and third approximations to the quantity M_1 are found in Table I. Again, the fractional change from the first to the third approximations is of the order 30% so that one may expect the third approximation to be fairly reliable. It is also seen from Table I that the experimental value of M_1 can be obtained by a cutoff between 5 and 6 μ, in agreement with Miyazawa.

We discuss finally the contribution

$$M_2=(e/4m)[(\psi_1,\sigma_3\psi_1)-1] \tag{4.26}$$

to the magnetic moment. There is no analog to this quantity in the theory of the point nucleus, so that we are left without systematic guidance. Formally, the calculation is easy; one can use the analog of the normalization equation (4.13) for the total angular momentum S_i alone. After taking expectation values with respect to the state (1,1) and multiplying by f_0, one obtains $\{\sigma_3\}$ as a function of curly brackets $\{c^\dagger c\}$ etc., which are calculated by perturbation calculus. The expectation value itself appears then as a quotient of two power series like M_0. However, it was not possible to find a reasonable sequence of approximations. In view of Miyazawa's unsatisfactory results for this (the "scalar") contribution to the magnetic moment, our result is not surprising. The model is evidently too crude to give even correct order-of-magnitude values for this contribution.

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Dispersion Relations for Scattering in the Presence of a Coulomb Field*

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A dispersion relation is derived for short-range potential scattering in the presence of a Coulomb field, and on the basis of this result a conjecture is made as to the Coulomb modifications of the Goldberger relations for pion-nucleon scattering. The new relations contain in addition to Coulomb phase shifts only amplitudes that are directly measurable experimentally, the assumption of charge independence not being required. Estimates are made to show that the Coulomb phases that appear explicitly are of no practical importance.

I. INTRODUCTION

1

IT has been pointed out by Puppi and Stanghellini[1] that the experimentally determined meson-nucleon forward-scattering amplitudes do not quantitatively satisfy the Goldberger dispersion relations.[2] Since if this discrepancy is real it constitutes the first concrete evidence against the validity of local field theory, the most careful scrutiny of both theory and experiment here is required.

The Goldberger relations are incomplete in that they take account only of strong interactions that satisfy charge independence. The very weak Fermi interactions may be safely ignored, but the electromagnetic interaction, which is only moderately weak, requires a closer study. Agodi, Cini, and Vitale[3] have estimated the corrections due to the production of photons both in the physical and nonphysical region. They find nothing large enough to account for the Bologna discrepancy. In this paper we address ourselves to another possible source of trouble: the Coulomb field.

2

The conventional approach to the Coulomb problem is to analyze the experimental angular distributions so as to extract the so-called "nuclear scattering amplitude," which is defined as the difference between the complete amplitude and the pure Coulomb amplitude. Except for some fairly trivial phases this amplitude is then assumed to be identical with the amplitude one would obtain in the absence of the Coulomb field. Actually it is not identical, and at very low kinetic energies the deviation is large.

Experience with the Coulomb effect in nucleon-nucleon scattering suggests that the energy at which important Coulomb corrections to the meson-nucleon interaction appear is sufficiently small that Puppi and Stanghellini were justified in ignoring them. The importance of the Bologna discrepancy is so great, however, that this possibility of trouble, even if small,

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[1] G. Puppi and A. Stanghellini, Nuovo cimento 5, 1257 (1957).
[2] Goldberger, Miyazawa, and Oehme, Phys. Rev. 99, 986 (1955).
[3] Agodi, Cini, and Vitale, Phys. Rev. 107, 630 (1957).

should be pursued. One way to study the effect we are interested in is to calculate the phase shifts with and without the Coulomb field on the basis of some model for the pion-nucleon interaction. This was the approach used by Noyes.[4] The difficulty here is that no good model exists for the S-wave part of the interaction, which is dominant at low energies, so that the results of such calculations can never be completely convincing.

In this paper a different approach is adopted. We make no attempt to find a "pure" meson-nucleon amplitude to insert into the Goldberger relation, but instead seek a dispersion relation involving quantities that are more or less directly measurable. That such a relation should exist seems *a priori* likely, since the electromagnetic interaction as well as the strong interaction is microscopically causal. However, we do not pretend to derive the relation we shall write down. To do so would require at least the complex procedures of Bogoliubov[5] and no doubt in addition theorems not yet discovered. We shall instead arrive at our relation on the basis of plausibility arguments, starting with the nonrelativistic potential scattering problem, in which fairly rigorous statements can be made.

3

The form for our conjectured dispersion relations turns out to be identical with that of Goldberger,[2] so long as the latter is expressed in terms of real and imaginary parts of a "forward" scattering amplitude. What we propose to change is the definition of the appropriate amplitude. In particular, if the complete forward amplitude in the presence of the (screened) Coulomb field is decomposed according to angular momentum,[6]

$$f=\frac{k}{q^2}\sum_l\left[le^{i\delta_{l-}}\sin\delta_{l-}+(l+1)e^{i\delta_{l+}}\sin\delta_{l+}\right],\quad(3.1)$$

where k is the laboratory wave number of the pion and q the wave number in the barycentric system, and the subscripts $(l+)$ and $(l-)$ refer to states of total angular momentum $l+\frac{1}{2}$ and $l-\frac{1}{2}$, respectively, then the partial nuclear phase shifts $\delta_{l\pm}{}^N$ may be defined as the difference between the full phase shifts $\delta_{l\pm}$ and the corresponding Coulomb phase shifts $\delta_{l\pm}{}^c$. We conjecture that the correct "forward" amplitude to use in the Goldberger relation is then

$$f'=\frac{k}{q^2}\sum_{l=0}e^{2i\sigma_l}[l\exp(i\delta_{l-}{}^N)\sin\delta_{l-}{}^N+(l+1)\exp(i\delta_{l+}{}^N)\sin\delta_{l+}{}^N],\quad(3.2)$$

where the Coulomb phase shifts σ_l are sufficiently well given by

$$\sigma_l\approx\arg\Gamma(l+1+i\eta),\quad(3.3)$$

for $\eta=e^2/v$, with v the laboratory velocity of the pion.

4

The presence of the Coulomb phase shifts in Eq. (3.2) produces more complicated expressions for the real and the imaginary parts of f' than are normally used, the difference being important whenever $2\sigma_l$ is comparable in size to the corresponding nuclear phase shifts. From zero up to 5- or 10-Mev pion energy, $2\sigma_l$ is of the same order of magnitude or larger than the S-wave nuclear phase shifts, so that this energy region should be studied with care. In particular the zero-kinetic-energy scattering lengths, introduced by Goldberger when he makes subtractions in his equations, must be re-examined.

We shall find it desirable not to use the scattering-length concept at all but to make the necessary subtraction at zero total energy rather than zero kinetic energy, thus avoiding emphasis of a point which from the Coulomb point of view is singular. That is, we propose to use dispersion relations for the positive- and negative-pion forward amplitudes in the form

$$\mathrm{Re}f'^{(\pm)}(\nu)=\mp\frac{2f^2}{\nu\mp1/2M}+\frac{\nu^2}{\pi}\mathrm{P}\int_1^\infty\frac{d\nu'}{\nu'^2}\times\left\{\frac{\mathrm{Im}f'^{(\pm)}(\nu')}{\nu'-\nu}+\frac{\mathrm{Im}f'^{(\mp)}(\nu')}{\nu'+\nu}\right\}+C_1\pm\nu C_2,\quad(4.1)$$

where ν is the laboratory pion energy in units of the pion rest mass and C_1 and C_2 are two constants. The symbol "P" means principal value. This is essentially the form used by Haber-Schaim[7] to determine f^2, the Yukawa coupling constant; it has the advantage of treating the experimental information on low-energy S-wave scattering on the same basis as the rest of the data. The original Goldberger form can be reached from Eq. (4.1) by straightforward manipulation.

In Part II of this paper the expression (3.2) is derived for potential scattering in the presence of a screened Coulomb field. In Part III plausibility arguments are given for the extension to relativistic field theory, while numerical estimates of Coulomb effects in the dispersion relation (4.1) will occupy Part IV.

II. POTENTIAL SCATTERING

5

It has been shown by Khuri that the scattering amplitude for all potentials that fall off sufficiently rapidly at large distances and are not too singular at the origin satisfy a simple dispersion relation.[8] Con-

[4] H. P. Noyes, Phys. Rev. **101**, 320 (1956).
[5] Bogoliubov, Medvedev, and Polivanov, Institute for Advanced Study Notes, Princeton, 1956 (unpublished).
[6] We do not assume charge independence. The dispersion relations for negative and positive meson scattering depend on crossing symmetry but not on charge independence, as pointed out by Agodi, Cini, and Vitale in reference 3.
[7] U. Haber-Schaim, Phys. Rev. **104**, 1113 (1956).
[8] N. N. Khuri, Phys. Rev. **107**, 1148 (1957).

sequently, if we assume that the Coulomb potential vanishes beyond some screening radius r_s, we can immediately say that the forward Coulomb scattering amplitude $f^c(E)$ for like charges satisfies the dispersion relation

$$\mathrm{Re}f^c(E)=-\frac{M}{2\pi}\tilde{V}_c+\frac{\mathrm{P}}{\pi}\int_0^\infty dE'\frac{\mathrm{Im}f^c(E')}{E'-E},\qquad(5.1)$$

where $\tilde{V}_c$ is the volume integral of the potential, E is the relative kinetic energy, and M is the reduced mass. Similarly, if there is a short-range nuclear interaction present in addition, which gives rise to no bound states, the scattering amplitude for this case obeys

$$\mathrm{Re}f(E)=-\frac{M}{2\pi}(\tilde{V}_c+\tilde{V}_N)+\frac{\mathrm{P}}{\pi}\int_0^\infty dE'\frac{\mathrm{Im}f(E')}{E'-E},\qquad(5.2)$$

where $\tilde{V}_N$ is the volume integral of the added nuclear potential. Therefore we can *define* a "nuclear" scattering amplitude $f^N\equiv f-f^c$, which [subtracting Eq. (5.1) and (5.2)] obeys

$$\mathrm{Re}f^N(E)=-\frac{M}{2\pi}\tilde{V}_N+\frac{\mathrm{P}}{\pi}\int_0^\infty dE'\frac{\mathrm{Im}f^N(E')}{E'-E}.\qquad(5.3)$$

If the charges are of opposite sign, there will be a sum of terms $R_j^c/(E_j^c-E)$, where E_j^c are the bound-state energies of the (screened) Coulomb field, appearing in both Eqs. (5.1) and (5.2), but if the nuclear potential introduces no appreciable level shifts in these states, the poles do not appear in Eq. (5.3). One may perhaps worry that the coefficients R_j^c may be changed by the presence of a short-range interaction even though the energy levels are not. It can be shown, however, that these coefficients are even less sensitive to modifications of the potential at short distances than are the binding energies.

We can also define "nuclear" phase shifts by $\delta_l^N=\delta_l-\delta_l^c$, and hence

$$f^N(E)=\sum_l\frac{(2l+1)}{2ik}\{[\exp(2i\delta_l)-1]-[\exp(2i\delta_l^c)-1]\}$$

$$=\sum\frac{(2l+1)}{k}\exp(2i\delta_l^c)\exp(i\delta_l^N)\sin\delta_l^N,\qquad(5.4)$$

where k is the wave number. Note that instead of the usual identification

$$\mathrm{Im}f(E)=\sum\,(2l+1)\frac{\sin^2\delta_l}{k}=\frac{k}{2\pi}\sigma_{\mathrm{total}}(E),\qquad(5.5)$$

we have

$$\mathrm{Im}f^N(E)=\sum\,(2l+1)\frac{\sin(\delta_l^N+2\delta_l^c)\sin\delta_l^N}{k}$$

$$=\frac{k}{4\pi}\int d\Omega\left\{\frac{d\sigma(\theta)}{d\Omega}-\frac{d\sigma_c(\theta)}{d\Omega}\right\}.\qquad(5.6)$$

At first sight the last line, which tells us to remove the Coulomb scattering from the total cross section (but to leave the Coulomb-nuclear interference), looks like a simple prescription to apply. It can be used, however, only if the difference $(d\sigma/d\Omega)-(d\sigma_c/d\theta)$ can be extrapolated to the forward direction without ambiguity; in practice this means that except at high energy the alternate expression in terms of phase shifts must be used. Thus we need to know the Coulomb phase shifts δ_l^c explicitly to correct both the real part of the forward scattering amplitude and the imaginary part.

Provided that we are at a high enough energy to have $kr_s\gg1$, and do not have to analyze for partial waves such that $l\gtrsim kr_s$, the Coulomb phase shifts are

$$\delta_l^c=\sigma_l-\eta\ln(2kr_s),\qquad(5.7)$$

where

$$\eta=\pm e^2M/k\quad\text{and}\quad\sigma_l=\arg\Gamma(1+l+i\eta).\qquad(5.8)$$

Were we interested in energies or angles such that these conditions would be violated, we would have to explicitly evaluate δ_l^c for the charge distribution under consideration and use the dispersion relation (5.3) together with (5.4). However, such is not the case in the present application to pion-nucleon scattering, and all explicit reference to the screening may be removed, as we now show.

6

Note that at the energies and for the partial waves of interest the effect of the screening is simply to multiply the scattering amplitude f^N by an energy-dependent phase factor, and that this factor—although it has an essential singularity at the origin—is analytic in the upper half of the complex k plane and approaches unity for large k. The behavior at the origin is physically incorrect, since for low enough energy the S phase must go as $a(r_s)k$, where $a(r_s)$ is the scattering length for our screened Coulomb field. Consequently, it can do no harm to displace the essential singularity below the origin by an amount $k_0\sim1/r_s$. Then the function $\exp(2iS)(f^N-\tilde{V}_N)$, with

$$S=e^2M\frac{\ln[2(k+ik_0)r_s]}{k+ik_0},\qquad(6.1)$$

is analytic in the upper half-plane and goes to zero as k becomes large. Hence we have

$$\mathrm{Re}e^{2iS(E)}[f^N(E)-\tilde{V}_N]=\frac{\mathrm{P}}{\pi}\int_0^\infty dE'\frac{\mathrm{Im}e^{2iS(E')}[f^N(E')-\tilde{V}_N]}{E'-E}.\qquad(6.2)$$

But $e^{2iS}-1$ satisfies the same conditions, so that one obtains

$$\mathrm{Re}(e^{2iS(E)}-1)\tilde{V}_N=\frac{\mathrm{P}}{\pi}\int dE'\frac{\mathrm{Im}(e^{2iS(E')}-1)\tilde{V}_N}{E'-E}\qquad(6.3)$$

and by subtracting Eq. (6.3) from (6.2) we see that the dispersion relation (5.3) is also satisfied by the quantity

$$f'=e^{2iS}f^N=\sum_l \frac{(2l+1)}{k}e^{2i\sigma_l}\exp(i\delta_l^N)\sin\delta_l^N, \quad (6.4)$$

where σ_l is the usual Coulomb phase for $l\ll kr_s$ and otherwise is δ_l^c+S.

Since the Klein-Gordon equation leads to the same radial equation as the Schrödinger equation, if we interpret η as e^2/v and l in the centrifugal term as $[(l+\frac{1}{2})^2-e^4]^{\frac{1}{2}}-\frac{1}{2}$ (and the nuclear interaction is a world scalar), we can immediately extend our results to this case, if these modifications cause no difficulty. The modification of σ_l is clearly trivial, but the phase factor no longer goes to zero for large k. However if S is multiplied by $iK/(k+iK)$, where K is much larger than any wave number of interest, we have restored this property without destroying the analyticity in the upper half-plane, and the proof still stands.

III. EXTENSION TO RELATIVISTIC FIELD THEORY

7

The general procedure by which one might hope to approach the Coulomb problem in local field theory is quite analogous to the foregoing. First one would consider a hypothetical scattering problem, for particles of pionic and nucleonic mass but having only an electromagnetic interaction, and attempt to derive a forward dispersion relation. Probably one would want to give a small but nonzero mass to the photon in order to avoid the necessity of screening and to facilitate the extension of the scattering amplitude into the complex plane. The line of approach initiated by Bogoliubov[5] and developed further by Bremermann, Oehme, and Taylor[9] would presumably be appropriate here, although further development of the theory of many complex variables may be required.

If this first hurdle is overcome, the problem of deriving a forward dispersion relation in the combined presence of strong and electromagnetic interactions will present no additional difficulty, and if one takes a difference, the desired type of relation should follow. Of course, as pointed out by Agodi, Cini, and Vitale,[3] the nonphysical region, $-1<\nu<+1$, is now filled with contributions from intermediate states containing photons, but such effects are to be classified as radiative rather than Coulomb in nature and require a separate discussion, in practice if not in principle. The position and nature of the single-nucleon poles in the nonphysical region are determined by kinematical considerations, the only electromagnetic effect here being a unobservable renormalization of the Yukawa constant. Finally, there are Coulomb bound-state poles in the case of π^-, p scattering, but just as for potential scattering, these will be removed to a very good approximation when the difference is taken between the full amplitude and the electromagnetic amplitude.

In the physical region for $\nu>1$, it is hard to think of any possible modifications of the Goldberger relation which could occur, and the crossing relation tells us what to do for $\nu<-1$. We are therefore led to the conjectured relation (4.1) to be obeyed by the amplitude (3.2). Of course the Coulomb phase shifts are accurately given by Eq. (3.3) only for low pion velocities but at high velocities they are negligible in any case. If it is necessary to introduce a screening radius, we expect that it will be possible to remove the screening phase just as was done for potential scattering.

IV. NUMERICAL ESTIMATE OF COULOMB PHASE-SHIFT CORRECTIONS

8

If the argument in Sec. III is accepted, the only change introduced into the pion-nucleon dispersion relations by the presence of the Coulomb field, outside of tiny corrections associated with the mesonic-hydrogen-atom level shifts, is to replace the forward scattering amplitude by

$$f'^{(\pm)}=\frac{k}{q^2}\sum_l \exp(2i\sigma_l^{(\pm)})[l\exp(i\delta_{l-}^{N(\pm)})\sin\delta_{l-}^{N(\pm)} + (l+1)\exp(i\delta_{l+}^{N(\pm)})\sin\delta_{l+}^{N(\pm)}], \quad (8.1)$$

where the phase shifts for positive pions are real at low energies, but those for negative pions must be taken to be complex in order to account for charge-exchange scattering and radiative capture. (The superscripts $(\pm)$ distinguish between positive and negative pions.) Here in order to estimate the order of magnitude of the correction we may use the usual charge-independent real phase shifts $\delta_{l\pm}^{NI}$, where $I=1$ or 3,[10] so that we have

$$\delta_{l\pm}^{N(+)}=\delta_{l\pm}^{N3},$$
$$\exp(i\delta_{l\pm}^{N(-)})\sin\delta_{l\pm}^{N(-)}\approx\tfrac{1}{3}\exp(i\delta_{l\pm}^{N3})\sin\delta_{l\pm}^{N3} + \tfrac{2}{3}\exp(i\delta_{l\pm}^{N1})\sin\delta_{l\pm}^{N1}. \quad (8.2)$$

We see that for each term of the form $\exp(i\delta_l)\sin\delta_l$ in the uncorrected scattering amplitude, the (additive) correction to the real part is

$$-2\sin\sigma_l\sin(\delta_l+\sigma_l)\sin\delta_l \quad (8.3)$$

and the correction to the imaginary part is

$$2\sin\sigma_l\cos(\delta_l+\sigma_l)\sin\delta_l. \quad (8.4)$$

Evidently these corrections, being proportional to $\sin\sigma_l$, are important only when the Coulomb phase

[9] Bremermann, Oehme, and Taylor, Phys. Rev. (to be published).

[10] It should be said that we are not at all convinced that it is safe to assume charge independence in testing the dispersion relations. However, all we are doing in this section is estimating the order of magnitude of explicit Coulomb phase-shift effects. Any assumption about the nuclear phase shifts that gives them a reasonable size should suffice for this purpose.

becomes substantial in absolute value, that is, at very low energies where the nuclear phases may be approximated by

$$\delta_{l\pm}{}^{NI} \approx a_{l\pm}{}^{I} q^{2l+1}. \qquad (8.5)$$

9

Considering first the correction to the real part of the forward scattering amplitude, we find that so long as σ_l itself is also small, the fractional correction is

$$\frac{2\sin\sigma_l \sin(\delta_l+\sigma_l)}{\cos\delta_l} \approx 2\sigma_l(a_{l\pm}{}^{I} q^{2l+1}+\sigma_l), \qquad (9.1)$$

with

$$\sigma_l{}^{(\pm)} \approx \pm\left(1-\gamma+\sum_{p=1}^{l}\frac{1}{p}\right)\frac{e^2}{k} \qquad (9.2)$$

(using the pion rest mass as our energy unit). Since e^2/k is only 0.05 at 1.5 Mev, not rising to 0.5 until the energy is less than 15 kev, and the scattering amplitude is never measured directly at such a low energy, this approximation in effect can always be used. Further, since the S-wave scattering lengths are ~ -0.1, ~ 0.17 and the largest P-wave scattering length is 0.235, we see that the Coulomb correction to the real part of the amplitude can be safely ignored.

Turning to the imaginary part of the forward scattering amplitude, and using the dispersion relations in the form (4.1), we can again show the correction to be small. From Eq. (8.4), the fractional correction at low energies is

$$2\sigma_l/(a_{l\pm}{}^{I} q^{2l+1}), \qquad (9.3)$$

which becomes comparable to unity for S waves at about 5 Mev and for P waves at about 20 Mev. However, the contribution to the dispersion integral in Eq. (4.1) from the region below 50 Mev is generally less than 1%. Consequently, except for fine details such as finding the precise energy at which the real part of the forward scattering amplitude vanishes, the Coulomb phase-shift effects must be negligible. We have verified this fact by direct calculation.

10

If instead of using Eq. (4.1) we had made our subtraction at zero kinetic energy, that is, had used the dispersion relations in the Goldberger form

$$\mathrm{Re}f^{(\pm)}(\nu) = \pm\frac{2k^2f^2}{\nu\mp 1/2M}\frac{1}{1-(1/4M^2)}+\frac{k^2}{\pi}\int_1^\infty\frac{d\nu'}{k'^2}\times\left\{\frac{\mathrm{Im}f^{(\pm)}(\nu')}{\nu'-\nu}+\frac{\mathrm{Im}f^{(\mp)}(\nu')}{\nu'+\nu}\right\}+C_1'\pm\nu C_2', \qquad (10.1)$$

this circumstance would not have been quite so obvious. In fact, because of the k'^2 in the denominator of the dispersion integral of Eq. (10.1), the Coulomb correction appears to be enormous. However, the region of ν' affected is so close to the lower limit that we can clearly take the denominators $\nu'\pm\nu$ outside the integrals as $1\pm\nu$, and since the integrals are multiplied by $k^2=\nu^2-1$, we see that this Coulomb contribution effectively changes the constants C_1' and C_2' to C_1'' and C_2''. Of course, since the Bologna analysis identified these constants with "scattering lengths," one must check to see if the use of the data was correct. To make this check, note that at 2 Mev, for example, the Coulomb correction, Eq. (9.1), to the real part of the forward scattering amplitude is still negligible, while the integral and the f^2 term of Eq. (10.1), being multiplied by k^2, contribute less than 1% to the right-hand side; thus the identification of C_1'' and C_2'' with scattering lengths can indeed be made to the required accuracy. Except for this redefinition of the constants, then, the Coulomb corrections to Eq. (10.1) are no greater than to the form (4.1).

V. CONCLUSION

11

In a sense the conclusions of this paper are negative. We have been unable to find any Coulomb corrections to the pion-nucleon dispersion relations larger than 1% or 2%. However, there is also a positive aspect. If our conjectured modification of the Goldberger relations is correct (as we hope can ultimately be proved), one can stop worrying about the difference between a "pure" scattering amplitude, generated only by charge-independent strong interactions, and the actual amplitude that is measured. It is possible to use the measured amplitude directly as a test of microscopic causality.

It must be emphasized again that we have not shown (nor do we believe) that the failure of charge independence leads to negligible effects. The amplitudes used in the dispersion relations in a convincing test of microscopic causality must be obtained from experiment without the assumption of isotopic spin conservation. Puppi and Stanghellini[1] avoided the use of charge independence to a considerable extent but not completely.[11] It remains to be seen whether the Bologna discrepancy will persist when an analysis entirely free from the charge-independence assumption is carried out.

[11] For example, they used the Orear scattering lengths [J. Orear, Nuovo cimento 4, 856 (1956)], which depend on charge exchange as well as elastic scattering measurements.