Ionization Cross Section of Argon-Argon Collisions near Threshold*

PHILIP ROSEN
Hasbrouck Physics Laboratory, University of Massachusetts, Amherst, Massachusetts
(Received September 9, 1957)

The ionization cross section of argon-argon collisions near threshold is calculated by using a semiclassical method. The wave function of the eight electrons in the M shell is obtained approximately by considering the core as having an atomic number of eight. Products of hydrogenic functions are used for the eight M electrons with the screening constants of Slater. The agreement between theory and experiment is satisfactory.

RECENTLY the author developed an approximation method for determining low-energy inelastic collision cross sections.[1] For the ionizing collision of two atoms it was found that the ionization cross section was

$$\sigma=2\pi\int_{\Omega_e}\int_0^{(P_e)_{\max}}\int_0^{\infty}\frac{|V_{fi}(R)|^2}{[I+(P_e/2m)^2]^2}\frac{P_e^2dP_ed\Omega_e}{(2\pi\hbar)^3}pdp, \tag{1}$$

where $V_{fi}(R)$ is the matrix element between initial and final state of the perturbation energy between the atoms at the distance of closest approach R, p is the impact parameter, P_e is the electron momentum, $d\Omega_e$ is an element of the electron's solid angle, and I is the ionization energy. The relation between the distance of closest approach and the impact parameter is given by

$$1=\frac{p}{R}+\frac{U(R)}{\frac{1}{2}\mu v^2}, \tag{2}$$

where $U(R)$ is the interatomic potential energy at the distance of closest approach, and $\frac{1}{2}\mu v^2$ is the relative kinetic energy of collision (μ is the reduced mass of an atom). The maximum electron momentum is given approximately by

$$\frac{[(P_e)_{\max}]^2}{2m}\cong\tfrac{1}{2}\mu v^2-I\equiv\epsilon.$$

In the latter case the electron carries off most of the energy.

For the case of argon-argon collisions our problem resolves itself into evaluating the matrix element V_{fi}. In order to evaluate V_{fi}, we must decide upon an approximate wave function. We shall take the six ($3p$) electrons and the two ($3s$) electrons in the outer shell to surround a core of charge of atomic number equal to eight. For the latter eight electrons we shall use products of Slater orbitals:

$$\begin{aligned}\psi&=(2\zeta)^{n+\frac{1}{2}}[(2n)!]^{-\frac{1}{2}}r^{n-1}e^{-\zeta r}Y_l^m(\theta,\varphi),\\ \psi&=Cr^{n-1}e^{-\zeta r}Y_l^m,\end{aligned} \tag{3}$$

where $n=3$, $\zeta=(Z-S)/na_0$, and $Y_l^m(\theta,\varphi)$ are the spherical harmonics. Here S is the screening constant and a_0 is the first Bohr orbit radius. To evaluate the screening constant, we use Slater's rules.[2] We obtain for a $3p$ or $3s$ electron, $S=2.45$. Thus

$$\zeta=(8-2.45)/(3a_0)=3.5\times10^8\ \mathrm{cm}^{-1}.$$

Instead of using symmetrized functions of the Slater orbitals, we find it more convenient to use the approximation

$$|V_{fi}|^2\cong\sum_{p\ \text{electrons}}|V_{fi}^{(p)}|^2+\sum_{s\ \text{electrons}}|V_{fi}^{(s)}|^2, \tag{4}$$

where $V_{fi}^{(p)}$ is the matrix element with simple product functions with a p electron removed from a beam atom and $V_{fi}^{(s)}$ is the matrix element with an s electron removed. This approximation is possible because for the initial state individual orbitals are orthogonal and for the final state individual orbitals are almost orthogonal (i.e., the overlap integrals are small between orbitals in the final state). The latter is so because we use plane-wave functions for the emitted electron of the type $\exp(i\mathbf{P}_e\cdot\mathbf{r}/\hbar)$, which are normalized in a box of unit length.

We shall call the beam atoms A, and the target atoms B. We shall label electrons i as belonging to A and electrons j as belonging to B. The p electrons of type i we shall number 1 and 2 if the magnetic quantum number m is one; numbers 3 and 4 have $m=0$, and numbers 5 and 6 have $m=-1$. The s electrons of type i will be numbered 7 and 8. Similarly p electrons of the type j are such that electrons 9 and 10 have $m=1$, 11 and 12 have $m=0$, and 13 and 14 have $m=-1$. The s electrons of type j are labeled 15 and 16. We shall only remove electrons from the beam atoms A in evaluating the cross section since it is incorrect to count the ionization of the target atom also. Thus the

* This research was carried out when the author was consultant to the Department of Missile and Ordnance Systems of the General Electric Company which operates under contract with the U. S. Air Force. At the time the author was also associated with the University of Connecticut, Storrs, Connecticut.

[1] Philip Rosen, Phys. Rev. 109, 348 (1958), preceding paper.

[2] J. C. Slater, Phys. Rev. 36, 57 (1930).

perturbation potential is

$$V=\sum_{i=1}^{16}\sum_{j=9}^{8}\frac{e^2}{r_{ij}}-8\sum_{i=1}^{8}\frac{e^2}{B_i}-8\sum_{j=9}^{16}\frac{e^2}{A_j}+\frac{64e^2}{R}. \qquad (5)$$

Here B_i is the distance of electron i to nucleus B, A_j is the distance of electron j to nucleus A, and R is the internuclear distance.

EVALUATION OF MATRIX ELEMENTS

Before we begin the evaluation of the matrix elements involved in Eq. (4), we shall for convenience give the necessary spherical harmonics:

$$Y_1^1=(3/8\pi)^{\frac{1}{2}}\sin\theta e^{i\varphi},$$
$$Y_1^{-1}=(3/8\pi)^{\frac{1}{2}}\sin\theta e^{-i\varphi},$$
$$Y_1^0=(3/4\pi)^{\frac{1}{2}}\cos\theta,$$
$$Y_0^0=1/(4\pi)^{\frac{1}{2}}.$$

We shall consider the evaluation of $\sum_p|V_{fi}^{(p)}|^2$ first. Since $Y_1^{\pm1}(\theta,\varphi)$ appears as $|Y_1^{\pm1}|^2$ in all integrals which do not vanish, the electrons with $m=1$ are equivalent to $m=-1$. Thus we have

$$\sum_p|V_{fi}^{(p)}|^2=4|V_{fi}^{(1)}|^2+2|V_{fi}^{(3)}|^2,$$

where $V_{fi}^{(1)}$ is the matrix elements of V between product functions with electron 1 removed and $V_{fi}^{(3)}$ is the matrix element with electron 3 removed. Some of the integrals we shall encounter will be very difficult and so we shall make approximations.

ELECTRON 1 REMOVED

We shall first consider the matrix element V_{fi} with electron 1 removed. For the interaction of the bare cores we need the integral

$$\frac{64e^2}{R}C\int\exp\frac{(-i\mathbf{P}_{e1}\cdot\mathbf{A}_1)}{\hbar}A_1^2e^{-\zeta A_1}Y_1^1d\tau_1.$$

This vanishes because Y_1^1 contains $e^{i\varphi}$ and $\mathbf{P}_{e1}\cdot\mathbf{A}_1=P_{e1}A_1\cos\theta_i$. In addition we have 8 integrals of the type

$$C^3e^2\int\frac{1}{r_{1j}}\exp\left(\frac{-i\mathbf{P}_{e1}\cdot\mathbf{A}_1}{\hbar}\right)A_1^2e^{-2\zeta A_1}B_j^4e^{-2\zeta B_j}\times Y_1^1(\theta_1,\varphi_1)|Y_l^m(\theta_j,\varphi_j)|^2d\tau_1d\tau_j.$$

We can approximate the latter integral by replacing r_{1j} by B_1 because $e^{-2\zeta B_j}$ causes the j electron function to decay rapidly and thus giving us a δ function charge distribution about nucleus B. We also have for the interaction of nucleus B with electron 1 the integral:

$$-8e^1C\int\frac{1}{B_1}\exp\left(\frac{-i\mathbf{P}_{e1}\cdot\mathbf{A}_1}{\hbar}\right)A_1^2e^{-\zeta A_1}Y_1^1d\tau_1,$$

which will cancel the previous contribution if we use the former approximation. Integrals of the type

$$e^2C^4\int\frac{1}{r_{ij}}A_i^4e^{-2\zeta A_i}|Y_1^m(\theta_i,\varphi_i)|^2B_j^4e^{-2\zeta B_j}|Y_1^m(\theta_j,\varphi_j)|^2\times d\tau_id\tau_j$$

are always associated in a product with

$$\int\exp\left(-\frac{i\mathbf{P}_{e1}\cdot\mathbf{A}_1}{\hbar}\right)A_1^2e^{-2\zeta A_1}Y_1^1d\tau_1,$$

and thus we have no contribution from these. Thus $V_{fi}^{(1)}$ is very small and can be neglected.

ELECTRON 3 REMOVED

We now consider the matrix element with the $m=0$ electron (electron 3) removed. For Coulomb repulsion of the nuclei, we have

$$\frac{64e^2}{R}C\int\exp\left(-\frac{i\mathbf{P}_e\cdot\mathbf{A}_3}{\hbar}\right)A_3^2e^{-\zeta A_3}\left(\frac{3}{4\pi}\right)^{\frac{1}{2}}\times\cos\theta_3d\tau_3\equiv\frac{64e^2J_1}{R}.$$

For J_1, we easily find

$$J_1=(3\pi)^{\frac{1}{2}}C\left[\frac{6\zeta^4-6K^4-36K\zeta^3+12\zeta K^3}{K(\zeta^2+K^2)^4}-\frac{6K\zeta^2-2K^3}{K^2(\zeta^2+K^2)^3}\right],$$

where $K=P_e/\hbar$.

We also have integrals of the type $(i\neq3)$:

$$-8e^2C^2J_1\int\frac{1}{B_i}A_i^4e^{-2\zeta A_i}\frac{3}{8\pi}\sin^2\theta_id\tau_i\equiv-8e^2J_1J_2.$$

This integral is equal to the one with nuclei A and B interchanged. Consequently we have 8 integrals each equal to $-8e^2J_1J_2$. To evaluate J_2, we expand $1/B_i$ in Legendre polynomials and easily find

$$J_2=C^2\left\{\frac{720}{R\alpha^7}-\frac{8064}{R^3\alpha^9}+e^{-\alpha R}\left[\frac{6R^2}{\alpha^4}+\frac{96R}{\alpha^5}+\frac{744}{\alpha^6}+\frac{3312}{R\alpha^7}+\frac{8064}{R^2\alpha^8}+\frac{8064}{R^3\alpha^9}\right]\right\},$$

where $\alpha=2\zeta$.

Similarly we need, for $i\neq3$,

$$-8e^2C^2J_1\int\frac{1}{B_i}A_i^4e^{-2\zeta A_i}\frac{3}{4\pi}\cos^2\theta_id\tau_i\equiv-8e^2J_1J_3.$$

For J_3 we find

$$J_3=C^2\left\{\frac{720}{R\alpha^7}+\frac{16\,128}{R^3\alpha^9}-e^{-\alpha R}\left[\frac{3R^4}{\alpha^2}+\frac{30R^3}{\alpha^3}+\frac{192R^2}{\alpha^4}+\frac{912R}{\alpha^5}+\frac{3288}{\alpha^6}+\frac{8784}{R\alpha^7}+\frac{16\,128}{R^2\alpha^8}+\frac{16\,128}{R^3\alpha^9}\right]\right\}.$$

In the latter case we require a total of 3 integrals of this type.

For the interaction of an $l=0$ electron with a nucleus, we require

$$-8e^2C^2\int \frac{A_i^4e^{-2\zeta A_i}}{B_i}\frac{1}{4\pi}d\tau_i \equiv -8e^2Q,$$

where Q is easily evaluated:

$$Q=C^2\left\{\frac{720}{R\alpha^7}-e^{-\alpha R}\left[\frac{R^4}{\alpha^2}+\frac{10R^3}{\alpha^3}+\frac{60R^2}{\alpha^4}+\frac{240R}{\alpha^5}+\frac{600}{\alpha^6}+\frac{720}{R\alpha^7}\right]\right\}.$$

We need 4 integrals of the type $-8e^2Q$.

We now consider electron-electron interactions not involving electron 3. One of this type is

$$e^2J_1J_4\equiv e^2J_1C^4\int \frac{1}{r_{ij}}A_i^4e^{-2\zeta A_i}\frac{3}{8\pi}\sin^2\theta_i \times B_j^4e^{-2\zeta B_j}\frac{3}{8\pi}\sin^2\theta_j d\tau_i d\tau_j.$$

By expanding $1/r_{ij}$ in Legendre polynomials, we get

$$J_4=C^4\int A_i^4e^{-2\zeta A_i}\frac{3}{8\pi}\sin^2\theta_i d\tau_i\left[\frac{720}{B_i\alpha^7}-\frac{8064}{B_i^3\alpha^9}+e^{-\alpha B_i}\left(\frac{6B_i^2}{\alpha^5}+\frac{96B_i}{\alpha^5}+\frac{744}{\alpha^6}+\frac{3312}{B_i\alpha^7}+\frac{8064}{B_i^2\alpha^8}+\frac{8064}{B_i^3\alpha^9}\right)\right].$$

To evaluate J_4 we shall approximate the terms associated with $e^{-\alpha B_i}$ by replacing B_i by R. The $1/B_i$ term can be expanded in the usual Legendre polynomial expansion, and for $1/B_i^3$ we use

$$\frac{1}{B_i^3}=-\frac{1}{A_iR^2}\sum_{n=1}\left(\frac{A_i}{R}\right)^n\frac{n(n+1)}{2n+1}\frac{P_{n+1}-P_{n-1}}{\sin^2\theta_i},\quad A_i<R,$$

and A_i and R interchanged for $R<A_i$. For J_4 we then find, after integration,

$$J_4=C^2\left\{\frac{720}{R\alpha^7}-\frac{16\,128}{R^3\alpha^9}+e^{-\alpha R}\left(\frac{34.4R^2}{\alpha^4}+\frac{201.6R}{\alpha^5}+\frac{2563.2}{\alpha^6}+\frac{10\,387.2}{R\alpha^7}+\frac{24\,192}{R^2\alpha^8}+\frac{24\,192}{R^3\alpha^9}\right)\right\}.$$

We require 16 integrals of the type $e^2J_1J_4$.

Another of the electron-electron interactions is

$$e^2J_1J_5\equiv e^2J_1C^4\int \frac{1}{r_{ij}}A_i^4e^{-2\zeta A_i}\frac{3}{8\pi}\sin^2\theta_i \times B_j^4e^{-2\zeta B_j}\frac{3}{4\pi}\cos^2\theta_j d\tau_i d\tau_j.$$

By expanding $1/r_{ij}$ in Legendre polynomials, and by using the same approximation as in evaluating J_4 we find:

$$J_5=C^2\left[\frac{720}{R\alpha^7}+\frac{8064}{R^3\alpha^9}-e^{-\alpha R}\left(\frac{3R^4}{\alpha^2}+\frac{30R^3}{\alpha^3}+\frac{230.8R^2}{\alpha^4}+\frac{1219.2R}{\alpha^5}+\frac{4694.4}{\alpha^6}+\frac{12\,998.4}{R\alpha^7}+\frac{24\,192}{R^2\alpha^8}+\frac{24\,192}{R^3\alpha^9}\right)\right].$$

We require 12 integrals of the type $e^2J_1J_5$.

There are still several electron-electron interactions not involving electron 3. One of these is

$$e^2J_1J_6\equiv e^2J_1C^4\int \frac{1}{r_{ij}}A_i^4e^{-2\zeta A_i}\frac{3}{4\pi}\cos^2\theta_i \times B_j^4e^{-2\zeta B_j}\frac{3}{4\pi}\cos^2\theta_j d\tau_i d\tau_j.$$

By expanding $1/r_{ij}$ in an Legendre expansion, then letting $B_i=R$ as an approximation and integrating, we find

$$J_6=C^2\left[\frac{720}{R\alpha^7}+\frac{32\,256}{R^3\alpha^9}-e^{-\alpha R}\left(\frac{6R^4}{\alpha^2}+\frac{60R^3}{\alpha^3}+\frac{384R^2}{\alpha^4}+\frac{1824R}{\alpha^5}+\frac{6576}{\alpha^6}+\frac{17\,568}{R\alpha^7}+\frac{32\,256}{R^2\alpha^8}+\frac{32\,256}{R^3\alpha^9}\right)\right].$$

We also need the interactions

$$e^2C^4\int \frac{1}{r_{ij}}A_i^4e^{-2\zeta A_i}\frac{1}{4\pi}B_j^4e^{-2\zeta B_j}\frac{3}{8\pi}\sin^2\theta_j d\tau_i d\tau_j,$$

$$e^2C^4\int \frac{1}{r_{ij}}A_i^4e^{-2\zeta A_i}\frac{1}{4\pi}B_j^4e^{-2\zeta B_j}\frac{3}{4\pi}\cos^2\theta_j d\tau_i d\tau_j,$$

$$e^2C^4\int \frac{1}{r_{ij}}A_i^4e^{-2\zeta A_i}\frac{1}{4\pi}B_j^4e^{-2\zeta B_j}\frac{1}{4\pi}d\tau_i d\tau_j,$$

which are approximately $e^2J_1J_2$, $e^2J_1J_3$, and e^2J_1Q, respectively. We require 16 of the first of these, 6 of the second, and 4 of the last.

Finally we consider interactions of electron 3 with other electrons and also with nucleus B. We have four integrals of the type

$$e^2C^3\int \frac{1}{r_{3j}}\exp\left(\frac{-i\mathbf{P}_e\cdot\mathbf{A}_3}{\hbar}\right)A_3^2e^{-\zeta A_3}\left(\frac{3}{4\pi}\right)^{\frac{1}{2}}\cos\theta_3 \times B_j^4e^{-2\zeta B_j}|Y_1^1(\theta_j)|^2d\tau_3d\tau_j,$$

two of the type

$$e^2C^3\int \frac{1}{r_{3j}}\exp\left(\frac{-i\mathbf{P}_e\cdot\mathbf{A}_3}{\hbar}\right)A_3^2e^{-\zeta A_3}\left(\frac{3}{4\pi}\right)^{\frac{1}{2}}\cos\theta_3 \times B_j^4e^{-2\zeta B_j}|Y_1^0(\theta_j)|^2d\tau_3d\tau_j,$$

and two of

$$e^2C^3\int\frac{1}{r_{3j}}\exp\left(\frac{-i\mathbf{P}\cdot\mathbf{A}_3}{\hbar}\right)A_3{}^2e^{-\zeta A_3}\left(\frac{3}{4\pi}\right)^{\frac{1}{2}}\cos\theta_3 \times B_j{}^4e^{-2\zeta B_j}\frac{1}{4\pi}d\tau_3d\tau_j.$$

Finally we have

$$-8e^2C\int\frac{1}{B_3}\exp\left(\frac{-i\mathbf{P}_e\cdot\mathbf{A}_3}{\hbar}\right)A_3{}^2e^{-\zeta A_3}\left(\frac{3}{4\pi}\right)^{\frac{1}{2}}\cos\theta_3d\tau_3.$$

Using approximations similar to those used in evaluating J_6, we find for previous four integrals:

$$4e^2J_1\left(J_2-\frac{1}{R}\right)+2e^2J_1\left(J_3-\frac{1}{R}\right)+2e^2J_1\left(Q-\frac{1}{R}\right).$$

Thus, collecting all our results, we have (in the order of our analysis)

$$V_{fi}{}^{(3)}=\frac{64e^2J_1}{R}-64e^2J_1J_2-24e^2J_1J_3-32e^2J_1Q+16e^2J_1J_4 +12e^2J_1J_5+2e^2J_1J_6+16e^2J_1J_2+6e^2J_1J_3+4e^2J_1Q +4e^2J_1\left(J_2-\frac{1}{R}\right)+2e^2\left(J_3-\frac{1}{R}\right)+2e^2J_1\left(Q-\frac{1}{R}\right),$$

or

$$V_{fi}{}^{(3)}=e^2C^2J_1\left[\frac{e^{-\alpha R}}{\alpha}\left(\frac{26R^4}{\alpha}+\frac{260R^3}{\alpha^2}+\frac{1348.8R^2}{\alpha^3}+\frac{1555.2R}{\alpha^4}+\frac{6998.4}{\alpha^5}-\frac{11\,285.6}{R\alpha^6}-\frac{64\,512}{R^2\alpha^7}-\frac{64\,512}{R^3\alpha^8}\right)\right].$$

Consequently, for the cross section for removal of a p electron, we find

$$\sigma^{(p)}=\frac{\alpha^9e^4m^2\hbar^7}{30(2mI)^{11/2}}g_1(\epsilon)\int_0^\infty f_1{}^2(\alpha R)pdp,$$

where

$$g_1(\epsilon)=\int_0^{\sqrt{(\epsilon/I)}}\frac{(4x^3+36a^2+4a^2x-12ax^2)^2x^2dx}{(x^2+1)^2(x^2+a^2)^8},$$

with

$$\epsilon=\tfrac{1}{2}\mu v^2-I,\quad a^2=\zeta^2\hbar^2/2mI,\quad \alpha=2\zeta,$$

and

$$f_1(\alpha R)=\frac{e^{-\alpha R}}{\alpha R}\left[0.0361(\alpha R)^5+0.361(\alpha R)^4+1.873(\alpha R)^3+2.16(\alpha R)^2+9.72(\alpha R)-15.674-\frac{89.6}{\alpha R}-\frac{89.6}{(\alpha R)^2}\right].$$

Before we consider the possibility of removal of an s electron, we shall give the functional form of the repulsive potential of argon.[3] The potential is:

$$U(R)=3.22\times10^4e^{-3.68R}\text{ electron volts.}$$

[3] Hirschfelder, Curtiss, and Bird, *Molecular Theory of Gases and Liquids* (John Wiley and Sons, Inc., New York, 1954).

For the ionization potential we have 15.68 ev. We assume that the ionization potential is approximately the same for an s electron and a p electron.

REMOVAL OF ELECTRON 7

Most of the integrals involved here have been discussed, except the integral:

$$K_1=C\int A^2e^{-\zeta A}\frac{1}{(4\pi)^{\frac{1}{2}}}\exp\left(\frac{-i\mathbf{P}_e\cdot\mathbf{A}}{\hbar}\right)d\tau,$$

which replaces J_1 as a multiplicative factor. For K_1 we find

$$K_1=24(4\pi)^{\frac{1}{2}}C\left[\frac{\zeta^3-\zeta K^2}{(K^2+\zeta^2)^4}\right],\quad K=P_e/\hbar.$$

In an analogous manner we find $V_{fi}{}^{(0)}$ as we have found $V_{fi}{}^{(3)}$. Keeping the same ordering of terms we have:

$$V_{fi}{}^{(7)}=\frac{64e^2K_1}{R}-64e^2K_1J_2-32e^2K_1J_3-24e^2K_1Q +16e^2K_1J_4+16e^2K_1J_5+4e^2K_1J_6+12e^2K_1J_2 +6e^2K_1J_3+2e^2K_1Q+4e^2K_1\left[J_2-\frac{1}{R}\right] +2e^2K_1\left[J_3-\frac{1}{R}\right]+2e^2K_1\left[Q-\frac{1}{R}\right].$$

The last three terms are obtained from the interactions involving electron 7 (by approximation).

Substituting the expressions for the integrals in $V_{fi}{}^{(0)}$, we find

$$V_{fi}{}^{(7)}=e^2K_1C^2e^{-\alpha R}\left[\frac{20R^4}{\alpha^2}+\frac{200R^3}{\alpha^3}+\frac{1353.6R^2}{\alpha^4}-\frac{1497.6R}{\alpha^5}-\frac{5203.2}{\alpha^6}-\frac{53\,811.2}{R\alpha^7}-\frac{129\,024}{R^2\alpha^8}-\frac{129\,024}{R^3\alpha^9}\right].$$

Now because electrons 7 and 8 are equivalent, we have

$$\sum_{s\text{ electrons}}|V_{fi}{}^{(s)}|^2=2|V_{fi}{}^{(7)}|^2.$$

Consequently we find for the cross section for removal

TABLE I. The ionization cross section of argon-argon collisions *versus* energy and comparison with experiment.

Relative energy (ev)	Laboratory energy (ev)	σ (A²)	σ_{Rostagni} (A²)
15.68	31.36	0	...
100	200	0.207	0.396
150	300	0.696	0.588
250	500	2.23	0.93

of an s electron:

$$\sigma^{(s)}=\frac{25.6\alpha^9 m^2 e^4 \hbar^7}{(2mI)^{11/2}} g_2(\epsilon)\int_0^\infty f_2(\alpha R)pdp,$$

where

$$g_2(\epsilon)=\int_0^{\sqrt{(\epsilon/I)}}\frac{(a^3-ax^2)^2x^2dx}{(a^2+x^2)^8(x^2+1)^2},$$

and

$$f_2(\alpha R)=\frac{e^{-\alpha R}}{\alpha R}\bigg[0.0278(\alpha R)^2+0.278(\alpha R)^4+1.88(\alpha R)^3 -2.08(\alpha R)^2-7.23(\alpha R)-747-\frac{1792}{\alpha R}-\frac{1792}{(\alpha R)^2}\bigg].$$

The total cross section is nothing more than the sum,

$$\sigma=\sigma^{(p)}+\sigma^{(s)}.$$

In Table I we give the results of our computation. The integrals in $\sigma^{(s)}$ and $\sigma^{(p)}$ have been evaluated graphically.

We have compared our results with that of Rostagni.[4] The agreement is satisfactory when one considers the nature of the approximations.

[4] A. Rostagni, Nuovo cimento **11**, 621 (1934).

PHYSICAL REVIEW VOLUME 109, NUMBER 2 JANUARY 15, 1958

Charge Exchange Cross Sections of Hydrogen Particles in Gases at High Energies

C. F. Barnett and H. K. Reynolds*
Oak Ridge National Laboratory, Oak Ridge, Tennessee

(Received September 23, 1957)

Measurements are reported of the cross section for electron loss (σ_{01}) and electron capture (σ_{10}) for hydrogen atoms and ions traversing gases of hydrogen, helium, nitrogen, and argon. The kinetic energy of the particles was from 250 kev–1 Mev. Cross sections for electron loss were determined by measuring the attenuation of a neutral monatomic hydrogen beam in a gas cell in which a transverse electric field was present. The cross section for electron capture was measured by determining the fraction of the beam in each charge state after passing through a "thick" gas target. The fraction of the neutral component to the charge-one component is equal to the ratio of the cross sections for electron capture to electron loss. The velocity dependence of the cross section does not follow a strict power law. In hydrogen the cross section decreases approximately as v^{-10}, while for heavier target gases the velocity dependence is smaller, being $v^{-3.7}$ for argon at energies greater than 500 kev. The electron loss cross section has a $1/v^2$ dependence for gases of hydrogen and helium and a $1/v$ dependence for nitrogen and argon target gases.

INTRODUCTION

PREVIOUS papers[1] have included descriptions and results of a number of experiments in which were investigated the cross sections for electron capture and loss of hydrogen and helium ions and atoms in various gases. The measurements have been made in the energy interval of 3–220 kev. Investigations at other laboratories prior to 1953 have been summarized in the excellent review articles by Allison and Warshaw[2] and Burhop and Massey.[3] Several additional papers have appeared since these reviews. Stedeford and Hasted[4] and also Whittier[5] have extended the measurements of electron capture and loss of fast hydrogen atoms and ions in the energy range less than 100 kev. Several theoretical[6–10] papers have been published in which refinements have been made to the original calculations using the original Born approximation. At the present time there is very good agreement in the theoretical and experimental work at energies less than 200 kev. For energies greater than 200 kev very few previous experimental data are available for comparison with the theoretical calculations which should give much better agreement at the higher energies. The present paper extends the measurements of the electron loss cross section and the equilibrium charge distribution in the energy interval of 250 kev–1 Mev.

The differential equation for the transition between the various charge states has been formulated and solved by Allison and Warshaw[2] for the general case. If the moving particles have passed through sufficient

* Summer employee (1956) from the University of Houston, Houston, Texas.

[1] P. M. Stier and C. F. Barnett, Phys. Rev. **103**, 896 (1956), henceforth referred to as I. C. F. Barnett and P. M. Stier, Phys. Rev. **109**, 385 (1958), this issue. Stier, Barnett, and Evans, Phys. Rev. **96**, 973 (1954), henceforth referred to as II.

[2] S. K. Allison and S. D. Warshaw, Revs. Modern Phys. **25**, 779 (1953).

[3] H. S. W. Massey and E. H. S. Burhop, *Electronic and Ionic Impact Phenomena* (Clarendon Press, Oxford, 1952).

[4] J. B. H. Stedeford and J. B. Hasted, Proc. Roy. Soc. (London) **A227**, 466 (1955).

[5] A. C. Whittier, Can. J. Phys. **32**, 275 (1954).

[6] D. R. Bates and G. W. Griffing, Proc. Phys. Soc. (London) **A68**, 90 (1955).

[7] D. R. Bates and A. Dalgarno, Proc. Phys. Soc. (London) **A66**, 972 (1953).

[8] J. D. Jackson and H. Schiff, Phys. Rev. **89**, 359 (1953).

[9] T. Pradhan, Phys. Rev. **105**, 1250 (1957).

[10] Bransden, Dalgarno, and King, Proc. Phys. Soc. (London) **A67**, 1075 (1954).