## APPENDIX

The function $Y=F+iG$ is given by:

$$Y_l(\eta,\rho)=[\Gamma(l+1-i\eta)/\Gamma(l+1+i\eta)]^{\frac{1}{2}} \times e^{\frac{1}{2}\pi i(l+1-i\eta)}W_{i\eta,\,l+\frac{1}{2}}(2i\rho), \quad \text{(A-1)}$$

where $W$ is a Whittaker function. Thus,

$$A_{\eta,\,l}^2=YY^*=e^{\pi\eta}W_{i\eta,\,l+\frac{1}{2}}(2i\rho)W_{-i\eta,\,l+\frac{1}{2}}(-2i\rho). \quad \text{(A-2)}$$

However, we have the relation[15]

$$W_{\kappa,\,\mu}(z)W_{\lambda,\,\mu}(\zeta)=[\Gamma(1-\kappa-\lambda)]^{-1}(z\zeta)^{\mu+\frac{1}{2}}e^{-\frac{1}{2}(z+\zeta)} \times\int_0^\infty e^{-t}t^{-\kappa-\lambda}(z+t)^{-\frac{1}{2}+\kappa-\mu}(\zeta+t)^{-\frac{1}{2}+\lambda-\mu} \times{}_2F_1(\tfrac{1}{2}-\kappa+\mu,\tfrac{1}{2}-\lambda+\mu;1-\kappa-\lambda;\Theta)dt, \quad \text{(A-3)}$$

where

$$\Theta=\frac{t(z+\zeta+t)}{(z+t)(\zeta+t)}.$$

Introducing values for the various parameters, $\kappa$, $\lambda$, $\cdots$, and employing Kummer's relations for the hypergeometric function yield the result given in Eq. (9).

[15] W. Magnus and F. Oberhettinger, *Formulas and Theorems for the Special Functions of Mathematical Physics* (Chelsea Publishing Company, New York, 1949), p. 91.

PHYSICAL REVIEW VOLUME 109, NUMBER 2 JANUARY 15, 1958

# Resonance Scattering of Slow Neutrons on Indium*

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(Received May 29, 1957)

The scattering of slow neutrons by indium was observed by using a crystal spectrometer as a monoenergetic neutron source. Thin scattering samples were placed in the neutron beam with the plane of the samples at a small angle to the incident beam. The samples therefore appeared thick for the transmitted neutrons but thin for neutrons scattered at right angles to the direction of the incident beam. A measurement of the scattered and the transmitted neutrons gives $\sigma_s/\sigma_t$. The total cross section $\sigma_t$ was also measured in the energy range from 0.3 to 11 ev and the value of $\sigma_s$ then computed. Both the total cross section and $\sigma_s/\sigma_t$ results could be well matched to the Breit-Wigner formulas from 0.3 to 3 ev with the same set of parameters, assuming that the spin of the compound state in $In^{116}$ for the 1.456-ev level was 5. No determination of the spin state of the compound nucleus could be made for higher levels.

## I. INTRODUCTION

THE Breit-Wigner single-level equation gives the variation of scattering, capture, and total cross sections with the energy of the incident neutron. These formulas are given in Eqs. (1), (2), and (3).[1] In Eqs. (1′), (2′), and (3′) are presented alternative formulations which abbreviate certain groupings of the primary parameters by expressions which are convenient for analysis.

$$\sigma_s=4\pi gR_R^2+\frac{4\pi g\lambda\!\!\!^{-}_0{}^2\Gamma_n^2}{4(E-E_0)^2+\Gamma^2}+\frac{16\pi g\lambda\!\!\!^{-}_0\Gamma_nR_R(E-E_0)}{4(E-E_0)^2+\Gamma^2}+4\pi(1-g)R_{NR}^2, \quad (1)$$

$$\sigma_c=\left(\frac{E_0}{E}\right)^{\frac{1}{2}}\frac{4\pi g\lambda\!\!\!^{-}_0{}^2\Gamma_n\Gamma_\gamma}{4(E-E_0)^2+\Gamma^2}, \quad (2)$$

$$\sigma_t=\sigma_s+\sigma_c, \quad (3)$$

where the symbols represent the following: $\sigma_s$= scattering cross section, $\sigma_c$= capture cross section, $\sigma_t$= total cross section, $E$= energy of incident neutron, $E_0$= resonance energy, $2\pi\lambda\!\!\!^{-}_0$= neutron wavelength at resonance, $\Gamma_n$= partial width for neutron decay, $\Gamma_\gamma$= partial width for electromagnetic radiation, $\Gamma=\Gamma_n+\Gamma_\gamma$= total width of resonance, $R_R$= radius of nucleus for resonance interaction, and $R_{NR}$ radius of nucleus for nonresonance interaction.

$$\sigma_s=\sigma_p+\frac{\sigma_{s0}\Gamma^2}{4(E-E_0)^2+\Gamma^2}+\frac{I(E-E_0)}{4(E-E_0)^2+\Gamma^2}, \quad (1')$$

$$\sigma_c=\left(\frac{E_0}{E}\right)^{\frac{1}{2}}\frac{\sigma_{c0}\Gamma^2}{4(E-E_0)^2+\Gamma^2}, \quad (2')$$

$$\sigma_t=\sigma_s+\sigma_c, \quad (3')$$

where

$$\sigma_p=4\pi gR_R^2+4\pi(1-g)R_{nR}^2,$$

$$\sigma_{s0}\Gamma^2=4\pi g\lambda\!\!\!^{-}_0{}^2(\Gamma_n/\Gamma)^2\Gamma^2,$$

$$\sigma_{c0}\Gamma^2=4\pi g\lambda\!\!\!^{-}_0{}^2(\Gamma_n/\Gamma)(\Gamma_\gamma/\Gamma)\Gamma^2,$$

$$I=16\pi g\lambda\!\!\!^{-}_0R_R\Gamma_n.$$

* This work partially supported by the U. S. Atomic Energy Commission.

[1] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1951).

Another paper[2] describes an improved method for measuring the slow neutron scattering cross section of materials and results are given for a measurement of the cross section of gold. The present paper describes an extension of this technique to the much more difficult element, indium. The principal features of the measurement as described in detail in reference 2, henceforth referred to as I, are as follows:

(a) A beam of monoenergetic neutrons from a crystal spectrometer impinges at a small angle (about 6°) upon a target of the material under investigation. Part of the incident beam is transmitted through the target, the remainder is either captured or scattered. With the geometry used, the probability is high that a neutron which is scattered in the direction of the scattering counter (perpendicular to the incident beam) will leave the target without suffering another collision. Therefore the multiple-scattering correction will be small.

(b) The scattered and transmitted neutrons are observed for a standard and an unknown target. The relation between the observed counting rates, scattering and total cross sections is given by

$$\left(\frac{\sigma_s}{\sigma_t}\right)_x=\left(\frac{\sigma_s}{\sigma_t}\right)_{\mathrm{std}}\frac{N_x}{N_{\mathrm{std}}}\frac{(1-T_{\mathrm{std}}^{\alpha})}{(1-T_x^{\alpha})}\frac{C_{\mathrm{std}}}{C_x},\qquad(4)$$

where the subscripts "$x$" and "std" indicate the unknown and standard targets, respectively, the $N$'s are observed scattering rates, the $T$'s are observed transmissions, the $C$'s are multiple-scattering corrections, and $\alpha$ is a numerical factor equal to 1.106 determined in paper I. Lead, which has a ratio $\sigma_s/\sigma_t=1$, was the standard used. The multiple-scattering correction is given in I in graphical form as a function of $\sigma_s/\sigma_t$ and $T$.

(c) By using $\sigma_s/\sigma_t$ from (b) and $\sigma_t$ from transmission measurements, $\sigma_s$ is calculated.

(d) $\sigma_s$ as a function of neutron energy is fitted by a Breit-Wigner equation to determine the parameters of a level.

Indium was chosen as the second element to be investigated because (a) there is a considerable body of total cross-section information on indium and by using the scattering and total cross-section data available, it should be possible to evaluate all the Breit-Wigner parameters for at least the first level; (b) in the region to be investigated the source of neutrons was strong enough to yield a good counting rate; (c) since the spin of $In^{115}$ is 9/2, indium is a suitable element for investigating the sensitivity of the present experiment in determining the "$g$" value of the compound nucleus; (d) $In^{115}$ has three levels in the range up to 10 ev. Therefore the single-level formulas given by the Breit-Wigner equations will overlap. However, if the individual levels could be successfully separated in the analysis, a valuable check on the validity of summing single levels would be obtained.

TABLE I. Multiple-scattering correction for lead. The percentile difference between the two values of the correction is given in the last column. The agreement is satisfactory.

| Transmission | $N_s$ | $C_x/C_{\mathrm{std}}$ expt. | $C_x/C_{\mathrm{std}}$ corr. curve | Difference % |
|---|---|---|---|---|
| 0.8866 | 76.2 | 0.928 | 0.932 | 0.5 |
| 0.7946 | 141.3 | 0.957 | 0.955 | 0.3 |
| 0.6273 (std) | 265.1 | 1.000 | 1.000 | 0.0 |
| 0.4107 | 434.7 | 1.052 | 1.068 | 1.7 |

[2] H. E. Foote and J. A. Moore (to be published).

For the case of gold, where there is a single isolated level and the difference between possible $g$ values is fairly large, the cross section can be evaluated from Eq. (4) using a rough value of the multiple scattering correction. However, in order to obtain the higher accuracy required to achieve significant results from the analysis of indium data, many other factors involved in the measurements and the analysis of results had to

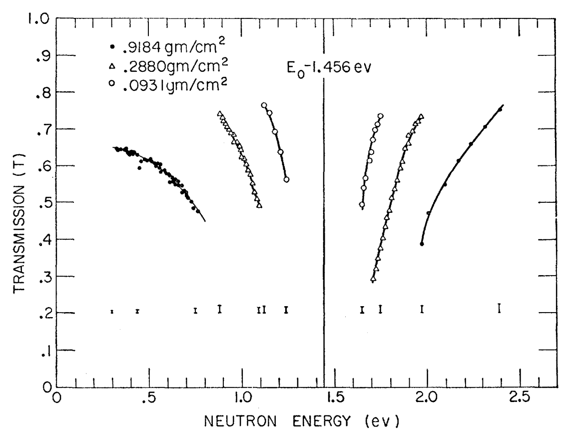


FIG. 1. Neutron transmission as a function of neutron energy for 1.456-ev level of indium for several different sample thicknesses.

be carefully investigated. The factors studied were (a) the effect of the energy loss of a neutron having an elastic collision; (b) instrument resolution; (c) a more accurate evaluation of the Doppler effect; (d) effect of a mixing of individual levels. These effects are discussed in separate appendices. All analysis was done for values of $E$ (the neutron energy) such that $|E-E_0|>3\Gamma$, where $\Gamma$ is the total level width. Fortunately, all the corrections are quite small in the wings of the indium 1.456-ev level where corrections for these effects are applied.

To check the validity of the multiple-scattering correction, which was calculated in I by using a ray-tracing technique, two simple experiments were performed. The first experiment was to measure the ratio of $\sigma_s/\sigma_t$ of carbon using lead as a standard and compare the value obtained with the known value which is $\sigma_s/\sigma_t=1$, for the energy range considered. The calculated multiple-scattering correction for both the lead and carbon was 1.10. The correction for energy loss on collision for carbon was 0.925 and for lead it was 1.00. The result was $\sigma_s/\sigma_t = 0.99 \pm 0.01$, after applying these corrections. The error is calculated from the statistical accuracy of the number of counts taken on the lead and carbon targets.

The second experiment was to determine the multiple-scattering correction for lead, using targets of varying thicknesses. Equation (4) can be rewritten in the form

$$\frac{C_x}{C_{\text{std}}} = \frac{(1-T_{\text{std}}{}^{\alpha})}{(1-T_x{}^{\alpha})} \frac{N_x}{N_{\text{std}}},$$

where

$$(\sigma_s/\sigma_t)_x = (\sigma_s/\sigma_t)_{\text{std}} = 1. \qquad (5)$$

The results of this experiment are shown in Table I.

## II. RESULTS AND ANALYSES

### Total Cross-Section Measurements

The transmissions of several indium samples were studied in the wings of the resonances at 1.456 ev,

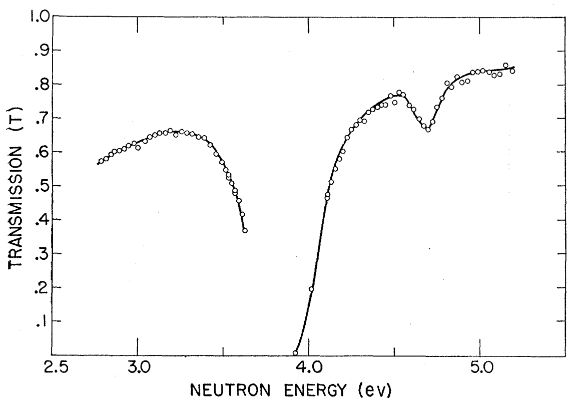


FIG. 2. Neutron transmission as a function of neutron energy for 3.85-ev level of indium.

3.85 ev, with the results as shown in Figs. 1, 2, and 3. Because the resonance energy was not well known, the central region as well as the wings was studied for the 9.01-ev level.

The experimental value of the total cross section is calculated from $T = e^{-n\sigma_t t}$.

The experimental cross section is corrected for the Doppler effect and the effect of other levels and the resultant curve fitted by a Breit-Wigner formula.

### *Analyais of the 1.456-ev Level*

To remove the effects of the 3.85-ev and 9.01-ev levels from the first level in indium, a set of Breit-Wigner parameters was calculated using the results of Landon[3] and Carter[4] together with an approximate value of $g$ of $\frac{1}{2}$ and a nuclear radius of $1.5 \times 10^{-13} A^{\frac{1}{3}}$ cm. The cross section predicted by these parameters was subtracted

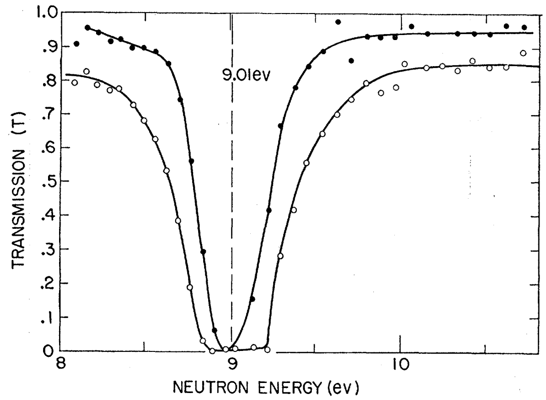


FIG. 3. Neutron transmission as a function of neutron energy for 9.01-ev level of indium.

from the measured cross section in the range 0.3 ev to 3 ev. The resulting cross section was fitted to a Breit-Wigner curve by means of least squares and the parameters for the 1.456-ev level obtained. To determine the effect of the weak 1.80-ev level of $In^{113}$ (abundance 3%) reported by Borst and Sailor,[5] the residuals, which are the measured cross sections minus the cross sections calculated from the parameters, were examined in the region from 1.7 to 1.9 ev. The residuals show consistent positive values so these points were excluded from the fitting procedure. In the fitting procedure statistical weights were assigned to each point in accordance with the statistical accuracy of the point as determined from counting. The parameters obtained were: $\sigma_{t0}\Gamma^2 = 213 \pm 4$ barn-ev$^2$; $I = 13.4 \pm 4$ barn-ev; $\sigma_p = 6.7 \pm 1$ barns. The errors quoted are those obtained from the sum of the residuals squared and the effect of varying the resonance energy $E_0$. Figure 4 shows the corrected total cross sections *vs* the neutron energy and the curve calculated from the parameters. If the value of the cross section calculated using the parameters is subtracted from the measured cross section in the vicinity of 1.8 ev the curve shown in Fig. 5 is obtained. The resonance

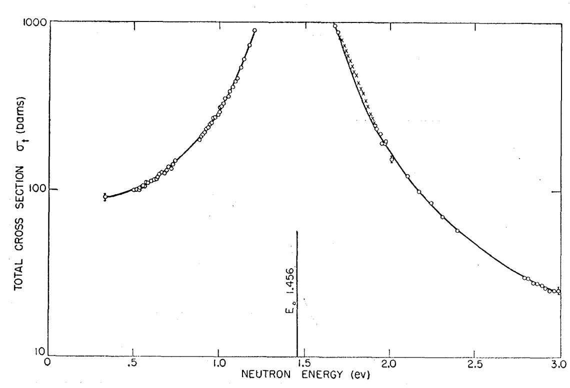


FIG. 4. Total cross section as a function of neutron energy for 1.456-ev level of indium.

[3] H. Landon and V. Sailor, Phys. Rev. 98, 1267 (1955).
[4] R. Carter (private communication).
[5] V. Sailor and L. Borst, Phys. Rev. 87, 161 (1952).

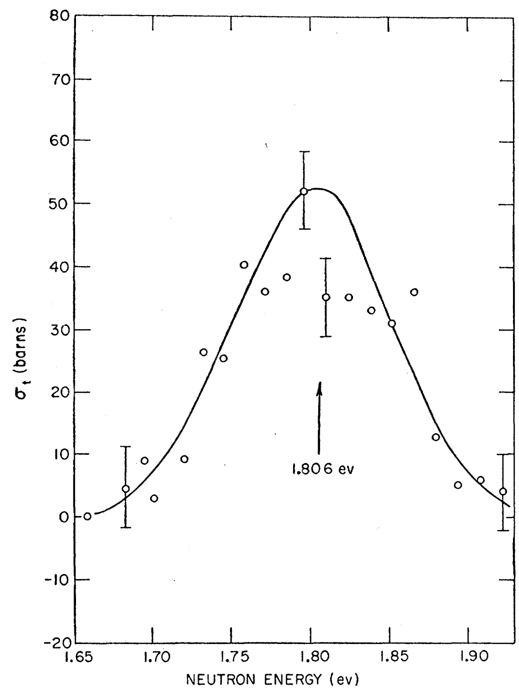


FIG. 5. Total cross section as a function of neutron energy for 1.8-ev level of indium.

energy is $1.81\pm0.02$ ev. From this figure the limits of $\Gamma<0.12$ ev and $\sigma_{t0}>50$ barns can be placed on the Breit-Wigner parameters for that level.

### *Analysis of the 9.01-ev Level*

Using the Breit-Wigner parameters determined above for the 1.456-ev level and the estimate of the 3.85-ev level from Landon's[3] results, levels were subtracted from the total cross section in the vicinity of 9.01 ev. The resulting cross section was further corrected for the Doppler effect and the following parameters obtained from a least squares fit: $\sigma_{t0}\Gamma^2=19.7\pm3.0$ barn-ev$^2$; $I=6.3\pm2.0$ barn-ev; $\sigma_p=4.8\pm1.0$ barns. The experimental and calculated cross-section curves are shown in Fig. 6.

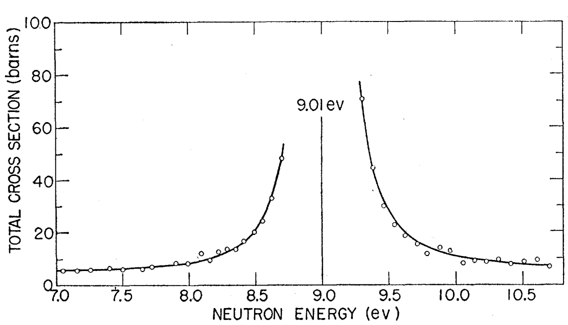


FIG. 6. Total cross section as a function of neutron energy for 9.01-ev level of indium.

### *Analysis of the 3.85-ev Level*

Proceeding as before, the cross section was corrected for the Doppler effect and the effects of the levels at 1.456 ev and 9.01 ev. The resulting cross sections are shown in Fig. 7. Omitting the points in the vicinity of 4.7 ev the remaining values were fitted and the following parameters obtained: $\sigma_{t0}\Gamma^2=8.2\pm0.1$, $I=0.28\pm0.16$, $\sigma_p=6.1\pm0.1$. The error associated with the interference term is large, however there is yet another effect which tends to make all the errors even larger. This is the effect on the 3.85-ev cross sections produced by varying the 1.456-ev resonance parameters within their limits of uncertainty. (The correction from the 9.01-ev level is so small that it need not be considered in this connection.) Upon taking $\sigma_{t0}\Gamma^2=217$, $I=17$, for the parameters of the 1.456-ev level and using the cross section predicted by these values to correct the 3.85-ev level, the values obtained by fitting are: $\sigma_{t0}\Gamma^2=8.3\pm0.1$,

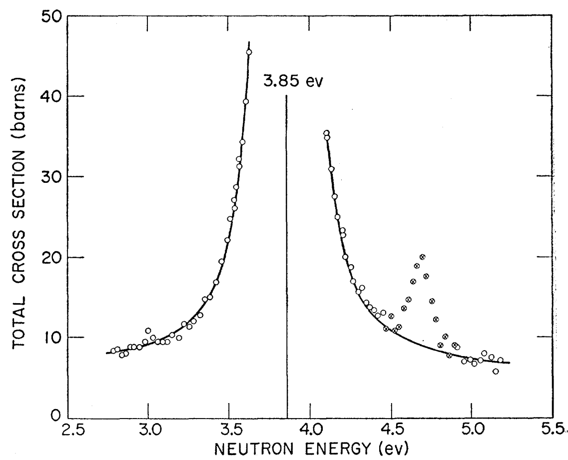


FIG. 7. Total cross section as a function of neutron energy for 3.85-ev level of indium.

$I=0.51\pm0.28$, $\sigma_p=5.5\pm0.2$. The parameters with their errors for the 3.85-ev level are therefore

$$\sigma_{t0}\Gamma^2=8.2\pm0.2,\quad I=0.3\pm0.3,\quad \sigma_p=6.1\pm0.6.$$

The error in the interference coefficient is so large that this quantity is essentially undetermined. The effect of this upon the scattering fit is considered below.

The observed cross sections near 4.7 ev after the values calculated from the parameters have been subtracted are shown in Fig. 8. These results show the resonance in the indium 113 isotope at $4.69\pm0.01$ ev.[6] The value of $\Gamma$ for this level must be less than 0.15 ev and $\sigma_0$ must be greater than 11 barns.

## Scattering Measurements

The scattering counting rate for indium as a function of the spectrometer arm angle is shown in Fig. 9. The

[6] Dabbs, Roberts, and Bernstein, Oak Ridge National Laboratory Report ORNL-CF-55 5 126 (unpublished).

discontinuity at 52° is occasioned by the introduction of a cadmium filter in the beam to remove thermal background. The oblique transmission of the scattering target from 0.3 to 3.0 ev is shown in Fig. 10(a). By using the values of the counting rates and oblique transmissions, the ratio $\sigma_s/\sigma_t$ was calculated. This was corrected for multiple scattering and the result for the 1.456-level is shown in Fig. 10(b).

By using the values of $\sigma_t$ from fitting the Breit-Wigner curve to the transmission data for the level, $\sigma_s$ is obtained. The significant contributions from the levels at 3.85 ev and 9.01 ev may be calculated by using the parameters obtained from the transmission analysis, the assumption of $g=\frac{1}{2}$ for both levels and the value of $\Gamma$ from the analyses of Landon[3] and Carter.[4] There is a considerable uncertainty in the value of $I$ for the 3.85 ev level. To ascertain the effect of this uncertainty, two fits were made to the scattering cross-section data. The first used a correction for the 3.85-ev level with $I=1.2$ (too large) and gave the results: $\sigma_{s0}\Gamma^2=8.6\pm0.2$, $I=17.42\pm0.27$, $\sigma_p=4.88\pm0.08$. The second fit used $I=0$ (too small) and gave $\sigma_{s0}\Gamma^2=8.52$, $I=17.13$, $\sigma_p=4.65$. When this uncertainty is included, the values of the parameters for the 1.456-ev level are $\sigma_{s0}\Gamma^2=8.6\pm0.2$, $I=17.4\pm0.4$, $\sigma_p=4.8\pm0.2$. The fit calculated from these parameters is shown in Fig. 10(c).

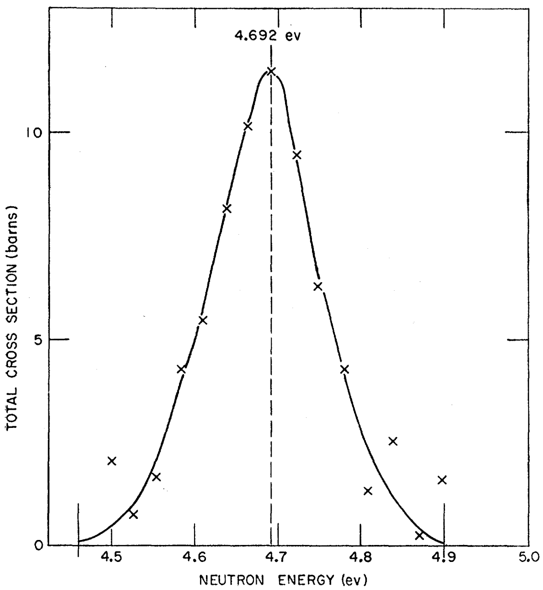


FIG. 8. Total cross section as a function of neutron energy for 4.7-ev level of indium.

There is another method[7] of analyzing the results of the scattering experiment which makes no use of the results of the transmission measurements except the value of the resonance energy. The basic data from the scattering measurement are the ratios $\sigma_s/\sigma_t$. This ratio has no simple analytical representation. However, if it is converted into $(E_0/E)^{\frac{1}{2}}(\sigma_s/\sigma_c)$, the following relation is obtained:

$$(E_0/E)^{\frac{1}{2}}(\sigma_s/\sigma_c)=a+bE+cE^2,$$

where $\sigma_p/\sigma_{c0}\Gamma^2=1/4C$; $I/\sigma_{c0}\Gamma^2=b+2cE_0$; $\Gamma_n/\Gamma_\gamma=a+bE_0+cE_0^2$.

The values of $(E_0/E)^{\frac{1}{2}}(\sigma_s/\sigma_c)$ calculated from the observed $\sigma_s/\sigma_t$ together with the standard deviation associated with each point are shown in Fig. 10(d). A weighted fit to these data gives the Breit-Wigner parameter ratios: $\Gamma_n/\Gamma=0.039\pm0.001$; $I/\sigma_{c0}\Gamma^2=0.0747\pm0.0015$; $\sigma_p/\sigma_{c0}\Gamma^2=0.0194\pm0.0005$.

Upon using a value of $\sigma_{t0}\Gamma^2=214$ barn-ev$^2$, which is an average of the result of the wing analysis, and Landon's[3] value, the following parameters are obtained: $\sigma_{s0}\Gamma^2=8.35\pm0.40$; $I=15.4\pm0.5$; $\sigma_p=4.0\pm0.2$. The values of $\sigma_p$ and $I$ are appreciably lower than those obtained from the other method of analysis to obtain $\sigma_s$. This occurs because the contribution of the higher levels cannot be removed from the ratio analysis. This is a fundamental shortcoming of the ratio analysis, which makes it applicable only to well-separated levels.

The results must also be corrected for the effects mentioned in Sec. I of this paper. The effect of the corrections which come from the Doppler correction and the interference between levels is to change the value of $I$ by 0.72 barn-ev. The effects of resolution and energy loss on collision were of the order of one percent and were neglected. One other correction is the net effect of the interference terms from the levels higher than 10 ev. Making some rough estimates of the level parameters, a contribution of 0.5 barn is to be added to the measured value of the potential cross section. Listed in Table II are the measured parameters obtained in this experiment and those obtained by Landon[3] in his central peak analysis.

By using a combination of Landon's parameters and the results of the $\sigma_s$ analysis, a complete set of Breit-

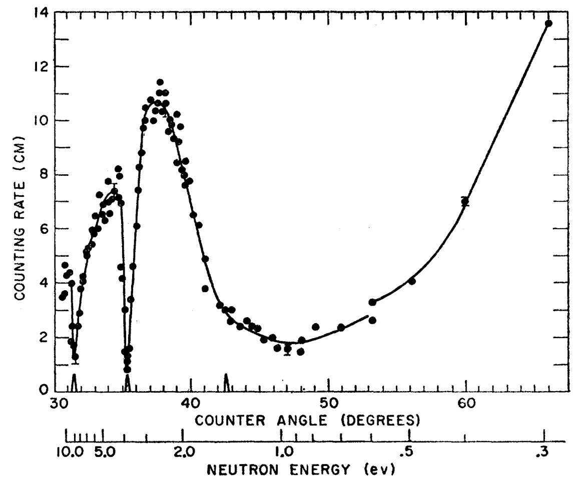


FIG. 9. Scattering rate of indium target as a function of spectrometer arm angle.

[7] C. Sheer and J. A. Moore, Phys. Rev. 98, 565 (1955).

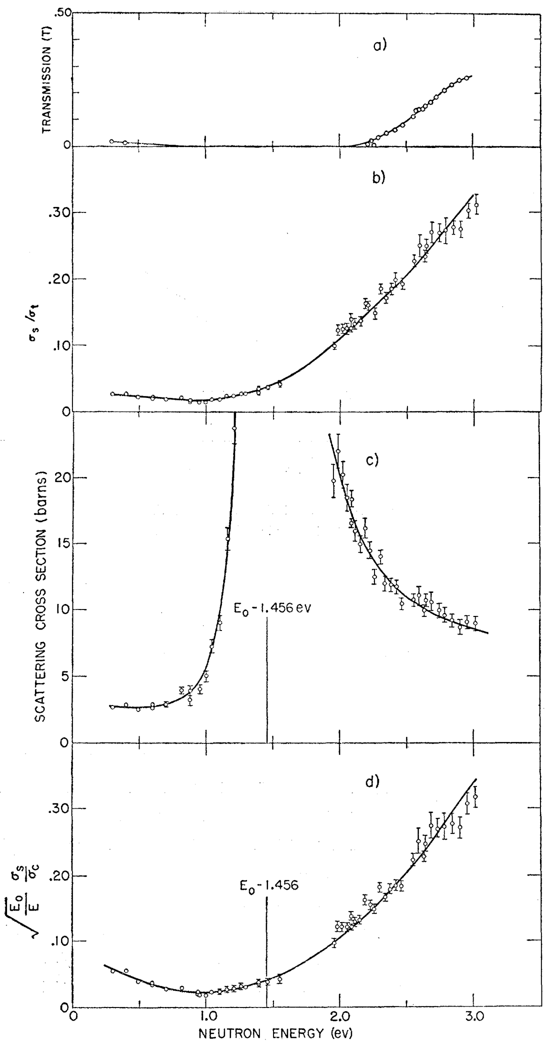


FIG. 10. (a) Oblique transmission of indium as a function of neutron energy. (b) Ratio of scattering cross section to total cross section for indium as a function of neutron energy. (c) Scattering cross section of indium as a function of neutron energy. (d) $(E_0/E)^{\frac{1}{2}}\sigma_s/\sigma_c$ for indium as a function of neutron energy.

Wigner parameters can be obtained. These are:

$E_0=1.456\pm0.002$ ev; $\Gamma=0.075\pm0.002$ ev;
$\Gamma_n=0.0030\pm0.0001$ ev; $\Gamma_\gamma=0.072\pm0.002$ ev;
$R_R=0.58\pm0.03$ barn$^{\frac{1}{2}}$; $R_{NR}=0.74\pm0.02$ barn$^{\frac{1}{2}}$;
$g=11/20$.

The $g$ is calculated by using Landon's measured peak cross section, of $\sigma_{t0}=38\,500\pm1000$ barns, with the value of $\Gamma_n/\Gamma$ determined by this experiment. The value of $g$ obtained was $g=0.54\pm0.02$. The possible values are $g=0.45$ or $g=0.55$. The fact that the $g$ value is determined within 4% indicates the accuracy of the experiment. There is direct experimental confirmation of this selection of the $g$ value in the work of Dabbs *et al.*[6]

Because the errors assigned to both $\sigma_{t0}\Gamma^2$ and $\sigma_{s0}\Gamma^2$ are important in determining the $g$ value, one additional source of error was considered besides the counting statistics. To determine the effect on the parameters of the uncertainty in the resonance energy (0.002 ev),[3] unweighted fits were made for both $\sigma_t$ and $\sigma_s$ with this change in the resonance energy. The change in the parameters was taken as indicative of the error associated with this effect. The amount of this error is contained in the quoted results.

Possibly a comment is merited concerning the inequality of the radii for the resonant and nonresonant interactions. Since the nuclear forces are spin-dependent, these radii should be different. It is difficult to make a theoretical estimate of how large this difference should be. Perhaps the wavelength of a neutron inside the compound nucleus would be a reasonable order-of-magnitude estimate of the uncertainty of the nuclear radius for the resonance case. This is of the order of $0.18\times10^{-12}$ cm, which is about the size of the difference between the two radii. The deviation of the nonresonance interaction from the value predicted by $0.14\times10^{-12}A^{\frac{1}{3}}$ cm might be attributed to the 0.52 barn addition to the potential scattering cross section from the levels higher than 10 ev. If this contribution is neglected, the radius is essentially equal to the value calculated from this formula. The slowly varying interference tails from distant resonances make the whole concept of potential scattering cross section difficult to understand in the region near 0 ev.

### *Scattering Analysis of 3.85 and 9.01-ev Levels*

An attempt was made to analyze the higher levels in indium using the scattering cross section obtained from the $\sigma_s/\sigma_t$ ratio. The value of $\sigma_{s0}\Gamma^2$ for both levels was found to be negative. The reason for these impossible values may be seen by inspecting Fig. 9, which shows the scattering rate as a function of the spectrometer angle. Indicated on the figure is the resolution triangle of the instrument. The dips corresponding to the levels at 3.85 ev and 9.01 ev are too steep to be resolved. With the resolution available not even an order of magnitude estimate could be made for $\Gamma_n/\Gamma$.

## III. CONCLUSION

The present experiment indicates clearly the nature of the principal difficulties of a scattering measurement. Dispersing the incident beam over $4\pi$ steradians

TABLE II. Parameters obtained in this experiment and those obtained by Landon in his central peak analysis. Scattering I are the results of ratio analysis, and Scattering II are the results of cross-section analysis.

| | $E_0$ ev | $\Gamma$ ev | $\sigma_{t0}\Gamma^2$ barn-ev$^2$ | $\sigma_{s0}\Gamma^2$ barn-ev$^2$ | $I$ barn-ev | $\sigma_p$ barns |
|---|---|---|---|---|---|---|
| Landon | 1.456 | 0.075 | 216 | ··· | ··· | ··· |
| Transmission | ··· | ··· | 213 | ··· | 14.1 | 7.2 |
| Scattering I | ··· | ··· | ··· | 8.4 | 16.1 | 4.5 |
| Scattering II | ··· | ··· | ··· | 8.6 | 18.1 | 5.4 |

decreases the intensity to such an extent that the resolution had to be decreased substantially, making significant analysis of results above 3 ev impossible.

The chief merits of this experiment were: a fairly small multiple scattering correction (10%); a relatively simple relation between $\sigma_s/\sigma_t$ and the scattering rates; simplicity in experimental technique, chiefly the ease of changing specimens; a small target size (4 cm$\times$5 cm $\times$0.1 cm).

The experiment indicates that the Breit-Wigner single-level equation with slight corrections for contributions for other levels gives an adequate description of the variation of the scattering cross section of indium in the energy range 0.3 ev to 3 ev, "adequate" meaning that the observed cross section can be fitted within its error of observation by a unique choice of Breit-Wigner parameters. However, the Breit-Wigner single-level equation contains six independent parameters: $E_0$, $\Gamma$, $\sigma_{t0}$, $\Gamma_n/\Gamma$, $I$, $\sigma_p$. The general shape of a cross-section variation is that of a resonance peak somewhat asymmetrical because of the interference term in the scattering and the $1/v$ variation in the capture cross sections. Some finite region is chosen over which the curve is to be fitted; this is dictated by experimental limitations or the presence of other levels. With this number of independent parameters available and the limited range to be fitted, it should not be surprising that an "adequate" fit may be obtained. Any small deviation of not too drastic nature (possibly caused by the presence of other levels) could be taken into consideration by slight variations in the six parameters. The point is not that the Breit-Wigner description should be suspect, quite the contrary since it admirably represents the shape of resonances (height, width, asymmetry), but that the parameters determined from any given fit may not be the parameters of the single level. One suspects that a considerable improvement in experimental technique must be effected before such quantities as the interference between levels can actually be observed.

## ACKNOWLEDGMENTS

The author wishes gratefully to acknowledge the assistance of the many individuals who aided him during the performance of this investigation. In particular he thanks Professor W. W. Havens, Jr., under whose guidance the work was performed; Dr. E. Melkonian of Columbia University and the members of Dr. V. Sailor's crystal spectrometry group at the Brookhaven National Laboratory. Finally, he acknowledges the friendly cooperation of Dr. H. L. Foote, with whom the experimental and analytical techniques of the thick-thin method were jointly devised.

## APPENDIX 1. ENERGY LOSS ON COLLISION

In deriving the Eq. (4) for $\sigma_s/\sigma_t$, the nuclei of the target were assumed to remain fixed, i.e., have infinite mass. The neutron after the collision has the same energy as the incident neutron. Because of the finite mass of the nucleus, the neutron after collision has an energy $E'$ which is $E'=E(1-m/M)/(1+m/M)$, where $E$ is the incident energy, $m$ is the neutron mass, and $M$ is the mass of the nucleus. Here the neutron comes off at 90° to its initial direction, the condition for the counted first order neutrons in this experiment. This loss of energy results in two effects on the scattering counting rate: first, the probability of the neutron leaving the target is altered, it decreases if $d\sigma_t/dE$ is negative (to the right of a resonance) and increases if $d\sigma_t/dE$ is positive (to the left of a resonance); second, because the efficiency of a $BF_3$ counter increases with decreasing energy, the probability of the neutron being counted increases. These effects will be considered in turn.

1. The effect of change of cross section will be greatest where the cross section changes most rapidly, i.e., near a resonance. The point of nearest approach to the resonance energy in the analysis was $3\Gamma$. Even at a distance of $3\Gamma$ the effect would be appreciable if it were not for the particular geometry of this experiment. Here the neutron has a path length in the sample before the collision about ten times the path length after collision. This tends to lessen any effects which occur after the collision. A straightforward calculation of the effect for the 1.456-ev level in indium gives an asymmetric change about $E_0$ which would tend to change $I$ by about 1%. All other terms would be affected by less than 1%.

2. To determine the increase in counting rate associated with a change in energy of the counted neutron, the variation of the scattering counter efficiency with energy was measured. The efficiency was found to vary as $E^{-0.45}$. The counting rate would therefore change by a factor $[(1-m/M)/(1+m/M)]^{-0.45}$. The only element for which this factor was greater than 1% was carbon for which the change amounted to 8%. This large effect occasioned by the small mass of the carbon nucleus was the principal reason for choosing lead rather than carbon as the standard scatterer.

## APPENDIX 2. INSTRUMENT RESOLUTION

The effect of resolution on the scattering counting rate may be represented as follows:

$$N_m(\theta_m)=\int N_A(\theta)R(\theta-\theta_m)d\theta, \qquad (6)$$

where $\theta_m$ is the spectrometer crystal angle, $N_m$ is the measured counting rate, $N_A$ is the counting rate with perfect resolution, and $R$ is the resolution function of the instrument.

An approximate expression for $R$ is obtained by taking a "rocking curve" for the spectrometer crystal. This resolution function has a bell-shaped variation with energy and is symmetric about the vertical axis of the bell. For convenience in analytical treatment it was represented by a normalized Gaussian function:

$$R(\theta-\theta_m)=\frac{1}{\sqrt{\pi}}\bar{\theta}\exp\left[-\frac{(\theta-\theta_m)^2}{\bar{\theta}}\right].$$

To find the first-order effect on the counting rate produced by resolution, $N_A(\theta)$ was expanded about $\theta_m$ in a Taylor series of powers of $(\theta-\theta_m)$. This was multiplied by $R(\theta)$ and then integrated over $\theta$. All even terms in the series were zero. The result to the first approximation was

$$N_m(\theta_m)\cong N_A(\theta_m)+C_3, \tag{7}$$

with

$$C_3=\left(\frac{\bar{\theta}}{2}\right)^2\left(\frac{d^2N_A}{d\theta^2}\right)_{\theta_m}\cong\left(\frac{\bar{\theta}}{2}\right)^2\frac{d^2N_m(\theta_m)}{d\theta_m{}^2}.$$

The quantity $d^2N_m/d\theta_m{}^2$ may be evaluated experimentally from the observed counting rate *vs* spectrometer angle curve. From this, $C_3$ can be calculated and the effect of resolution on the counting rate determined. The maximum value of $C_3$ for the 1.456-ev level in indium was only 0.01 $N_m$.

### APPENDIX 3. DOPPLER BROADENING

The effect of Doppler broadening on the scattering analysis is to change $\sigma_s/\sigma_t$ to a $\sigma_s'/\sigma_t'$:

$$\sigma_s'(E_m)=\int\sigma_s(E)w(E_m-E)dE, \tag{8}$$

$$\sigma_t'(E_m)=\int\sigma_t(E)w(E_m-E)dE, \tag{9}$$

where $w(E_m-E)=\pi^{-\frac{1}{2}}\exp[-(E_m-E)^2/\Delta^2]/\Delta$, with $\Delta=2(mE_0KT/M)^{\frac{1}{2}}$. See Bethe[8] for derivation and details.

Tables of the integrals Eq. (8) and Eq. (9) are presented by Rose *et al.*[9] in a form convenient for treating the resonance region. Because the Doppler effect on $\sigma_s$ is much the same as on $\sigma_t$ the overall effect for the ratio will be small. The actual change for the ratio was calculated for energies in the region of the 1.456-ev level of indium and represented a contribution to the cross section which could be absorbed by a change in the parameter $I$ by 2%.

[8] H. Bethe, Revs. Modern Phys. **9**, 140 (1937).

[9] Rose, Miranker, Leak, Rosenthal, and Hendrickson, Westinghouse Electric Corporation Atomic Power Division Report WAPD-SR-506, 1954 (unpublished).

### APPENDIX 4. INTERFERENCE BETWEEN LEVELS

If the level width is assumed small in comparison to the level spacing, the scattering cross section may be represented by[1,8]

$$\sigma_s=4\pi g\left|R_R+\sum\frac{\lambda\!\!\!^{-}_{0i}\Gamma_{ni}}{2(E-E_{0i})+i\Gamma_i}\right|^2+4\pi(1-g)R_{NR}{}^2. \tag{10}$$

For the following analyses, it is convenient to let $g$ be $\frac{1}{2}$ for all levels. As a first approximation, the mixed terms in the product are neglected. The result is a sum of Breit-Wigner single level cross sections. To determine the effect of the neglected terms consider the terms due to nearest neighbors. This may be expressed most simply in terms of two levels. The expanded quadratic consists of the sum of two single level cross sections plus a term corresponding to the effect of the interference between levels. This term is

$$\frac{8(\sigma_{s01}\Gamma_1{}^2\sigma_{s02}\Gamma_2{}^2)^{\frac{1}{2}}(E-E_{01})(E-E_{02})}{[4(E-E_{01})^2+\Gamma_1{}^2][4(E-E_{02})^2+\Gamma_2{}^2]}+\frac{4\pi\Gamma_1\Gamma_2\lambda\!\!\!^{-}_{01}\lambda\!\!\!^{-}_{02}\Gamma_{n1}\Gamma_{n2}}{[4(E-E_{01})^2+\Gamma_1{}^2][4(E-E_{02})^2+\Gamma_2{}^2]}. \tag{11}$$

The product of the two partial widths, $\Gamma_1\Gamma_2$, in the second term makes this negligible with respect to the first for the case of indium. To find an expression for the contribution to the first level because of the presence of the second, the interference term can be expanded in a Taylor series about the first resonance energy. Upon neglecting all but the first term, this becomes

$$\frac{2(\sigma_{s01}\Gamma_1{}^2\sigma_{s02}\Gamma_2{}^2)^{\frac{1}{2}}}{(E_{01}-E_{02})}\times\frac{(E-E_{01})}{4(E-E_{01})^2+\Gamma_1{}^2}. \tag{12}$$

To this approximation the net effect of the interference between levels is to increase the interference term for the first level by the coefficient of the energy-dependent factor. For the 1.456-ev level this change was 2.2%.