Center-of-Mass Motion in Brueckner Theory for a Finite Nucleus

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Center-of-mass motion, generally not considered in the Brueckner theory, can cause divergences in the case of a finite nucleus. A possible way of avoiding this difficulty is presented.

IN Brueckner's treatment of the many-body problem,[1] "model wave functions" and a "model Hamiltonian" are defined. These are used to calculate the energies and other properties of a real nucleus. The Hamiltonian for a system of nucleons with two-body forces is

$$H=\sum_i T_i+\sum_{i>j} v_{ij}, \tag{1}$$

where T_i is the kinetic energy of the ith particle and v_{ij} is the two-body interaction. The model Hamiltonian is

$$H_M=\sum_i T_i+\sum_i V_i, \tag{2}$$

where V_i is a single-particle potential defined in terms of the two-body potential v_{ij} by use of the t-matrix and a self-consistency requirement. The exact details of the definition of V_i are irrelevant to this discussion. The essential feature of the Brueckner method which is considered here is the use of the model Hamiltonian (2) as the unperturbed Hamiltonian in a perturbation treatment to solve the problem defined by the Hamiltonian (1).

The real Hamiltonian (1) is translation-invariant, and its energy spectrum is continuous because states of center-of-mass motion with arbitrary momentum are included. There is no reference to a fixed origin in space and the nuclear eigenfunctions have a uniform probability density throughout all space. The model Hamiltonian (2) is not translation-invariant in the case of a finite nucleus. The energy spectrum is discrete and the model wave functions are concentrated within a finite volume of space. As a result, center-of-mass motion is not treated properly in the nuclear shell model, as has been noted.[2-5] Furthermore, any perturbation treatment of the real problem starting from the model Hamiltonian (2) does not converge.[5] Thus, one should expect to encounter convergence difficulties in treating a finite nucleus by the Brueckner method. (This problem does not exist, of course, in an infinite nucleus.)

These difficulties can be avoided by the use of an alternative Hamiltonian to the real Hamiltonian (1):

$$H'=\sum_i T_i+\sum_{i>j} v_{ij}+M\omega^2(\sum_i x_i)^2/2A, \tag{3}$$

where M is the nucleon mass, ω is a parameter which can be set at a convenient value, x_i is the coordinate of the ith particle, and A is the number of particles. The Hamiltonian (3) differs from the original Hamiltonian (1) only by the addition of a function of the center-of-mass coordinate ($\sum_i x_i$). The internal motion of (3) is therefore identical to that of (1) and all quantities of physical interest in the original problem can be calculated directly from the Hamiltonian (3). The only difference is that the center of mass is bound by a harmonic oscillator potential instead of being free. Thus, instead of the continuous spectrum of states describing free translation of the nucleus there will be a discrete spectrum of states describing oscillation of the whole nucleus in the fictitious potential binding the center of mass to the origin of the coordinate system. Since this center-of-mass oscillation is completely decoupled from the other degrees of freedom of the system, these "spurious states"[2] should cause no difficulty. They need only be recognized and rejected.

Since the Hamiltonian (3) describes a system having a discrete spectrum and having wave functions concentrated in a finite volume of space, it may offer a more suitable point of departure for the Brueckner treatment, avoiding convergence difficulties while still giving all desired results.

The added term in (3) can be conveniently split into two parts

$$H'=\sum_i T_i+\sum_{i>j} v_{ij}+\sum_i \tfrac{1}{2}M\omega^2x_i^2-\sum_{i>j} M\omega^2(x_i-x_j)^2/A$$
$$=\sum_i T_i+\sum_i \tfrac{1}{2}M\omega^2x_i^2+\sum_{i>j} u_{ij}, \tag{4}$$

where

$$u_{ij}=v_{ij}-M\omega^2(x_i-x_j)^2/A \tag{5}$$

can be considered as a modified two-body interaction. The Hamiltonian (4) with a harmonic oscillator shell-model potential and modified two-body interaction is still *exactly* equivalent to the original problem except for center-of-mass effects.

The modified Hamiltonian (4) can be treated by the standard Brueckner methods by defining a modified

[1] R. J. Eden, *Proceedings of the Rehovoth Conference on Nuclear Structure* (North-Holland Publishing Company, Amsterdam, 1958), p. 3.
[2] J. P. Elliott and T. H. R. Skyrme, Proc. Roy. Soc. (London) **A232**, 561 (1955).
[3] Lipkin, de-Shalit, and Talmi, Nuovo cimento **2**, 773 (1955).
[4] F. Villars (unpublished).
[5] R. E. Peierls, *Proceedings of the Rehovoth Conference on Nuclear Structure* (North-Holland Publishing Company, Amsterdam, 1958), p. 135. See also R. E. Peierls and J. Yoccoz, Proc. Phys. Soc. (London) **A70**, 381 (1957).

model Hamiltonian:

$$H_M'=\sum_i T_i+\sum_i \tfrac{1}{2}M\omega^2 x_i^2+\sum_i U_i, \quad (6)$$

where the modified single-particle potential U_i is related to the modified two-body interaction u_{ij} in the same manner as V_i is related to v_{ij} in the usual treatment. The correspondence between Eqs. (1) and (2) on the one hand and Eqs. (4) and (6) on the other can be shown formally by writing

$$T_i'=T_i+\tfrac{1}{2}M\omega^2 x_i^2. \quad (7)$$

Equations (4) and (6) are then obtained from Eqs. (1) and (2) simply by replacing T_i by T_i', v_{ij} by u_{ij}, and V_i by U_i. Since the usual manipulations of Brueckner theory in a finite nucleus do not require explicitly that T_i be the kinetic energy operator, the same manipulations can be done with T_i'. Therefore all the formal treatment of Brueckner theory can be applied to Eqs. (4) and (6).

In conclusion, we can say that Eqs. (4) and (6) present a modified point of departure for Brueckner theory, with the following differences from the usual treatment:

1. There should be fewer convergence difficulties due to center-of-mass motion.
2. A harmonic oscillator shell-model potential appears in the Hamiltonian *from the beginning* before any approximations are made. It is only the deviation of the shell-model potential from the harmonic oscillator which appears in the perturbation treatment. This seems to be reasonable, since the harmonic oscillator potential has been used widely with good results in practical shell-model calculations.
3. The modified interaction u_{ij} is not a short-range interaction because of the term $(x_i-x_j)^2$. This may cause difficulty if short-range approximations are desirable in calculations.

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Angular Distribution of Photoprotons from Deuterium from 9 to 23 Mev*

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Photoprotons from a deuterated paraffin target irradiated with betatron x-rays have been detected with a NaI(Tl) scintillator. The angle and energy of the protons have been measured, and the data has been fitted to the angular distribution form $f(\theta)=(A+B\sin^2\theta)(1+2\beta\cos\theta)$. The ratio A/B rises from a value of 0.015 ± 0.041 for the 9- to 12-Mev photon group to a value of 0.133 ± 0.020 for the 20- to 23-Mev group. A/B increases in a complicated way suggesting several contributions to the isotropic component. The value determined for β agrees with the calculation of v_p/c. A Schiff thin-target spectrum is assumed for the incident photons, and the cross section obtained is consistent with the Marshall and Guth calculations, although the energy dependence of the data has slightly less slope than the calculated values.

I. INTRODUCTION

THE theoretical calculations[1] of the photodisintegration of the deuteron below photon energies of, say, 25 Mev predict a predominantly $\sin^2\theta$ angular distribution. The electric dipole, ED, transition from the ${}^3S_1$ part of the ground state to the ${}^3P_J$ states accounts for most of the disintegration process. The electric quadrupole absorption causes a fore-aft asymmetry modifying the distribution to $\sin^2\theta(1+2\beta\cos\theta)$. In addition a small isotropic component is predicted, the explanation of which has become one of the most interesting aspects of the deuteron photodisintegration problem at intermediate energies. The usual forms assumed for the angular distribution are

$$f(\theta)=A+B\sin^2\theta(1+2\beta\cos\theta), \quad (1)$$

and

$$f(\theta)=(A+B\sin^2\theta)(1+2\beta_1\cos\theta). \quad (2)$$

The recent experimental determinations of the angular distribution, that of Halpern and Weinstock[2] and that of Allen,[3] have shown the isotropic component in the region of 20 Mev to be considerably larger than that predicted by most of the theoretical work. Experimentally the ratio A/B is found to be about one-tenth at this energy. A brief review of the attempts to explain the observed isotropic component follows.

1. A small contribution comes from the magnetic dipole, MD, transition ${}^3S_1\rightarrow{}^1S_0$. Using the usual potentials, Yukawa or Hulthén, this is estimated to contribute 0.01 or 0.02 to the ratio A/B at these energies.[4,5]
2. The ${}^3S\rightarrow{}^3P_J$ transition in the presence of a tensor

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[1] For a comprehensive bibliography of the work see M. Elaine Toms, "Bibliography of Photonuclear Reactions," Naval Research Laboratory (1955, 1956, 1957).
[2] J. Halpern and E. Weinstock, Phys. Rev. **91**, 934 (1953).
[3] L. A. Allen, Jr., Phys. Rev. **98**, 705 (1955).
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