Radiative Meson-Nucleon Scattering

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In this paper the static-source model of the pion-nucleon interaction is applied to the problem of the bremsstrahlung emitted in pion-nucleon scattering. It is shown that the matrix element for radiative meson scattering can be expressed in terms of other experimentally determined quantities, such as the matrix element for elastic scattering, provided the general static-source model is valid. Therefore, comparison of experimental results with those predicted by the present investigation will indicate the extent to which this simple model is valid, and suggest the refinements which are needed.

I. INTRODUCTION

Most of our present knowledge of the interaction between π mesons and nucleons has been obtained from a series of experiments on meson-proton scattering and on the photoproduction of mesons. The principal features of these processes agree well with the predictions of the simple theory suggested by Chew.[1–3] In this paper we point out that radiative meson scattering—in which a gamma ray is given off while the pion is being scattered by a nucleon—can be used to examine further the nature of the scattering process and the range of validity of the static-source theory. It is important for this purpose that, using the simple theory, precise predictions be made as to the gamma radiation to be expected, so that deviations from the predicted intensity can be readily related to needed refinements of the model. The new formalism of Low[2–4]—in which the physical scattering states appear in a basic way—can be applied to this process in a natural manner, leading to relations between the radiative scattering cross section and other experimentally determined quantities, and obviating the need for calculations which would just reproduce the scattering amplitudes. The accuracy of the results obtained by using such an approach are expected to be greater than would be obtained from a perturbation calculation,[5] or a Tamm-Dancoff calculation.

Before proceeding with the calculation of the matrix element, it will be useful to orient ourselves by considering some of the qualitative features we expect. First we notice that, especially when the process is to be studied through the detection of the gamma rays, it will perhaps be most convenient to examine the bremsstrahlung of positive mesons scattered by protons. Fortunately, this is the case in which the cross section is largest and in which the resonant state dominates the scattering most completely. We also observe that the effect of Rutherford scattering is insignificant at energies near the P-wave resonance. As is well known, the intensity of long-wavelength gamma radiation is given exactly by a semiclassical calculation; the number of quanta of frequency K is proportional to K^{-1} and to the scattering cross section in the limit $K\rightarrow 0$. No new information about scattering can be obtained from a measurement of this part of the spectrum, but the number of gamma rays with a shorter period, comparable to the collision time, will give some additional knowledge of the currents set up during the scattering. An important simplification occurs when $K\rightarrow K_0$, the end point of the bremsstrahlung spectrum; the part of the matrix element which is dominant in this limit arises from the interaction current—an incident P-wave meson is scattered by the nucleon, and electric dipole radiation is generated by the very high-frequency currents which are induced inside the nucleon core during the process of scattering. This effect is the same as that which is dominant in the simultaneous photoproduction of an S-wave and a P-wave meson[6]; the Feynman graphs are equivalent, as can be seen by reversing the temporal direction of the S-wave meson. The calculation of this term is easy and unambiguous in the static P-wave theory, but recoil effects and S-wave interactions will affect it by an uncertain amount. The most interest lies, therefore, in the high-energy limit of the gamma spectrum, but its measurement will require knowledge of the spectrum at lower energies as well.[7]

The same notation that was used in reference 6 will be used throughout the calculation which follows, except as will be otherwise noted.

II. GENERALIZED STATIC MODEL

We use, as was remarked above, a model in which the nucleon is supposed to be a fixed source of finite extent. At the end, it will be possible to incorporate certain

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[3] G. C. Wick, Revs. Modern Phys. **27**, 339 (1955).
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[6] R. E. Cutkosky and F. Zachariasen, Phys. Rev. **103**, 1108 (1956).
[7] The bremsstrahlung emitted by extremely relativistic mesons scattered from nuclei has been discussed by L. D. Landau and I. Ya. Pomeranchuk, J. Exptl. Theoret. Phys. U.S.S.R. **24**, 505 (1953) and Iu. A. Vdovin, Doklady Akad. Nauk S.S.S.R. **105**, 947 (1955), using a semiphenomenological model which is quite different from that applied here.

recoil corrections in an obvious fashion; others, which are fundamentally uncertain, are left out. Chew, Low, and others, in their applications of the static model, have in addition assumed that the interaction Hamiltonian depends on the meson field variable $\phi(\mathbf{x})$ linearly.[1–3] In this section we discuss a slightly more general version of the static theory, in which it is assumed that the main terms in the interaction Hamiltonian depend only on the P-wave part of the field variable, but possibly through an arbitrary function $H'(\nabla\phi,\boldsymbol{\sigma},\boldsymbol{\tau})$.[8] We shall not discuss the nature of the source function, which may have a very complicated structure in a nonlinear theory; we only need to assume that it is flat for the momenta in the initial and final states of primary interest, and can be characterized by some over-all cutoff. Our reason for discussing such a model is partly that there is at present little evidence concerning the linearity of the interaction, and partly that calculation with a nonlinear theory is not much more difficult.

The Hamiltonian is written as H_0+H', where H_0 is the usual free field Hamiltonian. The interaction term H' appears in the results only through commutators of the form $V_p=[H',a_p^*]$, $V_p^*=[a_p,H']$, and further commutators such as $[V_p^*,a_p]$, etc. The identity $V_p=-V_p^*$ follows from the fact that H' does not depend on the conjugate momentum $\pi(\mathbf{x})$. A renormalized coupling constant f is defined by

$$(\psi_0,V_{p\alpha}\psi_0)=if(2\omega_p)^{-\frac{1}{2}}v(p)(\phi_0,\boldsymbol{\sigma}\cdot\mathbf{p}\tau_\alpha\phi_0) \qquad (1)$$

where the ψ_0 denote states of the physical nucleon and the ϕ_0 are simple spinor wave functions. It is assumed that $v(p)=1$ in the neighborhood of the resonance.

The proofs of the following identities, which will be used repeatedly, are obtained in the same way as in the linear theory[2,3]:

$$a_p\psi_0=-(H+\omega_p)^{-1}V_p^*\psi_0, \qquad (2a)$$

$$a_p(H-E)^{-1}=(H+\omega_p-E)^{-1}a_p -(H+\omega_p-E)^{-1}V_p^*(H-E)^{-1}, \qquad (2b)$$

$$\psi_p^{\pm}=a_p^*\psi_0-(H-\omega_p\mp i\epsilon)^{-1}V_p\psi_0. \qquad (2c)$$

A slight change in the derivation[3] is required for the proof of the identity:

$$a_q\psi_p^{\pm}=\delta_{pq}\psi_0-(H+\omega_q-\omega_p\mp i\epsilon)^{-1}V_q^*\psi_p^{\pm}. \qquad (2d)$$

It is first noted that

$$a_q\psi_p^{\pm}=a_qa_p^*\psi_0-a_q(H-\omega_q\mp i\epsilon)^{-1}V_p\psi_0,$$

and that

$$a_qa_p^*\psi_0=\delta_{qp}\psi_0-a_p^*(H+\omega_q)^{-1}V_q^*\psi_0. \qquad (3)$$

[8] If H' depended also on the conjugate momentum $\pi(x)$, the calculations which follow would be greatly complicated. For brevity, we may refer to the absence of $\pi(x)$ from the interaction Hamiltonian as being the property of "rigidity" of the source.

Furthermore, using (2b):

$$-a_q(H-\omega_p\mp i\epsilon)^{-1}V_p\psi_0=-(H+\omega_q-\omega_p\mp i\epsilon)^{-1}a_qV_p\psi_0 +(H+\omega_q-\omega_p\mp i\epsilon)^{-1}V_q^*(H-\omega_p\mp i\epsilon)^{-1}V_p\psi_0. \qquad (4)$$

Use of the Jacobi identity,

$$[a_q,V_p]+[a_p^*,V_q^*]=[[a_p^*,a_q],H']=0,$$

transforms (4) to

$$\begin{aligned}-a_q(H-\omega_p\mp i\epsilon)^{-1}V_p\psi_0 &=(H+\omega_q-\omega_p\mp i\epsilon)^{-1}(a_p^*V_q^*-V_q^*a_p^*)\psi_0\\ &\quad+(H+\omega_q-\omega_p\mp i\epsilon)^{-1}V_p(H+\omega_q)^{-1}V_q^*\psi_0\\ &\quad+(H+\omega_q-\omega_p\mp i\epsilon)^{-1}V_q^*(H-\omega_p\mp i\epsilon)V_p\psi_0.\end{aligned}$$

Taking the Hermitian conjugate of (2b) and applying it to the above equation, one obtains

$$-a_q(H-\omega_p\mp i\epsilon)^{-1}V_p\psi_0=a_p^*(H+\omega_q)^{-1}V_q^*\psi_0 -(H+\omega_q-\omega_p\mp i\epsilon)^{-1}V_q^*[a_p^* -(H-\omega_p\mp i\epsilon)^{-1}V_p]\psi_0. \qquad (5)$$

Addition of (3) to (5) proves (2d).

The derivation of the Low scattering equation is now very easy. The scattering matrix is given by either $(\psi_q^-,V_p\psi_0)$ or $(\psi_0,V_q^*\psi_p^+)$, the two expressions being equal when $\omega_q=\omega_p$. Applying, in order, identities (2c) and (2a), one obtains

$$(\psi_q^-,V_p\psi_0)=-(\psi_0,V_q^*(H-\omega_q-i\epsilon)^{-1}V_p\psi_0) -(\psi_0,V_p(H+\omega_q)^{-1}V_q^*\psi_0)+(\psi_0,[a_q,V_p]\psi_0). \qquad (6)$$

The one-meson approximation to Eq. (6) is:

$$\begin{aligned}(\psi_q^-,V_p\psi_0)&=(\psi_0,[a_q,V_p]\psi_0)+(\psi_0,V_q^*\psi_0)\omega_q^{-1}(\psi_0,V_p\psi_0)\\ &\quad-(\psi_0,V_p\psi_0)\omega_q^{-1}(\psi_0,V_q^*\psi_0)\\ &\quad-\textstyle\sum_l[(\psi_0,V_q^*\psi_l^-)(\omega_l-\omega_q-i\epsilon)^{-1}(\psi_l^-,V_p\psi_0)\\ &\quad+(\psi_0,V_p\psi_l^-)(\omega_l+\omega_q)^{-1}(\psi_l^-,V_q^*\psi_0)]. \qquad (7)\end{aligned}$$

which differs from the result of the linear model only in the addition of the term

$$(\psi_0,[a_{q\alpha},V_{p\beta}]\psi_0) =q_ip_j(4\omega_p\omega_q)^{-\frac{1}{2}}v(p)v(q)(\phi_0,Q_{ij,\alpha\beta}\phi_0). \qquad (8)$$

For simplicity it may be assumed that $Q_{ij,\alpha\beta}$ does not depend on p or q. Then, because of the form of the interaction Hamiltonian (the rigidity of the source[8]),

$$Q_{ij,\alpha\beta}=a\delta_{ij}\delta_{\alpha\beta}+b\epsilon_{ijk}\sigma_k\epsilon_{\alpha\beta\gamma}\tau_\gamma. \qquad (9)$$

It is evident that the crossing theorem still holds.

It is not easy to see how inclusion of the additional parameters a and b affects the solution of Eq. (7) since an exact solution of that equation when $a=b=0$ is unknown. It appears, however, that the effective-range treatment of δ_{33} is not changed if a and b are not too large (that is, a, $b\lesssim f^2$), but at higher energies there may be important modifications, particularly in the other phase shifts.[9] In the calculation of inelastic scattering, further inhomogeneous terms appear, making the justification of the one-meson approximation somewhat more obscure; nevertheless, this approximation will be used in many of the following calculations.

The current operator in the static model consists of three terms: $\mathbf{j}=\mathbf{j}_m+\mathbf{j}_n+\mathbf{j}_{nm}$. The meson current is

$$\mathbf{j}_m(\mathbf{x})=-e\epsilon_{3\alpha\beta}\phi_\alpha(\mathbf{x})\nabla\phi_\beta(\mathbf{x}). \tag{10}$$

The position of the source is fixed, but it is of course free to rotate, so the nucleon current $\mathbf{j}_n$ in this theory is given by the magnetic moment of a nucleon which is stripped of its outer clothing of pions. This magnetic moment is not necessarily that of a bare Dirac particle because the nucleon may possess an inner clothing consisting of pairs, strange particles, etc.[10] Finally, there is the interaction current; in order to conserve charge, instantaneous currents must flow inside the source during the emission or absorption of charged pions. Although the nature of this current is quite unknown, the electric dipole contribution for wavelengths much larger than the source radius is given uniquely if one replaces $\nabla\phi$ by $\nabla\phi\mp ie\mathbf{A}\phi$ (for mesons of charge $\pm e$) in the interaction Hamiltonian. If one writes

$$V_{p\alpha}=(2\omega_p)^{-\frac{1}{2}}\mathbf{p}\cdot\mathbf{U}_\alpha, \tag{11}$$

then, in this approximation,

$$\mathbf{j}_{nm}(\mathbf{x})=ie\epsilon_{3\alpha\beta}\phi_\alpha(0)\mathbf{U}_\beta\delta(\mathbf{x}). \tag{12}$$

Since $\phi_\beta(0)$ creates or annihilates S-wave mesons, it commutes with all terms in $\mathbf{U}_\beta$. The Kroll-Ruderman theorem,[2,11] as well as the multiple meson generalization,[6] follow directly from (12).

III. BREMSSTRAHLUNG MATRIX ELEMENT

The matrix element for emission of a quantum of momentum $\mathbf{K}$, polarization $\boldsymbol{\varepsilon}$, while the meson is scattered from a momentum p to a momentum q, is

$$M_{qp}=-(2K)^{-\frac{1}{2}}(\psi_q^-,J\psi_p^+), \tag{13}$$

where

$$J=\int d^3x e^{-i\mathbf{K}\cdot\mathbf{x}}\boldsymbol{\varepsilon}\cdot\mathbf{j}(\mathbf{x}).$$

We shall first evaluate the most interesting term, which is given by the interaction current (12):

$$M_{q\alpha,\,p\beta}^{(1)}=(\psi_{q\alpha}^-,\,-i(2K)^{-\frac{1}{2}}e\epsilon_{3\mu\nu}\phi_\mu(0)\mathbf{U}_\nu\cdot\boldsymbol{\varepsilon}\psi_{p\beta}^+). \tag{14}$$

[9] M. Cini and S. Fubini, [Nuovo cimento 3, 764 (1956); Phys. Rev. **102**, 1687 (1956)], have derived sum rules for the cross sections, using the linear theory, and found that they do not agree well with experimental results. These sum rules do not exist in the generalized theory.

[10] It can be shown that introduction of nonlinearities does not change the general nature of the results of H. Miyazawa [Phys. Rev. **101**, 1597 (1956)].

[11] N. M. Kroll and M. A. Ruderman, Phys. Rev. **93**, 233 (1954).

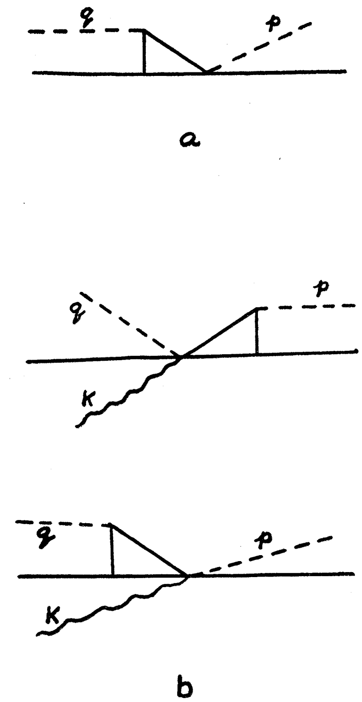

FIG. 1. Feynman graphs for the scattering matrix $(\psi_q^-,V_p\psi_0)$, and for the interaction current term $M_{qp}^{(1)}$.

Evidently $M_{qp}^{(1)}=0$ unless either the initial or final meson is in an S-state. We introduce the notation

$$(\psi_{q\alpha}^-,U_{j\beta}\psi_0)=(2\omega_q)^{-\frac{1}{2}}q_i(\phi_0,T_{ij,\,\alpha\beta}(q)\phi_0). \tag{15}$$

Then, since for an S-state $\psi_p^\pm=a_p{}^*\psi_0$,

$$M_{q\alpha,\,p\beta}^{(1)}=ie(8K\omega_p\omega_q)^{-\frac{1}{2}}[\epsilon_{3\alpha\gamma}\epsilon_i p_j T_{ij,\,\gamma\beta}(p)-\epsilon_{3\beta\gamma}\epsilon_j q_i T_{ij,\,\alpha\gamma}(q)]. \tag{16}$$

In order to picture this process a little more clearly, it may be convenient to refer to the Feynman graphs in Fig. 1. A nonsymmetrical symbol is used for the matrix element $(\psi_q^-,V_p\psi_0)$ in order to emphasize that it has a trivial dependence on one parameter. This symbol is to be considered as an abbreviation for the totality of all graphs which contribute to scattering.[3]

It is more difficult to evaluate the remaining contributions to the matrix element. The problem here is to separate these contributions into three parts: first, we must find a term which contains the result of the classical calculation, and which we shall call the "quasi-classical" term; secondly, we must try to combine certain terms which contain the mesonic contribution to the magnetic moment of the physical nucleon with the current of the unclothed nucleon in such a way that the matrix element can be expressed in terms of directly measurable electromagnetic properties of the nucleon; we can then interpret most of the remaining corrections as being rescattering corrections, that is, terms which correspond to processess such as that in which the meson is scattered, emits a photon, and then is scattered once more. The approach which seems to lead most directly to the desired result follows. By repeatedly applying the identities satisfied by the eigenstates of the theory—Eqs. (2a–d), (6), and (7)—we shall eventually deduce a nontrivial identity, in the form of an integral equation, satisfied by the matrix element.

We write $\mathbf{j}'(\mathbf{x})=\mathbf{j}_m(\mathbf{x})+\mathbf{j}_n(\mathbf{x})$, and

$$M_{qp}'=-(2K)^{-\frac{1}{2}}(\psi_q^-,J'\psi_p^+). \tag{17}$$

Expanding the initial and final states according to (2c), we obtain

$$\begin{aligned}M_{qp}'=&-(2K)^{-\frac{1}{2}}([a_q^*-(H-\omega_q+i\epsilon)^{-1}V_q]\psi_0,\ J'[a_p^*-(H-\omega_p-i\epsilon)^{-1}V_p]\psi_0)\\ =&(2K)^{-\frac{1}{2}}\{(\psi_0,[a_q,J']\psi_p^+)+(\psi_q^-,[J',a_p^*]\psi_0)+(\psi_0,\ J'a_q[a_p^*-(H-\omega_p-i\epsilon)^{-1}V_p]\psi_0)\\ &+(\psi_0,\ [a_q-V_q^*(H-\omega_q-i\epsilon)^{-1}]a_p^*J'\psi_0)-(\psi_0,a_qJ'a_p^*\psi_0)\\ &+(\psi_0,\ V_q^*(H-\omega_q-i\epsilon)^{-1}J'(H-\omega_p-i\epsilon)^{-1}V_p\psi_0)\}.\end{aligned} \tag{18}$$

We now use the identity

$$J'a_qa_p^*+a_qa_p^*J'-a_qJ'a_p^*=a_p^*J'a_q+J'\delta_{pq}+[a_q,[a_p^*,J']]$$

to transform Eq. (18) to

$$\begin{aligned}M_{qp}'=&-(2K)^{-\frac{1}{2}}\{(\psi_0,[a_q,J']\psi_p^+)+(\psi_q^-,[J',a_p^*]\psi_0)+(\psi_0,[a_q,[a_p^*,J']]\psi_0)+(\psi_0,J'\delta_{pq}\psi_0)+(a_p\psi_0,J'a_q\psi_0)\\ &+(\psi_0,\ J'(H+\omega_q-\omega_p-i\epsilon)^{-1}L(q,p)\psi_0)+(\psi_0,\ \bar{L}(q,p)(H+\omega_p-\omega_q-i\epsilon)^{-1}J'\psi_0)\\ &+(\psi_0,\ V_q^*(H-\omega_q-i\epsilon)^{-1}J'(H-\omega_p-i\epsilon)^{-1}V_p\psi_0)\}.\end{aligned} \tag{19}$$

In Eq. (19) we have used the relation

$$\begin{aligned}-a_q(H-\omega_p-i\epsilon)^{-1}V_p\psi_0&=(H+\omega_q-\omega_p-i\epsilon)^{-1}[V_q^*(H-\omega_p-i\epsilon)^{-1}V_p\\ &\quad+V_p(H+\omega_q)^{-1}V_q^*-[a_q,V_p]]\psi_0\\ &=(H+\omega_q-\omega_p-i\epsilon)^{-1}L(q,p)\psi_0,\end{aligned} \tag{20a}$$

and the related expression in which

$$\bar{L}(q,p)=V_q^*(H-\omega_q-i\epsilon)^{-1}V_p+V_p(H+\omega_p)^{-1}V_q^*-[V_q^*,a_p^*]. \tag{20b}$$

The nucleon current $\mathbf{j}_n(\mathbf{x})$ is independent of the meson field variables so we can evaluate the commutators in the first three terms on the right-hand side of Eq. (19) by using only the known expression for the meson current:

$$J_m=-ie\epsilon_{3\mu\nu}\sum_l \mathbf{l}\cdot\boldsymbol{\epsilon}(4\omega_l\omega_{l-K})^{-\frac{1}{2}}\times[a_{l-K,\mu}^*a_{-l,\nu}^*+a_{K-l,\mu}a_{l,\nu}+2a_{l-K,\mu}^*a_{l,\nu}]. \tag{21}$$

Therefore, we obtain

$$\begin{aligned}[a_{q,\alpha},J']&=-2ie\epsilon_{3\alpha\nu}\mathbf{q}\cdot\boldsymbol{\epsilon}(4\omega_q\omega_{q+K})^{-\frac{1}{2}}\times[a_{-q-K,\nu}^*+a_{q+K,\nu}],\\ [J',a_{p,\beta}^*]&=-2ie\epsilon_{3\mu\beta}\mathbf{p}\cdot\boldsymbol{\epsilon}(4\omega_p\omega_{K-p})^{-\frac{1}{2}}\times[a_{K-p,\mu}+a_{p-K,\mu}^*],\\ [a_{q\alpha},[a_{p\beta}^*,J']]&=2ie\epsilon_{3\alpha\beta}\mathbf{p}\cdot\boldsymbol{\epsilon}(4\omega_p\omega_q)^{-\frac{1}{2}}\delta_{q,p-K}.\end{aligned} \tag{22}$$

Now, using identities (2a) and (2d), we find that

$$\begin{aligned}(\psi_0,\ [a_{-q-K,\nu}^*+a_{q+K,\nu}]\psi_{p\alpha}^+)&=(\psi_0,\psi_0)\delta_{\alpha\nu}\delta_{p,q+K}-(\omega_{q+K}-\omega_p-i\epsilon)^{-1}(\psi_0,V_{q+K,\nu}^*\psi_{p\alpha}^+)\\ &\quad-(\omega_p+\omega_{q+K})^{-1}(\psi_0,V_{-q-K,\nu}\psi_{p\alpha}^+)\\ &=(\psi_0,\psi_0)\delta_{\alpha\nu}\delta_{p,q+K}-\frac{2\omega_{q+K}}{\omega_{q+K}^2-\omega_p^2-i\epsilon}(\psi_0,V_{q+K,\nu}^*\psi_{p\beta}^+);\end{aligned} \tag{23}$$

therefore,

$$\begin{aligned}-(2K)^{-\frac{1}{2}}(\psi_0,[a_{q\alpha},J']\psi_{p\beta}^+)&=2ie\epsilon_{3\alpha\beta}\delta_{p,q+K}\mathbf{p}\cdot\boldsymbol{\epsilon}(8K\omega_p\omega_q)^{-\frac{1}{2}}(\psi_0,\psi_0)\\ &\quad+\frac{2ie\epsilon_{3\alpha\nu}\mathbf{q}\cdot\boldsymbol{\epsilon}(2\omega_{q+K})}{(8K\omega_q\omega_{q+K})^{\frac{1}{2}}(\omega_p^2-\omega_{q+K}^2+i\epsilon)}\times(\psi_0,V_{q+K,\nu}^*\psi_{p\beta}^+).\end{aligned} \tag{24}$$

We treat in the same way the matrix element $(\psi_q^-,[J',a_{p\beta}^*]\psi_0)$; thus we obtain for the first three terms of Eq. (19):

$$\begin{aligned}-(2K)^{-\frac{1}{2}}\{&(\psi_0,[a_{q\alpha},[a_{p\beta}^*,J']]\psi_0)+(\psi_0,[a_{q\alpha},J']\psi_{p\beta}^+)+(\psi_{q\alpha}^-,[J',a_{p\beta}^*]\psi_0)\\ =&2ie\epsilon_{3\alpha\beta}\delta_{p,q+K}(8K\omega_p\omega_q)^{-\frac{1}{2}}\mathbf{p}\cdot\boldsymbol{\epsilon}(\psi_0,\psi_0)\\ &+\frac{2ie\epsilon_{3\alpha\nu}\mathbf{q}\cdot\boldsymbol{\epsilon}(2\omega_{q+K})^{\frac{1}{2}}}{(4K\omega_q)^{\frac{1}{2}}(\omega_p^2-\omega_{q+K}^2+i\epsilon)}(\psi_0,V_{q+K,\nu}^*\psi_{p\beta}^+)\\ &+\frac{2ie\epsilon_{3\mu\beta}\mathbf{p}\cdot\boldsymbol{\epsilon}(2\omega_{p-K})^{\frac{1}{2}}}{(4K\omega_p)^{\frac{1}{2}}(\omega_q^2-\omega_{p-K}^2+i\epsilon)}(\psi_{q\alpha}^-,V_{p-K,\mu}\psi_0).\end{aligned} \tag{25}$$

We have now succeeded in separating out the most important contribution of the meson current, the quasi-classical term; the last two terms in Eq. (25) can be related immediately to the scattering matrix, and evidently have the proper behavior as $K\to 0$. When $\omega_p=\omega_q+K$, we may rewrite these terms, after referring to Eqs. (11) and (15), as:

$$\begin{aligned}M_{\mathbf{q}\alpha,\,\mathbf{p}\beta}{}^{(2)}&=ie(8K\omega_p\omega_q)^{\frac{1}{2}}\\&\times\{\epsilon_{3\alpha\gamma}(K\omega_q-\mathbf{K}\cdot\mathbf{q})^{-1}\mathbf{q}\cdot\boldsymbol{\varepsilon}(q_i+K_i)p_jT_{ij,\,\gamma\beta}(p)\\&+\epsilon_{3\beta\gamma}(K\omega_p-\mathbf{K}\cdot\mathbf{p})^{-1}\mathbf{p}\cdot\boldsymbol{\varepsilon}q_i(p_j-K_j)T_{ij,\,\alpha\gamma}(q)\}.\end{aligned}\tag{26}$$

The structure of this term is described by the graphs in Fig. 2. The first term in Eq. (25) is a "disconnected graph" term, and does not give any contribution to a matrix element between states which conserve energy. However, as we shall see, we shall have use for the matrix element between arbitrary states. Another disconnected graph term is $(\psi_0,J'\psi_0)\delta_{\mathbf{qp}}$; we note that the interaction current does not contribute to such a matrix element, since the nucleon does not recoil, so $(\psi_0,J'\psi_0)$ equals $(\psi_0,J\psi_0)$, which is given by the magnetic moment of the physical nucleon.

We now make closure expansions in the remaining terms of Eq. (19), keeping only zero- and one-meson terms:

$$\begin{aligned}(\psi_0,a_p{}^*J'a_q\psi_0)&=(\psi_0,\,V_p(H+\omega_p)^{-1}J'(H+\omega_q)^{-1}V_q{}^*\psi_0)\\&=(\psi_0,V_p\psi_0)\omega_p{}^{-1}(\psi_0,J'\psi_0)\omega_q{}^{-1}(\psi_0,V_q{}^*\psi_0)\\&+\textstyle\sum_r(\psi_0,V_p\psi_r{}^-)(\omega_r+\omega_p)^{-1}(\psi_r{}^-,J'\psi_0)\omega_q{}^{-1}(\psi_0,V_q{}^*\psi_0)\\&+\textstyle\sum_r(\psi_0,V_p\psi_0)\omega_p{}^{-1}(\psi_0,J'\psi_r{}^+)(\omega_r+\omega_q)^{-1}(\psi_r{}^+,V_q{}^*\psi_0)\\&+\cdots.\end{aligned}\tag{27}$$

Again, $(\psi_0,J'\psi_0)$ is related to the magnetic moment of the nucleon, and $(\psi_r{}^-,J'\psi_0)$ is evidently the magnetic dipole and electric quadrupole part of the photoproduction amplitude. It should be remarked that among the terms represented by the dots at the end of Eq. (27) are some terms which are needed to demonstrate that the entire matrix element satisfies a sort of "crossing symmetry," but these terms are actually very small. Returning to Eqs. (20a,b), we write

$$\begin{aligned}L(q,p)&=-X_q(p)\\&\quad+V_p(H+\omega_p)^{-1}(\omega_p-\omega_q)(H+\omega_q)^{-1}V_q{}^*,\\X_q(p)&=-V_q{}^*(H-\omega_p-i\epsilon)^{-1}V_p\\&\quad-V_p(H+\omega_p)^{-1}V_q{}^*+[a_q,V_p];\\\bar{L}(q,p)&=-\bar{X}_p(q)\\&\quad+V_p(H+\omega_q)^{-1}(\omega_q-\omega_p)(H+\omega_p)^{-1}V_q{}^*,\\\bar{X}_p(q)&=-V_q{}^*(H-\omega_q-i\epsilon)^{-1}V_p\\&\quad-V_p(H+\omega_q)^{-1}V_q{}^*+[V_q{}^*,a_p{}^*].\end{aligned}\tag{28}$$

Using the above equations, we have

$$\begin{aligned}&(\psi_0,\,J'(H+\omega_q-\omega_p-i\epsilon)^{-1}L(q,p)\psi_0)\\&=-(\psi_0,J'\psi_0)[(\omega_q-\omega_p-i\epsilon)^{-1}(\psi_0,V_q{}^*\psi_p{}^+)\\&\quad+(\psi_0,\,V_p(H+\omega_p)^{-1}(H+\omega_q)^{-1}V_q{}^*\psi_0)]\\&\quad+\textstyle\sum_r(\psi_0,J'\psi_r{}^-)(\omega_r+\omega_q-\omega_p-i\epsilon)^{-1}\\&\qquad\times(\psi_r{}^-,L(q,p)\psi_0),\end{aligned}\tag{29a}$$

and

$$\begin{aligned}&(\psi_0,\,\bar{L}(q,p)(H+\omega_p-\omega_q-i\epsilon)^{-1}J'\psi_0)\\&=-[(\psi_q{}^-,V_p\psi_0)(\omega_p-\omega_q-i\epsilon)^{-1}\\&\quad+(\psi_0,\,V_p(H+\omega_p)^{-1}(H+\omega_q)^{-1}V_q{}^*\psi_0)](\psi_0,J\psi_0)\\&\quad+\textstyle\sum_r(\psi_0,\bar{L}(q,p)\psi_r{}^+)(\omega_r+\omega_p-\omega_q-i\epsilon)^{-1}\\&\qquad\times(\psi_r{}^+,J'\psi_0).\end{aligned}\tag{29b}$$

We remember that we are primarily interested in the scattering of positive mesons from protons, and therefore write for the last term of (19)[12]:

$$\begin{aligned}&(\psi_0,\,V_q{}^*(H-\omega_q-i\epsilon)^{-1}J'(H-\omega_p-i\epsilon)^{-1}V_p\psi_0)\\&=\sum_{rs}\frac{(\psi_0,V_q{}^*\psi_r{}^-)(\psi_r{}^-,J'\psi_s{}^+)(\psi_s{}^+,V_p\psi_0)}{(\omega_r-\omega_q-i\epsilon)(\omega_s-\omega_p-i\epsilon)}.\end{aligned}\tag{30}$$

We now collect our results in the form

$$\begin{aligned}M_{qp}&=M_{qp}{}^{(n)}+M_{qp}{}^{(m)}+M_{qp}{}^{(1)}+M_{qp}{}^{(2)}\\&\quad+M_{qp}{}^{(3)}+M_{qp}{}^{(4)}+M_{qp}{}^{(5)}.\end{aligned}\tag{31}$$

The two disconnected graph terms are obtained from Eqs. (19) and (25),

$$\begin{aligned}M_{qp}{}^{(n)}&=-(2K)^{-\frac{1}{2}}(\psi_0,J\psi_0)\delta_{qp},\\M_{qp}{}^{(m)}&=2ie\epsilon_{3\alpha\beta}\delta_{\mathbf{p},\,\mathbf{q}+\mathbf{K}}\mathbf{p}\cdot\boldsymbol{\varepsilon}(8K\omega_p\omega_q)^{-\frac{1}{2}}(\psi_0,\psi_0),\end{aligned}\tag{32}$$

while $M_{qp}{}^{(1)}$ is given by Eq. (16) and $M_{qp}{}^{(2)}$ by Eq. (25). Also, we define

$$\begin{aligned}M_{qp}{}^{(3)}&=(2K)^{-\frac{1}{2}}(\psi_0,J\psi_0)(\omega_q-\omega_p-i\epsilon)^{-1}(\psi_0,V_q{}^*\psi_p{}^+)\\&\quad+(2K)^{-\frac{1}{2}}(\psi_q{}^-,V_p\psi_0)(\omega_p-\omega_q-i\epsilon)^{-1}(\psi_0,J\psi_0);\end{aligned}\tag{33}$$

this [Eq. (33)] is what a semiclassical calculation would suggest for the contribution of the magnetic

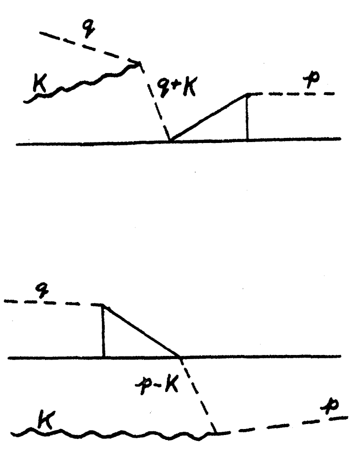

FIG. 2. Feynman graphs for the quasi-classical term $M_{qp}{}^{(2)}$.

[12] The "zero-meson" terms left out of Eq. (30), which contribute only when negative mesons are scattered, can be expressed in terms of the photomeson matrix element and the nucleon magnetic moment. They are less important than the magnetic moment term $M_{qp}{}^{(3)}$ of Eq. (33).

moment of the nucleon. The remaining terms of (29a,b) are combined with those of (27) into

$$
\begin{aligned}
M_{qp}^{(4)}=&-(2K)^{-\frac{1}{2}}\{-(\psi_0,J'\psi_0)\omega_q^{-1}(\psi_0,V_p\psi_0)\omega_p^{-1}(\psi_0,V_q{}^*\psi_0)-(\psi_0,V_p\psi_0)\omega_p^{-1}(\psi_0,V_q{}^*\psi_0)\omega_q^{-1}(\psi_0,J'\psi_0)\\
&+(\psi_0,V_p\psi_0)\omega_p^{-1}(\psi_0,J'\psi_0)\omega_q^{-1}(\psi_0,V_q{}^*\psi_0)+\textstyle\sum_r[(\psi_0,V_p\psi_0)\omega_p^{-1}(\psi_0,J'\psi_r{}^+)(\omega_r+\omega_q)^{-1}(\psi_r{}^+,V_q{}^*\psi_0)\\
&+(\psi_0,V_p\psi_r{}^-)(\omega_p+\omega_r)^{-1}(\psi_r{}^-,J'\psi_0)\omega_q^{-1}(\psi_0,V_q{}^*\psi_0)+(\psi_0,J'\psi_r{}^-)(\omega_r+\omega_q-\omega_p-i\epsilon)^{-1}(\psi_r{}^-,L(q,p)\psi_0)\\
&+(\psi_0,\bar{L}(q,p)\psi_r{}^+)(\omega_r+\omega_p-\omega_q-i\epsilon)^{-1}(\psi_r{}^+,J'\psi_0)-(\psi_0,J'\psi_0)(\omega_r+\omega_q)^{-1}(\psi_0,V_p\psi_r{}^-)(\omega_p+\omega_r)^{-1}(\psi_r{}^-,V_q{}^*\psi_0)\\
&-(\psi_0,V_p\psi_r{}^-)(\omega_r+\omega_q)^{-1}(\omega_p+\omega_r)^{-1}(\psi_r{}^-,V_q{}^*\psi_0)(\psi_0,J'\psi_0)]\}. \qquad (34)
\end{aligned}
$$

The last term of Eq. (31) is given by (30):

$$M_{qp}^{(5)}=\sum_{rs}\frac{(\psi_0,V_q{}^*\psi_r{}^-)M_{rs}'(\psi_s{}^+,V_p\psi_0)}{(\omega_r-\omega_q-i\epsilon)(\omega_s-\omega_p-i\epsilon)}. \qquad (35)$$

As a consequence of this term, (31) is a linear integral equation for M_{qp}. [In Eq. (35), r and s must refer to P-states, for which $M_{rs}'\equiv M_{rs}$.] This integral equation clearly has a form which is related to the Low scattering Eq. (7), but while it is similar to that equation in involving only matrix elements between physical states, it differs from it in being linear and in that the solution must be obtained also for states which do not conserve energy. It is essential to notice that the integral equation involves—except of course for the solution itself—only quantities which can be determined from other experiments, and therefore the bremsstrahlung will serve as an especially good test of the applicability of the general static model to scattering. Note that none of the relations which are implied by Eq. (31) depend on the linearity of the P-wave interaction.

From purely classical arguments we can understand that the magnetic moment term $M_{qp}^{(3)}$ is, for the energies with which we are concerned, much smaller than the quasi-classical term $M_{qp}^{(2)}$. The different terms in $M_{qp}^{(4)}$, Eq. (34), are even smaller than $M_{qp}^{(3)}$; the first three contain the Born approximation to the phase shift δ_{33} instead of the actual experimental value of this phase shift (the energy denominators are also bigger in $M_{qp}^{(4)}$). The remaining terms in Eq. (34) are also very small, despite the appearance of matrix elements which are enhanced by the resonant state. The physical basis for this is that the resonance is a narrow one, so integrals over it are not large unless, of course, there are energy denominators which may vanish in the region of resonance, as in the Low scattering equation, or in $M_{qp}^{(5)}$ as given by expression (35). For preliminary comparisons with experiments near the resonance energy, it will be convenient to neglect the smaller phase shifts in comparison with δ_{33}, and if this is done it will also be appropriate to neglect $M_{qp}^{(4)}$. We cannot say that the effect of the kernel of the integral equation (31) will not be to amplify certain terms of $M_{qp}^{(4)}$; this possibility we shall discuss again later.

IV. RESCATTERING EFFECT

It is clear that the function of the kernel of the integral Eq. (31) is to determine what we have previously called the rescattering corrections to the matrix element. The kernel is of an unfamiliar and rather singular sort, and the inhomogeneous terms are very singular as well, so we shall perhaps be unable to solve this equation very accurately, but we must at least try to obtain a rough understanding of how the solution will depend on the energies, and a general idea of the importance of the rescattering effects. In order to do this, it is convenient to split the matrix element M_{qp}' into three parts:

$$M_{qp}'=M_{qp}^{(M)}+M_{qp}^{(N)}+\tilde{M}_{qp}, \qquad (36)$$

$$M_{qp}^{(M)}=M_{qp}^{(m)}+M_{qp}^{(2)}+\sum_{rs}\frac{(\psi_0,V_q{}^*\psi_r{}^-)M_{rs}^{(M)}(\psi_s{}^+,V_p\psi_0)}{(\omega_r-\omega_q-i\epsilon)(\omega_s-\omega_p-i\epsilon)}, \qquad (37)$$

$$M_{qp}^{(N)}=M_{qp}^{(n)}+M_{qp}^{(3)}+\sum_{rs}\frac{(\psi_0,V_q{}^*\psi_r{}^-)M_{rs}^{(N)}(\psi_s{}^+,V_p\psi_0)}{(\omega_r-\omega_q-i\epsilon)(\omega_s-\omega_p-i\epsilon)}. \qquad (38)$$

We may think of $M_{qp}^{(N)}$ as being the *entire* contribution of the nucleon moment, and $M_{qp}^{(M)}$ as being the corresponding contribution of the meson current; $\tilde{M}_{qp}$ represents the minor residual effects associated with $M_{qp}^{(4)}$.

Let us consider first Eq. (38) for $M_{qp}^{(N)}$ and attempt to write the solution in the following manner:

$$M_{qp}^{(N)}=-(2K)^{-\frac{1}{2}}\sum_{l,l'}\left[\delta_{ql}(\psi_0,\psi_0)-\frac{(\psi_q{}^-,V_l\psi_0)}{\omega_l-\omega_q-i\epsilon}\right]\times\delta_{ll'}(\psi_0,J\psi_0)\left[\delta_{l'p}(\psi_0,\psi_0)-\frac{(\psi_0,V_{l'}{}^*\psi_p{}^+)}{(\omega_{l'}-\omega_p-i\epsilon)}\right]+\bar{M}_{qp}^{(N)}. \qquad (39)$$

The first term on the right in Eq. (39) contains not only the inhomogeneous term of Eq. (38), but also part of the rescattering correction. We shall call this part of the rescattering correction $S_{qp}^{(N)}$:

$$S_{qp}^{(N)}=-(2K)^{-\frac{1}{2}}\sum_{l}\frac{(\psi_q{}^-,V_l\psi_0)(\psi_0,J\psi_0)(\psi_0,V_l{}^*\psi_p{}^+)}{(\omega_l-\omega_q-i\epsilon)(\omega_l-\omega_p-i\epsilon)}. \qquad (40)$$

We might interpret $S_{qp}^{(N)}$ as a Tamm-Dancoff approximation to the rescattering correction. Our purpose in separating out this part of the rescattering correction explicitly is that the integral equation for the remaining term $\bar{M}_{qp}^{(N)}$ takes a form which appears to be simpler, and which one might hope to understand more easily. It is remarkable that the term which we find most convenient to separate out is exactly what one would write down for the contribution of the nucleon magnetic moment to the bremsstrahlung matrix element if one were to make the very naive assumption that the meson-nucleon system could be represented by a simple wave function $\Phi_p(l)$ given by

$$\Phi_p(l)=(\psi_0,a_l\psi_p^+)=\delta_{pl}(\psi_0,\psi_0)-(\omega_l-\omega_p-i\epsilon)^{-1}(\psi_0,V_l^*\psi_p^+). \quad (41)$$

The correction $\bar{M}_{qp}^{(N)}$ satisfies the integral equation which we obtain by inserting the expression (39) into Eq. (38):

$$\bar{M}_{qp}^{(N)}=-(2K)^{-\frac{1}{2}}\left\{-\sum_l\frac{(\psi_q^-,V_l\psi_0)(\psi_0,J\psi_0)(\psi_0,V_l^*\psi_p^+)}{(\omega_l-\omega_q-i\epsilon)(\omega_l-\omega_p-i\epsilon)}+\sum_l\frac{(\psi_0,V_q^*\psi_l^-)(\psi_0,J\psi_0)(\psi_l^+,V_p\psi_0)}{(\omega_l-\omega_q-i\epsilon)(\omega_l-\omega_p-i\epsilon)}\right.$$
$$-\sum_{rl}\frac{(\psi_0,V_q^*\psi_r^-)(\psi_r^-,V_l\psi_0)(\psi_0,J\psi_0)(\psi_l^+,V_p\psi_0)}{(\omega_r-\omega_q-i\epsilon)(\omega_l-\omega_r-i\epsilon)(\omega_l-\omega_p-i\epsilon)}-\sum_{ls}\frac{(\psi_0,V_q^*\psi_l^-)(\psi_0,J\psi_0)(\psi_0,V_l^*\psi_s^+)(\psi_s^+,V_p\psi_0)}{(\omega_l-\omega_q-i\epsilon)(\omega_l-\omega_s-i\epsilon)(\omega_s-\omega_p-i\epsilon)}$$
$$\left.+\sum_{rls}\frac{(\psi_0,V_q^*\psi_r^-)(\psi_r^-,V_l\psi_0)(\psi_0,J\psi_0)(\psi_0,V_l^*\psi_s^+)(\psi_s^+,V_p\psi_0)}{(\omega_r-\omega_q-i\epsilon)(\omega_l-\omega_r-i\epsilon)(\omega_l-\omega_s-i\epsilon)(\omega_s-\omega_p-i\epsilon)}\right\}+\sum_{rs}\frac{(\psi_0,V_q^*\psi_r^-)\bar{M}_{rs}^{(N)}(\psi_s^+,V_p\psi_0)}{(\omega_r-\omega_q-i\epsilon)(\omega_s-\omega_p-i\epsilon)}. \quad (42)$$

The inhomogeneous term in Eq. (42) looks forbidding, but can be reduced to a more manageable form. We must consider sums such as

$$A_{ql}=\sum_r(\psi_0,V_q^*\psi_r^-)(\psi_r^-,V_l\psi_0)(\omega_r-\omega_q-i\epsilon)^{-1}(\omega_l-\omega_r-i\epsilon)^{-1}$$
$$=\sum_r(\psi_0,V_q^*\psi_r^-)(\psi_r^-,V_l\psi_0)[(\omega_r-\omega_q-i\epsilon)^{-1}(\omega_l-\omega_q-i\epsilon)^{-1}-(\omega_r-\omega_l+i\epsilon)^{-1}(\omega_l-\omega_q-i\epsilon)^{-1}]. \quad (43)$$

Similar sums appear in the Low scattering equation; reference to Eq. (7) shows that we may write

$$A_{ql}=(\omega_l-\omega_q-i\epsilon)^{-1}\{-(\psi_q^-,V_l\psi_0)+(\psi_0,V_q^*\psi_l^-)-\sum_r[(\omega_r+\omega_q)^{-1}-(\omega_r+\omega_l)^{-1}](\psi_0,V_l\psi_r^-)(\psi_r^-,V_q^*\psi_0)\}$$
$$+\omega_q^{-1}\omega_l^{-1}\{(\psi_0,V_q^*\psi_0)(\psi_0,V_l\psi_0)-(\psi_0,V_l\psi_0)(\psi_0,V_q^*\psi_0)\}. \quad (44)$$

It is clear that the summation in Eq. (44) is smaller than the first term, so as a first approximation it might be neglected. Note that Eq. (44) does not contain the unknown quadratic term of the Low equation (7) or (8). In the same way we prove that

$$B_{lp}=\sum_s(\psi_0,V_l^*\psi_s^+)(\psi_s^+,V_p\psi_0)(\omega_l-\omega_s-i\epsilon)^{-1}(\omega_s-\omega_p-i\epsilon)^{-1}$$
$$=(\omega_l-\omega_p-i\epsilon)^{-1}[(\psi_l^+,V_p\psi_0)-(\psi_0,V_l^*\psi_p^+)]-\sum_s(\psi_0,V_p\psi_s^+)(\psi_s^+,V_l^*\psi_0)(\omega_s+\omega_l)^{-1}(\omega_s+\omega_p)^{-1}$$
$$+\omega_p^{-1}\omega_l^{-1}\{(\psi_0,V_l^*\psi_0)(\psi_0,V_p\psi_0)-(\psi_0,V_p\psi_0)(\psi_0,V_l^*\psi_0)\}. \quad (45)$$

If we were to insert the expressions (44) and (45) into the inhomogeneous term in Eq. (42), neglecting all but the first term in (44) and (45), we would find that the inhomogeneous term, and hence also $\bar{M}_{qp}^{(N)}$, vanished identically. More exactly, we obtain:

$$\bar{M}_{qp}^{(N)}=-(2K)^{-\frac{1}{2}}\left\{\sum_l\left[\frac{\{(\psi_0,V_l\psi_0)(\psi_0,V_q^*\psi_0)-(\psi_0,V_q^*\psi_0)(\psi_0,V_l\psi_0)\}}{\omega_l\omega_q}\right.\right.$$
$$\left.+\sum_r\frac{(\psi_0,V_l\psi_r^-)(\psi_r^-,V_q^*\psi_0)}{(\omega_l+\omega_r)(\omega_q+\omega_r)}\right]\frac{(\psi_0,J\psi_0)(\psi_0,V_l^*\psi_p^+)}{\omega_l-\omega_p-i\epsilon}+\sum_l\frac{(\psi_q^-,V_l\psi_0)(\psi_0,J\psi_0)}{\omega_l-\omega_q-i\epsilon}\left[\sum_s\frac{(\psi_0,V_p\psi_s^+)(\psi_s^+,V_l^*\psi_0)}{(\omega_l+\omega_s)(\omega_p+\omega_s)}\right.$$
$$\left.+\omega_l^{-1}\omega_p^{-1}\{(\psi_0,V_p\psi_0)(\psi_0,V_l^*\psi_0)-(\psi_0,V_l^*\psi_0)(\psi_0,V_p\psi_0)\}\right]+\sum_l[\omega_l^{-1}\omega_q^{-1}\{(\psi_0,V_l\psi_0)(\psi_0,V_q^*\psi_0)$$
$$-(\psi_0,V_q^*\psi_0)(\psi_0,V_l\psi_0)\}+\sum_r(\omega_l+\omega_r)^{-1}(\omega_q+\omega_r)^{-1}(\psi_0,V_l\psi_r^-)(\psi_r^-,V_q^*\psi_0)](\psi_0,J\psi_0)$$
$$\times[\omega_l^{-1}\omega_p^{-1}\{(\psi_0,V_p\psi_0)(\psi_0,V_l^*\psi_0)-(\psi_0,V_l^*\psi_0)(\psi_0,V_p\psi_0)\}$$
$$\left.+\sum_s(\omega_l+\omega_s)^{-1}(\omega_p+\omega_s)^{-1}(\psi_0,V_p\psi_s^+)(\psi_s^+,V_l^*\psi_0)]\right\}+\sum_{rs}\frac{(\psi_0,V_q^*\psi_r^-)\bar{M}_{rs}^{(N)}(\psi_s^+,V_p\psi_0)}{(\omega_r-\omega_q-i\epsilon)(\omega_s-\omega_p-i\epsilon)}. \quad (46)$$

The advantage of Eq. (46), compared with Eq. (38), lies mainly in the fact that the inhomogeneous term is a smoother function of ω_p and ω_q, not necessarily in the supposed smallness of $\bar{M}_{qp}{}^{(N)}$.

Only if both the initial and final mesons form the $(\frac{3}{2},\frac{3}{2})$ resonant state with the nucleon is $\bar{M}_{qp}{}^{(N)}$ likely to be significant. Supposing this to be the case, let us examine the first two terms in curly brackets in Eq. (46): we write these terms as

$$+(2K)^{-\frac{1}{2}}\sum_l\{T_0(q,l)F(\omega_q,\omega_p,\omega_l)(\psi_0,V_l{}^*\psi_p{}^+) + (\psi_q{}^-,V_l\psi_0)F(\omega_p,\omega_q,\omega_l)T_0(l,p)\},$$

where $T_0(q,l)$ is the second-order perturbation approximation to the $(\frac{3}{2},\frac{3}{2})$ part of the scattering matrix, and $F(\omega_q,\omega_p,\omega_l)$ we may suppose to be a slowly varying function of ω_q and ω_p, at least when ω_l is large. Making use of the fact that [see Eq. (7)],

$$\sum_r(\psi_0,V_q{}^*\psi_r{}^\pm)(\psi_r{}^\pm,V_p\psi_0)(\omega_r-\omega_q-i\epsilon)^{-1} \approx T_0(q,p)-(\psi_q{}^-,V_p\psi_0),$$

we see that an approximate solution of Eq. (46) is given by

$$\bar{M}_{qp}{}^{(N)}\approx+(2K)^{-\frac{1}{2}}\sum_l(\psi_q{}^-,V_l\psi_0) \times F(\omega_q,\omega_p,\omega_l)(\psi_0,V_l{}^*\psi_p{}^+). \quad (47)$$

We shall use the above approximation merely in an illustrative way, not to obtain numerical values. Since $(\psi_q{}^-,V_l\psi_0)\sim(4f^2q^3)^{-1}3\omega_q\sin\delta_{33}(q)T_0(q,l)\exp(i\delta_{33})$, the effect of the kernel in Eq. (46) is to amplify the inhomogeneous term by a factor of about 3. Therefore, it may not be possible to neglect $\bar{M}_{qp}{}^{(N)}$ in comparison with the rest of the rescattering correction; nevertheless, we learn that the dependence on the energies ω_q and ω_p of the entire rescattering correction to $M_{qp}{}^{(N)}$ can probably be well represented by the factors $(\psi_q{}^-,V_l\psi_0)$ and $(\psi_0,V_l{}^*\psi_p{}^+)$.

It is clear that the exact form of the part of the matrix element $M_{qp}{}^{(N)}$ which we split off explicitly in Eq. (39) is arbitrary, and is to be chosen to suit our convenience. For instance, if we modify $S_{qp}{}^{(N)}$ (as given in Eq. (40)) by introducing an arbitrary function $f(l^2)$, nothing is affected so long as we put this factor $f(l^2)$ in the proper places in Eq. (42) as well. Now the passage from Eq. (42) to Eq. (45), which depends on the identities (44) and (45), is not greatly affected by such a factor $f(l^2)$, provided the factor does not differ from unity for energies which lie in the resonance region, so that if, for example, the effect of $f(l^2)$ is to reduce the contribution of energies $\omega_l>\Lambda$ in the sum in Eq. (40), the principal result will be the insertion of the same cutoff function in the sums over l in the inhomogeneous term of Eq. (46), assuming that Λ is sufficiently larger than the resonance energy and the energies ω_p and ω_q. We see from Eq. (40) that $S_{qp}{}^{(N)}$ depends quadratically upon the cutoff, but we realize that we are not required to choose this cutoff to be that which was imposed at the beginning in order to give the theory a physical meaning. The cutoff can, instead, be chosen in such a way as to facilitate solution of Eq. (46), for example, by making the inhomogeneous term as small as possible. In other words, the structure of the scattering equations, in the 1-meson approximation, provides an automatic cutoff which appears to be more powerful than that which is associated with the size of the source, and which tends to reduce the magnitude of the rescattering effect.[13]

We can discuss the integral equation (37) for $M_{qp}{}^{(M)}$ in the same way that we have discussed Eq. (38) for $M_{qp}{}^{(M)}$, using the same "wave function" $\Phi_p(l)$ to separate out the part of $M_{qp}{}^{(M)}$ that is a singular function of ω_p and ω_q (although not all of the quasi-classical term $M_{qp}{}^{(2)}$ can be obtained from the wave function). All of our conclusions about the qualitative behavior of the rescattering correction to the radiation from the nucleon magnetic moment hold equally well for the entire rescattering correction. The quantity $\tilde{M}_{qp}$, which is the solution of our basic integral equation with $M_{qp}{}^{(4)}$ as the inhomogeneous term, does not require any special consideration. This quantity, which is important only in so far as the $(\frac{3}{2},\frac{3}{2})$ part of $M_{qp}{}^{(4)}$ is amplified by the kernel of the integral equation, and is even then a relatively small term, will have the same general dependence on the energies as the rest of the rescattering correction.

The rescattering correction to the matrix element can be split into magnetic dipole and electric quadrupole terms; the magnetic dipole term appears to be several times larger than the electric quadrupole term, just as is the case in the photoproduction of P-wave mesons; not only is the magnetic dipole part of the rescattering correction obtained from $M_{qp}{}^{(M)}$ larger than the electric quadrupole part of the same term, but it interferes constructively with the rescattering part of the nucleon current term $M_{qp}{}^{(N)}$, as can be understood from the alignment of the angular momenta of the positive meson and the proton in the $(\frac{3}{2},\frac{3}{2})$ resonant state. We may express the magnetic dipole rescattering correction in the form

$$M_{q\alpha,\,p\beta}{}^{(a)}=ie\epsilon_{3\mu\nu}Y(12\pi)^{-1}(8K\omega_p\omega_q)^{-\frac{1}{2}} \times q_ip_jT_{ia,\,\alpha\mu}(q)[\epsilon_aK_b-\epsilon_bK_a]T_{bj,\,\nu\beta}(p), \quad (48)$$

where $Y=Y(\omega_q,\omega_p)$ is, according to the foregoing discussion, a complex function which is more nearly constant than are the other energy-dependent factors in Eq. (48). This function, Y, can, at least in principle, be calculated as accurately as one might desire, by solving the integral Eq. (31) in a manner such as has been outlined above. In this way one could express Y in terms of quantities obtained from other experiments.

[13] For large ω_l, $F(\omega_q,\omega_p,\omega_l)\approx+(1/\omega_l{}^2)(\psi_0,J\psi_0)$; thus, the cutoff-dependent parts of Eqs. (40) and (47) actually tend to cancel.

We express the electric quadrupole rescattering correction in the form

$$M_{\mathbf{q}\alpha,\,\mathbf{p}\beta}{}^{(b)}=ie\epsilon_{3\mu\nu}Z(12\pi)^{-1}(8K\omega_p\omega_q)^{-\frac{1}{2}}\times q_ip_jT_{ia,\,\alpha\mu}(q)[\epsilon_aK_b+\epsilon_bK_a]T_{bj,\,\nu\beta}(p),\quad(49)$$

where $Z=Z(\omega_q,\omega_p)$. Again, the theory is capable of predicting Z. It is convenient to write the two unknown functions in the form

$$Y=ye^{i\eta},\quad Z=ze^{i\zeta}.\quad(50)$$

An examination of the integral equation indicates that the magnitude, y, is perhaps ~ 3 (in our units), while the phase η is probably positive but not large. It is harder to say anything about Z, except that it is smaller than Y in magnitude, and seems to depend more strongly on the energy.

It does not seem reasonable at present to attempt to calculate the rescattering effect precisely. It is perhaps possible to evaluate this quantity experimentally, using its angular and energy dependence to help separate it from parts of the cross section which can be related more directly to the model. At any event, it would be premature to calculate this quantity before it has been ascertained which energy region is of greatest experimental interest.

V. CONCLUSIONS

It has been shown that the static source theory predicts uniquely the matrix element for radiative meson-nucleon scattering. Only experimentally determined quantities enter into the expression for the matrix element; knowledge of the cutoff parameter, of the "bare" nucleon magnetic moment, and of the importance of nonlinear effects is unnecessary.

For a preliminary investigation, it will be convenient to neglect all phase shifts but that in the resonant $(\frac{3}{2},\frac{3}{2})$ state. Then we may write

$$T_{ij,\,\alpha\beta}(p)=-12\pi(\delta_{ij}-\tfrac{1}{3}\sigma_i\sigma_j)(\delta_{\alpha\beta}-\tfrac{1}{3}\tau_\alpha\tau_\beta)h_pe^{i\delta_p},\quad h_p=p^{-3}\sin\delta_p,\quad\delta_p=\delta_{33}(p).\quad(51)$$

Using the above expression, we calculate for the matrix element for radiative π^+-p scattering (remembering that $-i\epsilon_{3\alpha\beta}\to+1$ and $\delta_{\alpha\beta}-\frac{1}{3}\tau_\alpha\tau_\beta\to+1$ in this case):

$$M_{qp}=M_1+M_2+M_3+M_4,$$

$$M_1=12\pi e(8K\omega_p\omega_q)^{-\frac{1}{2}}\{(\mathbf{p}\cdot\boldsymbol{\varepsilon}-\tfrac{1}{3}\boldsymbol{\varepsilon}\cdot\boldsymbol{\sigma}\boldsymbol{\sigma}\cdot\mathbf{p})h_pe^{i\delta_p}+(\mathbf{q}\cdot\boldsymbol{\varepsilon}-\tfrac{1}{3}\mathbf{q}\cdot\boldsymbol{\sigma}\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon})h_qe^{i\delta_q}\},$$

$$M_2=12\pi e(8K\omega_p\omega_q)^{-\frac{1}{2}}\{(K\omega_q-\mathbf{K}\cdot\mathbf{q})^{-1}\mathbf{q}\cdot\boldsymbol{\varepsilon}(q_i+K_i)\times(p_i-\tfrac{1}{3}\sigma_i\boldsymbol{\sigma}\cdot\mathbf{p})h_pe^{i\delta_p}-\mathbf{p}\cdot\boldsymbol{\varepsilon}(K\omega_p-\mathbf{K}\cdot\mathbf{p})^{-1}\times(q_i-\tfrac{1}{3}\boldsymbol{\sigma}\cdot\mathbf{q}\sigma_i)(p_i-K_i)h_qe^{i\delta_q}\},$$

$$M_3=12\pi e\mu_p(8K\omega_p\omega_q)^{-\frac{1}{2}}(2MK)^{-1}\times\{i\boldsymbol{\sigma}\cdot(\mathbf{K}\times\boldsymbol{\varepsilon})(\mathbf{q}\cdot\mathbf{p}-\tfrac{1}{3}\boldsymbol{\sigma}\cdot\mathbf{q}\boldsymbol{\sigma}\cdot\mathbf{p})h_pe^{i\delta_p}-(\mathbf{q}\cdot\mathbf{p}-\tfrac{1}{3}\boldsymbol{\sigma}\cdot\mathbf{q}\boldsymbol{\sigma}\cdot\mathbf{p})h_qe^{i\delta_q}i\boldsymbol{\sigma}\cdot(\mathbf{K}\times\boldsymbol{\varepsilon})\},\quad(52)$$

$$M_4=-12\pi e(8K\omega_p\omega_q)^{-\frac{1}{2}}\{h_ph_qye^{i\delta_p+i\delta_q+i\eta}\times(q_i-\tfrac{1}{3}\boldsymbol{\sigma}\cdot\mathbf{q}\sigma_i)(\epsilon_iK_j-\epsilon_jK_i)(p_j-\tfrac{1}{3}\sigma_j\boldsymbol{\sigma}\cdot\mathbf{p})\}+h_ph_qe^{i\delta_p+i\delta_q+i\zeta}z(q_i-\tfrac{1}{3}\boldsymbol{\sigma}\cdot\mathbf{q}\sigma_i)\times(\epsilon_iK_j+\epsilon_jK_i)(p_j-\tfrac{1}{3}\sigma_j\boldsymbol{\sigma}\cdot\mathbf{p})\}.$$

In the term M_3, μ_p represents the magnetic moment of the proton, expressed in nuclear magnetons, and M denotes the mass of the proton.

It must be emphasized that the matrix element we have obtained is based on a model that is at best incomplete. Recoil corrections and S-wave interactions are minor effects, but should, nevertheless, be considered as well as other even more uncertain effects. However, the above calculation should be useful as a guide to experimental studies, and will enable one to relate discrepancies between predicted and observed spectra to deficiencies in the model. Certain refinements, such as recoil corrections and S-wave corrections, can be incorporated in an obvious way for $K\to 0$ into the "quasi-classical" term, but for $K\neq 0$, uncertainties arise; in particular, the recoil corrections to the interaction current term, $M_{qp}{}^{(1)}$, cannot be obtained in such a natural fashion. The interaction current term also depends somewhat on the S-wave interactions, and is further modified if the γ-ray wavelength is comparable to the effective radius of the source. Therefore, the greatest interest lies in experimental study of the hardest γ rays which are emitted in scattering, as they can determine the importance of corrections to the static model.

The calculation of bremsstrahlung cross sections, based on the present work, but incorporating certain corrections in a semiphenomenological way, will be presented in a subsequent paper.

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