_{rs}, where σ_{rs} is proportional to $\nu_i+\nu_m$. For electron-ion collisions, ν_i is expected to vary as $T_e^{-\frac{3}{2}}$; whereas for electron-molecule collisions in helium gas, ν_m varies as $T_e^{\frac{1}{2}}$. Evaluation of Ginsburg's results, Eq. (8), for $\nu_i+\nu_m$ as a function of T_e in helium at 77.3°K is shown in Fig. 3 for several combinations of ion and molecule concentrations. Here, $N_m=1.25\times10^{17}$ per cm³ per mm Hg, and $q=6.75\times10^{-16}$ cm² (reference 11). N_i and p (gas pressure) are selected to give $\nu_i+\nu_m=3.0\times10^9$ per sec for $T_e=77.3$°K in each instance. For $N_i=4$, 3, and 2×10^{11} per cm³, $\nu_i+\nu_m$ at first *decreases* with increasing T_e, then reaches a minimum value and subsequently in-

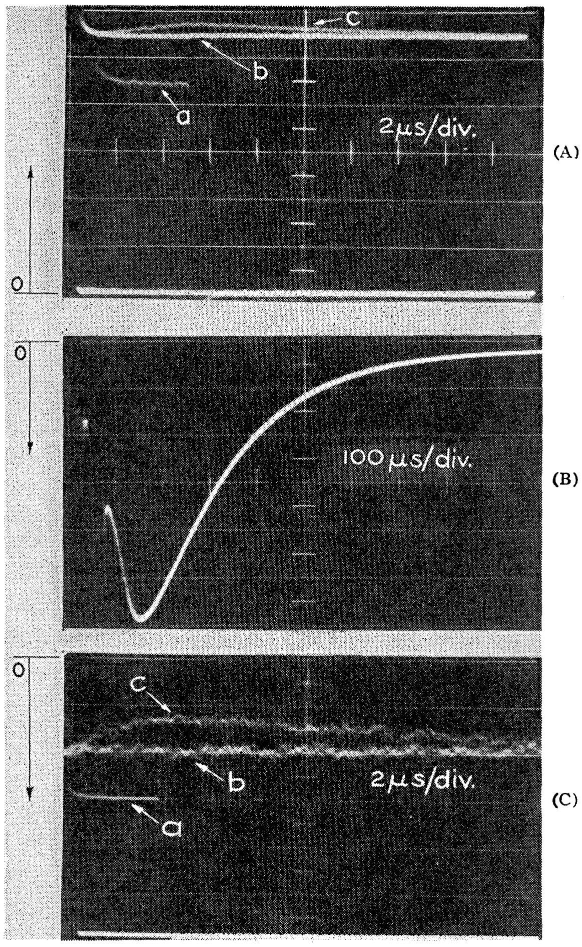

FIG. 5. Experimental observations of microwave interaction and plasma luminous intensity for determination of τ.

creases. This shows the transition from ν_i predominating to ν_m predominating as T_e increases; it is to be contrasted with $\nu_i+\nu_m$ for $N_i=5\times10^{10}$ per cm^3, and $p=3.86$ mm Hg, which uniquely *increases* as T_e increases from 77°K, where the electron-ion collisions are no longer predominant for the smaller ion concentration. Microwave interaction in a plasma produced in helium gas cooled to 77°K, at 0.54 mm Hg, is shown in Fig. 4. Interaction between the incident heating wave a and the transmitted sensing wave without b and with c the heating wave is shown for a time interval of 20 μsec, starting at 420 μsec after the discharge pulse. At this time in the disintegrating plasma, the electron (and ion) concentration is 1.82×10^{11} per cm^3, and it does not change appreciably throughout the 20 μsec interval. In Fig. 4(A), the transmitted sensing wave increases in amplitude as the electron gas is heated to temperatures above 77°K by the heating wave, which corresponds to a decrease in the absorption of the sensing wave. This decrease in absorption is characteristic of the predominant contribution to σ_{rs} of the electron-ion collisions. Upon termination of the heating wave, the transmitted sensing wave returns to its original amplitude as the electron temperature decreases, the excess mean electron energy being transferred to the ions and molecules. In Figs. 4(B) and 4(C), incident heating wave power 5.1 and 13 times higher, respectively, than in 4(A) increases the electron temperature more rapidly, and to higher values. This results in the transmitted sensing wave increasing in amplitude at first, then its slope changes and it begins to decrease. Its amplitude decreases to less than its initial value in 4(C), prior to termination of heating. Then upon termination of heating, its amplitude increases again through its original value, maximizes, and subsequently returns to its original value. Decrease in transmission of the sensing wave during heating occurs due to a transition from ν_i predominating at low electron temperatures to ν_m becoming predominant at higher electron temperatures, in agreement with theory.

Experimental observations for the measurement of τ, the characteristic time for equipartition of excess mean electron energy with the ion and molecule gases are shown in Fig. 5. The plasma in this instance was produced in a gas mixture of 99.9% neon plus 0.1% argon at a gas temperature of 300°K and 2.25 mm Hg pressure. Both methods for measurement of τ are employed here. Figures 5(A) and 5(C) show the microwave interaction and luminous intensity, respectively, for a time interval of 20 μsec, starting at 800 μsec after the discharge pulse. Figure 5(B) shows the luminous intensity as a function of time from 0 to 1000 μsec after the discharge pulse; after 160 μsec the intensity decreases as a smooth function of time, presumably upon T_e approaching $T_i=T_m$ in the disintegrating plasma. The incident heating wave a in 5(A) and 5(C), has a pulse duration of 4 μsec, during which time it deviates the electron gas temperature from its initial 300°K temperature. Trace c in 5(A) and 5(C) shows the re-

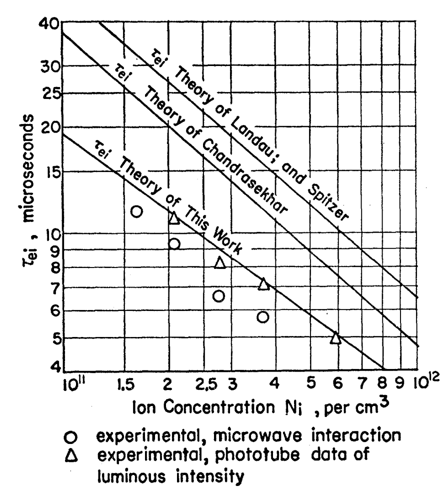

FIG. 6. Comparison of experimental and theoretical τ_{ei} *versus* ion concentration in ionized neon plus 0.1% argon gas at 300°K. $p=2.25$ mm Hg.

sultant effect on the transmitted sensing wave and luminous intensity, respectively. The transmitted sensing-wave amplitude *increases* during heating, which is again characteristic of electron-ion collisions predominating; upon termination of heating, its amplitude returns to the original value which is in accordance with Eq. (17). The luminous intensity *decreases* during heating, and returns to its original value upon termination of heating, as predicted by Eq. (19). The times required for the amplitudes to return to $(1/e)=0.37$ of the deviations existing after termination of heating give the experimental values of τ; in 5(A), $\tau=8.8$ μsec and in 5(C), $\tau=10.2$ μsec. These agree within 16%, which is within the probable cumulative experimental accuracy.

The experimental T_{ei} is calculated from τ through use of Eq. (12), upon evaluation of τ_{em} in Eq. (3). For the neon-argon mixture in Fig. 5, where the low-percentage argon content does not contribute significantly to electron-molecule collisions, $N_m=3.22\times10^{16}p$ per cm^3, $(M/2m)=1.85\times10^4$, and $q=1.12\times10^{-16}$ cm^2. Therefore $\tau_{em}=363/p$ μsec, where p is the gas pressure in mm Hg, at 300°K. For $p=2.25$ mm Hg, $\tau_{em}=161$ μsec. The experimental τ_{ei}'s are plotted in Fig. 6 as a function of the experimentally measured ion concentration, and compared to the various theoretical τ_{ei}'s. The experimental τ_{ei} is in favorable comparison with the theory of this work; the theoretical τ_{ei} of Chandrasekhar is approximately 1.7 times larger, and that of Landau, and Spitzer, is approximately 2.3 times larger. The theoretical computation assumes singly charged, atomic neon ions. The time τ_{ei} decreases with increasing ion concentration, in accordance with the theory for energy exchange through Coulomb collisions. The electron temperature returns to equilibrium in 5 to 10 μsec, whereas a time of 161 μsec would be required in the absence of electron-ion collisions.

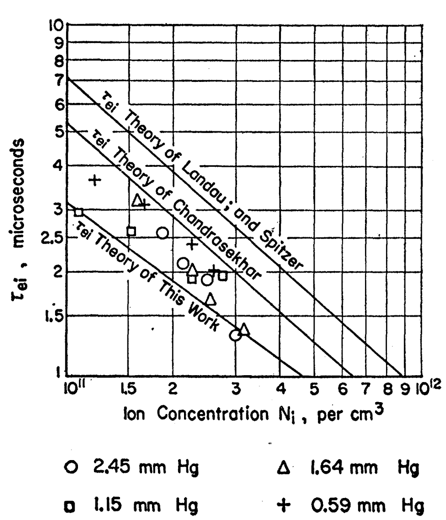

FIG. 7. Comparison of experimental and theoretical τ_{ei} *versus* ion concentration in ionized helium gas at 225°K.

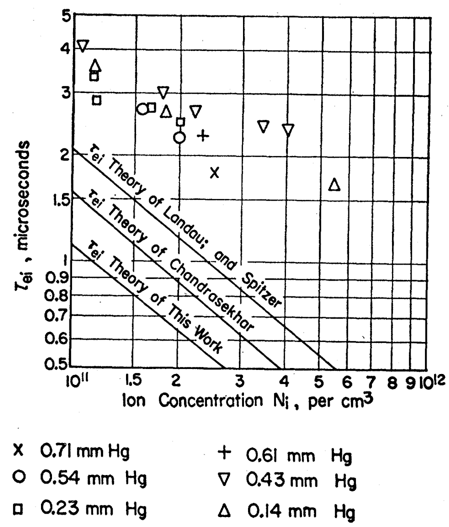

FIG. 8. Comparison of experimental and theoretical τ_{ei} *versus* ion concentration in ionized helium gas at 77°K.

Additional experimental results for τ_{ei} are shown in Figs. 7 and 8 for plasmas produced in helium gas cooled to 225°K and 77°K, respectively. We have used $N_m=4.29\times10^{16}$ and 1.25×10^{17} per cm^3 per mm Hg at 225 and 77°K, respectively, $q=6.75\times10^{-16}$ cm^2, and $(2m/M)=2.72\times10^{-4}$ to evaluate τ_{em}. The theoretical τ_{ei}'s are calculated under the assumption of single charged atomic helium ions. At 225°K, the experimental τ_{ei}'s are intermediate between the theoretical τ_{ei} of Chandrasekhar and that of this work. In Fig. 8, at 77°K, the experimental τ_{ei}'s also decrease with increasing ion concentration, but are well above the theoretical values. They are approximately twice the theoretical predictions of Landau, and of Spitzer.

Experiments were also performed in plasmas produced in neon at 300°K, neon plus helium at 300°K, neon at 225°K, 99.9% neon plus 0.1% argon at 77°K, and neon at 77°K, which yielded results for τ_{ei} comparable to those reported above. The experimental τ_{ei} in neon, and neon plus argon, at 77°K was also approximately twice the theoretical value of Landau, and of Spitzer, as was found in helium at 77°K.

DISCUSSION AND CONCLUSIONS

Coulomb collisions between electrons and ions are determined to contribute significantly to electron collision phenomena in the gaseous discharge plasmas investigated in this work, although the ionization never exceeds 0.001%. The method of microwave interaction between the heating and sensing waves, which are propagated through the plasma, permitted quantitative determination of the role of these Coulomb collisions. The characteristic time, τ_{ei}, for equipartition of energy through electron collisions with ions, determines the time rate of energy transfer between the electron and ion gases with mean thermal energies of U_e and U_i, respectively, through the relation $(U_e - U_i)/\tau_{ei}$. These experimental results show that τ_{ei} varies inversely with the ion concentration, in accordance with theory. Further, the experimental dependence of τ_{ei} on the ion mass M_i agrees reasonably well with the linear dependence predicted by theory. The ratio of neon to helium atomic ion masses is 5.02; the ratio of the experimental τ_{ei}'s in neon and helium plasmas varied from 4 to 6 for equivalent ion concentrations and electron temperatures. Whereas the experimental τ_{ei} has the expected dependence with respect to the ion concentration and ion mass, the trend with respect to electron temperature is not in agreement with theory. The results in Figs. 6, 7, and 8 at gas (and presumably) electron temperatures of 300, 225, and 77°K, respectively, show that τ_{ei} varies from values slightly less than the theory of this work at 300°K, to values approximately twice the theory of Landau, and of Spitzer, at 77°K. If the electron temperature dependence of τ_{ei} were that of $T_e^{-\frac{3}{2}}$, in the absence of secondary processes, it would be expected that τ_{ei} would come into consistent agreement with one or another of the theories, which it does not do.

Several secondary processes occur within the experimental plasmas which may intervene and impede a more refined comparison between these experimental results and the theories. Molecular ions, such as Ne_2^+ and He_2^+, are known to form in appreciable quantities in the post-discharge period. Dependent on the relative proportions of atomic and molecular ions present, the average ion mass will be intermediate between one to two times the atomic ion mass, which results in a similar indeterminancy in τ_{ei}. Another secondary process is the possible heating of the electron gas through long-life metastable atom collisions with electrons in which the metastable atom energy is transferred to kinetic energy of the electron. Such a source of energy could prevent the electron gas from reaching thermal equilibrium with the ion and molecule gases, which results in an indeterminancy in T_e. To eliminate possible electron heating through this process, experiments were performed with 0.1% argon impurity in the neon gas. Destruction of the neon metastable states occurs through the familiar Penning effect whereby the potential energy of the metastable atom is adequate to ionize an argon atom. In addition, transport of excess electron energy through thermal conduction in the electron gas to the boundaries, although expected to be negligibly small, may nevertheless result in some residual electron energy loss. Therefore, a more refined comparison between theory and experiment for energy exchange through Coulomb collisions between electrons and ions will require techniques to further remove these indeterminancies. The experimental results reported here do, however, attest to the approximate correctness of the theoretical predictions.

ACKNOWLEDGMENTS

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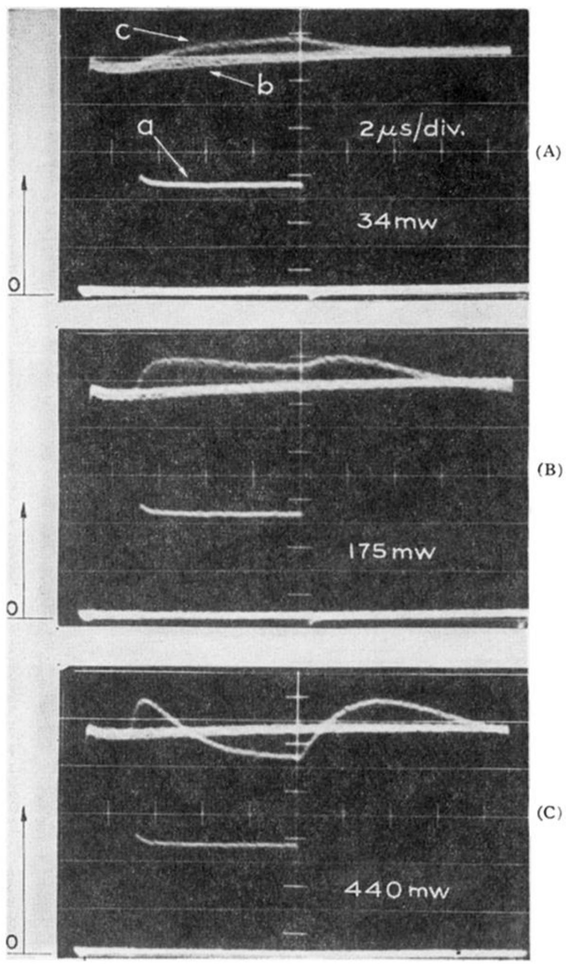

Fig. 4. Microwave interaction in ionized helium gas at 77°K.

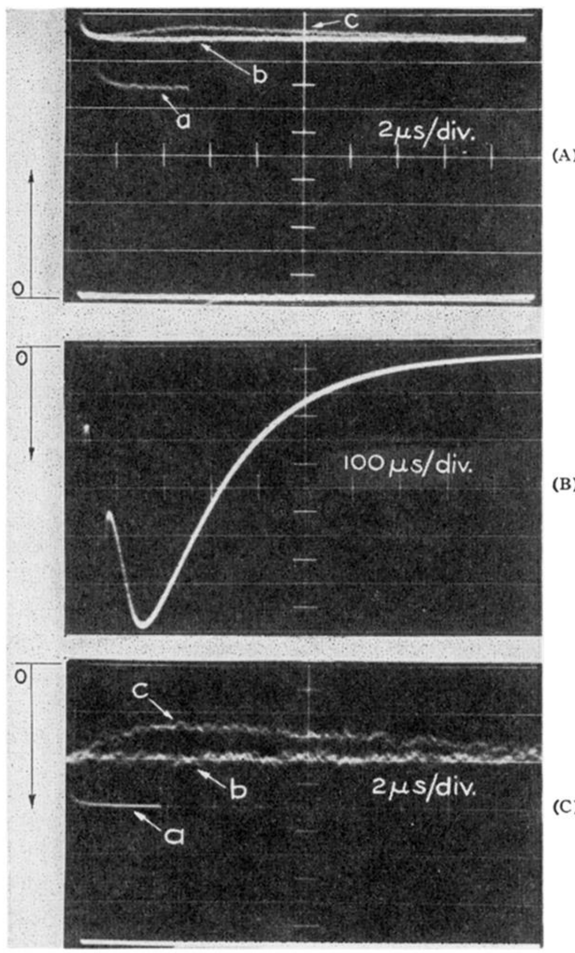

FIG. 5. Experimental observations of microwave interaction and plasma luminous intensity for determination of τ.