

$(h_{9/2})_P$ ($\phi_{1/2}g_{9/2}$) $_N$ character as the result of configuration mixing. Such mixing would permit a single-particle transition to take place from the $h_{9/2}$ proton state to the $g_{9/2}$ neutron state with normal speed for a transition of the $\Delta I=0$ (yes) type.

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Beta-Gamma Circular Polarization Correlation Experiments*

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A circular polarization analyzer for γ rays is described. Measurements on circular polarization of bremsstrahlung due to β particles from P^{32} and Tm^{170} agree with previous and concurrent measurements and, together with Co^{60} circular polarization correlation studies, establish a check on the calculated calibration curve of the analyzer. We have studied the β - γ circular polarization correlation in the following allowed j - j transitions: Na^{24} , Sc^{44} , Sc^{46} , V^{48} , and Co^{58} . The result for Sc^{46} indicates the presence of a strong interference between Gamow-Teller and Fermi couplings. The β interaction, therefore, should contain a combination of S and T , or (and) V and A , interactions with small or no phase difference between the interaction constants. If one assumes maximum interference, our experiments give information on the ratio of Gamow-Teller to Fermi interaction. The once-forbidden transition Au^{198} gives the maximum possible asymmetry. All results are in agreement with the 2-component neutrino theory and the assumption of V, A or alternatively S, T, P interaction.

I. INTRODUCTION

RECENTLY Lee and Yang¹ have proposed that parity is not conserved in beta-decay processes. One of the consequences of nonconservation of parity, the anisotropy in the angular distribution of electrons emitted by polarized nuclei, has been verified by Wu *et al.*,² Ambler *et al.*,³ and Postma *et al.*⁴ with cryogenically polarized Co^{60} and Co^{58} nuclei. Another consequence, the prediction that nuclear electrons are longitudinally polarized, has been confirmed by Frauenfelder *et al.*,⁵ Goldhaber *et al.*,⁶ and de Waard and Poppema.⁷

A different experimental approach to study the consequences of nonconservation of parity is the measurement of the circular polarization of γ rays in

coincidence with preceding β particles.⁸ This method was described and employed in earlier communications.⁹⁻¹¹ The same method has been used by Schopper.¹² The β - γ circular polarization correlation experiments furnishes essentially the same information on the relative magnitude of parity-conserving and parity-nonconserving parts in the β interaction as the cryogenic polarization experiments mentioned above.⁸ But they have the advantage of being extendible to a larger number of different nuclei.

After emission of electrons the initially unoriented nuclei are polarized in or opposite to the direction of the electron emission. The β - γ circular polarization correlation method makes use of the fact that subsequent γ quanta, if emitted in or opposite to the direction of the electrons, must be circularly polarized. The degree and sense of circular polarization, which can be measured experimentally, gives information on the relative magnitude of parity-conserving and parity-

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¹ T. D. Lee and C. W. Yang, *Phys. Rev.* **104**, 254 (1956).

² Wu, Ambler, Hayward, Hoppes, and Hudson, *Phys. Rev.* **105**, 1413 (1957).

³ Ambler, Hayward, Hoppes, Hudson, and Wu, *Phys. Rev.* **106**, 1361 (1957).

⁴ Postma, Huiskamp, Miedema, Steenland, Tolhoek, and Gorter, *Physica* **23**, 259 (1957).

⁵ Frauenfelder, Bobone, Von Goeler, Levine, Lewis, Peacock, Rossi, and De Pasquali, *Phys. Rev.* **106**, 386 (1957).

⁶ Goldhaber, Grodzins, and Sunyar, *Phys. Rev.* **106**, 826 (1957).

⁷ H. de Waard and O. J. Poppema, *Physica* **23**, 597 (1957).

⁸ Alder, Stech, and Winther, *Phys. Rev.* **107**, 728 (1957), and University of Illinois Report (unpublished).

⁹ F. Boehm and A. H. Wapstra, *Phys. Rev.* **106**, 1364 (1957).

¹⁰ F. Boehm and A. H. Wapstra, *Phys. Rev.* **107**, 1202 (1957).

¹¹ F. Boehm and A. H. Wapstra, *Phys. Rev.* **107**, 1462 (1957).

¹² H. Schopper, *Phil. Mag.* **2**, 710 (1957), and private communications.

nonconserving parts as well as on the interference between different β interactions.⁸

II. EXPERIMENTAL PROCEDURES

Circular polarization of γ rays can be studied with the help of Compton scattering on atoms whose electrons are polarized, because the Compton scattering cross section depends on the spin direction of electrons and photons.¹³ We have designed a circular polarization analyzer where the scattering occurs in a nearly forward direction. It consists of a hollow Armco iron cylinder (Fig. 1) which is magnetized to saturation with the help of a coil. The wall thickness of the inner cylinder is 2 cm. A lead cone between source and counter absorbs the direct γ rays. The average scattering angle is 42° . The NaI(Tl) γ scintillation crystal and the anthracene β crystal are mounted on 11-in. long Lucite light pipes. The Dumont 6292 photomultipliers are surrounded by magnetic shields. As a result, the single counting rates showed only small influences of the direction of the magnetic field in the analyzer. We give two typical results: in our first measurements (Co^{60}), reversing the magnetic field changed the single β - and γ -counting rates by $(0.02 \pm 0.01)\%$ and $(0.01 \pm 0.04)\%$ respectively; in our last experiments (V^{48}) these differences were $(0.02 \pm 0.02)\%$ and $(0.07 \pm 0.02)\%$.

The γ rays are detected in coincidence with electrons emitted into the β counter. The β and γ pulses, therefore, are fed, after suitable pulse-shaping and amplification, to a fast ($\tau = 2 \times 10^{-8}$ sec) coincidence circuit.

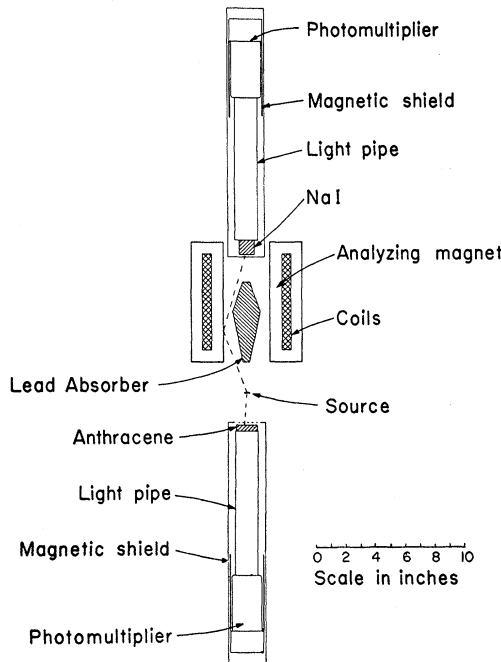


Fig. 1. Experimental arrangement for the study of β - γ circular polarization correlation.

¹³ S. B. Gunst and L. A. Page, Phys. Rev. **92**, 970 (1953).

Slow lower pulse height discriminators are provided on the β and γ side and are connected, together with the fast coincidence output, to a slow triple-coincidence circuit. The β discriminator is adjusted to accept pulses down to about $\frac{1}{3}$ of the maximum electron energy; the γ discriminator accepts pulses down to about $\frac{1}{2}$ of the maximum scattered quantum energy. High voltage supplies in connection with the photomultipliers, amplifiers and discriminators were adjusted for extreme stability.

Scattering of β particles in the sources or the source backings decreases the experimental asymmetries. The sources were therefore prepared as thin as possible, mostly by the Insulin spread technique and in some cases by vacuum evaporation. Mylar foils, 0.8 mg/cm² thick, vacuum-coated with aluminum, were used as source backings; in some cases thin mica backings were used. Corrections for scattering in the source backings were made; they were smaller than the statistical errors in the results. The following isotopes were obtained from the Oak Ridge National Laboratory: Na^{24} , P^{32} , Sc^{46} , Co^{58} , and Co^{60} . Hg^{203} , Au^{198} , and Tm^{170} were prepared by neutron irradiation in the Materials Testing Reactor in Arco, Idaho. Sc^{44} was prepared by deuteron irradiation of natural Ca in Berkeley¹⁴ and V^{48} by deuteron irradiation of Ti in the Philips synchro-cyclotron in Amsterdam.¹⁵

Single counting rates and coincidence counting rates showed small long-time fluctuations. In order to correct for these variations the following procedure was adopted. Singles and coincidences were counted in 20-minute intervals with alternate directions of the magnetic field in the analyzer. Corrections were made for accidental coincidences, γ - γ coincidences and, in the case of positron emitters, γ -annihilation coincidences, and, if necessary, for decay of the source. Care was taken that the corrections for accidental coincidences were not larger than about 15%. This corresponds to single β and coincidence counting rates of the order of 30 000 and 10 counts per second, respectively. The corrected coincidence rates were then divided by the product of the corrected single counting rates. The results were averaged over runs of about 10 hours. At least 10 runs were made for every nuclide. The results for different runs were always statistically consistent, which indicates that the procedure above eliminates the influences of shifts in the amplification, discriminator settings, and coincidence resolving times.

III. CALIBRATION

The efficiency of the analyzer can be defined as the percentage difference in the γ -counting rate for opposite field directions for a source of completely polarized

¹⁴ We thank Mr. W. B. Jones for an irradiation at the Crocker Cyclotron and Mr. B. Kraus for the chemical purification of the Sc^{44} .

¹⁵ We thank Professor A. H. W. Aten and Mrs. A. C. Funke-Klopper for the preparation of the V^{48} source.

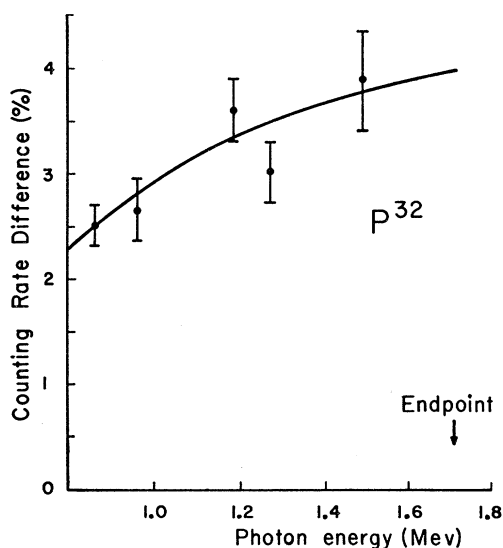


FIG. 2. Relative γ -counting rate difference for opposite magnetic fields of the analyzer as a function of the average photon energy for the bremsstrahlung of P^{32} . The drawn line gives the theoretical value computed by assuming an electron polarization of $-v/c$.

γ rays. This efficiency, as computed by Alder,¹⁶ is

$$\epsilon = 2.90k(1 + 0.13k)/(1 + 0.36k + 0.09k^2) \quad (1)$$

(where k is the γ -ray energy in units mc^2), with an estimated error of 15%. For β - γ angular correlation, this efficiency has to be multiplied by the cosine of the average angle between β and γ rays.

The expression (1) has been checked by measuring the circular polarization of bremsstrahlung of β particles emitted by P^{32} . The resulting difference in counting-rate for alternate magnetic field directions are shown in Fig. 2 as a function of the average energy of the bremsstrahlung quanta. The solid line in Fig. 2, is computed from Eq. (1). The degree of circular polarization of the bremsstrahlung quanta is estimated from the figures of McVoy¹⁷ under the assumption that the polarization of the electrons is $-v/c$. The comparison of the experimental points with the theoretical curve indicates a longitudinal electron polarization of $(-0.97 \pm 0.06)v/c$. Here the possible error in the calibration is not included. This result agrees with the value $(-1.00 \pm 0.10)v/c$ found by Frauenfelder *et al.*¹⁸

We have also measured the circular polarization of bremsstrahlung of Tm^{170} electrons. The results are shown in Fig. 3. If the experimental points are compared with the corresponding theoretical curve for full $-v/c$ polarization a longitudinal polarization of $(-0.93 \pm 0.07)v/c$ is obtained. A considerably lower polarization, $(-0.56 \pm 0.18)v/c$, has been reported by de Waard and Poppema.^{7,†}

¹⁶ We thank Dr. K. Alder for computing this efficiency.

¹⁷ K. W. McVoy, Phys. Rev. **106**, 828 (1957).

¹⁸ Frauenfelder, Hanson, Levine, Rossi, and De Pasquali, Phys. Rev. **107**, 643 (1957).

† Later investigations by these authors showed that due to systematic errors their values might be low by $\sim 40\%$ (de Waard,

Another check on our calibration has been obtained by measuring the β - γ circular polarization correlation in Co^{60} . A value of -0.41 ± 0.08 was found for the asymmetry coefficient A defined below. A preliminary result was reported earlier.⁹ This result agrees well with the other experimental values, $A = -0.34 \pm 0.04$ ¹² and -0.32 ± 0.07 ¹⁹ and with the theoretical value $A = -0.33$.⁸

The results of these measurements indicate that the accuracy of 15% quoted for the calibration formula Eq. (1) can indeed be considered to be a very conservative error limit.

IV. RESULTS AND DISCUSSION

The results of our measurements on allowed j - j transitions have been collected in Table I. The errors quoted are statistical errors only and do not include the uncertainty in calibration as discussed in the preceding section.

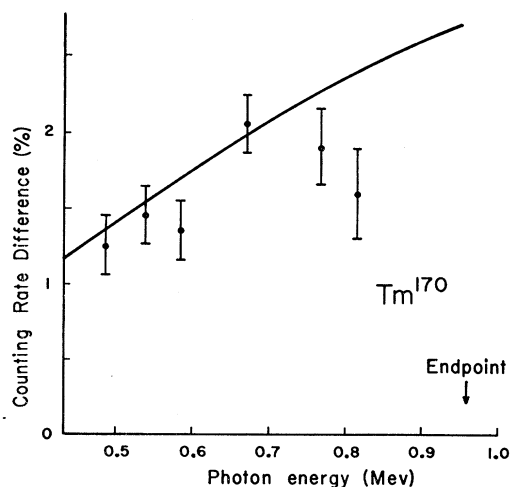


FIG. 3. Relative γ -counting rate difference as a function of the average photon energy for bremsstrahlung of Tm^{170} .

The anisotropy coefficient A listed in the last three columns is defined by

$$W(\theta, \tau) = 1 + \tau A (v/c) \cos\theta, \quad (2)$$

where $W(\theta, \tau)$ is the angular distribution of circularly polarized γ rays emitted under an angle θ with the preceding beta particle, v is the electron velocity, $\tau = +1$ for right-hand and $\tau = -1$ for left-hand circular polarization.⁸ Accordingly, $A(v/c) \cos\theta$ is the degree of polarization of the γ ray. The theoretical expression for A is given in the Appendix. Among the five allowed j - j transitions studied, Sc^{46} is particularly interesting because it shows a very large anisotropy coefficient and thus proves the existence of a large interference⁸ between Gamow-Teller and Fermi interaction. This nucleus will be discussed in some detail.

communication at the Israel Conference on Nuclear Physics, 1957).

¹⁹ Lundby, Patro, and Stroot, Nuovo cimento **6**, 745 (1957).

TABLE I. Summary of the experimental results. The following nuclear decay schemes (energies in kev) have been adopted (see Nuclear Data Cards, National Research Council, Washington 25, D. C.).

Na ²⁴	4 ⁺ (β^- 1390)4 ⁺ (γ 2750)2 ⁺ (γ 1370)0 ⁺
Sc ⁴⁴	2 ⁺ (β^+ 1460)2 ⁺ (γ 1160)0 ⁺
Sc ⁴⁶	4 ⁺ (β^- 360)4 ⁺ (γ 1120)2 ⁺ (γ 890)0 ⁺
V ⁴⁸	4 ⁺ (β^+ 690)4 ⁺ (γ 1330)2 ⁺ (γ 990)0 ⁺
Co ⁵⁸	2 ⁺ (β^+ 472)2 ⁺ (γ 805)0 ⁺
Au ¹⁹⁸	2 ⁻ (β^- 970)2 ⁺ (γ 411)0 ⁺

Type of experiment	Type of transition	Nuclide	Difference in coincidence counting rate δ in %	Anisotropy coefficient A^a		Theoretical for pure GT ^b
				This work	Experimental Others	
Longitudinal polarization	$\Delta j=1$; no	P ³²	+3.1 \pm 0.2	-0.97 \pm 0.06	-1.00 \pm 0.10 ^c -0.78 \pm 0.18 ^d	-1.00
	$\Delta j=0, 1$; yes	Tm ¹⁷⁰	+1.9 \pm 0.15	-0.93 \pm 0.07	(-0.56 \pm 0.18) ^d	-1.00
$\beta-\gamma$ circular polarization correlation	$\Delta j=1$; no	Co ⁶⁰	-0.97 \pm 0.17	-0.41 \pm 0.08	-0.34 \pm 0.04 ^e -0.32 \pm 0.07 ^f	-0.33
	$\Delta j=0$; no	Na ²⁴	+0.23 \pm 0.12	+0.07 \pm 0.04	-0.07 \pm 0.05 ^e	+0.08
		Sc ⁴⁴	-0.06 \pm 0.12	-0.02 \pm 0.04		-0.17
		Sc ⁴⁶	+0.73 \pm 0.09	+0.33 \pm 0.04		+0.08
		V ⁴⁸	+0.17 \pm 0.16	+0.06 \pm 0.05		-0.08
		Co ⁵⁸	-0.27 \pm 0.13	-0.14 \pm 0.07	(-0.20) ^g (-0.21 \pm 0.03) ^h	-0.17
	$\Delta j=0$; yes	Au ¹⁹⁸	+0.75 \pm 0.12	+0.52 \pm 0.09		
$\Delta j=?$; yes	Hg ²⁰³	-0.04 \pm 0.16	-0.06 \pm 0.22			

^a The definition for A is given by Eq. (2) except for the first two lines where A_{\parallel}/c denotes the longitudinal electron polarization.
^b Theoretical values for A according to Eq. (5) if isotopic spin selection rule holds strictly (Fermi matrix element vanishes).
^c See reference 13.
^d See reference 7.
^e See reference 12.
^f See reference 19.
^g See reference 4.
^h See reference 3.

The interference term which enters in the expression for A is

$$I = \frac{\text{Re}(C_S C_T'^* + C_S' C_T^* - C_V C_A'^* - C_V' C_A^*) + (Z\alpha/p) \text{Im}(C_S C_A'^* + C_S' C_A^* - C_V C_T'^* - C_V' C_T^*)}{|C_T|^2 + |C_T'|^2 + |C_A|^2 + |C_A'|^2} \quad (3)$$

Here C and C' are the coupling constants for parity-conserving and parity-nonconserving parts of the β interaction, Z the nuclear charge, α the fine structure constant, and p the electron momentum. The terms in I containing the imaginary part of the interaction constants are small or vanishing. If the two-component neutrino theory with time-reversal invariance is valid and if either (S and T) or (V and A) interactions occur, then the absolute value for I is $|I| = (C_{GT}/C_F)^{-1} = a^{-1/2}$. This is the maximum value which $|I|$ can assume. The value of a has been determined by Kofoed-Hansen and Winther²⁰ from studies of the ft values in mirror transitions. They find $a=1.3$.

In the transitions considered the Fermi matrix element can only be different from zero through a violation of the isotopic spin selection rule. One, therefore, expects a large ratio of the Gamow-Teller to the Fermi matrix element. Exceptions might occur if by accident the Gamow-Teller matrix element is very small (large ft value). Sc⁴⁶ (see below) seems to be an example of this kind.

Sc⁴⁶.—The anisotropy coefficient A (see Appendix)

²⁰ O. Kofoed-Hansen and A. Winther, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. 30, No. 20 (1956).

for a transition relating spin states 4-4-2-0, such as Sc⁴⁶ or Na²⁴, takes the simple form

$$A = (0.0834x^2 + 0.745a^3Ix)/(1+x^2), \quad (4)$$

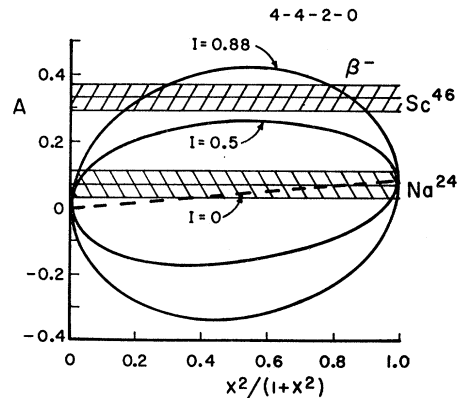


FIG. 4. Asymmetry coefficient for a 4⁺(β^-)4⁺(γ)2⁺(γ)0⁺ transition. The experimental values for Sc⁴⁶ and Na²⁴ are indicated. The curves are theoretical values corresponding to three parameters of the interference term I . If $I > 0$, the upper branch of the curve refers to the choice of $x = a^3 M_{GT}/M_F > 0$ and the lower branch to the case $x < 0$. If $I < 0$ the opposite assignment is true. The figure shows that a large value for I is required to fit the experimental data.

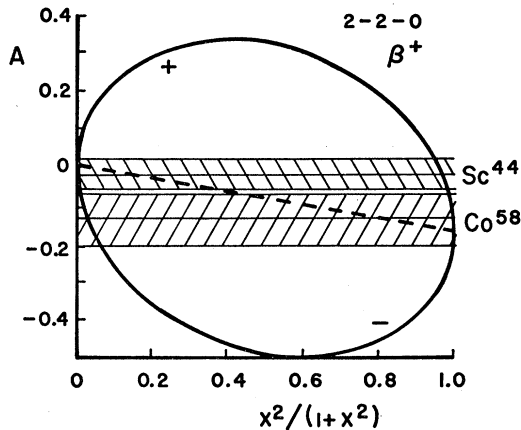


FIG. 5. Experimental and theoretical asymmetry coefficients for Sc^{44} and Co^{58} (see caption to Fig. 4).

where $x = a^{\frac{1}{2}} M_{\text{GT}}/M_{\text{F}}$, M_{GT} and M_{F} being the matrix elements for Gamow-Teller and Fermi transitions, respectively. In Fig. 4, A has been plotted versus $x^2/(1+x^2)$, the fractional contribution of Gamow-Teller and Fermi interaction. Curves have been drawn for three parameters of I , $I = I_{\text{max}} = 0.88$, $I = 0.5$ and $I = 0$. The experimental values are indicated. In the case of Sc^{46} it can be seen that independent of the unknown value of x the interference term must be large. The figure shows that in the case of Sc^{46} the following lower limit can be stated¹⁰:

$$|I| \geq 0.5,$$

the symbol \geq meaning that there is only 1% statistical chance for a smaller value than the lower limit. Assuming the maximum amount of interference, namely $a^{\frac{1}{2}}I = 1$, the ratio of Gamow-Teller to Fermi matrix element turns out to be $|M_{\text{GT}}/M_{\text{F}}| \cong 2.2$.

Na^{24} .—The Na^{24} result is in slight disagreement with the value reported by Schopper¹² (Table I). Our value (Fig. 4) does not permit a conclusion on the size of the interference term because of the unknown ratio of

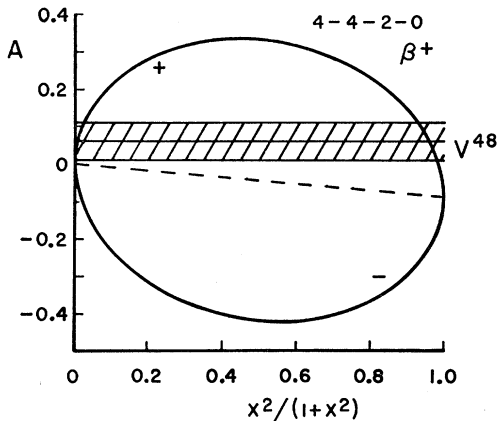


FIG. 6. Experimental and theoretical asymmetry coefficients for V^{48} (see caption to Fig. 4).

Gamow-Teller to Fermi matrix elements. If we accept the maximum value of I as suggested by the Sc^{46} experiment, it follows for the ratio of matrix elements $|M_{\text{GT}}/M_{\text{F}}| > 25$.

Co^{58} .—The result of Co^{58} is presented in Fig. 5. From the anisotropy in Co^{58} no conclusion can be drawn on the interference term.¹¹ If maximum interference is assumed, it would follow from our experiment that the matrix-element ratio must be rather large ($|M_{\text{GT}}/M_{\text{F}}| > 8$). This is in agreement with measurements of the positron distribution from oriented Co^{58} nuclei.^{3,4} For comparison anisotropy values derived from cryogenic orientation experiments^{3,4} are expressed in terms of our A and listed in column 6 of Table I. If large interference is present, these results are in disagreement with the ratio $|M_{\text{GT}}/M_{\text{F}}| \cong 2.8$ found by Griffing and Wheatley²¹ from a study of γ -ray distribution from aligned Co^{58} nuclei.

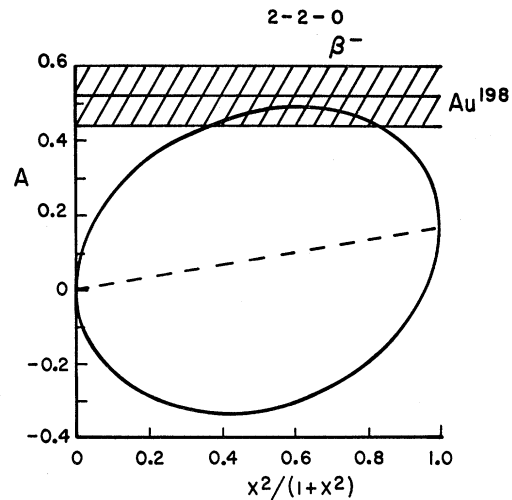


FIG. 7. Experimental and theoretical asymmetry coefficients for Au^{198} . (See caption to Fig. 4. For the definition of x in the case of forbidden transitions, see text.)

Sc^{44} and V^{48} .—The results on Sc^{44} and V^{48} are represented in Fig. 5 and Fig. 6, respectively. Both nuclei show an asymmetry parameter larger than that for pure Gamow-Teller interaction and, therefore, indicate the presence of an interference. Assuming again the maximum value for I , the matrix-element ratio for both Sc^{44} and V^{48} is of the order $|M_{\text{GT}}/M_{\text{F}}| \cong 5$.

Au^{198} .— Au^{198} is a first forbidden transition. The interpretation of the result for this nucleus (see Table I) is in general more involved. A discussion of this case has been given in a previous communication.¹¹ In Fig. 7 the Au^{198} result is compared with the theoretical values obtained under the assumption of the validity of the two-component theory and S, T, P or V, A interaction. This assumption is consistent with recent measurements on the longitudinal electron polarization in

²¹ D. F. Griffing and J. C. Wheatley, Phys. Rev. **104**, 389 (1956).

this nucleus.²² The definition of x in this case is given in the Appendix. Our result agrees with the maximum possible value for A and corresponds to $2.4 > |x| > 0.7$, with negative sign of x for I positive and vice versa. (In this case there is no reason to suppose that x should be large.)

V. CONCLUSION

This work, particularly the experiment on Sc^{46} , establishes the presence of a large interference between Gamow-Teller and Fermi couplings. It therefore excludes a pure V, T and a pure S, A interaction. It also disproves the validity of the twin neutrino theory.²³ Furthermore, if combined with the two-component theory and the result on longitudinal electron and positron polarization²⁴ it rules out a large breakdown of time-reversal invariance. The recent determination of the electron distribution in the neutron decay also indicates the presence of an interference term but its magnitude is smaller and not entirely in agreement with our value.

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APPENDIX

In the case of a pure $E2$ γ ray following the β decay the following expression for the anisotropy coefficient A [Eq. (2)] can be derived from the work of Alder, Stech, and Winther³:

²² Benczer, Koller, Schwarzschild, Vise, and Wu, Phys. Rev. (to be published); Cavanagh, Turner, Coleman, Gard, and Ridley, Phil. Mag. 21, 1105 (1957).

²³ M. Goeppert Mayer and V. L. Telegdi, Phys. Rev. 107, 1445 (1957).

²⁴ See, for example, Boehm, Novey, Barnes, and Stech [Phys. Rev. 108, 1947 (1957)], and reference 23.

$$A = -\frac{2}{3}F_1(2, 2, j_f, j) \{F_1(1, 1, j_i, j)Gx^2 + F_1(0, 1, j_i, j)a^3Ix\} / (1+x^2), \quad (5)$$

where

$$G = \{ \text{Re}(2C_T C_T'^* - 2C_A C_A'^*) + (Z\alpha/p) \text{Im}(C_T C_A'^* + C_T' C_A^*) \} / \{ |C_T|^2 + |C_T'|^2 + |C_A|^2 + |C_A'|^2 \}, \quad (6)$$

j_i and j are the spins of the initial and the final state in the β -decay process, respectively, and j_f is the spin of the final nucleus after γ emission. The coefficients F_1 can be obtained from the reference.⁸ The constants I and a and the matrix-element ratio x are defined in the text. In the expression (6) the small or zero Fierz term has been neglected. Parity measurements on $\text{Co}^{60, 2, 5, 7, 9, 12, 19}$ and $\text{P}^{32, 7, 18}$ yield an average value $G = (-0.96 \pm 0.05)v/c$. For the analysis we have chosen $G = -1$. For the special case of Sc^{46} , Eq. (5) leads to the expression (4) reproduced in the text as an example.

For the analysis of the first forbidden transitions with allowed spectrum shape such as Au^{198} the two-component theory has been assumed. The expression (5) then can be used in a good approximation together with the following substitutions: $a^3 I = 1$, $G = -1$, x is redefined in the case of S, T, P interaction as follows^{8, 9}:

$$x = + \left\{ C_S \xi \left| \int i\beta \mathbf{r} \right| + C_T \left| \int \beta \boldsymbol{\alpha} \right| + C_T \xi \left| \int \beta \boldsymbol{\sigma} \times \mathbf{r} \right| \right\} / \left\{ C_P \left| \int \beta \gamma_5 \right| + C_T \xi \left| \int \frac{\beta}{i} \boldsymbol{\sigma} \cdot \mathbf{r} \right| \right\}. \quad (7)$$

In the case of V, A interaction x is given by

$$x = + \left\{ C_V \xi \left| \int i\mathbf{r} \right| + C_V \left| \int \boldsymbol{\alpha} \right| + C_A \xi \left| \int \boldsymbol{\sigma} \times \mathbf{r} \right| \right\} / \left\{ C_A \left| \int \gamma_5 \right| + C_A \xi \left| \int \frac{1}{i} \boldsymbol{\sigma} \cdot \mathbf{r} \right| \right\}, \quad (8)$$

where $\xi = \alpha Z / 2R$, R being the nuclear radius; for Au^{198} , $\xi \cong 17$.