TABLE I. Summary of experimental data, containing information on "target" nuclei, run lengths, and count rates.

Target	Number of atoms H	D	O	Cd	Cl	Run length (hr)	Counts	Rate (hr^{-1})	Calculated acc. bkgd. (hr^{-1})
1	4.0×10^{27}	3.6×10^{27}	3.8×10^{27}	1.5×10^{26}	3.1×10^{26}	112.2	313	2.79 ± 0.16	0.23
2	1.1×10^{28}	...	5.5×10^{27}	...	...	72.0	173	2.40 ± 0.18	0.24

detectable gamma rays but we do not consider this possibility further here. Two experiments, one with an H_2O target and one with a target containing Cd to capture and detect the neutron by means of its capture gamma rays, were performed to enable a measurement to be made of the postulated neutron-associated delayed coincidences. The targets chosen were 165 kg of H_2O and 168 kg of a mixture of H_2O, D_2O, and $CdCl_2$. Table I shows the pertinent data from these two experiments. The neutron detection efficiency for the system with Cd is about 0.3 and charged particles emanating from the target should be seen with an efficiency approaching 100% since edge effects are small. Without Cd the neutron efficiency is effectively zero.

From the table it is seen that the net neutron-associated rate $=0.39\pm0.24$ hr^{-1} (<0.63 hr^{-1}). Although a partial cosmic-ray anticoincidence blanket in the form of liquid scintillation detectors was placed above the detectors it was not possible to rule out nucleon modes of decay which would not trigger the anticoincidence. The observed rate is not inconsistent with that expected from cosmic rays at the detector location. It seems quite conservative to assume that at most one-half of the events are due to nucleon decay. We interpret the data by assuming that less than half the neutron-associated rate (i.e., <0.32 hr^{-1}) is due to the reaction

$$D\rightarrow n+\text{decay products},$$

where the choice of decay products is limited by the laws of conservation of electrical charge and mass-energy-momentum. The nucleon lifetime based on this reaction is in excess of 4×10^{23} yr.

In view of the enormously great lifetimes under discussion, pions are assumed to be excluded as decay products by virtue of their strong interaction with nucleons, an assumption which precludes consideration in these estimates of the other nuclei comprising the target. This follows because the assumption makes improbable an interaction between the residual nucleus and the decay products and the consequent emission of a neutron which could give rise to a delayed coincidence. An estimate of the nuclear excitation produced by the sudden, noninteracting removal of a nucleon based on considerations of nuclear compressibility indicates that only a fraction of a Mev is available. This conclusion also rules against the possibility of neutron emission due to nucleon decay of the other nuclei which make up the target.

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[1] Reines, Cowan, and Goldhaber, Phys. Rev. **96**, 1157 (1954).
[2] These detectors designed for the free-neutrino studies are pictured in an article by F. Reines and C. L. Cowan, Phys. Today **10**, No. 8, 12 (1957).

Relations between the Hyperon Polarizations in Associated Production*

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IN view of the recent discovery of a large up-down asymmetry in $\Lambda^0\rightarrow p+\pi^-$,[1] we report in this note on some results on the polarizations of the produced hyperons, which will be useful in the interpretation of the experiments. We first show that in the decay $\Sigma^0\rightarrow\Lambda^0+\gamma$ from a polarized Σ^0 the Λ^0 is longitudinally polarized in the Σ^0 rest frame, and the value of its polarization is the same, except for having opposite sign, as the component of the Σ^0 polarization along the Λ^0 line of flight. It is assumed that both Λ^0 and Σ^0 have spin $\frac{1}{2}$, but the result is independent of their relative parity. Denoting by $\mathbf{u}$ the unit vector along the direction of emission of the Λ^0 in the Σ^0 rest frame, and by $\langle\boldsymbol{\sigma}\rangle_\Sigma$ and $\langle\boldsymbol{\sigma}\rangle_\Lambda$ the polarizations of the Σ^0 and of the Λ^0, one finds, after summing over the photon polarizations,

$$\langle\boldsymbol{\sigma}\rangle_\Lambda=-(\langle\boldsymbol{\sigma}\rangle_\Sigma\cdot\mathbf{u})\mathbf{u}. \quad (1)$$

The angular distribution of the pion emitted in the subsequent Λ^0 decay is given by

$$W(\mathbf{v})=1+\alpha(\langle\boldsymbol{\sigma}\rangle_\Lambda\cdot\mathbf{v})=1-\alpha P_\Sigma(\mathbf{n}\cdot\mathbf{u})(\mathbf{u}\cdot\mathbf{v}), \quad (2)$$

where $\mathbf{v}$ is the unit vector along the direction of emission of the π^- from Λ^0 decay in the Λ^0 rest frame, α is the asymmetry parameter for $\Lambda^0\rightarrow p+\pi^-$, and we have written $\langle\boldsymbol{\sigma}\rangle_\Sigma=P_\Sigma\mathbf{n}$, where, in a two-body production process, such as $\pi^-+p\rightarrow\Sigma^0+K^0$, $\mathbf{n}$ is a unit vector normal to the production plane. If α is known,[2] from

Eq. (2) one can determine P_Σ at a given angle. One sees from Eq. (1) that the polarization for the larger sample including Λ^0's emitted in all directions from the polarized Σ^0 is

$$[\langle\boldsymbol{\sigma}\rangle_\Lambda]_{\text{Av}} = -\tfrac{1}{3}\langle\boldsymbol{\sigma}\rangle_\Sigma,$$

and for such a sample Eq. (2) reduces to

$$[W(\mathbf{v})]_{\text{Av}} = 1 - \tfrac{1}{3}\alpha P_\Sigma(\mathbf{n}\cdot\mathbf{v}).$$

Therefore the only asymmetry expected for the larger sample is an up-down asymmetry with respect to the production plane (defined by the incoming π^- and the outgoing K^0), and its measurement will permit a determination of P_Σ. In a smaller sample, for which the Λ^0 direction is observed, the asymmetry will not in general be up-down, but relative to the vector $\mathbf{u}$.[3]

To derive Eq. (1) we observe that in the decay $\Sigma^0 \to \Lambda^0 + \gamma$, from the three available vectors—$\boldsymbol{\sigma}$, Pauli spin operator, e, polarization vector of the γ, and $\mathbf{u}$, the last two satisfying $(\mathbf{e}\cdot\mathbf{u}) = 0$ imposed by gauge invariance—we can form only one pseudoscalar, $(\boldsymbol{\sigma}\cdot\mathbf{e})$, and only one scalar, $(\boldsymbol{\sigma}\cdot\mathbf{e}\times\mathbf{u})$, which contain $\mathbf{e}$ linearly. Therefore the transition matrix T is, apart from constants, $(\boldsymbol{\sigma}\cdot\mathbf{e})$, if Σ^0 and Λ^0 have opposite (relative) parity ($E1$ transition), and $(\boldsymbol{\sigma}\cdot\mathbf{e}\times\mathbf{u})$ if they have same parity ($M1$ transition). The polarization of the emitted Λ^0 is given by

$$\langle\boldsymbol{\sigma}\rangle_\Lambda = \text{Tr}[T(1+\langle\boldsymbol{\sigma}\rangle_\Sigma\cdot\boldsymbol{\sigma})T^\dagger\boldsymbol{\sigma}]/\text{Tr}[T(1+\langle\boldsymbol{\sigma}\rangle_\Sigma\cdot\boldsymbol{\sigma})T^\dagger],$$

which reduces to $\text{Tr}[T\langle\boldsymbol{\sigma}\rangle_\Sigma\cdot\boldsymbol{\sigma}T^\dagger\boldsymbol{\sigma}]/\text{Tr}[TT^\dagger]$ in the present case, giving $\langle\boldsymbol{\sigma}\rangle_\Lambda = -\langle\boldsymbol{\sigma}\rangle_\Sigma + 2(\langle\boldsymbol{\sigma}\rangle_\Sigma\cdot\mathbf{e})\mathbf{e}$ in the case of opposite parity and $\langle\boldsymbol{\sigma}\rangle_\Lambda = -\langle\boldsymbol{\sigma}\rangle_\Sigma + 2(\langle\boldsymbol{\sigma}\rangle_\Sigma\cdot\mathbf{e}\times\mathbf{u})\times(\mathbf{e}\times\mathbf{u})$ in the case of equal parity of Σ^0 relative to Λ^0. It is evident from these last expressions that, after averaging over the γ polarization, one has $\langle\boldsymbol{\sigma}\rangle_\Lambda$ the same for both cases of relative parity, and one immediately derives Eq. (1).

We now discuss the restrictions imposed by charge independence on the hyperon polarizations in the reactions (a): $\pi^- + p \to \Sigma^- + K^+$; (b): $\pi^- + p \to \Sigma^0 + K^0$; (c): $\pi^+ + p \to \Sigma^+ + K^+$; and examine what information on the production matrix can be obtained from measurement of the intensities and of the polarizations. We express the angular distribution and the Σ polarization in each of such reactions in the forms $I = \frac{1}{2}\,\text{Tr}[MM^\dagger]$ and $\langle\boldsymbol{\sigma}\rangle_\Sigma = P_\Sigma\mathbf{n} = I^{-1}\,\text{Tr}[MM^\dagger\boldsymbol{\sigma}]$, in which M is the transition matrix for the particular reaction. If the relative parity of K with respect to ΣN is -1 the matrices M are of the form $E + F(\boldsymbol{\sigma}\cdot\mathbf{n})$; if such relative parity is $+1$ they are of the form $G(\boldsymbol{\sigma}\cdot\mathbf{m}) + H(\boldsymbol{\sigma}\cdot\mathbf{m}')$, where $\mathbf{m}$ and $\mathbf{m}'$ are unit vectors in the direction of the ingoing and of the outgoing momentum, respectively, and E, F, G, H are, for each reaction, functions of the energy and of $\cos\theta = \mathbf{m}\cdot\mathbf{m}'$. We introduce, for each reaction, the quantities $f^\pm$, defined by $f^\pm = (1/\sqrt{2})(E \pm F)$, or by $f^\pm = (1/\sqrt{2})(Ge^{\pm i\theta} + H)$, according to the two cases of relative parity. One verifies that $|f^+|^2$ and $|f^-|^2$ are the probabilities for production at a given angle of Σ with spin up or down, respectively: $|f^\pm|^2 = \frac{1}{2}\,\text{Tr}[MM^\dagger\Lambda(\pm\mathbf{n})] = \frac{1}{2}I(1 \pm P_\Sigma)$, where $\Lambda(\pm\mathbf{n})$ are the projection operators $\frac{1}{2}(1 \pm \boldsymbol{\sigma}\cdot\mathbf{n})$. The quantities $f^\pm$ are not properly quantum-mechanical amplitudes, since an average on the proton-spin orientation is involved in their definition. However, they can be expressed in terms of corresponding quantities for definite isotopic spin, $f_3^\pm$ for $I = \frac{3}{2}$, and $f_1^\pm$ for $I = \frac{1}{2}$, in exactly the same way as the amplitudes themselves are expressed in terms of the amplitudes for definite isotopic spin. Namely: $|f_a^\pm|^2 = |f_3^\pm|^2 + 4\,\text{Re}(f_3^{\pm *}f_1^\pm) + 4|f_1^\pm|^2$, $|f_b^\pm|^2 = 2|f_3^\pm|^2 - 4\,\text{Re}(f_3^{\pm *}f_1^\pm) + 2|f_1^\pm|^2$, and $|f_c^\pm|^2 = 9|f_3^\pm|^2$, where, according to the two cases of parity, $f_j^\pm = (1/\sqrt{2})\times(E_j \pm F_j)$ or $f_j^\pm = (1/\sqrt{2})(G_je^{\pm i\theta} + H_j)$, in which $j = 1, 3$ and in which E_j, F_j, G_j, and H_j refer to definite isotopic spin. Calling $|f_a^\pm|^2 = x_1^\pm$, $2|f_b^\pm|^2 = x_2^\pm$, $|f_c^\pm|^2 = x_3^\pm$, these equations imply, as is well-known, the existence of a triangle with sides $\sqrt{x_1^+}$, $\sqrt{x_2^+}$, $\sqrt{x_3^+}$ and of a triangle with sides $\sqrt{x_1^-}$, $\sqrt{x_2^-}$, $\sqrt{x_3^-}$.[4] Such conditions are all the conditions imposed on the process by charge independence. The triangle condition for the intensities I follows as a consequence. Provided the two triangle conditions are satisfied, the above equations can be solved, giving the six real quantities $|f_3^\pm|^2$, $|f_1^\pm|^2$, $\text{Re}(f_3^{+*}f_1^+)$, and $\text{Re}(f_3^{-*}f_1^-)$ in terms of the six observed $|f_a^\pm|^2$, $|f_b^\pm|^2$, $|f_c^\pm|^2$. Since the total transition matrix at a given angle is defined in terms of four complex numbers (E_3, F_3, E_1, F_1, or G_3, H_3, G_1, H_1), one relative phase in the transition matrix still remains undetermined from a measurement of the intensities and of the Σ polarizations at that angle. Such an undetermined phase (it can be taken as that between f_3^+ and f_3^-, or that between f_1^+ and f_1^-) is a relative phase between an amplitude for spin up and an amplitude for spin down—it is measurable, in principle, by more complicated experiments such as a measurement of the Σ polarizations when the initial protons are polarized.

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† On leave of absence from Istituto di Fisica dell'Università di Roma, Italy.

[1] Crawford, Cresti, Good, Gottstein, Lyman, Solmitz, Stevenson, and Ticho, Phys. Rev. **108**, 1102 (1957); F. Eisler *et al.*, Phys. Rev. **108**, 1353 (1957); L. Leipuner and R. Adair, Phys. Rev. (to be published).

[2] Measurements of the up-down asymmetries in $\Lambda^0 \to p + \pi^-$ (for directly produced Λ^0) cannot determine the sign of α, but only its magnitude—and that, presumably, not accurately. It has been pointed out that a measurement of the polarization of the nucleon emitted in Λ^0 decay would provide a direct determination of α. R. Gatto, University of California Radiation Laboratory Report UCRL-3795, June, 1957 (unpublished); T. D. Lee and C. N. Yang, Phys. Rev. (to be published).

[3] In particular these Λ^0's produced through intermediate Σ^0 may exhibit a forward-backward asymmetry, simulating parity doublets.

[4] Necessary and sufficient condition for the existence of a triangle with sides $\sqrt{x_1}$, $\sqrt{x_2}$, $\sqrt{x_3}$ is the inequality $x_1^2 + x_2^2 + x_3^2 - 2(x_1x_2 + x_1x_3 + x_2x_3) \leq 0$, which is equivalent to any of the three conditions $|\sqrt{x_r} - \sqrt{x_s}| \leq \sqrt{x_t} \leq \sqrt{x_r} + \sqrt{x_s}$, rst being a permutation of 123.