We estimate that this is not important since the Coulomb potential at a radius of $1/m_\pi$ is about 1 Mev, whereas we anticipate a nucleon-baryon interaction of dimensions similar to the nucleon-nucleon potential, say a well depth of 25 Mev. Alternatively the parameter involved is

$$(e^2/\hbar v),$$

where v is the relative velocity of the baryons. v/c varies between 1/7 and 1/20 in the region 1–10 Mev. The effects are thus not likely to be more than 10% on the K-meson spectrum and the distortion of the K spectrum gives direct evidence on the scattering length of the Σ^+p interaction.

Neglecting Coulomb effects, the remarks on bound states and parity apply also to this process near threshold, and it should be possible to obtain information on the Σ^+p interaction and the relative parity of K and Σ.

If four-dimensional or global symmetry in isotopic-spin space is assumed for the K-meson baryon interaction, as proposed by Schwinger and by Gell-Mann,[4] the Σ^+-proton and proton-proton interactions should be identical if K-meson interactions are neglected. The main effect of the K-meson interaction is to give the baryons their observed masses. The increased reduced mass of the Σp system relative to the pp system has the effect of lowering the virtual level known to exist in the singlet state. Thus in this state a very pronounced final-state interaction is predicted by these theories.

(c) Other Σ Processes

The same remarks apply here as have already been made for Λ processes near the threshold for Σ production. There is no possibility of a bound state since the (ΣN) system can transform to the (Λ,N) system and the distortion of the spectrum depends in a rather complicated way on the interaction.

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Phenomenological Analysis of μ Decay*

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The spectrum, asymmetry, and helicity of the electrons from μ decay are calculated from the most general form of the two-component neutrino theory with lepton conservation. In addition to the nonlocal interactions considered recently by Lee and Yang, terms appearing phenomenologically as derivative couplings may occur. In particular, a spectrum that is, aside from a statistical factor, linear in momentum is possible with $\rho\neq\frac{3}{4}$. This could be interpreted as evidence that fermions of baryonic mass are responsible for the nonlocality.

I. INTRODUCTION

THE two-component neutrino theory[1] with lepton conservation allows some rather definite predictions about processes involving neutrinos. At present, only the shape of the μ-decay spectrum is in apparent disagreement with experiment. The simple, local theory predicts for the decay

$$\mu^-\rightarrow e^-+\nu+\bar{\nu} \quad (1)$$

an electron spectrum described by a Michel parameter $\rho=\frac{3}{4}$, while the published experimental values, including radiative corrections, are about 0.68 ± 0.04.[2] Lee and Yang have shown recently that the lower ρ value may be explained by a four-fermion interaction taking place over a small space-time region rather than at a single point as is assumed in the usual Fermi theory of β decay.[3] In the present paper we will show that the complete analysis of μ decay must include consideration of certain other logically distinct possibilities in addition to those considered by Lee and Yang. It will turn out

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[1] A. Salam, Nuovo cimento **5**, 299 (1957); T. D. Lee and C. N. Yang, Phys. Rev. **105**, 1671 (1957); and L. Landau, Nuclear Phys. **3**, 127 (1957).

[2] Sargent, Rinehart, Lederman, and Rogers, Phys. Rev. **99**, 885 (1955); L. Rosenson, Phys. Rev. (to be published); and K. Crowe, Bull. Am. Phys. Soc. Ser. II, **2**, 206 (1957).

[3] T. D. Lee and C. N. Yang, Phys. Rev. **108**, 1611 (1957).

that these possibilities may be distinguished experimentally.

A ρ value different from $\frac{3}{4}$ can also be interpreted as evidence against the two-component lepton-conserving theory.[4] The approach followed in this paper as in reference 3 is, deferring the question of its confirmation or rejection to the low-energy phenomena of β decay, to assume the theory in its simple and restrictive form, and to look to the energetic processes like μ decay or K decay for increasing information about the structure of the weak interactions. After all, the apparent universality in strength of these interactions together with the symmetry laws they all seem to violate, suggests some common origin of weak interactions. Such a universal interaction, involving bosons as well as fermions, must give rise to somewhat complex intermediate states, so that from this point of view a nonlocal interaction would be expected. The last section of this paper touches briefly on some weak decays, other than μ decay, which could yield information about a size structure for weak interactions.

The main point of this paper is that at the present stage these experiments must be analyzed in full generality, without restriction to any preconceived dynamical model. For that reason, without reference to any underlying form of the theory, we proceed to write down directly the most general form of matrix element for μ decay admissible under the proper Lorentz group, assuming the two-component neutrino theory with lepton conservation.

II. GENERAL FORMULATION

In this theory the neutrino field operators always occur as the projection $\frac{1}{2}(1-\gamma_5)\psi_\nu$ of the four-component spinor ψ_ν. The vector[5] $\bar{\psi}_\nu\gamma_\mu\frac{1}{2}(1-\gamma_5)\psi_\nu$ is then the only covariant that can be formed out of two neutrino field operators. The most general form of matrix element for the decay (1) is

$$M=\bar{u}(p)O_\mu u(P)\bar{u}(q)\gamma_\mu\tfrac{1}{2}(1-\gamma_5)v(\bar{q}), \qquad (2)$$

where P, p, q, $\bar{q}$ refer to the four-momenta of muon, electron, neutrino, and antineutrino, respectively, and u and v designate free-particle spinors of positive and negative four-momentum. The most general four-vector that can be constructed from the available four-momenta and Dirac matrices is

$$\begin{aligned}O_\mu=(g_V-g_A\gamma_5)\gamma_\mu-(i/M)(G_V-G_A\gamma_5)p_\mu\\ -(i/M)(G_V'-G_A'\gamma_5)(\boldsymbol{q}-\bar{\boldsymbol{q}})\gamma_\mu\\ -(1/M^2)(G_T-G_{AT}\gamma_5)(\boldsymbol{q}-\bar{\boldsymbol{q}})p_\mu,\end{aligned} \qquad (3)$$

where M is the muon mass. Other covariants, such as $(\boldsymbol{q}+\bar{\boldsymbol{q}})\gamma_\mu$, do not occur because they are not linearly independent of those recorded, in virtue of momentum conservation and the Dirac equation.

[4] G. Bouchiat and L. Michel, Phys. Rev. **106**, 170 (1957); and T. Kinoshita and A. Sirlin, Phys. Rev. **108**, 844 (1957).

[5] Our choice of Dirac matrices is that of Lee and Yang, reference 1.

The g's and G's in Eq. (3) may be functions of the invariants

$$\begin{aligned}Q_{\rm I}&=(q+\bar{q})^2/M^2=2(q\cdot\bar{q})/M^2,\\ Q_{\rm II}&=(p+\bar{q})^2/M^2=2(p\cdot\bar{q})/M^2,\\ Q_{\rm III}&=(p+q)^2/M^2=2(p\cdot q)/M^2,\end{aligned} \qquad (4)$$

formed from the independent momenta in the problem. Only two of these invariants are independent, however, since from momentum conservation we have

$$1+Q_{\rm I}+Q_{\rm II}+Q_{\rm III}=0. \qquad (5)$$

The original two-component theory leading to $\rho=\frac{3}{4}$ is obtained by omitting the last three terms of Eq. (3) and making g_V and g_A constants. The effect of letting g_V and g_A vary with momentum will be considered in Sec. II, and the consequences of including G terms will be studied in Sec. III. The three cases in which $g_{V,A}$ are functions of but one of the Q's correspond to simple nonlocal interactions between fermion pairs, each of which are at the same point. This kind of nonlocality, which was treated by Lee and Yang,[3] originates where the nonlocality is propagated by virtual bosons or fermion pairs emitted in local momentum-independent interactions.

The last three terms in Eq. (3) appear phenomenologically as derivative couplings on the electron, neutrino, and electron and neutrino fields respectively. This second kind of nonlocality can occur even where the fundamental point interactions are momentum-independent, if the virtually propagating particles include fermions. This second kind of nonlocality must be considered on an equal footing with the first kind, treated by Lee and Yang.

The two kinds of nonlocality are both expected to be small on theoretical grounds. Experimentally, the μ spectrum is, in fact, described in good approximation by the original local theory without derivative couplings. The effect of the terms in p_μ, q_μ, and $\bar{q}_\mu$, and of the momentum dependence in the coefficient g's and G's can for this reason be treated additively. In Sec. III the derivative terms will be omitted and the dependence of $g_{V,A}$ on the Q considered only in lowest order. In Sec. IV the momentum dependence of the g's and G's will be ignored.

In writing the interaction in form (2), a particular ordering $(e\mu)(\nu\bar{\nu})$ has been chosen for the four fermions. Results can be transcribed for a different ordering by using standard formulas of the Dirac algebra.

III. NONLOCALITY OF FIRST KIND

Our general formula (3) is first specialized by setting the G's equal to zero and expanding

$$\begin{aligned}g_{V,A}(Q_{\rm I},Q_{\rm II},Q_{\rm III})=g_{V,A}[1-\epsilon_{V,A}Q_{\rm I}\\ -\zeta_{V,A}Q_{\rm II}-\zeta_{V,A}'Q_{\rm III}].\end{aligned} \qquad (6)$$

The ϵ, ζ, ζ' are small dimensionless parameters measuring the nonlocal effect. Further specialization reproduces the examples discussed by Lee and Yang.[3] When only ϵ_V and ϵ_A are nonvanishing, Eq. (6) coincides with their case I, in which $(\nu,\bar{\nu})$ and (μ,e) interact at different points and momentum $(q+\bar{q})$ is propagated between the pairs chosen. If only ζ_V and ζ_A are nonzero, then case II is obtained, in which $(e,\bar{\nu})$ and (μ,ν) interact at different points and momentum $(p+\bar{q})$ is thus exchanged. Finally, the case in which only ζ_V' and ζ_A' are finite is identical with their case III in which (e,ν) and $(\mu,\bar{\nu})$ interact at different points and the relevant momentum transfer is $(p+q)$. Because of relation (5), only two of these three cases are linearly independent in a phenomenological sense; Lee and Yang found indeed that the observables for case I were, in a certain well-defined sense, the sum of those for cases II and III.

The distinctive feature of all these cases is that they lead to spectra and asymmetries that are, aside from the statistical factor, quadratic in electron momentum. The deviation of the ρ value from $\frac{3}{4}$ then is fitted by a nonlocal interaction spread over a distance of the order 0.6×10^{-13} cm, corresponding to an intermediate boson of mass 600 times the electron mass.

IV. NONLOCALITY OF SECOND KIND: DERIVATIVE COUPLING

If the g's and G's in Eq. (3) are now treated as constants, the matrix element (2) leads to the following spectrum of electrons of momentum $p=x(M/2)$ emitted into solid angle $d\Omega$ at angle θ with respect to the polarization axis of the parent μ^-:

$$(2\pi)^{-3}(M/2)^5(\tfrac{1}{3})x^2dxd\Omega(4\pi)^{-1}(\mathcal{P}_1+\mathcal{P}_2), \tag{7}$$

where

$$\begin{aligned}\mathcal{P}_2=&|g_2|^2[(3-2x)-\cos\theta(1-2x)]\\&+\mathrm{Re}(g_2G_2^*)[1-\cos\theta]x+|G_2|^2[1+\cos\theta]\tfrac{1}{4}x^2\\&+\mathrm{Re}(g_2G_2'^*)[3-\cos\theta]2(1-x)+|G_2'|^2\\&\times[(3-x)-\cos\theta(1+x)](1-x)+\mathrm{Re}(g_2G_2''^*)\\&\times[-1+\cos\theta]x(1-x)+|G_2''|^2\\&\qquad\times[(5-4x)-\cos\theta(3-4x)](x^2/20)\end{aligned} \tag{8}$$

is proportional to the square of the matrix element for emitting electrons polarized along their direction of propagation. The corresponding quantity for electrons polarized antiparallel to their direction of propagation is given by $\mathcal{P}_1$, which is obtained from $\mathcal{P}_2$ by the substitutions

$$g_2,\ G_2\rightarrow g_1,\ G_1 \text{ and } G_2',\ G_2'',\ \cos\theta\rightarrow -G_1',\ -G_1'',\ -\cos\theta.$$

In these formulas, we have

$$g_1=g_A+g_V,\quad G_1=G_A+G_V,\quad G_1'=G_A'+G_V',$$
$$G_1''=G_T+G_{AT};\quad g_2=g_A-g_V,\quad \text{etc.}$$

Cross terms between derivative couplings and also terms proportional to the electron mass have been omitted.

The total transition rate for μ decay is

$$\begin{aligned}\tau^{-1}=&(2\pi)^{-3}(M/2)^5(1/6)\{(|g_1|^2+|g_2|^2)\\&+\tfrac{1}{2}\mathrm{Re}(g_1G_1^*+g_2G_2^*)+(1/10)(|G_1|^2+|G_2|^2)\\&+\mathrm{Re}(g_1G_1'^*-g_2G_2'^*)+(2/5)(|G_1'|^2+|G_2'|^2)\\&+(1/10)\,\mathrm{Re}(g_1G_1''^*-g_2G_2''^*)\\&\qquad+(1/30)(|G_1''|^2+|G_2''|^2)\}.\end{aligned} \tag{9}$$

The helicity (or spirality) of electrons depends on direction and momentum as follows:

$$S=(\mathcal{P}_2-\mathcal{P}_1)/(\mathcal{P}_2+\mathcal{P}_1). \tag{10}$$

After the numerator and denominator are integrated over electron angle θ, the resulting expression can be momentum-dependent only if there is some structure of either the first or second kind. However, even if there is structure to the μ decay, the helicity is still momentum-independent if either $\mathcal{P}_1$ or $\mathcal{P}_2$ vanishes, i.e., if the interactions always produce electrons of only one polarization.

The formulas for μ^+ decay are obtained from those above by interchanging subscripts 1 and 2 on the g's and G's. This only changes the sign of the asymmetry and of the angle-integrated helicity.

A. Discussion of Spectrum Evidence

In $\mathcal{P}_1+\mathcal{P}_2$, the various terms of Eq. (3) contribute as follows: (a) The terms in $g_{1,2}$ alone or crossed with $G_{1,2}$ or $G_{1,2}'$ contribute linearly in x. (b) The terms in $g_{1,2}$ crossed with $G_{1,2}''$ and in $G_{1,2}$ or $G_{1,2}'$ by themselves contribute quadratically in x. (c) The terms in $G_{1,2}''$ by themselves contribute cubically in x.

The first question to be answered by the experiments is whether, aside from the statistical factor $x^2dxd\Omega$, the electron spectrum is linear in x or contains higher momentum dependence. None of the present experiments answer this question to anything like the same accuracy with which the ρ value is determined. Nevertheless, it seems likely[6] that the spectrum is over-all linear to within about 15%. This indicates that in the phenomenological description the amount of electron or neutrino derivative coupling must be small compared to the amount of local nonderivative coupling, and that the amount of electron-neutrino tensor coupling present is smaller still:

$$g_{V,A}\gg G_{V,A},\ G_{V,A}'\gg G_{T,AT}. \tag{11}$$

This is what one would expect if the apparent derivative coupling was in fact due to heavy fermions in intermediate states. This also justifies our omitting from Eq. (6) any cross terms between derivative couplings, or any momentum dependence in the g's and G's where they appear multiplied into derivative couplings.

The momentum dependence of the third and fifth terms of Eq. (8) is quadratic, like that in the nonlocal interaction due to virtual bosons. If the μ-meson spec-

[6] We are grateful to K. Crowe for discussion on the experimental situation.

trum, aside from statistical factors, does turn out to be quadratic, it will be difficult to decide between ascribing this to virtual bosons or to virtual fermions producing a moderate amount of effective electron or neutrino derivative coupling.

If the spectrum turns out to be linear, then the interpretation is simplified. In this case the nonlocality cannot be of the type considered by Lee and Yang but must be ascribed to a small amount of electron or neutrino derivative coupling. Under these circumstances, the spectrum is strictly of the Michel form (linear in x):

$$\tau^{-1}x^2dxd\Omega(4\pi)^{-1}\{[12(1-x)+8\rho(\tfrac{4}{3}x-1)] \\ -\xi\cos\theta[4(1-x)+8\delta(\tfrac{4}{3}x-1)]\}, \quad (12)$$

where the total transition rate is

$$\tau^{-1}=(2\pi)^{-3}(M/2)^5(1/6)D, \quad (13)$$

and we have

$$\begin{aligned} D&=(|g_1|^2+|g_2|^2)+\tfrac{1}{2}\,\mathrm{Re}(g_1G_1^*+g_2G_2^*) \\ &\quad+\mathrm{Re}(g_1G_1'^*-g_2G_2'^*), \\ \rho&=\tfrac{3}{4}D^{-1}[(|g_1|^2+|g_2|^2)+\mathrm{Re}(g_1G_1^*+g_2G_2^*)], \\ \xi&=D^{-1}[(|g_1|^2-|g_2|^2)-\tfrac{3}{2}\,\mathrm{Re}(g_1G_1^*-g_2G_2^*) \\ &\quad-\mathrm{Re}(g_1G_1'^*+g_2G_2'^*), \\ \xi\delta&=\tfrac{3}{4}D^{-1}[(|g_1|^2-|g_2|^2)-\mathrm{Re}(g_1G_1^*-g_2G_2^*)]. \end{aligned} \quad (14)$$

The electron helicity is

$$S=\frac{3\bar{A}(1-x)+2\bar{\rho}(\tfrac{4}{3}x-1)-\bar{\xi}\cos\theta[(1-x)+2\bar{\delta}(\tfrac{4}{3}x-1)]}{[3(1-x)+2\rho(\tfrac{4}{3}x-1)]-\xi\cos\theta[(1-x)+2\delta(\tfrac{4}{3}x-1)]}, \quad (15)$$

where

$$\begin{aligned} \bar{A}&=D^{-1}[(|g_2|^2-|g_1|^2)+\tfrac{1}{2}\,\mathrm{Re}(g_2G_2^*-g_1G_1^*) \\ &\quad-\mathrm{Re}(g_1G_1'^*+g_2G_2'^*)], \\ \bar{\rho}&=\tfrac{3}{4}D^{-1}[(|g_2|^2-|g_1|^2)+\mathrm{Re}(g_2G_2^*-g_1G_1^*)], \\ \bar{\xi}&=D^{-1}[-(|g_1|^2+|g_2|^2)+\tfrac{3}{2}(g_1G_1^*+g_2G_2^*) \\ &\quad+\mathrm{Re}(g_1G_1'^*-g_2G_2'^*)], \\ \bar{\xi}\bar{\delta}&=\tfrac{3}{4}D^{-1}[-(|g_1|^2+|g_2|^2)+\mathrm{Re}(g_1G_1^*+g_2G_2^*)]. \end{aligned} \quad (16)$$

B. Asymmetry Evidence

The preliminary evidence gives[7] $\xi\cong-1$, which suggests $|g_1|^2\ll|g_2|^2$. If we adopt this as an assumption and write

$$\eta=\mathrm{Re}(G_2/g_2), \quad \eta'=\mathrm{Re}(G_2'/g_2), \quad (17)$$

the above formulas simplify to

$$\begin{aligned} \rho&=\tfrac{3}{4}(1+\tfrac{1}{2}\eta+\eta'), \\ \xi&=-(1+2\eta+2\eta'), \\ \delta&=\tfrac{3}{4}(1+\tfrac{1}{2}\eta-\eta'), \\ S&=1. \end{aligned} \quad (18)$$

[7] D. H. Wilkinson, Nuovo cimento **6**, 516 (1957).

The present experimental values $\rho=0.68\pm0.04$,[2] $\xi=-0.87\pm0.12$,[7] lead to a fit

$$\eta=0.0\pm0.2, \quad \eta'=-0.08\pm0.06, \quad (19)$$

and the prediction

$$\delta=0.81\pm0.07. \quad (20)$$

The entirely tentative value of η', for example, corresponds to a range of nonlocality $\sim0.1\ \hbar/Mc=0.2\times10^{-13}$ cm, or an intermediate fermion mass of about 2000 electron masses. This is to be contrasted with the corresponding fit to the Yang-Lee kind of nonlocality[3] which gave a range about 0.6×10^{-13} cm corresponding to an intermediate boson mass of about 600 electron masses.

V. CONCLUSIONS AND DISCUSSION

The principal conclusion of this paper is that the detailed shape of the electron spectrum, asymmetry, and helicity may distinguish a different kind of structure in the μ decay than that suggested in reference 3. In particular, a ρ value different from $\frac{3}{4}$ can be obtained in the two-component lepton-conserving theory with a spectrum that remains in the linear Michel form. Assuming the basic interactions to be local nonderivative couplings, such a result suggests the presence in intermediate states of fermions of baryonic mass.

Additional evidence on the interaction between pairs of leptons will be obtained from the corresponding analysis of the processes[8]

$$K\rightarrow\begin{Bmatrix} e \\ \mu \end{Bmatrix}+\pi+\nu. \quad (21)$$

These decays involve even greater momentum transfer than does the μ decay. Since Pais and Treiman, and Werle assumed that the leptons are produced at the same point, the failure of experiment to corroborate their simple predictions would be unambiguous evidence for a nonlocal interaction between the two leptons involved.

Finally the very presence or absence of certain decays in nature gives information on the fermion pairings actually occurring. For example, one might ask whether there is any evidence for a $(\nu\bar{\nu})$ coupling, corresponding to case I of reference 3. Such an interaction would allow the decay

$$K^{\pm}\rightarrow\pi^{\pm}+\nu+\bar{\nu}, \quad (22)$$

whose pion spectrum is peaked at the maximum kinetic energy, 127 Mev, about midway between the maximum energies of mesons from $K_{\mu2}$ and $K_{\pi2}$.

[8] A. Pais and S. B. Treiman, Phys. Rev. **105**, 1616 (1957); J. Werle, Nuclear Phys. **4**, 171 (1957); and Furuichi, Kodama, Sugahara, and Yonezawa, Progr. Theoret. Phys. (Japan) **16**, 64 (1956).