reference 19.[20] A detailed treatment of the Dirac-particle case is given in that paper.†

[20] The velocity vectors $\mathbf{V}_a$ and $\mathbf{V}_b$ that occur in Eq. (48) of this reference are the relativistic or covariant velocities $(d\mathbf{x}/d\tau) = \gamma(d\mathbf{x}/dt)$, where t, τ, and γ are time, proper time, and relativistic contraction factor, respectively. This fact is not made sufficiently clear in the reference.

†*Note added in proof.*—If in the production of a spin-$\frac{3}{2}$ hyperon the initial nucleon is unpolarized and only S and P waves contribute in the final state then the angular distribution of the decay products, when averaged azimuthally, takes the form

$$I_D(\theta,\Theta')=\frac{1}{4\pi}\left[\left(\tfrac{3}{2}\cos^2\Theta'+\tfrac{1}{2}\right)-\frac{\chi\sin^2\theta}{I(\theta)}\left(\tfrac{3}{2}\cos^2\Theta'-\tfrac{1}{2}\right)\right].$$

Here θ is the center-of-mass production angle, $I(\theta)$ is the production cross section, χ is a positive constant, and $\cos\Theta'=(V\cdot K')$. Relativistically Θ' is the angle in the hyperon rest frame between the direction of the decay products and the direction lying in the production plane that makes an angle of $(\pi-\theta)$ with the line of flight (see from the hyperon) of the production center of mass. The azimuthal average is about this direction from which Θ' is measured. The constant χ is restricted by the inequality

$$\chi\leq\frac{\lambda^2}{4}\{A'-C'+[(A'+C')^2-B'^2]^{\frac{1}{2}}\}\leq\frac{\lambda^2}{4}(2A'),$$

where A', B', and C' are the constants given in Eq. (2.7). The form of $I_D(\theta,\Theta')$ given above is an extension to all production angles of a relationship given by Adair,[4] and it will be useful in determining whether the hyperon spin is $\frac{3}{2}$. Its derivation, which is based upon Eqs. (2.5), (2.6), (2.11), and (2.12) and Table I together with the general expressions for the τ_i in terms of the g_i (or h_i) obtained in reference 3, is given by the present authors in the University of California Radiation Laboratory Report UCRL-8005 (unpublished).

ACKNOWLEDGMENTS

We would like to thank Dr. Warren Heckrotte for an extended discussion during the initial phase of this work.

PHYSICAL REVIEW VOLUME 109, NUMBER 2 JANUARY 15, 1958

Hyperon Production in Nucleon-Nucleon Collisions*

GORDON FELDMAN,† *University of Wisconsin, Madison, Wisconsin*

AND

P. T. MATTHEWS,‡ *University of Rochester, Rochester, New York*

(Received August 5, 1957)

The hyperon production process in nucleon-nucleon collisions is analyzed in terms of isotopic spin, and angular momentum near threshold, to deduce certain tests for global symmetry, charge independence, and the parities of Σ, Λ, and K mesons.

Information about the hyperon-nucleon interaction may be obtained from the K-meson spectrum. However, the Σ's and Λ's may interchange owing to this interaction, and Watson's theory of final-state interactions is generalized to allow for this effect. The result is applied to the Σ processes and its significance for the recent calculations of Henley, on the Λ-N final-state interaction is discussed.

1. INTRODUCTION

WE consider the process

$$N+N\rightarrow N+Y+K. \tag{1.1}$$

In Sec. 2 this is analyzed in terms of isotopic spin; certain predictions are made about production ratios, and a relation between cross sections of the type familiar in pion physics is derived.

In Sec. 3 an angular momentum analysis is made near the threshold for Λ production, and near threshold for the process

$$p+p\rightarrow p+\Sigma^++K^0, \tag{1.2}$$

which has the advantage that it does not interfere with the Λ production process. We assume that the final baryons are in s states and that the K meson is emitted in an s or p state according to whether it has the same, or opposite, parity as the accompanying hyperon.[1] The angular distribution of the K meson relative to the initial momentum is either isotropic, or of the form $a+b\cos^2\theta$, according to the two possibilities. Thus, the angular distributions should determine the relative parities of the Σ's, Λ's, and K mesons.

In the corresponding pion process,

$$N+N\rightarrow N+N+\pi, \tag{1.3}$$

the spectrum of the pion is strongly affected by the interaction between the final nucleons.[1] In that case, the two-nucleon potential was already phenomenologically well understood. At present little is known of the Y-N interaction and this is a fairly direct way of getting information. In particular, if bound states of Λ^0N or Σ^+p exist, this should be an efficient way of producing them. However, in this problem the effect is complicated by the fact that the final state inter-

* Supported in part by the U. S. Atomic Energy Commission.

† Now at the Department of Physics, Johns Hopkins University, Baltimore, Maryland.

‡ Now at the Department of Mathematics, Imperial College, London, England.

[1] K. M. Watson and K. N. Brueckner, Phys. Rev. 83, 1 (1951); A. H. Rosenfeld, Phys. Rev. 96, 139 (1954).

action has two channels. The hyperon emerging from the "primary" interaction may be, say, Σ, which has the possibility of being turned into a Λ by the "final-state" interaction with the nucleon. Watson's theory[2] is generalized in 4 to allow for this possibility. We show that sufficiently near the threshold for Λ production (say 20 Mev in c. m. system) it is reasonable to neglect the coupling to the (Σ,N) system. The final-state interaction for this process has recently been discussed by Henley,[3] who has not considered explicitly the effect of the two channels. Our results show that the K spectrum should not be qualitatively altered, but that only near threshold can it be interpreted in terms of a scattering length corresponding to a static Λ-nucleon potential. We also show that the excitation function will give information on the relative parities of the Λ and K mesons. Similar information about Σ may be obtained from the production of Σ^+ and K^0 near threshold since for this interaction the baryons are in a $T=\frac{3}{2}$ state, and there is no interference with the (Λ,N) system. The four-dimensional isotopic symmetry—or "global" symmetry—suggested by Schwinger and by Gell-Mann[4] has rather specific implications at this point, which may be tested by experiment. Λ production more than about 20 Mev above threshold and Σ production, when the baryons are in a $T=\frac{1}{2}$ state, have a final-state interaction effect which depends in a rather complicated way on the hyperon-hyperon potential and does not appear to be a useful source of information.

2. ISOTOPIC SPIN ANALYSIS

The initial nucleons are in an isotopic spin state of $T=0$, 1. There are five possible final isotopic spin states, which may be classified according to total isotopic spin $(T=0,1)$, the nature of the hyperons $(Y=\Sigma,\Lambda)$ and the isotopic spin combination of the hyperon and nucleon in the final state ($\frac{3}{2}$ or $\frac{1}{2}$). The corresponding transition amplitudes are given in Table I.

For Σ's in the final state there are in all fourteen different processes, if one considers differential cross sections in which, as pointed out by Van Hove, Marshak, and Pais,[5] one can distinguish between pn and np initial states. However, under charge symmetry

$$p\leftrightarrow n,\quad K^+\leftrightarrow K^0,\quad \Sigma^+\leftrightarrow\Sigma^-,\quad \Sigma^0\leftrightarrow\Sigma^0.$$

There are then seven pairs of amplitudes not related by charge symmetry, which can be expressed in terms of three complex amplitudes (or five real parameters).

[2] K. M. Watson, Phys. Rev. **88**, 1163 (1952).

[3] E. M. Henley, Phys. Rev. **106**, 1083 (1957).

[4] J. Schwinger, *Proceedings of the Seventh Annual Rochester Conference on High-Energy Nuclear Physics, 1957* (Interscience Publishers, Inc., New York, 1957), Sec. IX; M. Gell-Mann, *Proceedings of the Seventh Annual Rochester Conference on High-Energy Nuclear Physics, 1957* (Interscience Publishers, Inc., New York, 1957), Sec. IX.

[5] Van Hove, Marshak, and Pais, Phys. Rev. **88**, 1211 (1952).

TABLE I. Possible transitions and corresponding amplitudes; e.g., $(\Sigma,N)_{\frac{3}{2}}$ implies that in the final state the hyperon is a Σ and that the Σ and nucleon are in an isotopic spin $\frac{3}{2}$ state, which combines with the K to give total isotopic spin $T=1$.

Total isotopic spin	Final state	Amplitude
1	$(\Sigma,N)_{\frac{3}{2}}$	A_3
1	$(\Sigma,N)_{\frac{1}{2}}$	A_1
1	$(\Lambda,N)_{\frac{1}{2}}$	A_Λ
0	$(\Sigma,N)_{\frac{1}{2}}$	B_Σ
0	$(\Lambda,N)_{\frac{1}{2}}$	B_Λ

These are

$$\begin{aligned}
&(pp|p\Sigma^0K^+)=-2^{-\frac{1}{2}}A_3-A_1=(nn|n\Sigma^0K^0),\\
&(pp|n\Sigma^+K^+)=-\tfrac{1}{2}A_3+2^{\frac{1}{2}}A_1=(nn|p\Sigma^-K^0),\\
&(pp|p\Sigma^+K^0)=\tfrac{3}{2}A_3=(nn|n\Sigma^-K^+),\\
&(pn|p\Sigma^-K^+)=-\tfrac{1}{2}A_3-2^{-\frac{1}{2}}A_1+2^{\frac{1}{2}}B_\Sigma\\
&\qquad\qquad\qquad=(np|n\Sigma^+K^0),\qquad (2.1)\\
&(np|p\Sigma^-K^+)=-\tfrac{1}{2}A_3-2^{-\frac{1}{2}}A_1-2^{\frac{1}{2}}B_\Sigma=(pn|n\Sigma^+K^0),\\
&(pn|n\Sigma^0K^+)=-2^{-\frac{1}{2}}A_3+\tfrac{1}{2}A_1-\tfrac{1}{2}B_\Sigma=(np|p\Sigma^0K^0),\\
&(np|n\Sigma^0K^+)=-2^{-\frac{1}{2}}A_3+\tfrac{1}{2}A_1+\tfrac{1}{2}B_\Sigma=(pn|p\Sigma^0K^0).
\end{aligned}$$

From charge symmetry, it follows immediately that in np collisions, the production ratios are

$$K^+/K^0=1, \qquad (2.2)$$

$$\Sigma^+/\Sigma^-=1, \qquad (2.3)$$

and that K^+ and K^0 should appear in equal numbers associated with Σ^0.

From charge independence, in pp collisions the production ratio

$$\frac{K^+}{K^0}=\frac{|A_3|^2+4|A_1|^2}{3|A_3|^2}\geq\tfrac{1}{3}, \qquad (2.4)$$

which provides a rough check on the theory and, if one accepts the theory, measures the relative strength of the amplitude in the two $T=1$ states. If either amplitude is found to dominate, the ratios of the three possible reactions in pp collisions should then be determined by the coefficients in Eqs. (2.1). Also the obvious linear relation between the amplitudes for the three processes gives rise to the usual triangle inequalities between the square roots of the three differential cross sections.

The seven equations (2.1) imply two relations between the amplitudes. One of these involves the relative phases; the other is

$$\begin{aligned}
\sigma_T(pn|n\Sigma^0K^+)-\tfrac{1}{2}\sigma_T(pn|p\Sigma^-K^+)=\tfrac{1}{2}\sigma_T(pp|n\Sigma^+K^+)\\
+(1/18)\sigma_T(pp|p\Sigma^+K^0)-\sigma_T(pp|p\Sigma^0K^+). \qquad (2.5)
\end{aligned}$$

For the Λ particles in the final state, one has, similarly,

$$\begin{aligned}
&(pp|p\Lambda^0K^+)=A_\Lambda=(nn|n\Lambda^0K^0),\\
&(pn|n\Lambda^0K^+)=\tfrac{1}{2}A_\Lambda-B_\Lambda=(np|p\Lambda^0K^0),\qquad (2.6)\\
&(pn|p\Lambda^0K^0)=\tfrac{1}{2}A_\Lambda+B_\Lambda=(np|n\Lambda^0K^+);
\end{aligned}$$

from which it follows that

$$\sigma_T(pn|n\Lambda^0K^+)=\sigma_T(pn|p\Lambda^0K^0), \tag{2.7}$$

giving

$$K^+/K^0=1, \tag{2.8}$$

for the pn interaction.

3. ANGULAR MOMENTUM ANALYSIS AND PARITIES

Only the relative parities of Σ, Λ, and K are of physical significance. Let us adopt the convention that the Λ has the same parity as the nucleons. Then, as in the case for π mesons we assume that, in the Λ processes well below threshold for Σ production, the K meson will be emitted in an s state if it is scalar, and in a p state if it is a pseudoscalar. We further assume that near threshold the final baryons are in a relative s state. For pure $T=1$ processes (i.e., pp processes) the angular distribution of the K particle is isotropic for the scalar case, and of the form $a+b\cos^2\theta$ for the pseudoscalar case.[3]

Exactly the same analysis may be applied to the states with final Σ^+ and p, which do not interfere with the Λ processes, and at energies near enough to threshold to make the assumptions reasonable. If the K appears to be scalar with respect to the Λ, but pseudoscalar with respect to the Σ (or vice versa) this implies that Λ and Σ have opposite parities. If Λ and Σ have the same parity, then the K's should be isotropic (scalar) or nonisotropic (pseudoscalar) in all the pp initiated processes near threshold.

4. FINAL-STATE INTERACTION

We now consider the process in somewhat more detail.

The interaction Hamiltonian is

$$H_{\text{int}}=H_\pi+H_K, \tag{4.1}$$

where H_π and H_K are the interactions of the baryons with the pions and K meson, respectively.

$$H_\pi=\int J\phi d^3x, \tag{4.2}$$

where J is the baryon current (a pseudoscalar in configuration space and vector in isotopic-spin space). The exact transition amplitude for $N+N\rightarrow Y+N+K$ is

$$T_K={}_\pi(YNK^{(-)}|H_K|NN^{(+)})_{\pi K}\delta(E_i-E_f), \tag{4.3}$$

where $|\ )_{\pi K}$ denotes an eigenstate of the complete Hamiltonian, $|\ )_\pi$ denotes an eigenstate with H_π as interaction and neglecting H_K; the superscripts $(+)$ and $(-)$ denote the outgoing and incoming scattered waves, as defined by Lippmann and Schwinger.[6] Introduce a representation in configuration space $x_a x_p\cdots$), where x_a is the coordinate of the particle a, etc. Then

$$T_K=\sum_{x_a}{}_\pi(YNK^{(-)}|x_a\cdots)\times(x_a\cdots|H_K|NN^{(+)})_{\pi K}\delta(E_i-E_f). \tag{4.4}$$

The summation $\sum_{x_a}$ is over all states which are accessible to the state ${}_\pi(YNK^{(-)}|$ through the interaction H_π. Note that in the final state the K particle is not coupled to the baryons and travels free. The summation $\sum_{x_a}$ will be restricted to states which involve Σ or Λ, a nucleon and a K meson, since the remaining states include pions and/or antibaryons and differ considerably in energy from the final state. Thus the states $(x_a\cdots|$ may be replaced by $(\alpha,\mathbf{r},\boldsymbol{\xi})$ where α denotes Σ or Λ, $\mathbf{r}$ is the relative position vector of the nucleon and the hyperon, and $\boldsymbol{\xi}$ the vector between the K meson and the center of mass of the baryons.

$$T_K=\sum{}_\pi(NYK^{(-)}|\alpha,\mathbf{r},\boldsymbol{\xi})\times(\alpha,\mathbf{r},\boldsymbol{\xi}|H_K|NN^{(+)})_{\pi K}\delta(E_i-E_f). \tag{4.5}$$

The sum is over α, $\mathbf{r}$, and $\boldsymbol{\xi}$. If the calculation is made in the c. m. system the factor involving the K meson and $\boldsymbol{\xi}$ is just $\exp(i\mathbf{p}\cdot\boldsymbol{\xi})$ where $\mathbf{p}$ is the K-meson momentum. We can consider the $\boldsymbol{\xi}$ integration to be carried out and write

$$T_K=\sum{}_\pi(NY^{(-)}|\alpha,\mathbf{r})\times(\alpha,\mathbf{r},K|H_K|NN^{(+)})_{\pi K}\delta(E_i-E_f). \tag{4.6}$$

The final-state effect is now concentrated in the first term ${}_\pi(NY^{(-)}|\alpha\mathbf{r})$ on the right-hand side, which depends only on the relative momentum, $\mathbf{k}$ of the two baryons in the final state, and contains all the dependence of T_K on this variable, apart from some rather weak restrictions imposed by the conservation of energy.

Consider this factor. Since it is invariant under the time-reversal operator T, we have

$$(NY^{(-)}|\alpha\mathbf{r})=(NY^{(-)}|T^{-1}T|\alpha\mathbf{r})=(\alpha\mathbf{r}|NY^{(+)}). \tag{4.7}$$

The latter is the outgoing-scattered-wave eigensolution of

$$|H_0+H_\pi-E|NY^{(+)})=0. \tag{4.8}$$

If the interaction H_π is replaced by an effective "potential" V which only has nonvanishing matrix elements between nucleon and hyperon states, this may be written

$$\sum_\beta\{[H_0(\alpha,\mathbf{r})-E]\delta_{\alpha\beta}+V_{\alpha\beta}(\mathbf{r})\}(\beta,\mathbf{r}|NY^{(+)})=0. \tag{4.9}$$

The free energy is

$$H_0(\alpha,\mathbf{r})=m_n+m_\alpha+(1/2\mu_\alpha)k^2, \tag{4.10}$$

where μ_α is the reduced mass of the nucleon and the α hyperon (Λ or Σ). The total energy is

$$E=m_n+m_\alpha+(1/2\mu_\alpha)k_\alpha^2. \tag{4.11}$$

Thus (4.9) becomes

$$\sum_\beta\left[\delta_{\alpha\beta}\left(\frac{1}{2\mu_\alpha}k^2-\frac{1}{2\mu_\alpha}k_\alpha^2\right)+V_{\alpha\beta}(\mathbf{r})\right](\beta,\mathbf{r}|NY^{(+)})=0. \tag{4.12}$$

[6] B. A. Lippmann and J. Schwinger, Phys. Rev. 79, 469 (1950).

These are two coupled differential equations for the two components of $|NY^{(+)})$. In the asymptotic region the S-wave solution is

$$(\alpha,\mathbf{r}|N,\beta^{(+)})=\frac{\delta_{\alpha\beta}\exp(-ik_\alpha r)-S_{\alpha\beta}\exp(ik_\alpha r)}{k_\alpha^{\frac{1}{2}}k_\beta^{\frac{1}{2}}r}, \quad (4.13)$$

where the normalization corresponds to

$$(\alpha,\mathbf{r}|N\beta^{(+)})=\delta_{\alpha\beta}\exp(ik_\alpha z)+f_{\alpha\beta}\exp(ik_\alpha r)/r. \quad (4.14)$$

If the energy E is such that the state $\alpha=\Sigma$ is not physically possible,

$$\exp(ik_\Sigma r)\rightarrow\exp(-|k_\Sigma|r). \quad (4.15)$$

The Λ-Σ mass difference is about 80 Mev and we anticipate N-Y potentials of the same order of magnitude as the N-N potential. Near threshold for Λ production one may neglect $V_{\Sigma\Sigma}$ compared with $|k_\Sigma^2|2\mu_\Sigma$ ($\simeq m_\Sigma-m_\Lambda$). In this approximation, and in momentum space, the equation for

$$(\Lambda,\mathbf{k}|N\Lambda^{(+)})=\psi(\mathbf{k}), \quad (4.16)$$

is

$$(k^2-k_\Lambda^2)\psi(\mathbf{k})+2\mu_\Lambda V_{\Lambda\Lambda}(\mathbf{k},\mathbf{k}')\psi(\mathbf{k}')=4\mu_\Sigma\mu_\Lambda\frac{V_{\Lambda\Sigma}(\mathbf{k},\mathbf{k}')V_{\Sigma\Lambda}(\mathbf{k}',\mathbf{k}'')}{k'^2+|k_\Sigma^2|}\psi(\mathbf{k}''), \quad (4.17)$$

where summation over all repeated variables is implied. Taking the threshold value

$$|k_\Sigma^2|/2\mu_\Sigma\simeq 80 \text{ Mev}, \quad V\simeq 20 \text{ Mev},$$

the effect of the coupling to the ΣN system is to modify the $V_{\Lambda\Lambda}$ potential by a factor which may be very roughly estimated as $\frac{1}{4}$. As a first approximation it seems reasonable to neglect this effect within say 20 Mev of threshold.

Near Σ threshold in $(\Sigma,N)_{\frac{1}{2}}$ states, ($T_K=A_1$), one has precisely the same equation with $\Sigma\leftrightarrow\Lambda$, and the denominator in the final term replaced by $k^2-k_\Lambda^2$. This is not obviously small and the coupling to the Λ system must be included. For the $(\Sigma,N)_{\frac{3}{2}}$ states ($T_K=A_3$) there is no coupling to the (Λ,N) system.

In the regions where the coupling between the (ΣN) and (Λ,N) systems may be neglected, the final-state theory of Watson may be applied directly and the distortion in the K-meson spectrum expressed in terms of the scattering length. This in turn gives direct information about the appropriate diagonal part of the potential, say, $V_{\Lambda\Lambda}$.

If the coupling of the two systems cannot be neglected, Watson's theory may be generalized with the use of (4.6). Consider, for example, the production of Σ's near threshold in the $T=\frac{1}{2}$ state. The summation in (4.6) must be taken over $\alpha=\Sigma$, Λ. Using an argument very similar to that of Wigner,[7] it can be shown that

$$(S-1)_{\Sigma\Sigma}\sim k_\Sigma, \quad S_{\Sigma\Lambda}\sim k_\Sigma^{\frac{1}{2}}. \quad (4.18)$$

Thus from (4.13), if the range of interaction is a and $k_\Sigma a\ll 1$, the wave function in the region of the primary interaction may be taken to be

$$(\Sigma,\mathbf{r}|N,\Sigma^{(+)})\sim\frac{(1-S)_{\Sigma\Sigma}}{k_\Sigma}f(\mathbf{r}),$$
$$(\Lambda,\mathbf{r}|N,\Sigma^{(+)})\sim\frac{S_{\Sigma\Lambda}}{k_\Sigma^{\frac{1}{2}}}g(\mathbf{r}), \quad (4.19)$$

where f and g are functions of $\mathbf{r}$ (and k_Λ). The dependence on k_Σ thus cancels in both terms to this approximation, and the amplitude for say $(pp|n\Sigma^+K^+)$ can be written as the sum of three terms (two from A_1 and one from A_3) all of which have similar k_Σ dependence, with unknown relative phases. Thus, although the final-state interaction will certainly play a part in these processes, it does not appear possible to interpret it in any simple way in terms of the scattering matrix or equivalent interaction potential.

V. CONCLUSIONS

(a) Λ Processes

These have recently been discussed by Henley[3] who has used Watson's[2] "one-channel" theory. Since we have shown that the momentum dependence to be expected is not altered qualitatively by the inclusion of two channels, these effects should not modify the conclusions in any important way. However, only close to threshold for Λ production (say kinetic energy $T<20$ Mev in c. m. system) can the parameter a which occurs be interpreted as the scattering length corresponding to the static $V_{\Lambda\Lambda}$ potential.

If a hyperdeuteron ${}_\Lambda H^2$ exists, this should give rise to a line at the high-energy limit of the spectrum. The double event

$$p+p\rightarrow{}_\Lambda H^2+K^+, \quad {}_\Lambda H^2\rightarrow p+p+\pi^-, \quad (5.1)$$

would be very easily found in a bubble chamber since all the particles are charged.

It has been remarked above[3] that the angular distribution of the K meson gives an indication of the relative K,Λ parity. With the same assumptions and including the effect of the final-state interaction the cross section should behave like

$$\sigma\sim T,\ T^2, \quad (5.2)$$

for scalar, or pseudoscalar mesons respectively, where T is the kinetic energy in the c.m. system.

(b) $p+p\rightarrow\Sigma^++p+K^0$

This is the only reaction in which the final baryons are in the $T=\frac{3}{2}$ state and for which there is no final-state interaction between the (Λ,N) and (Σ,N) systems. The problem is thus exactly analogous to that of pion production by nucleons. The only possible complication is the Coulomb repulsion between the final baryons.

[7] E. P. Wigner, Phys. Rev. 73, 1002 (1948).

We estimate that this is not important since the Coulomb potential at a radius of $1/m_\pi$ is about 1 Mev, whereas we anticipate a nucleon-baryon interaction of dimensions similar to the nucleon-nucleon potential, say a well depth of 25 Mev. Alternatively the parameter involved is

$$(e^2/\hbar v),$$

where v is the relative velocity of the baryons. v/c varies between 1/7 and 1/20 in the region 1–10 Mev. The effects are thus not likely to be more than 10% on the K-meson spectrum and the distortion of the K spectrum gives direct evidence on the scattering length of the Σ^+p interaction.

Neglecting Coulomb effects, the remarks on bound states and parity apply also to this process near threshold, and it should be possible to obtain information on the Σ^+p interaction and the relative parity of K and Σ.

If four-dimensional or global symmetry in isotopic-spin space is assumed for the K-meson baryon interaction, as proposed by Schwinger and by Gell-Mann,[4] the Σ^+-proton and proton-proton interactions should be identical if K-meson interactions are neglected. The main effect of the K-meson interaction is to give the baryons their observed masses. The increased reduced mass of the Σp system relative to the pp system has the effect of lowering the virtual level known to exist in the singlet state. Thus in this state a very pronounced final-state interaction is predicted by these theories.

(c) Other Σ Processes

The same remarks apply here as have already been made for Λ processes near the threshold for Σ production. There is no possibility of a bound state since the (ΣN) system can transform to the (Λ,N) system and the distortion of the spectrum depends in a rather complicated way on the interaction.

VI. ACKNOWLEDGMENTS

Most of this work was carried out while both authors were enjoying the hospitality of the University of Rochester. One of the authors (P. T. M.) was on a leave of absence from the Department of Mathematical Physics, The University, Birmingham, England. We would like to acknowledge helpful discussions with Charles Goebel, George Salzman, and George Rawitscher.

PHYSICAL REVIEW VOLUME 109, NUMBER 2 JANUARY 15, 1958

Phenomenological Analysis of μ Decay*

Sidney Bludman, *Radiation Laboratory, University of California, Berkeley, California*

AND

Abraham Klein,† *Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania*

(Received September 9, 1957)

The spectrum, asymmetry, and helicity of the electrons from μ decay are calculated from the most general form of the two-component neutrino theory with lepton conservation. In addition to the nonlocal interactions considered recently by Lee and Yang, terms appearing phenomenologically as derivative couplings may occur. In particular, a spectrum that is, aside from a statistical factor, linear in momentum is possible with $\rho\neq\frac{3}{4}$. This could be interpreted as evidence that fermions of baryonic mass are responsible for the nonlocality.

I. INTRODUCTION

THE two-component neutrino theory[1] with lepton conservation allows some rather definite predictions about processes involving neutrinos. At present, only the shape of the μ-decay spectrum is in apparent disagreement with experiment. The simple, local theory predicts for the decay

$$\mu^-\rightarrow e^-+\nu+\bar{\nu} \quad (1)$$

an electron spectrum described by a Michel parameter $\rho=\frac{3}{4}$, while the published experimental values, including radiative corrections, are about 0.68±0.04.[2] Lee and Yang have shown recently that the lower ρ value may be explained by a four-fermion interaction taking place over a small space-time region rather than at a single point as is assumed in the usual Fermi theory of β decay.[3] In the present paper we will show that the complete analysis of μ decay must include consideration of certain other logically distinct possibilities in addition to those considered by Lee and Yang. It will turn out

* This work was supported by the U. S. Atomic Energy Commission.

† Supported in part by both the U. S. Atomic Energy Commission and by the Air Force Research and Development Command.

[1] A. Salam, Nuovo cimento **5**, 299 (1957); T. D. Lee and C. N. Yang, Phys. Rev. **105**, 1671 (1957); and L. Landau, Nuclear Phys. **3**, 127 (1957).

[2] Sargent, Rinehart, Lederman, and Rogers, Phys. Rev. **99**, 885 (1955); L. Rosenson, Phys. Rev. (to be published); and K. Crowe, Bull. Am. Phys. Soc. Ser. II, **2**, 206 (1957).

[3] T. D. Lee and C. N. Yang, Phys. Rev. **108**, 1611 (1957).