Recoil Protons from Meson Photoproduction in Hydrogen and Deuterium*

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The spectrum of photoprotons produced by 500-Mev bremsstrahlung on hydrogen and deuterium has been measured at laboratory angles of 29.7°, 41.2°, and 51.8° with a telescope of three scintillation counters. From the hydrogen data the π^0 cross section of hydrogen was obtained, which agrees with the previous results of this laboratory. The contribution of the deuterium photodisintegration reaction was subtracted from the deuterium counting rate and the deuterium-to-hydrogen ratio of recoil protons from meson photoproduction was obtained. This ratio was also calculated from the deuterium π^- and hydrogen π^0 photoproduction cross sections, by means of the "spectator model" of the deuteron. In this model photons interact with only one of the deuterium nucleons and the initial momentum of the target nucleon is taken into account. The ratio predicted by this model is significantly larger than the experimental ratio.

I. INTRODUCTION

IN order to compare the current theories of pion photoproduction with experiment, one would like to bombard free neutrons as well as free protons with photons. Since the deuteron is the simplest system involving neutrons, it is important to understand how the observed pion production from a bound nucleon in deuterium is related to pion production from a free nucleon. One may obtain some insight into this problem by studying positive pion production from the free proton and from the bound proton in deuterium.[1] Additional experiments have measured the deuterium-to-hydrogen ratio of neutral pion production,[2] and the negative to positive ratio of pion production from deuterium.[3] These last two experiments involve pion production from the bound neutron. From them one would like to calculate the cross sections for pion production from free neutrons.

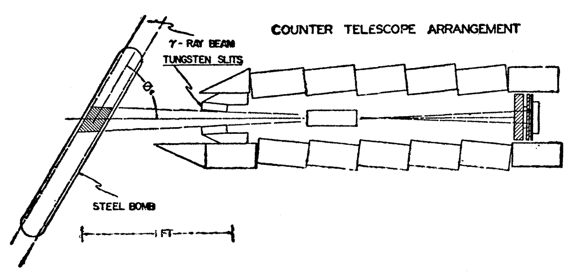

FIG. 1. Proton telescope and gas target geometry. The lead plug shown in the center of the telescope was used during background runs, to stop the true proton counts, but permitted the protons scattered from the shielding into the counter to be counted.

To obtain the free nucleon cross sections from the bound nucleon cross sections, a model of pion photoproduction is needed. One model is the "spectator model",[4] in which the deuteron is considered to be so loosely bound that the photon interacts with only one nucleon which is considered as free, except for its initial momentum. The deuterium photoproduction reactions become:

$$\begin{aligned}\gamma+p_D &\rightarrow \pi^+ + n\\ &\rightarrow \pi^0 + p,\\ \gamma+n_D &\rightarrow \pi^- + p\\ &\rightarrow \pi^0 + n,\end{aligned}$$

where the subscript D means that the target nucleons are moving with the momentum distribution of deuterium nucleons.

This paper reports the results of an experimental test of this model. The deuterium-to-hydrogen ratio of recoil protons from pion production was measured. This ratio can be calculated from previous pion photoproduction experiments by means of the spectator model. The comparison of the calculated and experimental ratios is a check on the validity of the spectator model. As an additional test of the model, we have investigated the effects of nucleon motion on the deuterium-to-hydrogen ratio of pion photoproduction and on the negative-to-positive ratio of pions from deuterium.

II. EXPERIMENTAL METHOD

The synchrotron beam collimation and monitoring equipment as well as the high-pressure, low-tempera-

TABLE I. The maximum and average values of the correction applied to the telescope data.

Correction	Maximum value	Average value
Background	−33%	−17%
Absorption	16%	6.5%
Accidentals	−8%	−2%
Deadtime	5.8%	2%
Slit edge	−5.2%	−2%

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[1] R. M. Littauer and D. Walker, Phys. Rev. **86**, 838 (1952); Jenkins, Luckey, Palfrey, and Wilson, Phys. Rev. **95**, 179 (1954); Lebow, Field, Frisch, and Osborne, Phys. Rev. **85**, 681 (1952); Crowe, Friedman, and Hagerman, Phys. Rev. **100**, 1799 (1955); White, Jakobson, and Schulz, Phys. Rev. **88**, 836 (1952); Sands, Teasdale, Walker, Querzoli, and Bloch (private communication).

[2] G. Cocconi and A. Silverman, Phys. Rev. **88**, 1230 (1952); Keck, Tollestrup, and Bingham, Phys. Rev. **103**, 1549 (1956).

[3] Sands, Teasdale, and Walker, Phys. Rev. **95**, 592 (1954).

[4] J. Keck and R. Littauer, Phys. Rev. **88**, 139 (1952).

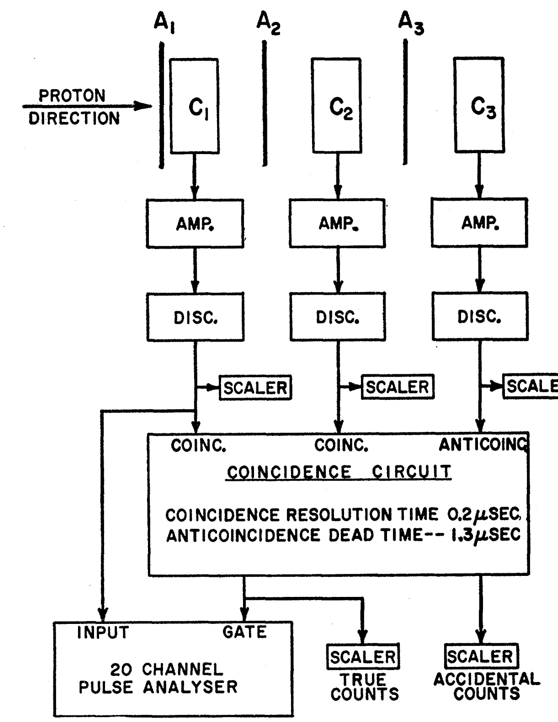

FIG. 2. Schematic of the proton telescope electronic system. The absorbers A_1, A_2, and A_3 determined the range of the particle observed. The pulse height from counter C_1 was recorded in the 20-channel pulse-height analyzer, which was gated by the coincidence circuit. Protons were identified by their pulse height in C_1.

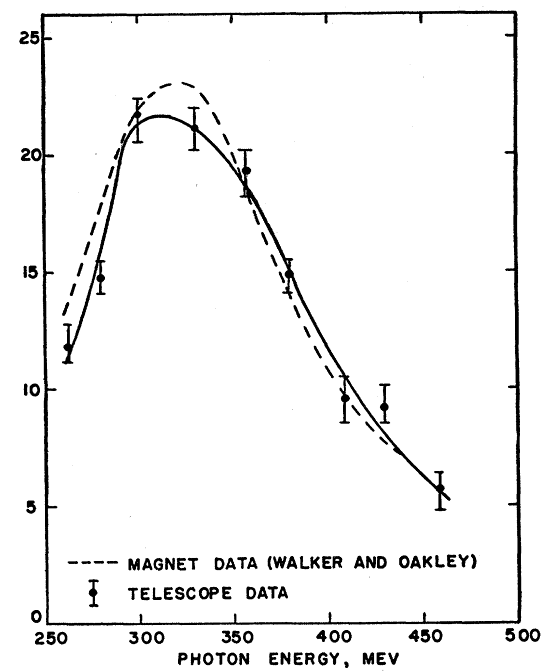

FIG. 3. Differential cross section for photoproduction of π^0 mesons from hydrogen, at a proton angle of 29.7°. Units are 10^{-30} cm^2 per steradian.

ture target have been previously described.[5,6] The proton telescope (Figs. 1 and 2) consisted of three plastic scintillation counters and a tungsten slit, similar to the four-counter telescope previously described[6] with the front counter replaced by a tungsten slit. The angular resolution function of the telescope was an isosceles triangle with a maximum width of 4.7° After runs with gas in the target, a $1\frac{1}{4}$ in. by 4 in. by 4 in. lead plug was inserted in the telescope (Fig. 1) and background runs were taken to count protons which were scattered into the counters from the telescope shielding. Empty target runs were taken in the same way and subtracted from the gas runs. This method eliminated all counts from sources other than the hydrogen. The only undesired events which could affect the counting rate are: (A) untrue counts from hydrogen which are stopped when the plug is inserted, (B) untrue counts from the hydrogen which are produced by the plug, and (C) true counts which somehow get through the plug and into the counter. From a study of these cases it is concluded that they are small, and further, that they tend to cancel.

Table I gives the average and maximum values of the corrections applied to the data. In addition the following possible sources of error were examined and found to be negligible: absorption of higher energy protons while in the sensitive region of the telescope; star production in C_1; recoil deuterons from elastic π^0 production in deuterium; recoil protons from meson pair production; Compton protons from hydrogen; multiple scattering of the protons in the absorbers.

III. HYDROGEN

The photoprotons from hydrogen can come from three reactions:

$$\gamma+p=p+\pi^0,\quad \gamma+p=p+\gamma',\quad \gamma+p=p+\pi+\pi.$$

The second of these is the Compton scattering of photons from protons and this reaction is negligible as is shown by the Illinois experiment.[7] The contribution of the π pair production was shown to be negligible by lowering the synchrotron energy below the π pair threshold. Since the second two of the above reactions make negligible contributions to the photoproton spectrum, the photoprotons must come from the first reaction. We can calculate the π^0-meson production cross section from the photoproton spectrum.

The cross section for π^0 production from hydrogen is shown in Figs. 3, 4, and 5. The dashed curves were drawn by Oakley and Walker[8] through their data at proton angles of 29°, 40.5°, and 50.3°. It is seen that there is good agreement between the two experiments. The sources of error common to both experiments are: the beam monitoring equipment; the measurement of

[5] Tollestrup, Keck, and Worlock, Phys. Rev. **99**, 220 (1955).
[6] J. C. Keck and A. V. Tollestrup, Phys. Rev. **101**, 360 (1956).
[7] T. Yamagata, thesis, University of Illinois (unpublished).
[8] D. C. Oakley and R. L. Walker, Phys. Rev. **97**, 1283 (1955).

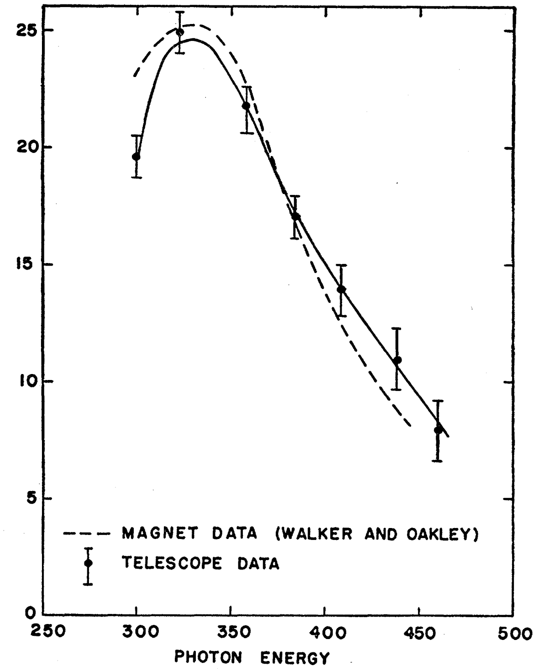

FIG. 4. Differential cross section for photoproduction of π^0 mesons from hydrogen, at a proton angle of 41.2°. Units are 10^{-30} cm^2 per steradian.

the synchrotron energy; and the hydrogen density measuring equipment.

Absolute errors which apply to all points, and which are not shown on the cross section points are: beam calibration 7%; telescope geometry 5%; hydrogen density 3%; synchrotron energy 0.4%. These combine to give a 9% probable error in absolute cross section, but these errors do not apply to the photoproton ratios. The sources which contribute to the errors shown on the individual points are: hydrogen and background counting statistics; uncertainty in proton peak limits; absorption correction (±5% of the correction); telescope angle (±0.3°); ΔR (2%, 3%, or 5% depending on A_3); range-energy relations (1%+0.01 cm of copper).

The angular distribution coefficients obtained from this experiment are given in Table II. The statistical errors overlap the Oakley-Walker[8] coefficients, except for the C coefficient at 460 Mev. Our C coefficient is more negative than theirs, but the statistical errors on C are large because our data are all obtained within 26° of 90° in the center-of-mass system. Since we have data at only three angles the angular distribution coefficients are not overdetermined, and a least-squares fit was not possible.

IV. DEUTERIUM

A. Interpretation of Deuterium Data

Since we are interested only in protons from meson-producing reactions, we have subtracted from the deuterium counting rate the protons produced by the photodisintegration of the deuteron.[6,9] The fraction of the deuterium photoprotons from photodisintegration for each of the three telescope angles is as follows: for 29.7 degrees, 11% to 20% except for the highest energy point which was 25%; for 41.2 degrees, 16% to 23%; for 51.8 degrees; 32% to 39%. By the spectator model the remaining proton counts are attributed to the following reactions: $\gamma+p_{\rm D}\to p+\pi^0$, $\gamma+n_{\rm D}\to p+\pi^-$. If the simplifying assumption is made that the motion of the target nucleons is not important, then the deuterium-to-hydrogen proton ratio is related to the meson ratio experiments in the following way:

$$\frac{\sigma_n^-+\sigma_p^0}{\sigma_{\rm H}^0}=\frac{\sigma_p^0}{\sigma_{\rm H}^0}+\frac{\sigma_n^-}{\sigma_p^+}\frac{\sigma_p^+}{\sigma_{\rm H}^+}\frac{\sigma_{\rm H}^+}{\sigma_{\rm H}^0},$$

where σ is the differential cross section for production of a pion (whose charge is indicated by the superscript) from the nucleon indicated by the subscript. The subscripts n and p denote the deuterium nucleons and H denotes the free proton. The π^-/π^+ and π^+/π^+ ratios have been measured by Sands *et al.*[1,3] The hydrogen π^+ cross section has been measured by Tollestrup *et al.*[5] and by Walker *et al.*[10] The hydrogen π^0 cross section has been measured by this experiment, by Oakley and Walker,[8] and by Corson *et al.*[11] The deuterium-to-hydrogen π^0 ratio[2] gives some indication of $\sigma_p^0/\sigma_{\rm H}^0$,

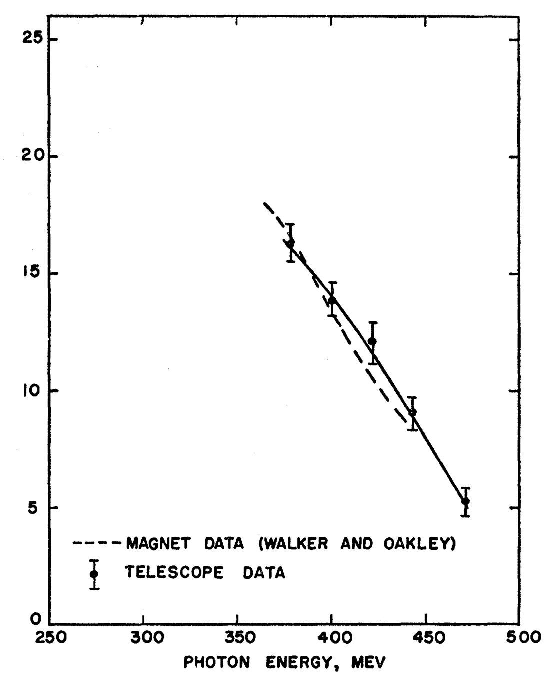

FIG. 5. Differential cross section for photoproduction of π^0 mesons from hydrogen, at a proton angle 51.8°. Units are 10^{-30} cm^2 per steradian.

[9] Whalin, Schriever, and Hanson, Phys. Rev. **101**, 377 (1956).
[10] Walker, Teasdale, Peterson, and Vette, Phys. Rev. **99**, 210 (1955).
[11] McDonald, Peterson, and Corson, Phys. Rev. **107**, 577 (1957).

since π^0 production from the neutron is nearly equal to production from protons, according to Watson *et al.*[12] A comparison of the experimental deuterium-to-hydrogen proton ratio with the meson ratio experiments mentioned showed some discrepancies which could not be ignored. For this reason the effect of the motion of the nucleons in deuterium was investigated.

B. Deuterium Dynamics Calculation

In the case of a proton telescope looking at a hydrogen target bombarded by photons, the counting rate is proportional to the appropriate differential cross section. If the target nucleons are moving, as in deuterium, then the counting rate is no longer dependent only on one particular value of the cross section, but it depends on a weighted average of the cross section over neighboring energies and angles. The problem now is to find this weighting function. It was decided to do this by considering two-hundred equally probable initial momenta for the target nucleon. For each momentum the following quantities were calculated, using the condition that the recoil proton be counted by the telescope. The laboratory energy of the incident photon k, the apparent photon energy as seen by the target nucleon k_0, the center-of-mass angle between the incident photon and the recoil proton δ, and a weighting function which is described in the appendix. The quantities k_0 and $\cos\delta$ determine the point where the differential cross section is sampled (the dots in Figs. 6 and 7). The single-meson production cross section can be described as a function of two variables, for example; $d\sigma/d\Omega' = d\sigma/d\Omega'(k_0,\cos\delta)$ where k_0 is the photon energy as observed from the target nucleon and δ is the angle between the photon and the recoil proton in the center-of-mass system (in general, primes denote center-of-mass system). A contour map of cross section can be made as shown in Figs. 6 and 7. If our telescope is set to observe the cross section at some point when the target nucleon is at rest, then as the nucleon assumes each of the two-hundred values of

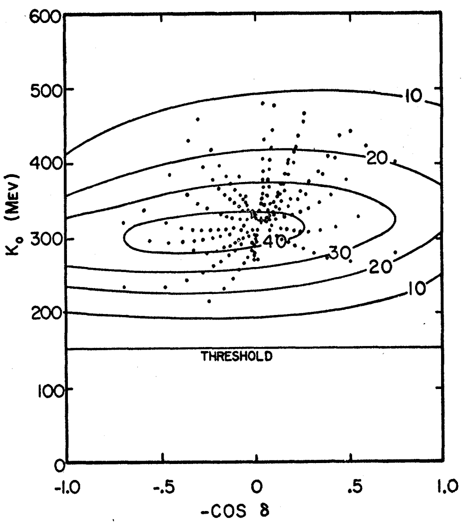

FIG. 6. Contour map of center-of-mass cross section obtained by adding the cross-section coefficients for the reactions $\gamma+p=p+\pi^0$, $\gamma+n=p+\pi^-$. These are the two meson-production reactions in deuterium which produce recoil protons. The coordinates are $-\cos\delta$, minus the cosine of the center-of-mass meson angle, and k_0, the incident photon energy as seen by the target nucleon. If recoil protons are observed at a fixed angle and energy, the cross section is sampled at each of the points shown as the target nucleon assumes each of the two-hundred momenta selected to represent the motion of the nucleon in deuterium.

momentum the cross section will be sampled at each of the points shown. There are less than two-hundred points shown because of some momenta it is impossible to produce the required recoil with a photon of 500 Mev or less. The two-hundred momenta were chosen to represent the motion of the nucleons in deuterium as follows. Twenty spherically symmetric directions were chosen. These are the perpendiculars to the faces of a regular icosahedron. Ten values of momentum were

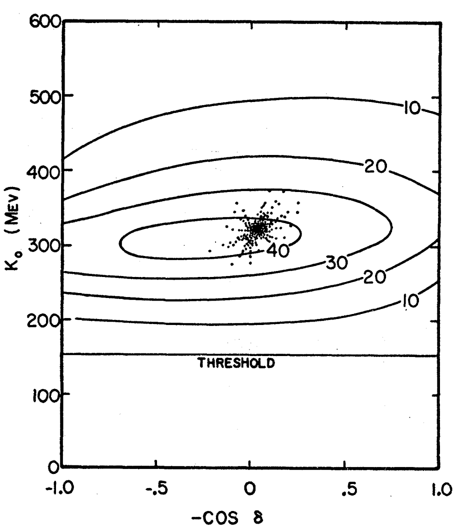

FIG. 7. This figure shows the reduced effect of the nucleon motion in the hypothetical case where the cross section shown in Fig. 6 is measured by observing the mesons.

TABLE II. Angular distribution coefficients obtained for the photoproduction of neutral pions from hydrogen. In parentheses are the coefficients given by McDonald, Peterson, and Corson,[a] obtained by combining their photographic plate data with the data of Oakley and Walker.[b] Units are microbarns per steradian. $d\sigma/d\Omega' = A + B\cos\theta_\pi' + C\cos^2\theta_\pi'$.

Coefficient	Photon energy 380 Mev	420 Mev	460 Mev
A	17.8±0.9	12.6±1.2	8.35±1.3
	(18.0±0.7)	(11.7±0.6)	(7.3 ±0.6)
B	1.7±2.0	2.7±2.4	0.9 ±2.6
	(2.6±0.9)	(2.7±0.7)	(2.7 ±0.7)
C	−14.0±4.8	−11.2±5.9	−13.2 ±6.3
	(−10.9±1.2)	(−7.3±1.1)	(−4.2 ±1.0)

[a] See reference 11.
[b] See reference 8.

[12] Watson, Keck, Tollestrup, and Walker, Phys. Rev. **101**, 1159 (1956).

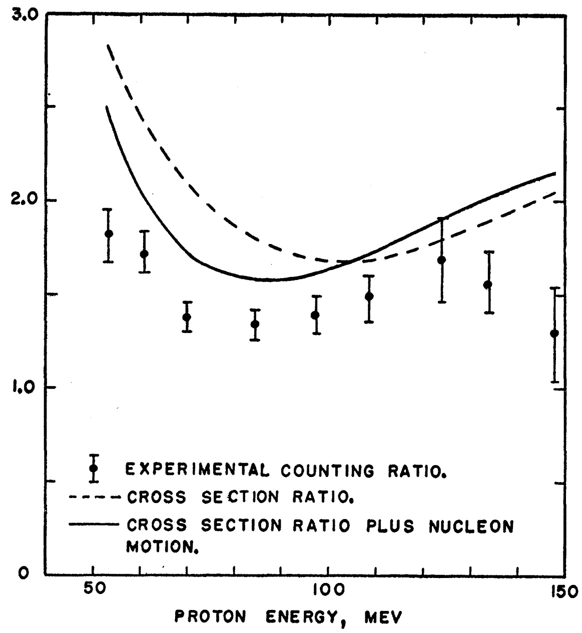

FIG. 8. Deuterium-to-hydrogen photoproton ratio. Protons from deuterium photodisintegration have been subtracted leaving the ratio of protons from meson-producing reactions. Proton angle is 29.7° from the photon direction.

chosen to represent the momentum spectrum. They were selected so that there was a 5% probability of a nucleon with momentum less than the first value, 15% for the second value, and so on up to 95% probability of a nucleon with momentum less than the tenth value. Two deuteron wave functions were tried and both gave practically the same ten values of momentum. One was the familiar Hulthén wave function and the other was the Gartenhaus[13] wave function, derived from the cutoff Yukawa meson theory. A digital computer program was developed which would perform the above operations. The computer input consisted of a table of cross-section coefficients for the reaction, and the energy and angle of the observed particle (proton or meson). The computer output showed the effect of measuring a cross section of a nucleon moving with the deuterium momentum distribution relative to the same cross section measured with the nucleon at rest. It is interesting to observe that the smearing effect of deuterium (Figs. 6 and 7) is much greater when recoil protons are observed than when the mesons are observed.

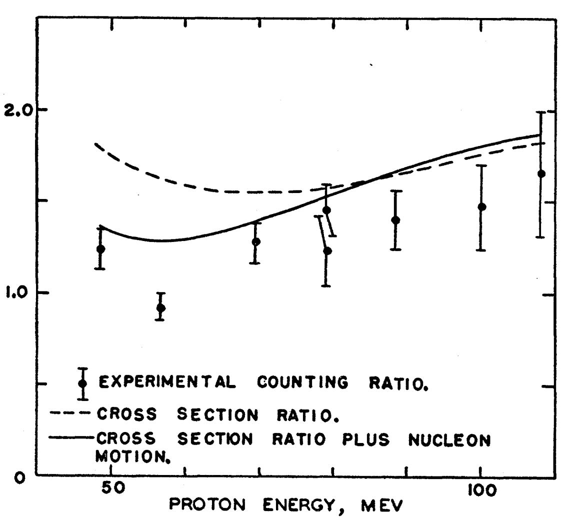

FIG. 9. Deuterium-to-hydrogen photoproton ratio. Protons from deuterium photodisintegration have been subtracted leaving the ratio of protons from meson-producing reactions. Proton angle is 41.2° from the photon direction.

[13] S. Gartenhaus, Phys. Rev. 100, 900 (1955).

C. Deuterium Results

The photoproton counting rate was adjusted by subtracting the photodisintegration contribution, as has been discussed, to obtain the photoproton counting rate which was divided by the hydrogen counting rate and multiplied by twice the hydrogen to deuterium density ratio, to obtain the photoproton ratio per nucleus. This ratio is plotted in Figs. 8, 9, and 10. Also shown (dashed curve) is the deuterium to hydrogen photoproton ratio as calculated from the π^- and π^0 cross section of a free proton and neutron at rest. The failure of this simple calculation led to the calculation which included the effect of the nucleon motion in the deuteron. This calculation has been described in the preceding section and the results are shown in Figs. 8, 9, and 10 (solid curve). The agreement between the predicted and measured deuterium-to-hydrogen photoproton ratio has been improved, especially in regard to the energy dependence of the ratio. However, there is still an amplitude discrepancy of about 15%. This discrepancy and its probable cause will be discussed.

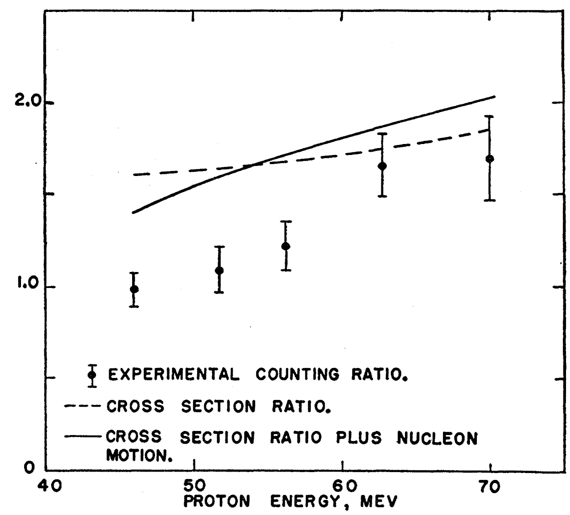

FIG. 10. Deuterium-to-hydrogen photoproton ratio. Protons from deuterium photodisintegration have been subtracted leaving the ratio of protons from meson-producing reactions. Proton angle is 51.8° from the photon direction.

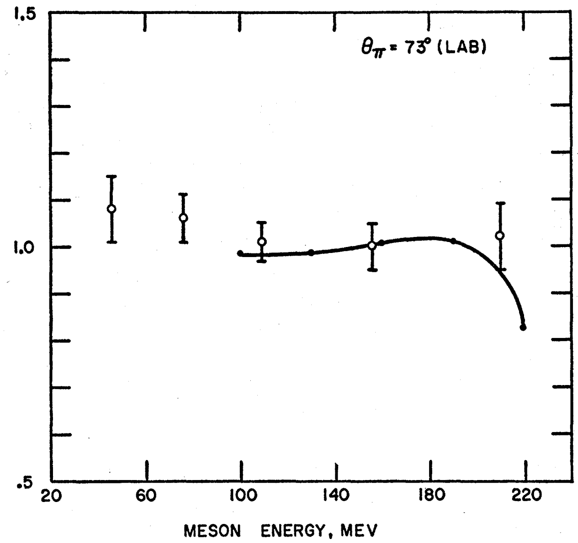

FIG. 11. Deuterium-to-hydrogen π^+ photoproduction ratio. The experimental points are from Sands *et al.*[1] The theoretical curve does not include the effect of meson absorption.

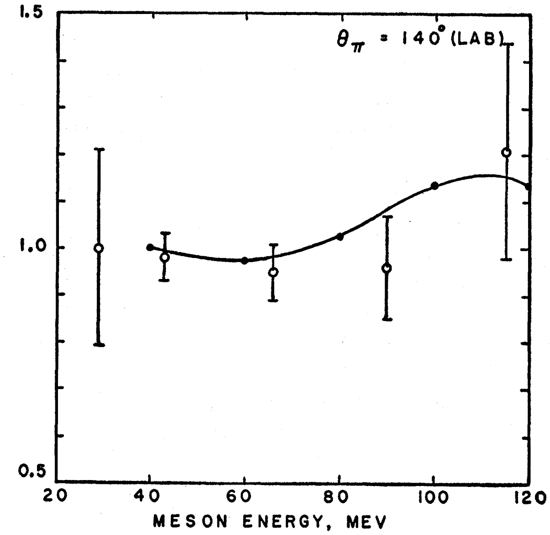

FIG. 13. Deuterium-to-hydrogen π^+ photoproduction ratio. The experimental points are from Sands *et al.*[1] The theoretical curve does not include the effect of meson reabsorption.

V. OTHER DEUTERIUM EXPERIMENTS

Sands *et al.*[1] have measured the ratio of positive-meson production from deuterium and hydrogen. This experiment should be a good subject for the deuteron dynamics calculation in the range where the recoil neutron has high momentum (>300 Mev/c) so that the exclusion principle and the density of final states do not restrict positive-meson production from deuterium. This calculation has the advantage that only one cross section enters, where in the photoproton case there were two cross sections involved and one (the negative-pion) was not as thoroughly known as is the positive-pion photoproduction cross section. The positive-meson ratio is shown in Figs. 11, 12, and 13 with the experimental points of Sands *et al.*

The effect of nucleon motion on negative-meson production is of interest for two reasons. First, we assumed that the effect was small enough to permit us to use the deuterium π^- photoproduction cross section as a free-neutron cross section to calculate the photoproton yield. Figure 14 shows that the effect of the nucleon motion on the π^- cross section is 6% or less in the region where the π^- cross section was measured, except for the 210-Mev point at 73°. Second, we can use the ratio of π^- photoproduction from neutrons in motion to neutrons at rest together with the previously

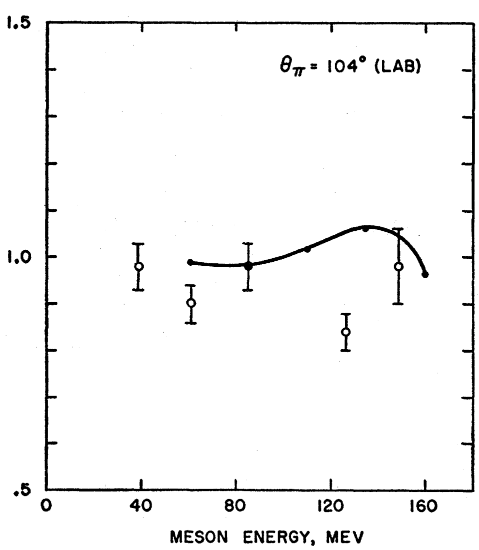

FIG. 12. Deuterium-to-hydrogen π^+ photoproduction ratio. The experimental points are from Sands *et al.*[1] The theoretical curve does not include the effect of meson reabsorption.

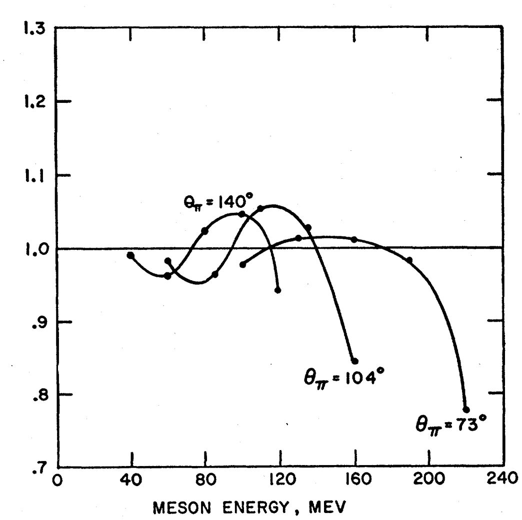

FIG. 14. Negative-meson photoproduction from neutrons moving with the momentum distribution of a deuterium nucleon, relative to production from neutrons at rest.

discussed π^+ ratio to find the effect of nucleon motion on the deuterium π^- to π^+ ratio. In general the effect is small. In particular, nucleon motion does not account for the discrepancy between theory and experiment at a meson angle of 140° reported by Watson *et al.*[12]

VI. DISCUSSION OF RESULTS

The results of the neutral-pion photoproduction cross section measurement (Figs. 3, 4, and 5) are not as extensive as those of Oakley and Walker,[7] but the agreement provides a useful consistency check, since the two methods employed different means of detecting the recoil protons.

The calculated deuterium-to-hydrogen photoproton ratio is larger than the experimental ratio by about 15%. A discrepancy of this type might be expected because the spectator model does not include the possibility of a meson being reabsorbed by the other nucleon. Since the reabsorption of a meson will lead to photodisintegration of the deuteron, the cross section of this reaction provides a measurement of the reabsorption probability. Wilson[14] has obtained a very good fit to the total cross section for deuterium photodisintegration using a meson reabsorption probability of 0.11. The deuterium-to-hydrogen positive pion ratio[1] and neutral pion ratio[2] were examined for evidence of this 11% reabsorption in deuterium. The experimental values of the π^+ deuterium-to-hydrogen ratio from the 300-Mev machines are inconclusive as regards evidence for or against the 11% reabsorption figure. The π^+ deuterium-to-hydrogen ratio from 500-Mev bremsstrahlung is shown in Figs. 11, 12, and 13. It is seen that if the curves were decreased by 11% the agreement with experiment would remain about the same.

Averaging the deuterium to hydrogen π^0 ratio from 310-Mev bremsstrahlung over meson energies and angles, we find that the π^0 production per nucleon in deuterium is 10% less than from hydrogen, with statistical errors of about 5%. The 500-Mev data are consistent with this 10% reduction at meson (actually decay-photon) angles of 73° and 140°. However, the ratio is larger at 30°, a fact which Keck and Tollestrup[6] suggest may be an interference effect. Typical errors are 5%.

From these π^+ and π^0 deuterium-to-hydrogen ratios, it can be said that they are consistent with the 11% reabsorption probability, but that they do not provide strong evidence for it. The best evidence comes not from the meson ratio experiments where one must look for an 11% difference between measured quantities, but from the photodisintegration of the deuteron at the higher energies, which is almost entirely caused by this effect.

The reabsorption correction applies only to the π^0 cross section used in the calculation, as the π^- cross section employed was obtained from measurements on deuterium. The actual correction to the ratio is a 5% to 6% decrease, which still leaves a significant discrepancy between the calculation and the experiment. Since this was a ratio-type experiment, and the cross sections used in the calculation were all measured at this laboratory using the same beam-monitoring equipment, the discrepancy between the calculated ratio and the measured ratio appears to be a real shortcoming of the spectator model. The model was successful, however, in yielding the proper shape of the energy dependence of the deuterium-to-hydrogen photoproton ratio, but predicts a value that is everywhere higher than is experimentally observed. The worst discrepancy is at 51.8° (Fig. 10) where the predicted ratio is about 40% high, whereas the errors on the calculated ratios due to the uncertainties in the meson production experiments are only about 15%. Thus, the discrepancy does not seem to be an experimental one. The quandary posed by this situation is that one is predicting the number of recoil protons that should be observed from the mesons actually observed. If these recoils are not observed, at the expected places, then either the momentum distribution within the deuteron, or interactions between the outgoing particles, is causing the protons to appear at different angles than expected. The deficiency observed at 51.8° would be expected to be largest on this basis as it is nearest the kinematic limit in angle for recoil protons to be observed from pion production from stationary nucleons. Any effects omitted in the spectator model that tend to cause the recoils to be scattered through a larger angle than predicted will deplete the protons most seriously from this region. Thus, further experiments to observe recoil protons in the regions kinematically forbidden from stationary nucleons should show more protons than expected on the basis of the model used here. In these regions the resonance enhances the importance of the higher momentum components of the deuteron, and thus such experiments would offer further information on the deuteron wave function.

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APPENDIX

To find the weighting function used in Sec. IV, let us first consider the contribution to the counting rate of the particular initial target-nucleon momentum.

$$\text{Counts} = N\int\int (d\sigma/d\Omega')d\Omega'(dn/dk_0)dk_0.$$

[14] Robert R. Wilson, Phys. Rev. 104, 218 (1956).

where $d\Omega'$ is the differential of solid angle in the center-of-mass system. The integral is carried out over the aperture of the telescope in k_0, Ω' space. N is the number of target nucleons/cm^2 with the particular value of initial momentum. $N(d\sigma/d\Omega')$ is the probability per unit solid angle of a photon with energy k_0 producing a meson at an angle $\pi-\delta$ in the center-of-mass system. The product $(dn/dk_0)dk_0$ is the number of photons in the energy range $k_0 \pm \frac{1}{2}dk_0$. If the aperture of the telescope is sufficiently small, as it was designed to be, then we may replace the integrand by its value at the center of the aperture and take it outside of the integral. Making successive transformations of the integral from k_0,Ω' space to k,Ω' space, to k,Ω space and finally to T_p,Ω space, where the integral becomes $\Delta T_p \Delta\Omega$, we obtain

$$\text{Counts/beam integrator pulse} = (d\sigma/d\Omega')\Delta T_p \Delta\Omega N G/E_0 \times \{(\partial \cos\alpha'/\partial \cos\alpha)(1-\beta_n \cos\theta_n)(\partial k/\partial T_p) f(k)/k\},$$

where ΔT_p is the width of the proton energy interval accepted by the telescope, $\Delta\Omega$ is the solid angle subtended by the telescope, G is the total energy in the photon beam per beam integrator pulse, E_0 is the maximum bremsstrahlung energy, $\partial \cos\alpha'/\partial \cos\alpha$ is the solid angle transformation function (photon energy held constant), β_n is the initial velocity of the target nucleon, (θ_n,ϕ_n) is the initial direction of the target nucleon (the photon direction is $\theta=0$; the plane containing photon and recoil proton defines $\phi=0$), $\partial k/\partial T_p$ is the partial derivative of photon energy with respect to recoil proton energy with the laboratory proton angle held constant, and $f(k)$ is the bremsstrahlung function defined so that the number of photons/beam integrator pulse in the Δk is $Gf(k)\Delta k/kE_0$.

In deriving the above equation, account was taken of the fact that at arget nucleon moving toward the synchrotron sees a larger photon flux than if it were moving away from the synchrotron. It is now seen that the cross section at each of the dots in Fig. 4 should be weighted by the expression inside the braces in the foregoing equation.

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Scattering of μ^- Mesons by Nuclei*

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The effect of finite nuclear size on the scattering of μ^- mesons is calculated for a nucleus $Z=80$, $R=1.2A^{\frac{1}{3}} \times 10^{-13}$ cm. The nucleus is considered as a uniformly charged sphere. The μ^- mesons have $v/c=0.2$ corresponding to an energy $E=2.1$ Mev. It is found that the left-right asymmetry in the scattering of polarized μ^- mesons is decreased from that for a point nucleus. However, considerable asymmetry remains at large angles.

INTRODUCTION

THE scattering of negative μ mesons by a point Coulomb field is similar to that for electrons if the μ meson is taken to be a Dirac particle. For the same $\beta=v/c$, the only difference is that the μ cross section is smaller in the ratio of the squares of the masses of the two particles. The angular distributions and polarizations are exactly the same in the two cases.

Recent experiments[1] on the decay of π mesons, $\pi\rightarrow\mu+\nu$, have indicated that the μ mesons are polarized in the direction of their momentum in the center-of-mass system. It is possible, by using this effect, to obtain a beam of transversely polarized μ mesons. The scattering of such a beam by a point Coulomb field will have a left-right asymmetry,[2] S, which has been calculated by Sherman.[3] Measurement of this asymmetry would indicate the original spin direction (parallel or antiparallel to the momentum) of the μ meson and this could be used as a check on the hypothesis of conservation of leptons.

The purpose of this paper is to examine the effect of the finite size of a charge distribution such as is found in heavy nuclei on the scattering cross section and on S for μ^- mesons. For electrons the finite size of the nucleus does not become important until highly relativistic velocities are reached, at which point there are no polarization effects. μ mesons, because of the larger mass, have smaller wavelengths than electrons for a corresponding β and in fact the finite size affects the scattering strongly already at the lowest energies.

We derive in the next section a general method for correcting the Dirac particle point Coulomb scattering results for finite nuclear size for a uniformly charged nucleus of radius R. We then apply the results to the

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[1] Garwin, Lederman, and Weinrich, Phys. Rev. **105**, 1415 (1957).
[2] H. A. Tolhoek, Revs. Modern Phys. **28**, 277 (1956).
[3] N. Sherman, Phys. Rev. **103**, 1601 (1956).