Relaxation Time Anisotropy in *n*-Type Germanium

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The anisotropy parameter $K=K_m/K_\tau=(m_{||}/m_\perp)(\tau_{||}/\tau_\perp)^{-1}$ is determined from magnetoconductance measurements in the temperature range 45°K to 300°K. If one uses the cyclotron resonance value for the mass ratio K_m, the magnetoconductance measurements give $K_\tau=1.0$ for lattice scattering and $K_\tau>1$ for moderate amounts of impurity scattering. For strong impurity scattering, the data show that the scattering cannot be represented by a simple relaxation-time tensor model.

1. INTRODUCTION

THE observed anisotropy of magnetoresistance in *n*-type germanium[1–5] is due to the anisotropy of the energy surfaces[6,7] and the anisotropy of the relaxation time.[8] Herring and Vogt[8] have shown that under some circumstances the anisotropy can be described by a single anisotropy parameter which can be obtained from magnetoresistance measurements. This anisotropy parameter is the ratio of a mass anisotropy parameter to a relaxation-time anisotropy parameter. Since the mass anisotropy parameter is known for *n* germanium from low-temperature cyclotron resonance experiments,[9,10] low-temperature magnetoresistance measurements can be used for an experimental determination of the relaxation-time anisotropy parameter.

In this paper we report measurements of certain galvanomagnetic effects in the temperature range 45°K to 300°K. These measurements permit the calculation of the anisotropy parameter for the case where lattice scattering is the only significant scattering mechanism and also for the case where both lattice and ionized-impurity scattering are important. In order to make quantitative conclusions about the relaxation-time anisotropy parameter it is necessary to assume that the mass anisotropy parameter in this temperature range is the same as that measured at liquid helium temperature. With this assumption we find from our data that, if one uses the relaxation-time tensor of Herring and Vogt, (a) the relaxation time is isotropic for lattice scattering; (b) the relaxation time becomes anisotropic for moderate amounts of impurity scattering; and (c) the Herring and Vogt model fails for strong impurity scattering, i.e., the data cannot be described by a single anisotropy parameter of the Herring-Vogt type.

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2. MAGNETOCONDUCTANCE AND MAGNETORESISTANCE

The relation between current density and electric field in the presence of a magnetic field can be given by

$$\mathbf{i}(\mathbf{H},\mathbf{E})=\mathbf{S}(\mathbf{H})\cdot\mathbf{E}, \tag{2.1}$$

or

$$\mathbf{E}(\mathbf{H},\mathbf{i})=\mathbf{T}(\mathbf{H})\cdot\mathbf{i}, \tag{2.2}$$

where the elements of the conductivity tensor $\mathbf{S}$ and the resistivity tensor $\mathbf{T}$ are functions of the direction and magnitude of the magnetic field and, obviously, $\mathbf{S}=\mathbf{T}^{-1}$.

Whether one measures conductivity or resistivity depends on sample geometry. If current is passed through a long thin sample so that the current is parallel to the long dimension, then measurements of different components of the electric field give directly the elements of the resistivity tensor $\mathbf{T}$. This long, thin sample geometry is what is used for measurement of the Hall, planar Hall, and magnetoresistance effects. Conversely, if the electric field is applied across a short, wide sample so that the electric field remains parallel to the short dimension, then the measurements of the different components of the current give directly the elements of the conductivity tensor $\mathbf{S}$. Such a geometry is shown in Fig. 1. In such a sample the field will always be normal to the electrodes if the sample is thin compared to its other dimensions.

In the next section it will be shown that for the analysis of our data, we shall be interested in the determination of the diagonal elements of $\mathbf{S}$. If the magnetoresistance sample geometry is used, we can

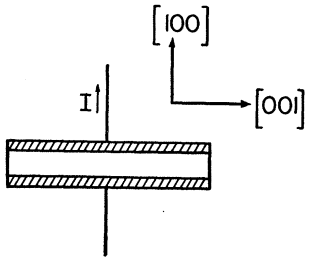

FIG. 1. Sample geometry used for direct measurement of magnetoconductance. The shaded areas indicate the electrodes. (General discussion is in Sec. 2 and the details of sample preparation are in Sec. 4.)

determine the elements of **T** and invert mathematically to obtain the desired element of **S**. Alternatively, using the other sample geometry we may determine the desired element of **S** directly. Determination of an element of **S** by direct measurement is more accurate than by mathematical inversion of elements of **T** simply because the latter technique usually requires two or more experimental measurements, while the former is, of course, determined by one measurement. Further, in performing the mathematical inversion of **T** it is necessary to take combinations of elements of **T**, and the measurements of the different elements of **T** do not depend upon the same sample dimensions. Thus, relative errors in the determination of sample dimensions can cause an error in the determination of the conductivity-tensor element that is different for each element of the conductivity tensor. On the other hand, an error in the measurement of the sample dimensions will cause the same relative error in all the diagonal elements of **S** if these elements are measured directly and the ratio of one element to another will not be affected by the accuracy of the measurement of the dimensions.

A sample with the geometry shown in Fig. 1 can be used only to measure the diagonal elements of **S**. The off-diagonal elements cannot be measured directly because the components of the current density transverse to the field cannot be measured. The most that can be determined is the set of elements of the symmetric part of **S**.

The coordinate system in which the elements of the tensors **S** and **T** are most conveniently described is one consisting of the cubic axes of the crystal with the subscripts 1, 2, and 3 referring to the [100], [010], and [001] axes, respectively. Superscripts will be used to indicate the crystallographic direction of the magnetic field. With this coordinate system, e.g., S_{12}^{110} refers to the effect of an [010] electric field and a [110] magnetic field upon the current density in the [100] direction.

3. MAGNETOCONDUCTANCE ACCORDING TO THE HERRING-VOGT MODEL

In order to use Boltzmann transport theory to evaluate the elements of the conductivity tensor, it is necessary to have a model that describes how the scattering mechanisms tend to restore thermal equilibrium when electric and magnetic fields are applied. The model described in this section is essentially that of Herring and Vogt[8] who represent the electron scattering by means of a relaxation-time tensor with the same symmetry as the energy surface.

The results of cyclotron resonance experiments[9,10] indicate that the energy surfaces for electrons in germanium are ellipsoids of revolution about the [111] axes. Near a given minimum the correct form for the electron energy in terms of electron momentum is

$$\epsilon=\tfrac{1}{2}[(P_{11}^2/m_{11})+(P_\perp^2/m_\perp)], \tag{3.1}$$

where the subscripts refer to the directions parallel and perpendicular to the appropriate [111] axis in momentum space. Equation (3.1) suggests the definition of the *mass anisotropy parameter*,

$$K_m\equiv m_{11}/m_\perp. \tag{3.2}$$

The most recently published cyclotron resonance data[10] give $K_m=20.0\pm0.4$.

Using an ellipsoid model, let us consider an energy shell in momentum space. When the crystal is in thermal equilibrium the electrons in this shell will carry no net current. When electric and magnetic fields are applied, the crystal will no longer be in thermal equilibrium and the electrons in this shell will carry a net current $d\mathbf{j}$. We define a tensor relaxation time so that τ_{11} is the time constant for the rate of decay (due to scattering) of the component of $d\mathbf{j}$ parallel to the axis of revolution and $\tau_\perp$ is the time constant for the rate of decay of the component of $d\mathbf{j}$ perpendicular to this axis of revolution. The above says that

$$(\partial d\mathbf{j}/\partial t)_{\text{scat}}=-\boldsymbol{\tau}^{-1}\cdot d\mathbf{j}, \tag{3.3}$$

where

$$d\mathbf{j}=\begin{bmatrix} dj_{11} \\ dj_{\perp 1} \\ dj_{\perp 2} \end{bmatrix}, \quad \boldsymbol{\tau}=\begin{bmatrix} \tau_{11} & 0 & 0 \\ 0 & \tau_\perp & 0 \\ 0 & 0 & \tau_\perp \end{bmatrix}. \tag{3.4}$$

Equation (3.4) suggests the definition of the *relaxation-time anisotropy parameter*,

$$K_\tau\equiv\tau_{11}/\tau_\perp. \tag{3.5}$$

In this paper it will be assumed that while τ_{11} and $\tau_\perp$ may be energy-dependent, the parameter K_τ is independent of energy.

It can be shown that with the Herring-Vogt scattering model there will be no longitudinal magnetoconductance in a single ellipsoid when the electric and magnetic fields are parallel to a principal axis of the ellipsoid. The conductivity of the crystal as a whole, however, consists of contributions from each of the ellipsoids. Since the ellipsoids are oriented along the [111] axes, there is no crystallographic direction which is a principal axis for all ellipsoids simultaneously and thus the crystal will exhibit some longitudinal magnetoconductance for any arbitrary orientation of the fields.

It can be shown[11] that the absence of longitudinal magnetoconductance for a single ellipsoid leads to the following weak magnetic-field relation when the conductivities of the individual ellipsoids are properly summed:

$$\lim_{H\to 0}\left|\frac{2(T_{12})^{110}}{(\Delta T_{11})^{001}}\right|=1, \tag{3.6}$$

where $\Delta T_{11}=T_{11}(0)-T_{11}(H)$. This relation, sometimes called the *symmetry relation*, has been previously expressed in the literature[3,4,6] as a relation between

[11] R. W. Keyes (private communication).

certain magnetoresistance coefficients and has been experimentally verified for fairly pure n-type germanium. If the data did not agree with (3.6), this would be evidence that the Herring-Vogt model was inapplicable.

If the Herring-Vogt model is used to evaluate the elements of the conductivity tensor, it is found[5,6] that for arbitrary values of magnetic field,

$$\frac{S_{11}(0)-S_{11}(H)^{100}}{S_{11}(0)-S_{11}(H)^{001}}=\frac{(\Delta S_{11})^{100}}{(\Delta S_{11})^{001}}=\frac{2(K-1)^2}{(2K+1)(K+2)}, \quad (3.7)$$

where K, the *anisotropy parameter*, is

$$K\equiv K_m/K_\tau. \quad (3.8)$$

The quantity $(\Delta S_{11})^{100}$ is the change in conductance when the electric and magnetic fields are parallel and thus is a *longitudinal* magnetoconductance. Similarly, $(\Delta S_{11})^{001}$ is a *transverse* magnetoconductance. Equation (3.7) thus indicates that the anisotropy parameter can be calculated from a measurement of the ratio of a longitudinal to a transverse magnetoconductance.

4. SAMPLE PREPARATION

The samples that were used were half-inch diameter disks of germanium. Figure 1 shows the cross section of such a disk. The measured magnetoconductance ratio was independent of the disk thickness for thicknesses varying from 0.5 to 1.0 mm, indicating that the diameter-to-thickness ratio was large enough to eliminate any edge effects.

When using a sample with the geometry of Fig. 1 to measure S_{11}, it is necessary to measure both the current and the electric field in the [100] direction since $S_{11}=i_1/E_1$. The current i_1 can be measured by an ammeter in the external circuit. It is necessary to measure the voltage across the whole crystal including the electrodes in order to find E_1 and for this reason it is necessary that the contact resistance between the electrode and the crystal be very low. Furthermore, the electrodes must be put on the crystal in such a way that the contact is uniform over the crystal face.

To meet the above electrode specifications, tin was put on the crystal faces by use of an ultrasonic soldering iron. The ultrasonic technique was used to obtain 100% wetting. The crystal was then heat treated at 450°C for about ten minutes. This results in a small amount of tin-germanium alloying and gives the desired low-resistance contact. We are convinced that the contact resistance is negligible since the measured resistance of the disks (which would include any contact resistance) had the same temperature dependence in the range 45°K to 350°K as the resistivity of germanium samples with conventional magnetoresistance geometry with which contact resistance is eliminated.

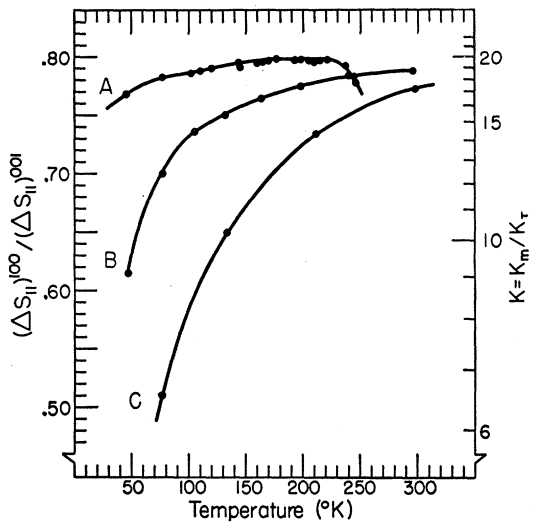

FIG. 2. Temperature dependence of the magnetoconductance ratio. The values of the anisotropy parameter shown on the right are calculated from Eq. (3.7). The number of carriers per cubic centimeter of samples A, B, and C are approximately 4×10^{12}, 3×10^{14}, and 6×10^{15}, respectively.

5. MAGNETOCONDUCTANCE RATIO MEASUREMENTS AND DISCUSSION

Figure 2 gives the measured values of the magnetoconductance ratio obtained for samples with three different impurity concentrations. On the right-hand side of the figure are given the values of the anisotropy parameter corresponding to the values of the magnetoconductance ratio on the left-hand side. [See Eq. (3.7).]

Sample A is of high purity and the effect of impurity scattering should be slight throughout most of the temperature range used. The effect of impurity scattering on the properties of sample B, which is of intermediate purity, should be slight at high temperatures and moderate at low temperatures. The effect of impurity scattering on the properties of sample C, which is the most impure, should be moderate at high temperatures and strong at low temperatures. (We realize that the adjectives *slight*, *moderate*, and *strong* are relative and have not been rigorously defined here. We are using these terms for convenience only, however, and rigorous definitions are not necessary for our present purposes.)

For sample A, the effect of intrinsic holes can be seen above 230°K. Below this temperature the magnetoconductance ratio varies by only a few percent. Starting at 48°K, the magnetoconductance ratio slowly increases with increasing temperature and decreasing importance of impurity scattering. Between 130°K and 230°K, the ratio is essentially independent of temperature. If there was any effect due to impurity scattering, it would be temperature-dependent so it is concluded that the value of the magnetoconductance ratio found for this pure sample in the range 130°K to 230°K is

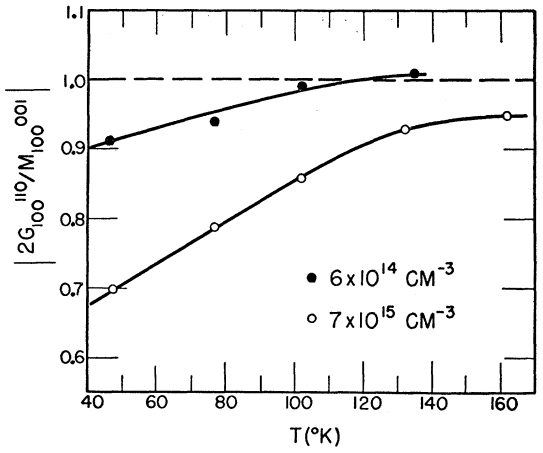

FIG. 3. Experimental test of the symmetry relation (3.6). The data indicate that (3.6) is approximately correct for moderate amounts of impurity scattering but is incorrect for strong impurity scattering.

characteristic of lattice scattering and is unaffected by impurity scattering.

In the temperature range from 130°K to 230°K all magnetoconductance ratio measurements on sample A are in the range $(\Delta S_{11})^{100}/(\Delta S_{11})^{001}=0.798\pm0.002$. Equation (3.7) gives $K\equiv K_m/K_\tau=19.8\pm0.2$ for this magnetoconductance ratio.

The cyclotron resonance value[10] of $K_m=20.0\pm0.4$ was obtained from measurements at liquid helium temperatures. If it can be assumed that this mass anisotropy parameter has the same value in the temperature range 130°K to 230°K, than $K_\tau=1.01\pm0.03$. This means that for lattice scattering the relaxation time is essentially isotropic. The theories of Herring and Vogt[8] and Dumke[12] indicate that for acoustical-mode scattering the relaxation time should be more isotropic than the energy surfaces. According to these theories, however, that the relaxation time should be exactly isotropic ($K_\tau=1.0$) would be fortuitous.

Our experimental results for samples B and C show that increasing the amount of impurity scattering increases the anisotropy of the relaxation time with $K_\tau>1$. This is in qualitative agreement with the theoretical work of Ham[13] who finds $K_\tau\sim12$ for impurity scattering.

However, our data on samples in regions of strong impurity scattering cannot be analyzed in terms of a relaxation-time tensor. The basis for this statement will be discussed in the next section.

Some previously published data[3,4] indicate that the anisotropy parameter K is relatively independent of temperature and has a value of approximately 12. The reasons for the discrepancy between these earlier results and those shown in Fig. 2 are not clear. However, the earlier measurements were magnetoresistance measurements and the magnetoconductance ratio was obtained by mathematical inversion, so that it would be expected that the data shown in Fig. 2 would be more accurate than the more indirect results of the earlier work. (See discussion of this point in Sec. 2.) Furthermore, the values of K reported in the earlier work depend on zero-field extrapolations of magnetoresistance and Hall coefficient measurements. These extrapolations can also cause appreciable error especially at the lower temperatures. Broudy and Venables[5] have calculated K from magnetoresistance and Hall coefficient measurements in a way that eliminated the necessity for zero-field extrapolations, and their data are in agreement with Fig. 2.

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[13] F. S. Ham, Phys. Rev. **100**, 1251(A) (1955).

6. APPLICABILITY OF THE HERRING-VOGT MODEL TO THE CASE OF STRONG IMPURITY SCATTERING

As indicated in Sec. 3, the symmetry relation (3.6) is a test of the applicability of the Herring-Vogt model. This relation can be checked experimentally by measuring the planar Hall coefficient[3] G_{100}^{110} and the transverse magnetoresistance coefficient[3] M_{100}^{001} since

$$\lim_{H\to0}\left|\frac{2(T_{12})^{110}}{(\Delta T_{11})^{001}}\right|=\left|\frac{2G_{100}^{110}}{M_{100}^{001}}\right|. \tag{6.1}$$

Previous measurements[3,4] indicated that Eq. (6.1) was valid from 77°K to 300°K for a sample with a carrier concentration of approximately 10^{14} cm^{-3}. Figure 3 shows measurements of $|2G_{100}^{110}/M_{100}^{001}|$ for less pure samples. It can be seen that at the higher temperatures where the impurity scattering is moderate, the data approximate the symmetry relation (3.6). As the temperature is lowered and the impurity scattering increases, the symmetry relation is invalid and, hence, the Herring-Vogt model is inapplicable.

As Keyes[11] has pointed out, if the energy surfaces are given by (3.1), deviation of the left-hand side of (3.6) from unity must be due to the existence of longitudinal magnetoconductance for a single ellipsoid when the fields are parallel to one of the principal axes of the ellipsoidal energy surfaces. The relaxation-time model used here could give no such longitudinal magnetoconductance.

From magnetoresistance measurements on samples with high carrier concentrations, Glicksman[14] has also found a failure of the symmetry relation (3.6). However, his measurements do not agree with ours since he finds that the ratio plotted in Fig. 3 is greater, not smaller, than unity. If this ratio is actually greater than unity and the energy surfaces are given by (3.1), the longitudinal magnetoconductance in an ellipsoid must be opposite in sign to that which one usually finds, i.e., Glicksman's data indicate that $S_{11}(H)>S_{11}(0)$.

[14] M. Glicksman, Phys. Rev. **108**, 264 (1957).

We do not yet understand the origin of longitudinal magnetoconductance in a single ellipsoid. However, our work provides clear-cut evidence for the failure of the Herring-Vogt model for the case of strong impurity scattering.*

* *Note added in proof.*—Recent magnetoresistance measurements by the author in collaboration with W. E. Howard indicate that the deviations from the symmetry relation are small for samples with carrier concentrations as high as 3×10^{16} and that the deviations reported in Sec. 6 are probably due to small systematic errors. A report of this new work is being prepared for publication.

7. ACKNOWLEDGMENTS

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X-Ray Study of Deuteron-Irradiated Copper near 10°K*†

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Precise measurements of lattice expansion of high-purity copper held near 10°K during deuteron bombardment were made using a rotating single-crystal method. An expansion of $(4.1\pm0.2)\times10^{-21}$ per 7-Mev deuteron/cm² was found. No broadening of the Laue-Bragg intensity around the (4,0,0) reciprocal lattice point occurred. These effects are broadly consistent with the introduction of small point centers of dilatation. On the assumption that the damage consists of Frenkel defects, published calculations for the volume expansion due to interstitial atoms and vacant lattice sites in copper and the observed expansion lead to a concentration of defects which is only 0.08 to 0.22 of that predicted by the simple theory of displacement. Several independent measurements of inhomogeneity of the damage indicated an E^{-1} variation of the probability of lattice-atom displacement with deuteron energy, E, in agreement with the simple theory. The ratio of resistivity increase (as determined by Cooper *et al.*) to lattice expansion is 7×10^{-4} μohm-cm for such deuteron irradiation. Use of the empirical defect concentrations then gives a value for the resistivity of 1% of Frenkel defects as 2.1 to 5.6 μohm-cm.

Thermal recovery of the expansion in the temperature range 10–302°K was measured. It was strikingly similar to the recovery of electrical resistivity changes produced by deuteron irradiation. About 55% of the recovery occurred in a range below 42°K and the recovery was essentially complete at 302°K. Whatever the activating mechanism may be in each stage of recovery, the observed recovery appears predominantly due to mutual annihilation of interstitials and vacancies.

I. INTRODUCTION

COPPER is a particularly interesting crystal for radiation damage study.[1,2] Extensive theoretical calculation has been carried out for this metal; most bulk physical properties are well known; it is readily worked; and it is obtainable in relatively pure form. On the other hand, liquid-helium temperatures are required in order to prevent immediate thermal recovery of the damage produced by irradiation.[3] The necessity of working at such low temperatures makes damage and annealing studies difficult.

In recent years a few helium-temperature investigations of irradiated copper have been carried out using electrons,[4] neutrons,[5] and deuterons.[3] While electron irradiation under suitable conditions is thought to produce the simplest disarrangement of the crystal lattice, the relative inefficiency of electron bombardment in producing atomic displacements has limited the variety of measurable effects and prevented the production of defect concentrations greater than chemical impurity concentrations. In nuclear reactors conditions are complicated by the incident-neutron energy spectrum, possible anisotropies in the neutron scattering cross section, gamma- and beta-ray flux, nuclear transmutations, and, most important, the large mean energy transmitted to the initially struck lattice atom. Cyclotron irradiations do not produce the simplest

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† Based upon a dissertation submitted by R. O. Simmons in partial fulfillment of the requirements for the Doctor of Philosophy degree at the University of Illinois.

[1] For a recent review of radiation effects in solids see F. Seitz and J. S. Koehler, in *Solid State Physics* (Academic Press, Inc., New York, 1956), Vol. 2, p. 305.

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