Single Configuration Analysis of Li^6†

WILLIAM T. PINKSTON,* *U. S. Naval Ordnance Laboratory, Silver Spring, Maryland*

AND

JAMES G. BRENNAN, *Catholic University, Washington, D. C. and U. S. Naval Ordnance Laboratory, Silver Spring, Maryland*

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The energy level and static moment data of Li^6 are analyzed in terms of the intermediate coupling model with the usual assumption of a $(1s)^4\ (1p)^2$ configuration. The matrix elements of the central, spin-orbit, and tensor forces are derived from the data. The potential implied by the data has a central force strength which is larger than that expected from the two-nucleon analysis and has a different spin dependence. The tensor part of the potential is found not to be necessary for a satisfactory energy-level structure, and an extremely weak tensor force is called for by the small quadrupole moment. The magnetic moment is too large by about six percent. An extensive numerical study has been made of two-body potentials with parameters adjusted to give the correct binding energy and quadrupole moment to the deuteron and no simultaneous fit of the lithium energy levels and quadrupole moment seems possible.

1. INTRODUCTION

THE original intermediate-coupling calculations of Inglis,[1] in which the perturbing Hamiltonian consisted of a central internucleon potential plus a one-body spin-orbit interaction, showed that the levels of Li^6 were consistent with a degree of intermediate coupling near the *LS* limit. Tauber and Wu[2] confirmed these results using a variety of central potential shapes, ranges and depths. In addition, they calculated the magnetic moment which they found to be off by six percent and insensitive to small changes in the wave function. Elliott[3] and Regge[4] have investigated the intermediate-coupling theory of Li^6 with tensor forces included in the internucleon potential. The energy levels were found to be unimproved, and Elliott found that the wave function was not sufficiently improved to reproduce the correct magnetic moment. Adkins and Brennan[5] made similar calculations, but in reverse order, choosing a wave function which gave the correct ground-state moments and finding sets of potential parameters which would reproduce the wave function. This was found to be possible for a wide variety of parameters, but none of the potential parameters approximate those deduced from the deuteron data. In addition, they had difficulty in placing the $J=2$, $T=0$ level correctly.

The energy levels of Li^6 are now sufficiently well known that it is possible to analyze them in terms of the intermediate-coupling model without making assumptions quite so restrictive as have been required by previous investigations, i.e., the energy matrices can be at least partially inverted to give relationships among the matrix elements. In the present investigation this is done and the matrix elements so derived are related to potentials deduced from the two-nucleon data. An attempt was made to improve the intermediate-coupling wave function by including tensor forces. It is shown that the quadrupole moment can be improved in this way but not the magnetic moment value. The potential which gives the best results to all the data has a central strength somewhat greater than one would like and a tensor part which is extremely small.

Data

The ground state of Li^6 is a $J=1$, $T=1$, even-parity state with a magnetic moment of 0.822 nm and a static electric quadrupole moment[6] of 2.3×10^{-28} cm^2. The accuracy of this latter value is about fifty percent. The sign of the quadrupole moment is unknown. Throughout this investigation we shall assume that Li^6 is adequately described by the single configuration, $(1s)^4(1p)^2$. The most general wave function which can be formed from this configuration with the symmetry of the Li^6 ground state is

$$\psi=C_1\,{}^3S_1+C_2\,{}^1P_1+C_3\,{}^3D_1. \tag{1}$$

The magnetic moment and the quadrupole moment can be used to fix C_1, C_2, and C_3. The magnetic moment alone requires that $0.85\leqslant|C_1|^2\leqslant0.90$; C_2^2 and C_3^2 much smaller. The quadrupole moment in terms of wave function Eq. (1) is given by

$$\langle Q\rangle=-\tfrac{4}{5}e\langle r^2\rangle[(1/\sqrt{5})C_1C_3+(7/40)C_3^2-(1/4)C_2^2]. \tag{2}$$

In order to evaluate this, one must first assume a value of $\langle r^2\rangle$, the mean square displacement of the p shell proton. Actually the value of the quadrupole moment is so small that the amplitudes, C_i, are not sensitive to this value. In what follows, a value[7] of

$$\langle r^2\rangle=7.8\times10^{-26}\text{ cm}^2$$

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* Present address: Palmer Physical Laboratory, Princeton University, Princeton, New Jersey.

[1] D. R. Inglis, Phys. Rev. **87**, 915 (1952); Revs. Modern Phys. **25**, 390 (1953).

[2] G. E. Tauber and T. Y. Wu, Phys. Rev. **93**, 295 (1954).

[3] J. P. Elliott, Proc. Roy. Soc. (London) **A218**, 345 (1953).

[4] T. Regge, Nuovo cimento **11**, 285 (1954).

[5] R. Adkins and J. G. Brennan, Phys. Rev. **99**, 706 (1955).

[6] T. P. Das (private communication).

[7] R. A. Ferrell and W. M. Visscher, Bull. Am. Phys. Soc. Ser. II, **1**, 17 (1956).

is taken. The magnetic moment, quadrupole moment, and normalization can now be utilized to give approximate values to the wave function coefficients, i.e.,

$$|C_1|^2\approx 0.85, \quad |C_2|^2\approx 0.14, \quad |C_3|^2\approx 0.01.$$

The exact values of neither the magnetic moment nor the quadrupole moment should be used to fix the amplitudes, because there are many theoretical and experimental uncertainties involved. Even though the usual meson currents cancel for a self-conjugate nucleus such as Li^6, there are contributions of unknown magnitude which arise from the spin-orbit interaction[8] and from relativistic effects. These effects are often estimated to be of the order of 0.1 nm; hence one should not necessarily reject a theoretical magnetic moment which differs from the experimental value by this amount. A magnetic moment larger than the value 0.879 (which is obtained in the Russell-Saunders limit $C_2=C_3=0$) cannot be obtained with wave function Eq. (1). It is seen, however, that the intermediate coupling model gives a wave function which is very close to the Russell-Saunders limit; hence any theoretical magnetic moment which is larger than the experimental value but less than 0.879 cannot be rejected. Accepting the estimated error of fifty percent in the quadrupole determination as exact, the quadrupole moment must be less than 3.5×10^{-28} cm^2. Dividing by the assumed value of the squared radius, we obtain

$$-0.0045\leqslant\langle Q\rangle/\langle r^2\rangle\leqslant+0.0045. \tag{3}$$

Since the quadrupole moment is sensitive to small details of the nuclear wave function, such as small admixtures of higher configurations, the above value should not be taken too seriously. Instead we shall arbitrarily take any theoretical value of the quadrupole moment as satisfactory which is within twice the above limits.

Information concerning the wave functions of the excited states of Li^6, principally the $(J,T)=(1,0)$ level at 3.57 Mev as well as information about the ground state, might be obtained from the ft value of $He^6\rightarrow Li^6$ beta decay and from the $Li^7(p,d)Li^6$ cross section.[9] Because of the experimental uncertainties involved however, these data indicate only that Li^6 is very near to the LS coupling limit. These data shall not be considered further.

The energy spectrum of Li^6 is given by Ajzenberg and Lauritsen.[10] The first six levels listed by these authors are assumed by us to belong to the lowest $(1s)^4(1p)^2$ configuration, and to be identified correctly by the diagram of Inglis.[1] The energy level diagram gives only energy spacings relative to the ground state; absolute energies may be obtained by calculating the absolute energy of the ground state from neutron and proton separation energies. If one measures from the ground state, the energies of Li^5+n and He^5+p are 5.50 Mev and 4.66 Mev, respectively. The energy of He^4+n+p is 3.70 Mev; hence the interaction between the alpha particle and the p shell particles is +6.46 Mev. Since the LS representation is to be used, the spin-orbit energy of $(P_{\frac{3}{2}})^2$, (−1.67 Mev), must be subtracted from the above energy value. This places the $(1p)^2$ configuration, without interparticle interactions, 8.13 Mev above the ground state; hence the lowest eigenvalue of the $(1p)^2$ matrix must be −8.13 Mev.

2. INTERMEDIATE COUPLING CALCULATIONS

For the present let us limit the potential to the form

$$\sum_{i,j} V_c(i,j)+\zeta\sum_i \mathbf{l}_i\cdot\mathbf{s}_i. \tag{4}$$

$V_c(i,j)$ is a central potential which is multiplied by some exchange operator. ζ is assumed to be a constant number. The $(1p)^2$ matrices of this potential are given by Tauber and Wu[2] and will not be repeated in their entirety here. The energies of the (2,0) and (3,0) excited levels are given by

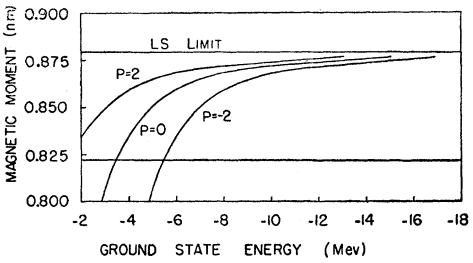

FIG. 1. Magnetic moment *vs* ground-state energy. The lower horizontal line is the experimental value of 0.822 nm. The upper line is the value 0.879 nm which would result if the ground state were pure $^3S_1$ state.

$$\begin{aligned} E(2,0)&=(^3D,\ V_c\,^3D)-\tfrac{1}{2}\zeta\equiv D-\tfrac{1}{2}\zeta,\\ E(3,0)&=(^3D,\ V_c\,^3D)+\zeta\equiv D+\zeta. \end{aligned} \tag{5}$$

The ground-state energy E_g and the coefficients C_i of the wave function (1) are the solutions of the following matrix equation:

$$\begin{aligned} (H_{11}-E_g)C_1+H_{12}C_2&=0,\\ H_{21}C_1+(H_{22}-E_g)C_2+H_{23}C_3&=0,\\ H_{32}C_2+(H_{33}-E_g)C_3&=0, \end{aligned} \tag{6}$$

$$H_{11}=(^3S,\ V_c\,^3S)\equiv S, \quad H_{22}=(^1P,\ V_c\,^1P)\equiv P,$$

$$H_{33}=D-\tfrac{3}{2}\zeta, \quad H_{12}=(\tfrac{2}{3})^{\frac{1}{2}}\zeta=-(\tfrac{4}{5})^{\frac{1}{2}}H_{23}.$$

If the experimental level spacing values, $E(2.0)-E_g=4.52$ Mev and $E(3,0)-E_g=2.19$ Mev, are used to eliminate $E(2,0)$ and $E(3,0)$, five equations in seven unknowns: S, P, D, E_g, C_1/C_3, C_2/C_3, and ζ, are obtained from Eqs. (5) and (6). Five of these unknowns can then be solved for in terms of the other two. In the

[8] J. H. D. Jensen and M. G. Mayer, Phys. Rev. **85**, 1040 (1952).
[9] T. Auerbach and J. B. French, Phys. Rev. **98**, 1276 (1955).
[10] F. Ajzenberg and T. Lauritsen, Revs. Modern Phys. **27**, 77 (1955).

present work all the other unknowns will be obtained in terms of E_g and P. The resulting expressions for S, D, and ζ are then inserted in the secular determinant of Eq. (6). A cubic equation results whose coefficients depend on the two parameters E_g and P. If $E-E_g$ is formally divided out, a quadratic equation remains, the roots of which are the energies of the excited (1,0) levels. The magnetic moment and the electric quadrupole moment can be calculated from the expressions for C_1/C_3 and C_2/C_3. In Figs. 1–3 the magnetic moment, the quadrupole moment, and the position of the first excited (1,0) level, E^*, are plotted as functions of E_g for three choices of P. Values of $P=-2$ Mev, 0, and $+2$ Mev are used. The energy curves for $P=+2$ and for $P=0$ are seen to be satisfactory since they yield a value, Figs. 1, 2, and 3, of $E^*-E_g=5.6$ Mev and 5.4 Mev respectively for values of E_g of about -8 Mev, the empirical value. For $P=-2$ Mev, the energy spacing is too low in the region where E_g is about -8 Mev. The quadrupole moment is seen to be always

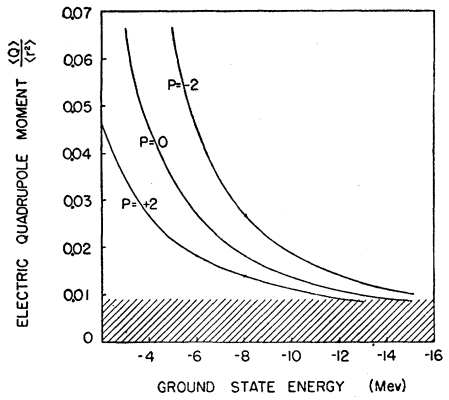

FIG. 2. Quadrupole moment *vs* ground-state energy. The shaded area represents the region of acceptable values.

positive and too high in the region of interest. The magnetic moment is too high and quite insensitive in the -8-Mev region, but the discrepancy is within the theoretical limits mentioned in the previous section. Table I gives approximate values of the potential matrix elements which will give the correct level structure for the $T=0$ levels. P may lie anywhere in the region, $0\leqslant P\leqslant 2$. L and K are the direct and exchange integrals of the central force.

While it is clear that the quadrupole moment is too large in this model, it is not clear just how sensitive it is to the degree of coupling. $\langle Q\rangle$ has therefore been calculated as a function of ζ by use of the central force matrix elements of Table I to see what the result of a small change in ζ will have on $\langle Q\rangle$. For this and all future considerations P is taken as zero. This simplifies the calculations and can be justified because of the insensitivity of the previous results to the exact value of P. The two-nucleon scattering data also favor a near-Serber force for the odd-parity singlet states. The

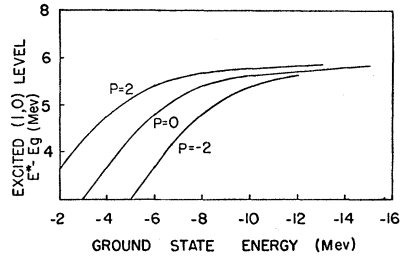

FIG. 3. Energy of the first excited (1,0) level minus the ground-state energy *vs* the ground-state energy.

dependence of $\langle Q\rangle$ on ζ is plotted in Fig. 4. The smallest (in magnitude) value of ζ which is consistent with the imposed limit on $\langle Q\rangle$ Eq. (3), is $\zeta=-1.0$ Mev. We consider such a small value as unsatisfactory, especially since tensor forces have been neglected.

The phenomenological matrix elements of Table I can be interpreted in terms of specific potential shapes and parameters which are in quite reasonable agreement with the deuteron requirements. Table II gives potential parameters for two shapes, Gaussian[11] and Yukawa,[12] which give the correct binding to the deuteron and whose matrix elements in the $(1p)^2$ configuration of the harmonic oscillator shell model are quite close to the matrix elements of Table I. The mean square radii values used to compute Table II are quite close to the value assumed previously in this work, 7.8×10^{-26} cm^2.

So far no mention has been made of the $T=1$ levels. In addition to the spin orbit parameter ζ, these levels depend on $(^1S, V_c\,^1S)$, $(^1D, V_c\,^1D)$, and $(^3P, V_c\,^3P)$. In order to determine these matrix elements numerically, we shall use $\zeta=-1.55$ and $E_g=-8.1$ Mev. Because of the simplicity of the potential (4), the following equality holds:

$$(^1D, V_c\,^1D)/(^1S, V_c\,^1S) = (^3D, V_c\,^3D)/(^3S, V_c\,^3S)=-0.55.$$

With the above assumptions and the observed spacings

TABLE I. Matrix elements deduced from the Li^6 level structure.

L	K	L/K	ζ	ζ/K	$(^3S, V_c\,^3S)$	$(^3D, V_c\,^3D)$
-5.6	-1.2	4.66	-1.55	1.42	-8	-4.4

TABLE II. Central potential parameters which have matrix elements approximating those of Table I.

Shape	Strength V_0 in Mev	Range in 10^{-13} cm	$\langle r^2\rangle$ in 10^{-26} cm^2	L	K
Yukawa	68	1.18	7.65	-5.86	-1.17
Yukawa	43	1.54	7.16	-6.90	-1.17
Gaussian	74.8	1.40	8.7	-5.56	-1.19

[11] E. Feenberg, Phys. Rev. **47**, 850 (1935); **48**, 906 (1935).
[12] R. G. Sachs and M. Goeppert-Mayer, Phys. Rev. **53**, 991 (1938).

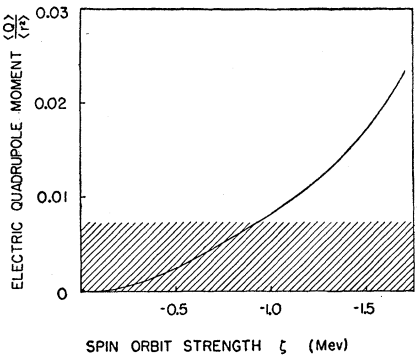

FIG. 4. Quadrupole moment *vs* spin-orbit strength ζ. The central force matrix elements of Table I are assumed. The shaded area represents the region of acceptable values.

of the $T=1$ levels, $E(0,1)-E_g=3.57$ Mev and $E(2,1)-E_g=5.31$ Mev, one can solve for the matrix elements to obtain

$$({}^3P, V_c\,{}^3P)\cong 0,$$
$$({}^1S, V_c\,{}^1S)\cong -3.85 \text{ Mev},$$
$$({}^1D, V_c\,{}^1D)\cong -2.12 \text{ Mev}.$$

The ratio of $({}^1S, V_c\,{}^1S)$ to $({}^3S, V_c\,{}^3S)$ is the ratio of singlet force strength to triplet force strength for even states. This ratio turns out to be 0.48, which is in disagreement with 0.7, the approximate value needed to obtain the observed singlet-triplet splitting of the deuteron for potentials such as listed in Table II. Since the tensor force is absent in those potentials, all the splitting must be the result of spin-exchange forces.

3. TENSOR FORCES

What has emerged so far is that the energy levels and magnetic moment are not inconsistent with a potential of the type Eq. (4) which excludes tensor forces. However, the quadrupole moment is not adequately explained by such a model. In this section we consider the possibility of reducing the theoretical value of the quadrupole moment by the addition to our interaction, Eq. (4), of a term $S_{12}U(1,2)$, in which $U(1,2)$ is a central well potential which may have a mixture of Wigner and Majorana exchange multiplying it and

$$S_{12}=\frac{3(\boldsymbol{\sigma}_1\cdot\mathbf{r}_{12})(\boldsymbol{\sigma}_2\cdot\mathbf{r}_{12})}{r_{12}^2}-\sigma_1\cdot\sigma_2. \tag{7}$$

In order to see how the addition of the tensor force can improve the quadrupole moment, consider the wave function which results from choosing the central force matrix elements as given in Table I but taking a value of the spin-orbit parameter of -1.7 Mev. The reason for choosing a larger ζ will become obvious later. The wave function amplitudes which result are $(C_1, C_2, C_3) = (0.983, 0.177, -0.0474)$. Substituting these values into the quadrupole expression, one obtains

$$\langle Q\rangle/\langle r^2\rangle=\tfrac{1}{5}C_2^2-(4C_1C_3/5\sqrt{5})-(7/50)C_3^2$$
$$=0.00627+0.01667-0.00031.$$

Clearly, if the wave function is brought closer to the LS limit by reducing the size of C_2 and C_3, the quadrupole moment can be made smaller. But the tensor force can effect the reduction by changing the phase of C_3 so that the larger terms, $-4C_1C_3/5\sqrt{5}$ and $\frac{1}{5}C_2^2$, tend to cancel. The ratio of C_1/C_3 is always negative for $\zeta<0$ in the absence of tensor forces. With tensor forces this is not necessarily so. This is exactly analogous to the famous $C^{14}\rightarrow N^{14}$ beta-decay problem.[1] In this problem, in order for the accidental cancellation of the beta-decay matrix element to occur, C_1 and C_2 must have the same phase. For $\zeta>0$ (as it is for nitrogen) and in the absence of tensor forces this is impossible. The tensor force is therefore invoked to explain the abnormally small beta-decay matrix element. It is also clear from the above that the quadrupole moment of Li^6 can be made negative through the inclusion of tensor force.

The tensor force matrix elements are now added to our $T=0$ matrices. These matrix elements are given by Regge.[4] Since the tensor force is considered to be a perturbation, we shall use the central force matrix elements of Table I and simply add the tensor contributions. The value for the spin-orbit parameter, $\zeta=-1.70$ Mev, is assumed because we know in advance that the tensor force pulls down the (2,0) level strongly and pushes up the other $T=0$ levels weakly; hence we want to start with the (2,0) level a little too high. Because of the difficulties encountered in the $A=14$ problem[13] when Yukawa forces are used in conjunction with harmonic oscillator radial wave functions, we shall consider only Gaussian shape potentials. In order that the tensor force will affect the wave functions as much as possible, but will affect the energy positions as little as possible, a potential is needed which has as large a ratio of off-diagonal to diagonal matrix elements as possible. A good measure of this in the present case is the ratio

$$({}^3S_1, S_{12}U\,{}^3D_1)/({}^3D_1, S_{12}U\,{}^3D_1).$$

This ratio is of the order of -1.5 for the Gaussian shape but only of the order of -0.3 for Yukawa shapes.

It is found that satisfactory agreement with the experimental limits of the quadrupole moment can be obtained with very weak tensor forces. The sign of C_3 becomes positive and the quadrupole moment can be made as small as desired, negative or positive; the energy levels remaining within one- or two-tenths of an Mev of the experimental values. The strength of the tensor force needed to effect this agreement depends upon the ratio, β/α_t. The parameter β determines the rate of fall off of the radial wave function; it is related to the mean square radius (for harmonic oscillator wave

[13] J. P. Elliott, Phil. Mag. **1**, 503 (1956).

functions) by

$$\langle r^2\rangle=\tfrac{5}{2}\beta^2. \quad (8)$$

The parameter α_t is the tensor range. For a ratio $\beta/\alpha_t=1$ the tensor potential strength should be about -6 Mev. For the value $\langle r^2\rangle=7.8\times10^{-26}$ cm^2, the above ratio implies $\alpha_t=1.77\times10^{-13}$ cm. The improvement in the magnetic moment is negligible. The reason for this is that the magnetic moment depends primarily on C_1^2, whereas $\langle Q\rangle$ depends sensitively upon the ratio C_2/C_3 both in magnitude and sign. With the inclusion of the tensor force as a perturbation, the new wave function has amplitudes (0.986, 0.163, 0.0190). The only large change over the unperturbed function is in the amplitude of the ^3D state.

The potential which emerges from this study has the following properties: (a) The triplet even-parity central force is strong enough to bind the deuteron. (b) The ratio of singlet to triplet even-parity state force strength is somewhat smaller than is required by the deuteron splitting. (c) The tensor force is quite small. This potential differs from the potentials obtained from the study of the two- and three-body problems.[14–16] These studies indicate that the central part of the potential should give approximately zero binding energy to the deuteron, that the tensor part of the potential should be strong enough to supply the additional 2.2 Mev of binding for the triplet state. For the accepted radius, such potentials usually have central force matrix elements much smaller $[(^3S_1|V_c|^3S)\approx-6$ Mev$]$ than those in Table I, and tensor forces much stronger than are required to reduce the lithium quadrupole moment.

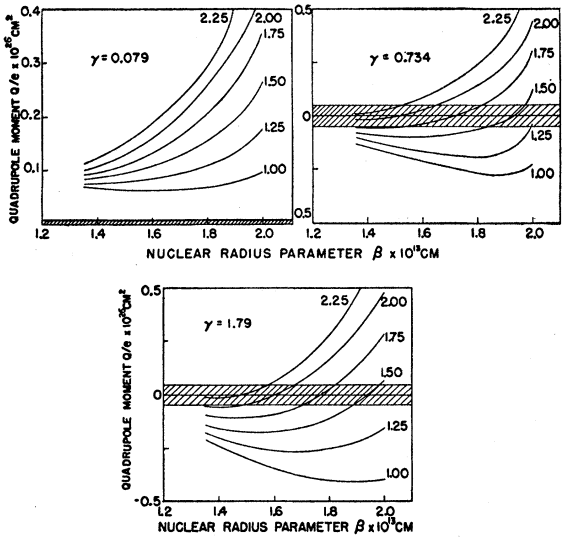

FIG. 5. The quadrupole moment *vs* the nuclear radius parameter β. The three graphs are for different values of γ=tensor strength/central strength. The value of γ is indicated on each graph. The numerical values labeling each curve in a graph are values of ζ. The shaded areas represent the region of acceptable values. These three potentials have the Yukawa shape and the potential parameters are taken from reference 17.

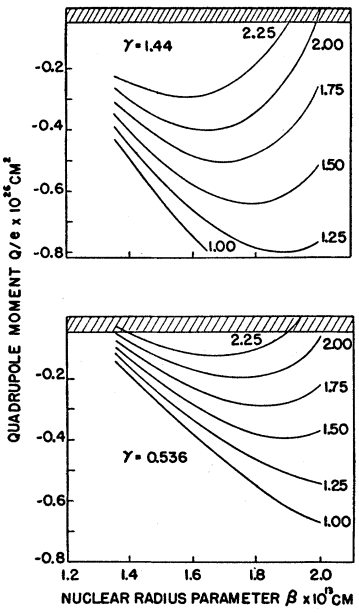

FIG. 6. The quadrupole moment *vs* the nuclear radius parameter β. The two graphs are for two different Gaussian-shaped potentials. The graphs are labeled by the value of γ of the potential used. The numerical values labeling each curve are the values of ζ assumed. The shaded areas represent the region of acceptable values. The potential parameters were taken from reference 18.

It is not obvious, however, from inspection of the energy matrices that the addition of a very weak tensor force to the matrix elements of Table I is the unique solution to the problem. Because of this we have tried to fit simultaneously the energy levels and the quadrupole moment of Li^6 using several potentials of Yukawa[17] and Gaussian[18] shape which yield the correct binding energy, quadrupole moment, and scattering properties of the deuteron system. The Naval Ordnance Laboratory IBM 650 Data Processing Machine was programed to calculate the energy levels and ground-state moments over a wide range of values of the parameters ζ and β. The resulting quadrupole moments are shown in Figs. 5 and 6 for five of the ten sets of parameters tried. It is clear from Fig. 5 that there are many possibilities for fitting the quadrupole moment using the Yukawa shape. But the energy level structure was unsatisfactory for all cases in which the quadrupole moment lay in the shaded area. The $J=2$ level always lay as low as or lower than the $J=3$ level, which was itself too close to the ground state. This difficulty is due to the unique problem with Yukawa shape already mentioned. In Fig. 6 the Gaussian shape is used. The Gaussian tensor potentials tend to overdo their role, making $\langle Q\rangle$ much too negative. This result is as expected because we have seen that the weak tensor force is capable of reducing the quadrupole moment adequately. For both Gaussian and Yukawa shapes, the tensor force is so strong that it forces the $J=2$ level too close to the ground state. For graphs and tables of the

[14] H. H. Hall and J. L. Powell, Phys. Rev. **90**, 912 (1953).
[15] R. L. Pease and H. Feshbach, Phys. Rev. **88**, 945 (1952).
[16] T. Hu and H. S. W. Massey, Proc. Roy. Soc. (London) **A196**, 135 (1949).
[17] H. Feshbach and J. Schwinger, Phys. Rev. **84**, 194 (1951).
[18] Kalos, Biedenharn, and Blatt, Nuclear Phys. **1**, 233 (1956).

behavior of the energy levels with the various potential parameters, consult Pinkston.[19]

4. CONCLUSION

The level structure of Li^6 is best fitted by a very strong central force with a great deal more spin exchange than is required by the deuteron, a spin-orbit force of strength about -1.7 Mev, and a tensor force so small that its effect on the Li^6 level structure is almost negligible. Of course this statement is made ignoring the effect of higher configurations. Recent calculations by Feingold[20] and Lyons[21] seem to indicate that central and tensor forces alone can account for the Li^6 energy level structure, and that there exists an effective potential which will give approximately the same results when used in a one-configuration calculation. This potential also has a spin dependence different from that of the actual two-body potential, a weaker tensor potential and a spin-orbit part. So far, however, the theory is not on a sufficiently quantitative basis to compare with our parameters. In addition this method of treating configuration interactions does not give a satisfactory quadrupole moment. Mottelson[22] introduced configuration interaction in a somewhat different fashion and got similar level spacings and astonishingly good theoretical moments. Unfortunately both the wave functions of Feingold and those of Mottelson contain unknown amounts of spurious states of center-of-mass excitation which might affect the moments.

[19] W. T. Pinkston, doctoral thesis, Catholic University, 1957 (unpublished).
[20] A. M. Feingold, Phys. Rev. **101**, 258 (1956); **105**, 944 (1957).
[21] D. H. Lyons, Phys. Rev. **105**, 936 (1957).
[22] Ben Mottelson, thesis, Harvard University, 1950 (unpublished).

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New Isotope, Sulfur-38†

D. R. NETHAWAY* AND A. A. CARETTO, JR.‡
Radiation Laboratory, University of California, Berkeley, California

(Received September 30, 1957)

A new isotope of sulfur, S^{38}, has been produced by the $(\alpha,3p)$ reaction on Cl^{37}. It was found to have a half-life of 172 ± 1 minutes, and to decay by the emission of two beta groups with end-point energies of 1.1 and 3.0 Mev. The 1.1-Mev beta was found to be in coincidence with a 1.88-Mev gamma ray. No other gamma rays were observed. The 3.0-Mev beta occurs in 5% of the disintegrations, and leads to the Cl^{38} ground state. The log (ft) values of the 1.1- and 3.0-Mev beta groups are 5.0 and 8.2, respectively. A comparison is made of the $(\alpha,3p)$ reactions on Al^{27}, Cl^{37}, and Cu^{65}.

INTRODUCTION

UNTIL now the only available radioactive isotope of sulfur that was suitable for tracer studies was S^{35}. The low decay energy of this isotope (167-kev β^-) leads to inaccuracy in absolute counting, so that its usefulness in many applications is greatly reduced. A new isotope of sulfur, S^{38}, has now been produced by alpha-particle bombardment of chlorine, and in some ways this nuclide is better suited for tracer studies than S^{35}.

An earlier search for S^{38} was made by Jones,[1] who tried to produce it by high-energy proton bombardment of scandium, but was unable to find direct evidence for its existence. He was able, however, to set exclusion limits for the half-life as less than 3 hours, or greater than 50 years. Roy and Kohman[2] had also obtained evidence that the half-life was about 6 minutes. The trend in the half-lives of the even-even isodiaspheres of the series including S^{38} indicated that its half-life might be long enough to be useful as a tracer. Table I gives the even-even isodiaspheric series with $N-Z=6$.

EXPERIMENTAL PROCEDURE

The S^{38} was produced by alpha-particle bombardment of NaCl, the reaction of interest being $Cl^{37}(\alpha,3p)S^{38}$. A similar reaction, $Al^{27}(\alpha,3p)Mg^{28}$ was known to produce Mg^{28} in good yield.[3] The target material was reagent-grade (99.9+%) NaCl crystals which had been ground into a powder. The powder was placed in an aluminum holder and covered with a piece of 0.001-inch aluminum foil. The target was exposed to the 48-Mev

TABLE I. Even-even isodiaspheric series with $N-Z=6$. (Isotopic abundances given.)

Cr^{54}	Ti^{50}	Ca^{46}	A^{42}	S^{38}
Stable (2%)	Stable (5%)	Stable (0.003%)	>3.5 years	···

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* Present address: Department of Chemistry, Washington University, St. Louis, Missouri.
‡ Present address: Department of Chemistry, Carnegie Institute of Technology, Pittsburgh, Pennsylvania.
[1] J. W. Jones, Ph.D. thesis, Carnegie Institute of Technology, Atomic Energy Commission Report NYO-6627, May, 1956 (unpublished).
[2] Jean-Claude Roy and T. P. Kohman (private communication).
[3] J. Hudis, J. Inorg. & Nuclear Chem. 4, 237 (1957).