Nuclear Potential and Symmetry Energy*

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A quadratic dependence on momentum is assumed for the two-nucleon interaction energy in the independent-particle model, and is used in a study of the nuclear binding energy and symmetry energy. The corresponding optical potentials for elastic nucleon scattering are discussed. The semiempirical interaction used is compared with the two-body potentials commonly used in shell-model calculations. These are found to be inadequate.

I. INTRODUCTION

It has been proposed by Brueckner and others[1] that the energy of the ground state of nuclear matter can be obtained from a model in which the nucleons move independently in a common (velocity-dependent) potential U arising from their mutual interactions. This potential is also the optical potential[2] used to describe elastic scattering of nucleons by nuclei. The actual two-body interactions v_{ij} between free nucleons are replaced in the model by effective interaction or scattering operators t_{ij}.[3] If the actual v_{ij} are known, the t_{ij} in principle may be calculated. An alternative approach[4] is to assume some simple form for the matrix elements of the t_{ij} and study its properties in a semi-phenomenological way. We describe here some simple calculations along these lines, chiefly using a quadratic dependence on the nucleon momenta. First we adjust the parameters of the nuclear potential U and two-body interaction energy t to give the observed binding and separation energies of nucleons in nuclear matter with equal numbers of neutrons and protons. Then we generalize to nuclei with unequal numbers of neutrons and protons, using the nuclear symmetry energy to place restrictions on the difference in interaction energy of like and unlike pairs of nucleons. Incidentally, we note a general relation between the symmetry energy and the difference in optical potential seen by neutrons and protons. Finally, we inquire how closely the interactions t_{ij} correspond to the effective central potentials used quite successfully in shell-model calculations of nuclear spectra.

II. NUCLEAR POTENTIAL AND BINDING ENERGY

Weisskopf has pointed out[5] that the equality of the separation energy S and the volume binding energy P ('packing fraction') requires the common potential U of an independent-particle model to be velocity-dependent. He finds relations between the mean potential $\bar{U}$, the potential felt by nucleons at the Fermi surface U_F, the Fermi kinetic energy T_F, and P.

$$U_F=-P-T_F;\quad \bar{U}=-2P-(6/5)T_F. \tag{1}$$

In particular, for a quadratic dependence of U on the momentum,

$$U(k)=-U_0+U_1(k/k_F)^2, \tag{2}$$

$$2U_0=7P+3T_F;\quad 2U_1=5P+T_F. \tag{3}$$

Now the potential U_a felt by a nucleon in a state $|a\rangle$ arises from its interaction with the nucleons in all other occupied states $|b\rangle$.

$$\begin{aligned}U_a&=\sum\nolimits_b \langle ab|t|ab\rangle-\langle ab|t|ba\rangle\\&=\sum\nolimits_b t_{ab}.\end{aligned} \tag{4}$$

The exchange term, of course, arises from antisymmetry, and the labels a, b include spin and charge. The pair interaction t_{ab} is easily evaluated in the Fermi gas approximation (infinite nuclear matter). If we assume equal numbers of neutrons and protons, and average over spin and charge states, $\bar{t}_{ab}$ can only depend on the relative momentum $\mathbf{q}=\mathbf{k}-\mathbf{k}'$ of the pair of states a, b. The sum over b is then replaced by an integral over $\mathbf{k}'$ in the usual way. Writing $\bar{t}_{ab}=t(q)$,

$$U(k)=(2\pi^3)^{-1}\int_0^{k_F} d\mathbf{k}'t(\mathbf{k}-\mathbf{k}'). \tag{5}$$

As a first trial we shall assume a quadratic dependence of $t(q)$ on q:

$$t(q)=t_0+t_1(q/k_F)^2. \tag{6}$$

This, of course, gives a corresponding quadratic dependence for the potential U as in (2), with

$$-U_0=(t_0+\tfrac{3}{5}t_1)\rho,\quad U_1=t_1\rho, \tag{7}$$

where ρ is the nuclear density. We then obtain t_0, t_1 from (3), using $P=15.75$ Mev found from the mass formula by Green.[6] The density ρ found in electron scattering experiments[7] gives T_F (assuming neutrons

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[1] K. A. Brueckner and J. L. Gammel, Phys. Rev. **105**, 1679 (1957), where earlier references are given; H. A. Bethe, Phys. Rev. **103**, 1353 (1956).
[2] Feshbach, Porter, and Weisskopf, Phys. Rev. **96**, 448 (1954); Brueckner, Eden, and Francis, Phys. Rev. **100**, 891 (1955).
[3] In recent papers by K. A. Brueckner *et al.*, the symbol K_{ij} is used; Bethe[1] uses G_{ij}.
[4] T. H. R. Skyrme, Phil. Mag. **1**, 1043 (1956).
[5] V. F. Weisskopf, Nuclear Phys. **3**, 423 (1957).
[6] A. E. S. Green, Phys. Rev. **95**, 1006 (1954).
[7] R. Hofstadter, Revs. Modern Phys. **28**, 214 (1956).

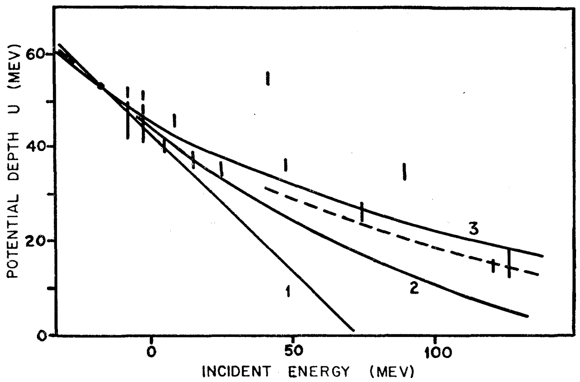

FIG. 1. Variation of optical potential U with incident energy, when adjusted to give nuclear binding energy. 1. Quadratic, Eq. (2); 2. Gaussian, Eq. (7a); 3. form of Eq. (7b). The experimental "points" are only intended to be representative. The appearance of experimental points at *negative* energy is due to surface effects. These raise the energy of a finite nucleus so that a zero-energy incident neutron is actually only about 8 Mev above the Fermi surface. In an infinite nucleus this would appear as a negative energy, $8-P\simeq-8$ Mev. The dashed curve was obtained by T. B. Taylor [Phys. Rev. **92**, 831 (1953)] from analysis of neutron total cross sections, but using a square well.

and protons occupy the same volume); the central densities correspond to T_F from 34 to 38 Mev. The results are given in Table I, together with the reduced mass ratio[1,5]

$$m/m^*=1+(U_1/T_F).$$

For comparison, the earlier value of $P=14$ Mev given by Feenberg[8] is also used.

We see the values of t_0 and t_1 are not very sensitive to the choice of P or T_F. But we require a value of t_1 of about $\frac{3}{2}T_F$, in contrast to the value $\frac{1}{2}T_F$ used by Skyrme[4] to explain the nuclear surface energy. Our value would lead to a surface thickness nearly twice that observed.[7] Inclusion of scattering in odd states (assumed negligible by Skyrme) is unlikely to produce much effect, and the value of the compressibility he uses is consistent with recent results of Brueckner.[1] A possible source of error lies in the assumption that, in a region of varying density, the interaction t is just that appropriate for the local density. Thus any possible dependence induced by the surface on $\mathbf{k}+\mathbf{k}'$ in addition to $\mathbf{q}=\mathbf{k}-\mathbf{k}'$ is neglected. This point will be investigated further.

Also, as already remarked by Weisskopf,[5] the reduced mass $m^*\simeq0.4m$ is somewhat smaller than is usually assumed. For example, the nuclear photoeffect[9] seems to require $m^*\simeq0.5m$ for nucleons near the Fermi surface. To see how much the discrepancy can be put down to the simple quadratic form we chose, we have also fitted the two more realistic two-parameter

TABLE I. Parameters for the nuclear potential well U and interaction energy t, with corresponding reduced masses. All energies are in Mev.

P	T_F	U_0	U_1	t_0	t_1	t_1/t_0	m/m^*
15.75	34	106.1	56.4	−140.0	56.4	−0.40	2.66
15.75	36	109.1	57.4	−143.6	57.4	−0.40	2.59
15.75	38	112.1	58.4	−147.2	58.4	−0.40	2.54
14.00	36	103.0	53.0	−134.8	53.0	−0.39	2.47
14.00	38	106.0	54.0	−138.4	54.0	−0.39	2.42

potentials:

$$U=-U_0\exp[-\alpha(k/k_F)^2], \tag{8a}$$

$$U=-U_0[1+\beta(k/k_F)^2]^{-1}. \tag{8b}$$

Using $P=15.75$, $T_F=38$, we find for (8a) that $U_0=124.1$ Mev, $\alpha=0.837$, while (8b) gives $U_0=150.5$ Mev, $\beta=1.80$.

The photoeffect is interpreted[9] as the raising of single nucleons from bound levels of energy E_a to states of the next shell at E_b. The difference in the single-particle energies is given by

$$\begin{aligned} E_b-E_a&=U_b-U_a+T_b-T_a \\ &=(T_b-T_a)m/m^{**}, \end{aligned} \tag{9}$$

say, where the "effective" reduced mass m^{**} over the range U_a to U_b is given by

$$\begin{aligned} (m/m^{**})-1&=(U_b-U_a)/(T_b-T_a) \\ &\simeq(dU/dT) \end{aligned} \tag{10}$$

evaluated at the Fermi surface. With the figures just given, $m/m^{**}=2.18$ for (8a), 1.91 for (8b), which are much closer to the experimental value.

It is also interesting to compare these potentials with those found necessary to describe elastic nucleon scattering. They are plotted in Fig. 1 for external incident energies up to 150 Mev. The experimental points scatter somewhat; those from an analysis using a Saxon potential are displayed in Fig. 1 in a schematic way. In general, they follow the curve for potential (8b); part of the scatter is due to the difference in potential seen by neutrons and protons. However, this has little significance other than showing that (8b) may be a convenient expression for the velocity dependence in this region. Apart from a certain amount of ambiguity in choosing potentials to fit the scattering data (uncertainty about the amount and distribution of the imaginary part, for example), no analysis has yet taken into account the velocity dependence. This introduces (in the case of a square well) a discontinuity in the logarithmic derivative[4,10] of the wave function, which tends to strengthen reflection from the surface.

In passing, it is of interest to note the degree of nonlocality this velocity dependence implies. In a coordinate representation, (8a) and (8b) lead to nonlocal

[8] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952), Chap. 6.

[9] D. H. Wilkinson, in *Proceedings of the 1954 Glasgow Conference* (Pergamon Press, London, 1954); S. Rand, Phys. Rev. **107**, 208 (1957).

[10] Ross, Mark, and Lawson, Phys. Rev. **104**, 401 (1956).

potentials[1] of the form

$$U(\mathbf{r},\mathbf{r}') = -A\exp[-(\rho/a)^2], \qquad a = 4\alpha/k_F, \quad (11a)$$

$$U(\mathbf{r},\mathbf{r}') = -B(b/\rho)\exp(-\rho/b), \quad b = \beta^{\frac{1}{2}}/k_F, \quad (11b)$$

where $\boldsymbol{\rho} = \mathbf{r}-\mathbf{r}'$. The above figures give $a = 1.4\times 10^{-13}$ cm, $b = 1.0\times 10^{-13}$ cm. Whatever the true form of $U(k)$, these results should remain qualitatively correct.

III. NUCLEAR SYMMETRY ENERGY

In the preceding section, we assumed equal numbers of neutrons and protons, and averaged over their interactions. We now relax this condition. However, first we make a few general remarks about the difference in potential seen by neutrons and protons. This arises in two ways, due to the Coulomb energy and to the nuclear symmetry energy. When we have equal numbers of neutrons and protons, each will experience the same potential, as a function of its kinetic energy T, provided the forces exhibit charge symmetry, $v_{nn} = v_{pp}$. However, because of the repulsive Coulomb energy U_c, protons will have a smaller kinetic energy in the medium than neutrons of the same *total* energy, and thus experience a different potential.[11] With our quadratic momentum dependence, the potential for both types of nucleon is $U(T) = -U_0 + \alpha T$, $\alpha = U_1/T_F$. But for neutrons we have a total energy $E = U(T)+T$, while for protons it is $E = U(T)+U_c+T$. Using these to express the optical potential as a function of total energy E, we have

$$\begin{aligned} U_n(E) &= (-U_0+\alpha E)/(1+\alpha), \\ U_p(E) &= (-U_0+\alpha E)/(1+\alpha) - \alpha U_c/(1+\alpha). \end{aligned} \quad (12)$$

Thus, even in this case, protons will see a nuclear potential $\alpha U_c/(1+\alpha)$ deeper than neutrons of the same incident energy. For Ca^{40}, for example, this is about 5 Mev, if one uses $U_c = 1.4(Z-1)A^{-\frac{1}{3}}$ Mev.[6]

With unequal numbers of neutrons and protons there is a further contribution to this difference from the symmetry energy. This arises partly because the ranges of relative momentum between one nucleon and those of the two groups are now different, and partly from any difference in the force between like and unlike pairs. Then, for example, the constants U_0 and U_1 (or α) for a proton will differ from those for a neutron.

A simple relation holds for nuclei which are β stable on the statistical model (i.e., ignoring shell effects). For these, the most energetic neutrons and protons (those at the Fermi surface) have the same total energy. Thus,

$$T_n + U_n(T_n) = T_p + U_p(T_p) + U_c,$$

where T_p, T_n are the respective proton and neutron Fermi kinetic energies; most medium and heavy nuclei depart little from this. It is well known[8] that

$$T_n - T_p = (4/3)T_F\epsilon, \quad \epsilon = (N-Z)/A,$$

if T_F is the Fermi energy appropriate to the mean of the neutron and proton densities, $\frac{1}{2}(\rho_n+\rho_p)$. Also, this stability condition is essentially the same as that used with the mass formula to extract values of the symmetry energy[8] u_τ from empirical nuclear masses. When the symmetry energy just balances the Coulomb force (or $\partial E/\partial\epsilon = 0$), we have (neglecting the neutron-proton mass difference)

$$U_c = 4\epsilon u_\tau.$$

Putting these results together, we obtain

$$U_n(T_n) - U_p(T_p) = 4\epsilon(u_\tau - \tfrac{1}{3}T_F). \quad (13)$$

Numerical values of this are included in Table II. For A about 60, $\epsilon \simeq 1/10$, and for A about 200, $\epsilon \simeq \frac{1}{5}$, giving a potential difference of 5 to 10 Mev. While this is only true at the Fermi surface, the change in going to low incident energies should be less than 10%. These values are consistent with the reported difference in neutron and proton potentials. (We note that Feenberg's value[8] for u_τ of 18 Mev leads to only half this difference. The experimental evidence, if significant, would seem to support Green's larger value.[6])

To obtain a more detailed insight into the symmetry effect, we continue to use the simple quadratic form for the interaction $t(q)$, but allow it to be different for like and unlike particles. A simple way to do this is to write the interaction, still averaged over spin orientations,

$$t(q) = t_0(1\pm\nu) + t_1(1\pm\eta)(q/k_F)^2. \quad (14)$$

The upper sign refers to like particles (nn or pp), the lower to unlike (np). Averaging over these leads again to (6), so t_0 and t_1 have the same meaning as in Sec. II.[12] Unfortunately we now have two new unknowns, and only one new datum (the symmetry energy), so we shall only be able to establish a relation between ν and η. This will be of interest, however, when we come to study the relation between $t(q)$ and two-body forces commonly used in shell-model calculations. We shall also point out further possible sources of information on ν and η separately.

The expression (5) for the potential must now be generalized to

$$U_x(k) = (4\pi^3)^{-1}\left\{\int_0^{k_n} d\mathbf{k}'\, t_{xn}(\mathbf{k}-\mathbf{k}') + \int_0^{k_p} d\mathbf{k}'\, t_{xp}(\mathbf{k}-\mathbf{k}')\right\}, \quad (15)$$

with $x = n$ or p for neutron or proton respectively. This is readily evaluated, giving in (2)

$$U_x(k) = -U_{x0} + U_{x1}(k/k_F)^2,$$

[11] This has also been pointed out by A. M. Lane, Revs. Modern Phys. **29**, 191 (1957).

[12] The expression (14) cannot be strictly true, since, even if the forces between free nucleons show charge symmetry, $v_{nn} = v_{pp}$, this property will not in general be retained by the t's. They have to be evaluated in the nuclear medium, and the differing densities of neutrons and protons will introduce some asymmetry. However, it is reasonable to suppose that this is a higher order effect than the one we are considering.

TABLE II. Empirical parameters in the relation $c\nu-\eta=d$, and the difference between neutron and proton potentials at the Fermi surface for "stable" nuclei. All energies in Mev.

P	T_F	u_τ	c	d	$(U_n-U_p)/\epsilon$
15.75	34	23.7	1.24	0.11	49
15.75	36	23.7	1.25	0.13	47
15.75	38	23.7	1.26	0.15	44
14.00	36	18.1	1.27	0.22	24
14.00	38	18.1	1.25	0.23	22

with

$$-U_{x0}=t_0\rho(1\pm\nu\epsilon)+t_1\rho(\tfrac{3}{5}\pm\eta\epsilon+\tfrac{1}{3}\epsilon^2),$$
$$U_{x1}=t_1\rho(1\pm\eta\epsilon). \qquad (16)$$

The $\pm$ now refers to $x=n$ or p. Using $k_x^2=k_F^2(1\pm\epsilon)^{\frac{2}{3}}$ and (13), or by explicitly calculating the total energy, we soon find the symmetry energy

$$u_\tau=\tfrac{1}{2}[t_0\nu+2t_1(\tfrac{1}{3}+\eta)]\rho+\tfrac{1}{3}T_F. \qquad (17)$$

Inserting the values of t_0, t_1 found in Sec. II we obtain a relation of the form $c\nu-\eta=d$. The c and d are included in Table II; we see c is about 5/4, and d is about 1/10 to 1/5. We shall return to this in the next section.

Equation (16) shows that the velocity dependence of U is sensitive to η, but not ν, in the k^2 approximation. That is, (dU/dT) at the Fermi surface depends on η and is different for neutrons and protons by perhaps 5% for medium weight nuclei. It follows from (10) that (γ,p) and (γ,n) will show giant resonances of slightly different energies. Figure 1 shows the k^2 approximation not to be very good above the Fermi surface, so the simple dependence of (16) will not hold exactly. However, we might hope to find some measure of η in this way.

Further, we may use (16) and the arguments at the beginning of this section to write down the difference between neutron and proton potentials at a total (or incident) energy E. We find to order ϵ^2

$$U_n(E)-U_p(E)\equiv\Delta_E=\Delta_0+2\gamma(1-\gamma)\epsilon\eta E, \qquad (18)$$

where

$$\gamma=t_1\rho/(t_1\rho+T_F),$$
$$\Delta_0=2\epsilon(1-\gamma)[t_0\rho(\nu-\gamma\eta)+t_1\rho\eta(1-\tfrac{3}{5}\gamma)]+\gamma[1-\epsilon\eta(1-\gamma)]U_c.$$

This reduces to (12) when $\epsilon=0$, and to (13) when $\epsilon=U_c/4u_\tau$. We see the energy dependence of Δ_E is proportional to η as well as ϵ, as already remarked above. With $P=15.75$, $T_F=38$, we have

$$\Delta_E=(99.4\eta-116.0\nu)\epsilon+(0.61-0.24\epsilon\eta)U_c+0.48\epsilon\eta E.$$

Thus more careful analyses of neutron and proton optical potentials and their variation at low energies could reveal further information on the symmetry effect.

IV. SHELL-MODEL TWO-BODY FORCES

In shell model calculations of nuclear spectra near closed shells,[13] a two-body central force v_{ij} with some exchange mixture, usually Rosenfeld or Serber, has been used quite successfully. We ask now how far such a force can be regarded as a reliable characterization of the true interaction t_{ij}. The usual central force with exchange is

$$v_{ij}=(w+mP^x+bP^\sigma+hP^\tau)J(r_{ij}),$$
$$w+m+b-h=-1.$$

The interaction between pairs is, analogous to (4),

$$t_{ab}=\langle ab|v|ab\rangle-\langle ab|v|ba\rangle.$$

Averaging over spin states, and using plane waves to evaluate the matrix elements,[14] we soon find

$$t(nn)=t(pp)=\tfrac{1}{2}(2w+2h+b-m)f(0)+\tfrac{1}{2}(2m-2b-h-w)f(q),$$
$$t(np)=\tfrac{1}{2}(2w+b)f(0)+\tfrac{1}{2}(2m-h)f(q), \qquad (19)$$

where

$$f(q)=\int_0^\infty d\mathbf{r}\exp(-i\mathbf{q}\cdot\mathbf{r})J(r) \qquad (20)$$

is the Fourier transform of the space dependence of the two-body potential. To compare (19) with our phenomenological quadratic expression (6) for $t(q)$, we can expand $f(q)$:

$$f(q)=4\pi\left\{\int r^2drJ(r)-\tfrac{1}{6}q^2\int r^4drJ(r)+\frac{1}{120}q^4\int r^6drJ(r)\cdots\right\}$$
$$=\alpha+\beta(q/k_F)^2+\gamma(q/k_F)^4\cdots,\quad\text{say.} \qquad (21)$$

This is closely connected to the technique[15] of expanding the potential in powers of its range. The quadratic approximation for $t(q)$ corresponds to retaining the first two terms of such an expansion. Collecting terms, we can identify

$$t_0=\tfrac{1}{4}\alpha(3w+3m);\qquad t_1=\tfrac{1}{4}\beta(4m-2h-2b-w),$$
$$t_0\nu=\tfrac{1}{4}\alpha(w+m+2);\quad t_1\eta=-\tfrac{1}{4}\beta(2b+w). \qquad (22)$$

We see that t_0 and ν depend only on the amounts of Wigner and Majorana force. We can use (22) to study the relation $c\nu-\eta=d$. Take $c=1.26$, corresponding to $T_F=38$, $P=15.75$ (Table II). The popular Rosenfeld mixture gives $d=-0.30$ instead of the empirical $+0.15$. A Serber force has $d=-0.09$, which is closer. In fact,

[13] M. G. Redlich, Phys. Rev. **99**, 1427 (1955); J. P. Elliott and B. H. Flowers, Proc. Roy. Soc. (London) **A229**, 536 (1955); M. J. Kearsley, Phys. Rev. **106**, 389 (1957).
[14] J. H. Van Vleck, Phys. Rev. **48**, 367 (1935).
[15] D. M. Brink, Proc. Phys. Soc. (London) **A67**, 757 (1954).

TABLE III. Force parameters for Serber (S), "almost Serber" (aS) and Rosenfeld (R) exchange mixtures, when $P=15.75$ Mev, $T_F=38$ Mev, and $t_1/t_0=-0.40$. For the Gaussian, $\alpha=Ba^3\pi^{3/2}$, $\beta/\alpha=-\frac{1}{4}(ak_F)^2$, $\gamma/\alpha=\frac{1}{2}(\beta/\alpha)^2$; for the Yukawa, $\alpha=4\pi Ba^3$, $\beta/\alpha=-(ak_F)^2$, $\gamma/\alpha=(\beta/\alpha)^2$.

				Gaussian			Yukawa		
	t_0/α	t_1/β	β/α	B (Mev)	$a(10^{-13}$ cm)	γ/α	B (Mev)	$a(10^{-13}$ cm)	γ/α
S	-0.750	-0.375	-0.80	92	1.32	0.32	326	0.66	0.64
aS	-0.750	-0.262	-1.14	54	1.57	0.65	191	0.79	1.30
R	-0.600	-0.600	-0.40	325	0.93	0.08	1156	0.46	0.16

if we restrict ourselves to Wigner and Majorana forces, the empirical values (for $T_F=38$, $P=15.75$) give $w=-0.59$, $m=-0.41$, which we might call an "almost-Serber" mixture.

The strength of the interaction t_0 and its velocity dependence t_1/t_0 are related by (21) and (22) to the shape of the potential $J(r)$ through α and β, as well as to the exchange character. It is difficult to disentangle the two influences, but in Table III we give the strength and range required for a Gaussian, $J=B\exp[-(r/a)^2]$, and Yukawa shape, $J=B(a/r)\exp[-(r/a)]$. We have chosen the Serber, Rosenfeld, and "almost-Serber" (mentioned above) exchange mixtures as representative. In addition, the corresponding coefficients of the q^4 terms in the expansion (21) are given. The striking thing is the large strengths B, and short ranges a, required for the Yukawa potential, compared with those used in the shell model.[12] The more "square" Gaussian shape shows a weaker velocity dependence than the long-tailed Yukawa, and hence yields more conventional values for range. In this way, the interaction energy averaged over the whole momentum range gives more information as to the potential shape than shell spectra, which are concerned with a narrow interval of momentum close to the Fermi surface. However, until more extensive calculations have been carried out, it is difficult to draw any definite conclusions on this. It is not clear, for example, how sensitive the shell-model results are to the choice of strength, range, and exchange mixture. In addition, it is quite plausible that the more energetic nucleons concerned in shell-model spectra are more strongly affected by surface restrictions than the average, more deeply imbedded, nucleons. Our model completely neglects these finite-size effects which, for example, will introduce elements of t off-diagonal in momentum space.

The large values of γ/α for the two Serber forces lead one to suspect that it is not a very good approximation to take just the first two terms of (21). Using the exact expression for the Fourier transform $f(q)$, and adjusting the constants to give the same average interaction over the momentum range of interest, increases the range a required, but also *increases* the strength B. In addition, the usual criterion that the parameters for different well shapes be adjusted to give the same (observed) low-energy free nucleon scattering is clearly not adequate. Thus there seems to be strong evidence against the conventional "shell-model" forces being an effective representation of the actual interaction operators t_{ij}.

V. CONCLUSIONS

A simple quadratic dependence on momentum, $t=t_0+t_1(q/k_F)^2$, has been assumed for the two-nucleon interaction energy, averaged over like and unlike pairs. At the normal density ρ of nuclear matter, the observed binding and separation energies require $t_0\rho$ to be about -145 Mev, and $t_1\rho$ to be about 58 Mev. The value of t_1 is about three times larger than that required by Skyrme to explain the nuclear surface energy; it is suggested the discrepancy may be due to the nonlocal nature of t near the surface. This interaction corresponds to an optical potential with a reduced mass m^*/m of 0.4. Agreement with the value 0.5 required by the nuclear photoeffect is improved by the use of more realistic velocity dependences for the potential U, which are also consistent with elastic nucleon scattering data. These potentials are nonlocal over a range of about 1×10^{-13} cm.

A general relation is noted between the nuclear symmetry energy and the difference in potential seen by slow neutrons and protons. The predicted value, U_n-U_p about 45 $(N-Z)/A$ Mev, is consistent with the scattering data. To study further the difference in interaction between like and unlike nucleon pairs, two new parameters have to be introduced in our approximation. We obtain a relation between then from the observed symmetry energy. It is possible to get further information from the different velocity dependence of slow neutron and proton optical potentials.

Finally, we compared the semiempirical $t(q)$ with the two-body interactions usually assumed in shell-model calculations. It was seen that these are not an adequate representation of t. However, it is possible that the interaction of the most energetic nucleons, concerned in shell spectra, is modified by surface effects.