First-Forbidden Transitions in Parity-Nonconserving Beta Decay*†

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Our previous work on beta-gamma directional correlations is extended to phenomena of nonconservation of parity in beta decay. The parameters, $b_{LL'}^{(n)}$, which express the beta-ray angular distributions, are given for first-forbidden transitions of a general interaction, *STPVA*, where we assume no interferences between *STP* and *VA*. All of the possible interferences among the six nuclear matrix elements, $\mathfrak{M}(\beta\boldsymbol{\sigma}\cdot\mathbf{r})$, $\mathfrak{M}(\beta\gamma_5)$, $\mathfrak{M}(\beta\mathbf{r})$, $\mathfrak{M}(\beta\boldsymbol{\alpha})$, $\mathfrak{M}(\beta\boldsymbol{\sigma}\times\mathbf{r})$, and $\mathfrak{M}(B_{ij}{}^\beta)$ for *STP* (and the corresponding matrix elements for *VA*) are taken into account. By using these $b_{LL'}^{(n)}$'s, it is easy to express the correction factor of beta spectra, the beta-ray angular distributions from oriented nuclei, and the angular correlations between beta rays and circularly polarized gamma rays from unoriented nuclei in double and triple cascade transitions. The experimental data on the beta decays of Sb^{124} and Au^{198} are analyzed.

1. INTRODUCTION

As a result of the discovery of nonconservation of parity in weak interactions,[1–3] much experimental data[4] on beta-ray angular distributions, polarizations of emitted beta particles, and angular correlations between beta rays and circularly polarized gamma rays are being accumulated. In these experiments, most of the measured beta decays are for allowed transitions, for which the theoretical formulas have been derived by many authors.[5] A few of them are for first-forbidden transitions, for which there are no adequate formulas, except for the longitudinal polarizations of the beta particles.[6,7]

The aim of this paper is to generalize our previous work on beta-gamma directional correlations[8–12] to the phenomena of first-forbidden transitions in beta decay with nonconservation of parity. In Sec. 2, we shall give formulas for (a) the correction factor for the beta spectrum, (b) the beta-ray angular distributions from oriented nuclei, and (c) the angular correlations between beta rays and circularly polarized gamma rays from unoriented nuclei in double and triple cascade transitions, in the general cases of first-forbidden beta decay. We take a general beta interaction, *STPVA*,[13] with the assumption of no interference between *STP* and *VA*. All the possible interferences among the six matrix elements, $\mathfrak{M}(\beta\boldsymbol{\sigma}\cdot\mathbf{r})$, $\mathfrak{M}(\beta\gamma_5)$, $\mathfrak{M}(\beta\mathbf{r})$, $\mathfrak{M}(\beta\boldsymbol{\alpha})$, $\mathfrak{M}(\beta\boldsymbol{\sigma}\times\mathbf{r})$, and $\mathfrak{M}(B_{ij}{}^\beta)$ for *STP* (and the corresponding matrix elements for *VA*) are considered. These interferences, especially between nuclear matrix elements of different rank, have a very important role in our problems and they should not be dropped without justification. In Sec. 3, angular correlation functions between beta rays and circularly polarized gamma rays, for special decay schemes which are interesting in experiments, are given explicitly. In Sec. 4, some remarks concerning applications are discussed. Two different approximations for the $b_{LL'}^{(n)}$'s are obtained in Appendices 1 and 2. The experimental data on the first-forbidden beta decays of Sb^{124} and Au^{198} are discussed in Appendix 3.

2. FORMULAS

Since detailed treatments of beta-ray angular distributions from oriented nuclei,[7,14] and of the angular correlations between beta rays and circularly polarized gamma rays from unoriented nuclei in double[7,14,15] and in triple

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2 Wu, Ambler, Hayward, Hoppes, and Hudson, Phys. Rev. **105**, 1413 (1957).
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4 For complete references, see C. S. Wu, Proceedings of the Israel Conference, September, 1957 (unpublished).
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7 Alder, Stech, and Winther, Phys. Rev. **107**, 728 (1957). Although they have calculated the formulas for beta-ray angular distributions from oriented nuclei, and for angular correlations between the beta rays and the circularly polarized gamma rays for first-forbidden transitions, their formulas are still restricted to cases of transitions with spin change or cases where a nuclear matrix element, $B_{ij}{}^\beta$ is very small. Besides this, they have assumed the beta interaction to be a combination of scalar, tensor, and pseudoscalar.
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11 M. Morita, Progr. Theoret. Phys. (Japan) **14**, 27 (1955).
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13 Abbreviation for a combination of scalar, tensor, pseudoscalar, vector, and axial vector.
14 M. Morita, Phys. Rev. **107**, 1729 (1957). For aligned nuclei, a formula for the beta-ray angular distribution was given in Eq. (6) of Morita, Ogata, and Sakai, Bull. Kobayasi Inst. Phys. Research **6**, 69 (1956).
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cascade transitions,[14] have been already given, we write here the final results only. The calculations of this section follow in a straightforward way from our previous work.[8–12] We use the following notation. The decay scheme is $j(\beta)j_1$, $j(\beta)j_1(\gamma_1)j_2$, or $j(\beta)j_1(\gamma_1)j_2(\gamma_2)j_3$. The quantity m is the magnetic quantum number of j. $W(abcd;ef)$ and $(j_1j_2m_1m_2|jm)$ are the Racah and the Clebsch-Gordan coefficients, respectively. $P_n(\cos\theta)$ is the Legendre polynomial with integral number, n. The $b_{LL'}^{(n)}$ are parameters which express the beta-ray angular distributions and are given in this section. The definition of the $b_{LL'}^{(n)}$ is given in Eqs. (3) and (4) in reference 11, where $2n$ is replaced by n. L means the rank of the nuclear matrix element for beta decay, or the multipolarity, 2^L, of the gamma rays. The dependence on the circular polarization of the gamma ray is $p^{\delta+\delta'+L+L'+n}$. Here p is $+1$ (-1) for left (right) circularly polarized gamma rays. δ is equal to 0 $(+1)$ for magnetic (electric) radiation.

(1) Correction factor for the beta spectrum.

$$\text{Correction factor for the beta spectrum}=b_{00}^{(0)}-(1/\sqrt{3})b_{11}^{(0)}+(1/\sqrt{5})b_{22}^{(0)}. \tag{1}$$

(2) Beta-ray angular distribution from oriented nuclei.

$$W(\theta:\beta)=\sum\nolimits_{n,\,L\leq L'}\bar{f}_n(j)(-)^{j_1-j+L+L'+n}W(jjLL';nj_1)b_{LL'}^{(n)}P_n(\cos\theta), \tag{2}$$

with

$$\bar{f}_n(j)=\sum\nolimits_m(-)^{j-m}(jjm-m|n0)a_m,$$

where the a_m are the relative populations of the initial magnetic substates.

(3) Angular correlation between beta rays and circularly polarized gamma rays.

$$W(\theta,p_1:\beta-\gamma_1)=\sum_n\Big\{\Big[\sum_{L\leq L'}(-)^{j_1-j}b_{LL'}^{(n)}W(j_1j_1LL';nj)(2j_1+1)^{\frac{1}{2}}\Big]\times\Big[\sum_{L_1L_1'}(-)^{L_1+L_1'}p_1^{\delta_1+\delta_1'+L_1+L_1'+n}(j_1\|L_1\|j_2)(j_1\|L_1'\|j_2)F_n(L_1L_1'j_2j_1)\Big]\Big\}P_n(\cos\theta), \tag{3}$$

with

$$F_n(LL'j_aj_b)=F_n(L'Lj_aj_b)=(-)^{j_b-j_a-1}\{(2j_b+1)(2L+1)(2L'+1)\}^{\frac{1}{2}}(LL'1-1|n0)W(j_bj_bLL';nj_a).$$

(4) Directional correlation between beta rays and gamma rays.

$$W(\theta:\beta-\gamma_1)=\text{terms involving } P_n(\cos\theta) \text{ with even } n \text{ in } W(\theta,p_1;\beta-\gamma_1). \tag{4}$$

(5) Angular correlation between beta rays and circularly polarized γ_2 rays without observing γ_1 rays in a triple cascade transition.

$$W(\theta,p_2:\beta-\gamma_2)=\sum_n\Big\{\Big[\sum_{L\leq L'}(-)^{j_1-j}b_{LL'}^{(n)}W(j_1j_1LL';nj)(2j_1+1)^{\frac{1}{2}}\Big]\Big[\sum_{L_1}(j_1\|L_1\|j_2)^2W(j_1nL_1j_2;j_1j_2)\times(2j_1+1)^{\frac{1}{2}}(2j_2+1)^{\frac{1}{2}}\Big]\Big[\sum_{L_2L_2'}(-)^{L_2+L_2'}p_2^{\delta_2+\delta_2'+L_2+L_2'+n}(j_2\|L_2\|j_3)(j_2\|L_2'\|j_3)F_n(L_2L_2'j_3j_2)\Big]\Big\}P_n(\cos\theta). \tag{5}$$

(6) Directional correlation between beta rays and γ_2 rays.

$$W(\theta:\beta-\gamma_2)=\text{terms involving } P_n(\cos\theta) \text{ with even } n \text{ in } W(\theta,p_2:\beta-\gamma_2). \tag{6}$$

We use Eq. (1) of reference 1 as the beta interaction. The parameters, $b_{LL'}^{(n)}$, have been given for allowed transitions.[15] For first-forbidden transitions, they are as follows:

STP

$$b_{00}^{(0)}=|\mathfrak{M}(\beta\boldsymbol{\sigma}\cdot\mathbf{r})|^2(|C_T|^2+|C_T'|^2)[(1/9)K^2L_0+\tfrac{2}{3}KN_0+M_0]+|\mathfrak{M}(\beta\gamma_5)|^2(|C_P|^2+|C_P'|^2)L_0-\{i\mathfrak{M}^*(\beta\boldsymbol{\sigma}\cdot\mathbf{r})\mathfrak{M}(\beta\gamma_5)\}2\,\mathrm{Re}(C_T^*C_P+C_T'^*C_P')(\tfrac{1}{3}KL_0+N_0). \tag{7}$$

$$\begin{aligned}b_{01}^{(1)}=&\{\mathfrak{M}^*(\beta\boldsymbol{\sigma}\cdot\mathbf{r})\mathfrak{M}(\beta\mathbf{r})\}[2\,\mathrm{Re}(C_T^*C_S'+C_T'^*C_S)2[(1/9)K^2\Lambda_1-\tfrac{1}{3}K\mathbf{L}_{12}+\tfrac{1}{3}K\mathbf{N}_{11}-\mathbf{N}_{12}-m_1)\\&+2\,\mathrm{Im}(C_T^*C_S'+C_T'^*C_S)2(\tfrac{1}{3}K\mathbf{H}_{12}+\mathbf{J}_{12})]-\{i\mathfrak{M}^*(\beta\boldsymbol{\sigma}\cdot\mathbf{r})\mathfrak{M}(\beta\boldsymbol{\alpha})\}4\,\mathrm{Re}(C_T^*C_T')(\tfrac{2}{3}K\Lambda_1+\mathbf{N}_{11})\\&+\{i\mathfrak{M}^*(\beta\boldsymbol{\sigma}\cdot\mathbf{r})\mathfrak{M}(\beta\boldsymbol{\sigma}\times\mathbf{r})\}4\,\mathrm{Re}(C_T^*C_T')[(2/9)K^2\Lambda_1-\tfrac{1}{3}K\mathbf{L}_{12}-\mathbf{N}_{12}+2m_1]+\{i\mathfrak{M}^*(\beta\gamma_5)\mathfrak{M}(\beta\mathbf{r})\}\\&\times[2\,\mathrm{Re}(C_P^*C_S'+C_P'^*C_S)(\tfrac{2}{3}K\Lambda_1+\mathbf{N}_{11}-2\mathbf{L}_{12})+2\,\mathrm{Im}(C_P^*C_S'+C_P'^*C_S)(\mathbf{J}_{11}+2\mathbf{H}_{12})]\\&+\{\mathfrak{M}^*(\beta\gamma_5)\mathfrak{M}(\beta\boldsymbol{\alpha})\}2\,\mathrm{Re}(C_P^*C_T'+C_P'^*C_T)2\Lambda_1+\{\mathfrak{M}^*(\beta\gamma_5)\mathfrak{M}(\beta\boldsymbol{\sigma}\times\mathbf{r})\}\\&\times[2\,\mathrm{Re}(C_P^*C_T'+C_P'^*C_T)(-\tfrac{2}{3}K\Lambda_1+\mathbf{L}_{12}+\mathbf{N}_{11})+2\,\mathrm{Im}(C_P^*C_T'+C_P'^*C_T)(\mathbf{J}_{11}-\mathbf{H}_{12})].\end{aligned} \tag{8}$$

$$\begin{aligned}b_{11}^{(0)}=&-\sqrt{3}\{|\mathfrak{M}(\beta\mathbf{r})|^2(|C_S|^2+|C_S'|^2)(\tfrac{1}{3}K^2L_0+\tfrac{2}{3}KN_0+2L_1+M_0)+|\mathfrak{M}(\beta\boldsymbol{\alpha})|^2(|C_T|^2+|C_T'|^2)L_0\\&+|\mathfrak{M}(\beta\boldsymbol{\sigma}\times\mathbf{r})|^2(|C_T|^2+|C_T'|^2)(\tfrac{1}{6}K^2L_0-\tfrac{2}{3}KN_0+\tfrac{1}{2}L_1+M_0)+\{\mathfrak{M}^*(\beta\boldsymbol{\alpha})\mathfrak{M}(\beta\boldsymbol{\sigma}\times\mathbf{r})\}\\&\times2(|C_T|^2+|C_T'|^2)(-\tfrac{1}{3}KL_0+N_0)-\{i\mathfrak{M}^*(\beta\mathbf{r})\mathfrak{M}(\beta\boldsymbol{\alpha})\}2\,\mathrm{Re}(C_S^*C_T+C_S'^*C_T')(\tfrac{1}{3}KL_0+N_0)\\&+\{i\mathfrak{M}^*(\beta\mathbf{r})\mathfrak{M}(\beta\boldsymbol{\sigma}\times\mathbf{r})\}2\,\mathrm{Re}(C_S^*C_T+C_S'^*C_T')(L_1-M_0)\}.\end{aligned}\quad(9)$$

$$\begin{aligned}b_{11}^{(1)}=&\sqrt{2}\{-|\mathfrak{M}(\beta\mathbf{r})|^2 2\,\mathrm{Re}(C_S^*C_S')2(\tfrac{1}{3}K\mathbf{L}_{12}+\tfrac{1}{3}K\mathbf{N}_{11}+\mathbf{N}_{12}+\Lambda_2-m_1)-|\mathfrak{M}(\beta\boldsymbol{\alpha})|^2 2\,\mathrm{Re}(C_T^*C_T')2\Lambda_1\\&+|\mathfrak{M}(\beta\boldsymbol{\sigma}\times\mathbf{r})|^2 2\,\mathrm{Re}(C_T^*C_T')(-\tfrac{1}{6}K^2\Lambda_1-\tfrac{1}{3}K\mathbf{L}_{12}+\tfrac{2}{3}K\mathbf{N}_{11}+\mathbf{N}_{12}-\tfrac{1}{2}\Lambda_2+2m_1)+\{\mathfrak{M}^*(\beta\boldsymbol{\alpha})\mathfrak{M}(\beta\boldsymbol{\sigma}\times\mathbf{r})\}\\&\times4\,\mathrm{Re}(C_T^*C_T')(\tfrac{2}{3}K\Lambda_1-\mathbf{N}_{11}+\tfrac{1}{2}\mathbf{L}_{12})+\{i\mathfrak{M}^*(\beta\mathbf{r})\mathfrak{M}(\beta\boldsymbol{\alpha})\}\{2\,\mathrm{Re}(C_S^*C_T'+C_S'^*C_T)(\tfrac{2}{3}K\Lambda_1+\mathbf{L}_{12}+\mathbf{N}_{11})\\&+2\,\mathrm{Im}(C_S^*C_T'+C_S'^*C_T)(-\mathbf{J}_{11}+\mathbf{H}_{12})\}+\{i\mathfrak{M}^*(\beta\mathbf{r})\mathfrak{M}(\beta\boldsymbol{\sigma}\times\mathbf{r})\}[2\,\mathrm{Re}(C_S^*C_T'+C_S'^*C_T)\\&\times(-\tfrac{1}{3}K^2\Lambda_1-\tfrac{1}{2}K\mathbf{L}_{12}+\tfrac{1}{2}\mathbf{N}_{12}-\Lambda_2-2m_1)+2\,\mathrm{Im}(C_S^*C_T'+C_S'^*C_T)(\tfrac{2}{3}K\mathbf{J}_{11}-\tfrac{1}{6}K\mathbf{H}_{12}+\tfrac{3}{2}\mathbf{J}_{12})]\}.\end{aligned}\quad(10)$$

$$\begin{aligned}b_{11}^{(2)}=&-6^{\frac{1}{2}}\{-|\mathfrak{M}(\beta\mathbf{r})|^2(|C_S|^2+|C_S'|^2)(\tfrac{2}{3}KL_{12}+L_1+2N_{12})+|\mathfrak{M}(\beta\boldsymbol{\sigma}\times\mathbf{r})|^2(|C_T|^2+|C_T'|^2)(-\tfrac{1}{3}KL_{12}-\tfrac{1}{4}L_1+N_{12})\\&+\{\mathfrak{M}^*(\beta\boldsymbol{\alpha})\mathfrak{M}(\beta\boldsymbol{\sigma}\times\mathbf{r})\}2(|C_T|^2+|C_T'|^2)\tfrac{1}{2}L_{12}+\{i\mathfrak{M}^*(\beta\mathbf{r})\mathfrak{M}(\beta\boldsymbol{\alpha})\}[2\,\mathrm{Re}(C_S^*C_T+C_S'^*C_T')L_{12}\\&-2\,\mathrm{Im}(C_S^*C_T+C_S'^*C_T')H_{12}]+\{i\mathfrak{M}^*(\beta\mathbf{r})\mathfrak{M}(\beta\boldsymbol{\sigma}\times\mathbf{r})\}[2\,\mathrm{Re}(C_S^*C_T+C_S'^*C_T')\tfrac{1}{2}(-KL_{12}-L_1+N_{12})\\&+2\,\mathrm{Im}(C_S^*C_T+C_S'^*C_T')(\tfrac{1}{6}KH_{12}-\tfrac{3}{2}J_{12})]\}.\end{aligned}\quad(11)$$

$$\begin{aligned}b_{02}^{(2)}=&6^{-\frac{1}{2}}\{\{i\mathfrak{M}^*(\beta\gamma_5)\mathfrak{M}(B_{ij}^{\beta})\}[2\,\mathrm{Re}(C_P^*C_T+C_P'^*C_T')3L_{12}+2\,\mathrm{Im}(C_P^*C_T+C_P'^*C_T')3H_{12}]\\&+\{\mathfrak{M}^*(\beta\boldsymbol{\sigma}\cdot\mathbf{r})\mathfrak{M}(B_{ij}^{\beta})\}2(|C_T|^2+|C_T'|^2)(KL_{12}+3N_{12})\}.\end{aligned}\quad(12)$$

$$\begin{aligned}b_{12}^{(1)}=&-\tfrac{1}{2}(5/3)^{\frac{1}{2}}\{\{\mathfrak{M}^*(\beta\mathbf{r})\mathfrak{M}(B_{ij}^{\beta})\}\{2\,\mathrm{Re}(C_S^*C_T'+C_S'^*C_T)[\tfrac{2}{3}K^2\Lambda_1-K\mathbf{L}_{12}-3\mathbf{N}_{12}+(6/5)\Lambda_2]\\&+2\,\mathrm{Im}(C_S^*C_T'+C_S'^*C_T)(K\mathbf{H}_{12}+3\mathbf{J}_{12})\}-\{i\mathfrak{M}^*(\beta\boldsymbol{\alpha})\mathfrak{M}(B_{ij}^{\beta})\}4\,\mathrm{Re}(C_T^*C_T')3\mathbf{L}_{12}\\&+\{i\mathfrak{M}^*(\beta\boldsymbol{\sigma}\times\mathbf{r})\mathfrak{M}(B_{ij}^{\beta})\}4\,\mathrm{Re}(C_T^*C_T')(\tfrac{1}{3}K^2\Lambda_1+K\mathbf{L}_{12}-3\mathbf{N}_{12}-\tfrac{3}{5}\Lambda_2)\}.\end{aligned}\quad(13)$$

$$\begin{aligned}b_{12}^{(2)}=&-\tfrac{1}{2}\{\{\mathfrak{M}^*(\beta\mathbf{r})\mathfrak{M}(B_{ij}^{\beta})\}[2\,\mathrm{Re}(C_S^*C_T+C_S'^*C_T')(KL_{12}-3L_1+3N_{12})\\&+2\,\mathrm{Im}(C_S^*C_T+C_S'^*C_T')(KH_{12}+3J_{12})]+\{i\mathfrak{M}^*(\beta\boldsymbol{\alpha})\mathfrak{M}(B_{ij}^{\beta})\}2(|C_T|^2+|C_T'|^2)3L_{12}\\&-\{i\mathfrak{M}^*(\beta\boldsymbol{\sigma}\times\mathbf{r})\mathfrak{M}(B_{ij}^{\beta})\}2(|C_T|^2+|C_T'|^2)(KL_{12}-\tfrac{3}{2}L_1-3N_{12})\}.\end{aligned}\quad(14)$$

$$b_{12}^{(3)}=(9/\sqrt{10})\{\{\mathfrak{M}^*(\beta\mathbf{r})\mathfrak{M}(B_{ij}^{\beta})\}2\,\mathrm{Re}(C_S^*C_T'+C_S'^*C_T)2\Lambda_2-\{i\mathfrak{M}^*(\beta\boldsymbol{\sigma}\times\mathbf{r})\mathfrak{M}(B_{ij}^{\beta})\}4\,\mathrm{Re}(C_T^*C_T')\Lambda_2\}.\quad(15)$$

$$b_{22}^{(0)}=|\mathfrak{M}(B_{ij}^{\beta})|^2(|C_T|^2+|C_T'|^2)[(1/12)K^2L_0+\tfrac{3}{4}L_1]\sqrt{5}.\quad(16)$$

$$b_{22}^{(1)}=|\mathfrak{M}(B_{ij}^{\beta})|^2 2\,\mathrm{Re}(C_T^*C_T')[(1/12)K^2\Lambda_1+(9/20)\Lambda_2]\sqrt{10}.\quad(17)$$

$$b_{22}^{(2)}=-|\mathfrak{M}(B_{ij}^{\beta})|^2(|C_T|^2+|C_T'|^2)\tfrac{3}{4}(\tfrac{7}{2})^{\frac{1}{2}}L_1.\quad(18)$$

$$b_{22}^{(3)}=-|\mathfrak{M}(B_{ij}^{\beta})|^2 2\,\mathrm{Re}(C_T^*C_T')(9/\sqrt{10})\Lambda_2.\quad(19)$$

$$b_{22}^{(4)}=0.\quad(20)$$

The numerical factor, 2 or 4, for $\mathrm{Re}(C_i^*C_j^{(\prime)})$ or $\mathrm{Re}(C_i^*C_i')$ is left in the above equations, for convenience in further calculation.

VA

The $b_{LL'}^{(n)}$'s for *VA* are easily deduced from those for *STP*. In the expressions for the $b_{LL'}^{(n)}$'s, the K^m term for $\mathfrak{M}^*(\beta\mathbf{X}_i)\mathfrak{M}(\beta\mathbf{X}_j)$ should be replaced by $(-)^{m+n}K^m$ for $\mathfrak{M}^*(\mathbf{X}_i)\mathfrak{M}(\mathbf{X}_j)$. Besides this, a few terms consisting of the imaginary part of the product of different coupling constants appear in *VA*, while the corresponding terms in *STP* vanish, and vice versa. The results are:

$$\begin{aligned}b_{00}^{(0)}=&|\mathfrak{M}(\boldsymbol{\sigma}\cdot\mathbf{r})|^2(|C_A|^2+|C_A'|^2)[(1/9)K^2L_0-\tfrac{2}{3}KN_0+M_0]+|\mathfrak{M}(\gamma_5)|^2(|C_A|^2+|C_A'|^2)L_0\\&+\{i\mathfrak{M}^*(\boldsymbol{\sigma}\cdot\mathbf{r})\mathfrak{M}(\gamma_5)\}2(|C_A|^2+|C_A'|^2)(\tfrac{1}{3}KL_0-N_0).\end{aligned}\quad(21)$$

$$\begin{aligned}b_{01}^{(1)}=&\{\mathfrak{M}^*(\boldsymbol{\sigma}\cdot\mathbf{r})\mathfrak{M}(\mathbf{r})\}\{2\,\mathrm{Re}(C_A^*C_V'+C_A'^*C_V)2[-(1/9)K^2\Lambda_1-\tfrac{1}{3}K\mathbf{L}_{12}+\tfrac{1}{3}K\mathbf{N}_{11}+\mathbf{N}_{12}+m_1]\\&+2\,\mathrm{Im}(C_A^*C_V'+C_A'^*C_V)2(\tfrac{1}{3}K\mathbf{H}_{12}-\mathbf{J}_{12})\}-\{i\mathfrak{M}^*(\boldsymbol{\sigma}\cdot\mathbf{r})\mathfrak{M}(\boldsymbol{\alpha})\}[2\,\mathrm{Re}(C_A^*C_V'+C_A'^*C_V)(\tfrac{2}{3}K\Lambda_1-\mathbf{N}_{11})\\&+2\,\mathrm{Im}(C_A^*C_V'+C_A'^*C_V)\mathbf{J}_{11}]+\{i\mathfrak{M}^*(\boldsymbol{\sigma}\cdot\mathbf{r})\mathfrak{M}(\boldsymbol{\sigma}\times\mathbf{r})\}4\,\mathrm{Re}(C_A^*C_A')[-(2/9)K^2\Lambda_1-\tfrac{1}{3}K\mathbf{L}_{12}+\mathbf{N}_{12}-2m_1]\\&-\{i\mathfrak{M}^*(\gamma_5)\mathfrak{M}(\mathbf{r})\}[2\,\mathrm{Re}(C_A^*C_V'+C_A'^*C_V)(-\tfrac{2}{3}K\Lambda_1+\mathbf{N}_{11}-2\mathbf{L}_{12})+2\,\mathrm{Im}(C_A^*C_V'+C_A'^*C_V)(\mathbf{J}_{11}+2\mathbf{H}_{12})]\\&-\{\mathfrak{M}^*(\gamma_5)\mathfrak{M}(\boldsymbol{\alpha})\}2\,\mathrm{Re}(C_A^*C_V'+C_A'^*C_V)2\Lambda_1-\{\mathfrak{M}^*(\gamma_5)\mathfrak{M}(\boldsymbol{\sigma}\times\mathbf{r})\}4\,\mathrm{Re}(C_A^*C_A')(\tfrac{2}{3}K\Lambda_1+\mathbf{L}_{12}+\mathbf{N}_{11}).\end{aligned}\quad(22)$$

$$b_{11}^{(0)}=-\sqrt{3}\{|\mathfrak{M}(\mathbf{r})|^2(|C_V|^2+|C_V'|^2)(\tfrac{1}{3}K^2L_0-\tfrac{2}{3}KN_0+2L_1+M_0)+|\mathfrak{M}(\boldsymbol{\alpha})|^2(|C_V|^2+|C_V'|^2)L_0$$
$$+|\mathfrak{M}(\boldsymbol{\sigma}\times\mathbf{r})|^2(|C_A|^2+|C_A'|^2)(\tfrac{1}{6}K^2L_0+\tfrac{2}{3}KN_0+\tfrac{1}{2}L_1+M_0)+\{\mathfrak{M}^*(\boldsymbol{\alpha})\mathfrak{M}(\boldsymbol{\sigma}\times\mathbf{r})\}$$
$$\times 2\,\mathrm{Re}(C_V^*C_A+C_V'^*C_A')(\tfrac{1}{3}KL_0+N_0)+\{i\mathfrak{M}^*(\mathbf{r})\mathfrak{M}(\boldsymbol{\alpha})\}2(|C_V|^2+|C_V'|^2)(\tfrac{1}{3}KL_0-N_0)$$
$$+\{i\mathfrak{M}^*(\mathbf{r})\mathfrak{M}(\boldsymbol{\sigma}\times\mathbf{r})\}2\,\mathrm{Re}(C_V^*C_A+C_V'^*C_A')(L_1-M_0)\}. \quad (23)$$

$$b_{11}^{(1)}=\sqrt{2}\{-|\mathfrak{M}(\mathbf{r})|^2 2\,\mathrm{Re}(C_V^*C_V')2(\tfrac{1}{3}K\mathbf{L}_{12}+\tfrac{1}{3}K\mathbf{N}_{11}-\mathbf{N}_{12}-\Lambda_2+m_1)+|\mathfrak{M}(\boldsymbol{\alpha})|^2 2\,\mathrm{Re}(C_V^*C_V')2\Lambda_1$$
$$+|\mathfrak{M}(\boldsymbol{\sigma}\times\mathbf{r})|^2 2\,\mathrm{Re}(C_A^*C_A')(\tfrac{1}{6}K^2\Lambda_1-\tfrac{1}{3}K\mathbf{L}_{12}+\tfrac{2}{3}K\mathbf{N}_{11}-\mathbf{N}_{12}+\tfrac{1}{2}\Lambda_2-2m_1)+\{\mathfrak{M}^*(\boldsymbol{\alpha})\mathfrak{M}(\boldsymbol{\sigma}\times\mathbf{r})\}$$
$$\times[2\,\mathrm{Re}(C_V^*C_A'+C_V'^*C_A)(\tfrac{2}{3}K\Lambda_1+\mathbf{N}_{11}-\tfrac{1}{2}\mathbf{L}_{12})+2\,\mathrm{Im}(C_V^*C_A'+C_V'^*C_A)(\mathbf{J}_{11}+\tfrac{1}{2}\mathbf{H}_{12})]$$
$$-\{i\mathfrak{M}^*(\mathbf{r})\mathfrak{M}(\boldsymbol{\alpha})\}4\,\mathrm{Re}(C_V^*C_V')(-\tfrac{2}{3}K\Lambda_1+\mathbf{L}_{12}+\mathbf{N}_{11})+\{i\mathfrak{M}^*(\mathbf{r})\mathfrak{M}(\boldsymbol{\sigma}\times\mathbf{r})\}[2\,\mathrm{Re}(C_V^*C_A'+C_V'^*C_A)$$
$$\times(\tfrac{1}{3}K^2\Lambda_1-\tfrac{1}{2}K\mathbf{L}_{12}-\tfrac{1}{2}\mathbf{N}_{12}+\Lambda_2+2m_1)+2\,\mathrm{Im}(C_V^*C_A'+C_V'^*C_A)(\tfrac{2}{3}K\mathbf{J}_{11}-\tfrac{1}{6}K\mathbf{H}_{12}-\tfrac{3}{2}\mathbf{J}_{12})]\}. \quad (24)$$

$$b_{11}^{(2)}=-6^{\frac{1}{2}}\{-|\mathfrak{M}(\mathbf{r})|^2(|C_V|^2+|C_V'|^2)(-\tfrac{2}{3}KL_{12}+L_1+2N_{12})+|\mathfrak{M}(\boldsymbol{\sigma}\times\mathbf{r})|^2(|C_A|^2+|C_A'|^2)(\tfrac{1}{3}KL_{12}-\tfrac{1}{4}L_1+N_{12})$$
$$+\{\mathfrak{M}^*(\boldsymbol{\alpha})\mathfrak{M}(\boldsymbol{\sigma}\times\mathbf{r})\}[2\,\mathrm{Re}(C_V^*C_A+C_V'^*C_A')\tfrac{1}{2}L_{12}+2\,\mathrm{Im}(C_V^*C_A+C_V'^*C_A')\tfrac{1}{2}H_{12}]$$
$$+\{i\mathfrak{M}^*(\mathbf{r})\mathfrak{M}(\boldsymbol{\alpha})\}2(|C_V|^2+|C_V'|^2)L_{12}+\{i\mathfrak{M}^*(\mathbf{r})\mathfrak{M}(\boldsymbol{\sigma}\times\mathbf{r})\}[2\,\mathrm{Re}(C_V^*C_A+C_V'^*C_A')\tfrac{1}{2}(KL_{12}-L_1+N_{12})$$
$$-2\,\mathrm{Im}(C_V^*C_A+C_V'^*C_A')(\tfrac{1}{6}KH_{12}+\tfrac{3}{2}J_{12})]\}. \quad (25)$$

$$b_{02}^{(2)}=(1/\sqrt{6})\{\{i\mathfrak{M}^*(\gamma_5)\mathfrak{M}(B_{ij})\}2(|C_A|^2+|C_A'|^2)3L_{12}$$
$$+\{\mathfrak{M}^*(\boldsymbol{\sigma}\cdot\mathbf{r})\mathfrak{M}(B_{ij})\}2(|C_A|^2+|C_A'|^2)(-KL_{12}+3N_{12})\}. \quad (26)$$

$$b_{12}^{(1)}=-\tfrac{1}{2}(5/3)^{\frac{1}{2}}\{\{\mathfrak{M}^*(\mathbf{r})\mathfrak{M}(B_{ij})\}[2\,\mathrm{Re}(C_V^*C_A'+C_V'^*C_A)(-\tfrac{2}{3}K^2\Lambda_1-K\mathbf{L}_{12}+3\mathbf{N}_{12}-(6/5)\Lambda_2)$$
$$+2\,\mathrm{Im}(C_V^*C_A'+C_V'^*C_A)(K\mathbf{H}_{12}-3\mathbf{J}_{12})]+\{i\mathfrak{M}^*(\boldsymbol{\alpha})\mathfrak{M}(B_{ij})\}[2\,\mathrm{Re}(C_V^*C_A'+C_V'^*C_A)3\mathbf{L}_{12}$$
$$-2\,\mathrm{Im}(C_V^*C_A'+C_V'^*C_A)3\mathbf{H}_{12}]+\{i\mathfrak{M}^*(\boldsymbol{\sigma}\times\mathbf{r})\mathfrak{M}(B_{ij})\}4\,\mathrm{Re}(C_A^*C_A')(-\tfrac{1}{3}K^2\Lambda_1+K\mathbf{L}_{12}+3\mathbf{N}_{12}+\tfrac{3}{5}\Lambda_2)\}. \quad (27)$$

$$b_{12}^{(2)}=-\tfrac{1}{2}\{\{\mathfrak{M}^*(\mathbf{r})\mathfrak{M}(B_{ij})\}[2\,\mathrm{Re}(C_V^*C_A+C_V'^*C_A')(-KL_{12}-3L_1+3N_{12})$$
$$+2\,\mathrm{Im}(C_V^*C_A+C_V'^*C_A')(-KH_{12}+3J_{12})]+\{i\mathfrak{M}^*(\boldsymbol{\alpha})\mathfrak{M}(B_{ij})\}[2\,\mathrm{Re}(C_V^*C_A+C_V'^*C_A')3L_{12}$$
$$+2\,\mathrm{Im}(C_V^*C_A+C_V'^*C_A')3H_{12}]+\{i\mathfrak{M}^*(\boldsymbol{\sigma}\times\mathbf{r})\mathfrak{M}(B_{ij})\}2(|C_A|^2+|C_A'|^2)(KL_{12}+\tfrac{3}{2}L_1+3N_{12})\}. \quad (28)$$

$$b_{12}^{(3)}=(9/\sqrt{10})\{-\{\mathfrak{M}^*(\mathbf{r})\mathfrak{M}(B_{ij})\}2\,\mathrm{Re}(C_V^*C_A'+C_V'^*C_A)2\Lambda_2+\{i\mathfrak{M}^*(\boldsymbol{\sigma}\times\mathbf{r})\mathfrak{M}(B_{ij})\}4\,\mathrm{Re}(C_A^*C_A')\Lambda_2\}. \quad (29)$$

$$b_{22}^{(0)}=|\mathfrak{M}(B_{ij})|^2(|C_A|^2+|C_A'|^2)[(1/12)K^2L_0+\tfrac{3}{4}L_1]\sqrt{5}. \quad (30)$$

$$b_{22}^{(1)}=-|\mathfrak{M}(B_{ij})|^2 2\,\mathrm{Re}(C_A^*C_A')[(1/12)K^2\Lambda_1+(9/20)\Lambda_2]\sqrt{10}. \quad (31)$$

$$b_{22}^{(2)}=-|\mathfrak{M}(B_{ij})|^2(|C_A|^2+|C_A'|^2)\tfrac{3}{4}(\tfrac{7}{2})^{\frac{1}{2}}L_1. \quad (32)$$

$$b_{22}^{(3)}=|\mathfrak{M}(B_{ij})|^2 2\,\mathrm{Re}(C_A^*C_A')(9/\sqrt{10})\Lambda_2. \quad (33)$$

$$b_{22}^{(4)}=0. \quad (34)$$

STPVA with Assumption of No Interference between *STP* and *VA*

Now, let us assume no interference between *STP* and *VA*. This is satisfied, for example, by the modified two-component neutrino theory, namely, $C_i=-C_i'$ with $i=S, T, P$, and $C_j=C_j'$ with $j=V, A$.[16] The $b_{LL'}^{(n)}$ for this *STPVA* interaction are:

$$b_{LL'}^{(n)}=(b_{LL'}^{(n)}\text{ for }STP)+(b_{LL'}^{(n)}\text{ for }VA). \quad (35)$$

In Eqs. (7)–(34), the $\mathfrak{M}(\mathbf{X}_i)$ are reduced nuclear matrix elements. Their definition is equal to that of Konopinski and Uhlenbeck[17] except for a constant common factor, e.g., $[(2j_1+1)/(2j+1)]\mathfrak{M}^*(\beta\boldsymbol{\alpha})\mathfrak{M}(\beta\mathbf{r})=(\int\beta\boldsymbol{\alpha})^*\cdot(\int\beta\mathbf{r})$. In the calculation of the $b_{LL'}^{(n)}$, we have assumed that the strong interactions are invariant under time reversal. Consequently, the products of two matrix elements in curly brackets, namely, $\{\mathfrak{M}^*(\mathbf{X}_i)\mathfrak{M}(\mathbf{X}_j)\}$ or $\{i\mathfrak{M}^*(\mathbf{Y}_i)\mathfrak{M}(\mathbf{Y}_j)\}$

[16] This choice of coupling constants was once introduced by the present authors (unpublished paper) to yield the Fierz terms equal to zero. One can explain the lack of the Fierz terms in several ways: (A) C_S or $C_V=0$ and C_T or $C_A=0$, and similar relations for coupling constants with primes; (B) $C_S=\pm C_S'$, $C_V=\mp C_V'$ and $C_T=\mp C_T'$, $C_A=\pm C_A'$; (C) $C_S=\pm C_S'$, $C_V=\mp C_V'$ and $C_T=\pm C_T'$, $C_A=\mp C_A'$; (D) $C_V/C_S=ig'$ and $C_A/C_T=ig$ with real g and g', and similar relations for coupling constants with primes: or different choices of (A), (B), (C), and (D) for Fermi and Gamow-Teller interactions. Here the assignment $C_T=C_T'$ (or $C_A=-C_A'$) is excluded by some data in allowed transitions with $\Delta J=\pm1$. The same choice of the relative sign of the coupling constants has been also considered by T. D. Lee and C. S. Wu [Proceedings of the Israel Conference, September, 1957 (unpublished)], to explain the cross section of neutrino capture by Cl^{37} [R. Davis, Jr. (to be published)] and other data on beta decay.

[17] E. J. Konopinski and G. E. Uhlenbeck, Phys. Rev. **60**, 308 (1941).

are always real numbers, and may be positive or negative. If the strong interactions are not invariant under time reversal, the expressions for the $b_{LL'}^{(n)}$'s should be slightly modified, and a few terms should be added, which do not appear in our case. W, p, and K are the energy and momentum of the electron and the energy of the neutrino, respectively. The other symbols are combinations of electron wave functions and are given below[18]:

$$L_0=(2p^2F)^{-1}[g_{-1}^2+f_1^2]\rightarrow 1. \quad (36)$$

$$L_1=(2p^2F)^{-1}\rho^{-2}[g_{-2}^2+f_2^2]\rightarrow p^2/9. \quad (37)$$

$$N_0=(2p^2F)^{-1}\rho^{-1}[f_{-1}g_{-1}-f_1g_1]\rightarrow -(p^2/3W)-V. \quad (38)$$

$$M_0=(2p^2F)^{-1}\rho^{-2}[f_{-1}^2+g_1^2]\rightarrow (p^2/9)+(2p^2/3W)V+V^2. \quad (39)$$

$$L_{12}=(2p^2F)^{-1}\rho^{-1}[g_{-1}f_2\cos(\delta_{-1}-\delta_2)-f_1g_{-2}\cos(\delta_1-\delta_{-2})]\rightarrow -p^2/3W. \quad (40)$$

$$N_{12}=(2p^2F)^{-1}\rho^{-2}[f_{-1}f_2\cos(\delta_{-1}-\delta_2)+g_1g_{-2}\cos(\delta_1-\delta_{-2})]\rightarrow (p^2/9)+(p^2/3W)V. \quad (41)$$

$$\mathbf{L}_{12}=(2p^2F)^{-1}\rho^{-1}[f_1f_2\sin(\delta_1-\delta_2)+g_{-1}g_{-2}\sin(\delta_{-1}-\delta_{-2})]\rightarrow \tfrac{1}{3}p. \quad (42)$$

$$\mathbf{N}_{11}=(2p^2F)^{-1}\rho^{-1}[f_{-1}f_1-g_{-1}g_1]\sin(\delta_{-1}-\delta_1)\rightarrow -\tfrac{1}{3}p-(p/W)V. \quad (43)$$

$$\mathbf{N}_{12}=(2p^2F)^{-1}\rho^{-2}[f_{-1}g_{-2}\sin(\delta_{-1}-\delta_{-2})-g_1f_2\sin(\delta_1-\delta_2)]\rightarrow -(p^3/9W)-\tfrac{1}{3}pV. \quad (44)$$

$$\Lambda_1=(2p^2F)^{-1}g_{-1}f_1\sin(\delta_{-1}-\delta_1)\rightarrow p/2W. \quad (45)$$

$$\Lambda_2=(2p^2F)^{-1}\rho^{-2}g_{-2}f_2\sin(\delta_{-2}-\delta_2)\rightarrow p^3/18W. \quad (46)$$

$$m_1=(2p^2F)^{-1}\rho^{-2}f_{-1}g_1\sin(\delta_{-1}-\delta_1)\rightarrow -(p^3/18W)-\tfrac{1}{3}pV-(p/2W)V^2. \quad (47)$$

$$H_{12}=(2p^2F)^{-1}\rho^{-1}[g_{-1}f_2\sin(\delta_{-1}-\delta_2)-f_1g_{-2}\sin(\delta_1-\delta_{-2})]\rightarrow -\tfrac{1}{4}p\alpha Z. \quad (48)$$

$$J_{12}=(2p^2F)^{-1}\rho^{-2}[f_{-1}f_2\sin(\delta_{-1}-\delta_2)+g_1g_{-2}\sin(\delta_1-\delta_{-2})]\rightarrow [(p^3/12W)+\tfrac{1}{4}pV]\alpha Z. \quad (49)$$

$$\mathbf{H}_{12}=(2p^2F)^{-1}\rho^{-1}[f_1f_2\cos(\delta_1-\delta_2)+g_{-1}g_{-2}\cos(\delta_{-1}-\delta_{-2})]\rightarrow -[(p^2/4W)+(1/3W)]\alpha Z. \quad (50)$$

$$\mathbf{J}_{11}=(2p^2F)^{-1}\rho^{-1}[f_{-1}f_1+g_{-1}g_1]\cos(\delta_{-1}-\delta_1)\rightarrow -(1/3W)\alpha Z. \quad (51)$$

$$\mathbf{J}_{12}=(2p^2F)^{-1}\rho^{-2}[f_{-1}g_{-2}\cos(\delta_{-1}-\delta_{-2})-g_1f_2\cos(\delta_1-\delta_2)]\rightarrow [(p^2/12)+(p^2/4W)V+(1/3W)V]\alpha Z. \quad (52)$$

Here $V\equiv\alpha Z/2\rho$. The arrow in each equation indicates the approximation $(\alpha Z)^2\ll 1$. When we need the correction to the $b_{LL'}^{(n)}$'s due to the finite de Broglie wavelength, we can obtain more accurate forms of $b_{LL'}^{(n)}$ from Eqs. (7)–(34), together with higher order expansions of the functions in Eqs. (36)–(52). To include the finite nuclear size correction, the expressions for the $b_{LL'}^{(n)}$ should be modified as for the beta-ray spectrum. See for example Matumoto and Yamada.[19] If one assumes invariance of beta interactions under time reversal, the $b_{LL'}^{(\mathrm{even})}$ are reduced to those for directional correlations given by Morita *et al.*[8–10]

In this section, the $b_{LL'}^{(n)}$ are given for an electron decay. In order to obtain the corresponding $b_{LL'}^{(n)}$ for a positron decay, the following substitution should be performed: $Z\rightarrow -Z$, $C_i\rightarrow -C_i^*$, $C_i'\rightarrow C_i'^*$ with $i=S, A, P$; and $C_j\rightarrow C_j^*$, $C_j'\rightarrow -C_j'^*$ with $j=V, T$.

3. EXPLICIT FORMS OF BETA-GAMMA ANGULAR CORRELATIONS

In some special decay schemes, Eqs. (2)–(6) are greatly simplified. For example, in the cases of $j(\beta:\text{allowed})j_1(\gamma: 2^L\text{ pole})j_1\pm L$, the angular correlation between the beta rays and the circularly polarized gamma rays is

TABLE I. $\mu(j,j_1,j_2)$ and $\nu(j,j_1,j_2)$.

Decay scheme $j(\beta)j_1(\gamma)j_2$	$\mu(j,j_1,j_2)$	$\nu(j,j_1,j_2)$
$j(\beta)j-1(\gamma)j-1-L$	1	0
$j(\beta)j(\gamma)j-L$	$-1/j$	$[(j+1)/j]^{\frac{1}{2}}$
$j(\beta)j+1(\gamma)j+1-L$	$-(j+2)/(j+1)$	0
$j(\beta)j-1(\gamma)j-1+L$	$(-j+1)/j$	0
$j(\beta)j(\gamma)j+L$	$1/j+1$	$-[j/(j+1)]^{\frac{1}{2}}$
$j(\beta)j+1(\gamma)j+1+L$	1	0

[18] Equations (36)–(47) were defined to calculate electron-neutrino angular correlations. See M. Morita, Phys. Rev. **90**, 1005 (1953); Progr. Theoret. Phys. (Japan) **9**, 345 (1953).
[19] Z. Matumoto and M. Yamada, Progr. Theoret. Phys. (Japan) (to be published). A. D. Dolginov and I. N. Topigin [Nuclear Phys. **2**, 147 (1956)] considered a similar effect for angular correlations with the old theory of beta decay.

expressed explicitly by

$$W(\theta,p_1:\beta-\gamma)=1\pm A\,(v/c)\cos\theta, \tag{53}$$

where the upper (lower) sign refers to the right (left) circularly polarized gamma rays. The asymmetry, A, is:

$$A=\frac{1}{L+1}\Bigg\{\pm\mu(j,j_1,j_2)\left[\mathrm{Re}(C_T^*C_T'-C_A^*C_A')\mp\frac{\alpha Z}{p}\,\mathrm{Im}(C_T^*C_A'+C_T'^*C_A)\right]M_{\mathrm{GT}}^2$$
$$+\nu(j,j_1,j_2)\bigg[\mathrm{Re}(C_T^*C_S'+C_T'^*C_S-C_A^*C_V'-C_A'^*C_V)$$
$$\mp\frac{\alpha Z}{p}\,\mathrm{Im}(C_T^*C_V'+C_T'^*C_V-C_A^*C_S'-C_A'^*C_S)\bigg]M_{\mathrm{F}}\cdot M_{\mathrm{GT}}\Bigg\}\frac{2}{\xi[1+(b/W)]}, \tag{54}$$

where

$$\xi=M_{\mathrm{F}}^2(|C_S|^2+|C_V|^2+|C_S'|^2+|C_V'|^2)+M_{\mathrm{GT}}^2(|C_T|^2+|C_A|^2+|C_T'|^2+|C_A'|^2),$$
$$\xi b=\pm2\gamma\,\mathrm{Re}[M_{\mathrm{F}}^2(C_S^*C_V+C_S'^*C_V')+M_{\mathrm{GT}}^2(C_T^*C_A+C_T'^*C_A')],$$
$$\gamma=[1-(\alpha Z)^2]^{\frac{1}{2}}.$$

In A, and ξb, the upper (lower) sign refers to the electron (positron) decay. $\mu(j,j_1,j_2)$ and $\nu(j,j_1,j_2)$ depend on the spin of the parent nucleus, j, and are given in Table I for relevant decay schemes. M_{F} and M_{GT} are taken to be real numbers. Therefore their product, $M_{\mathrm{F}}\cdot M_{\mathrm{GT}}$, may be positive or negative. Similar formulas for beta-ray angular distributions have been given by many authors [see, e.g., Eq. (1) of Ambler *et al.*,[20] or reference 4]. No restriction on the coupling constants is used in Eq. (54).

In the cases of first-forbidden transitions, the angular correlations between the beta rays and the circularly polarized gamma rays are more complicated. We give two examples, which are frequently encountered in experiments.

(1) $2^-(\beta)2^+(\gamma)0^+$.

$$W(\theta,p_1:\beta-\gamma)=(b_{00}^{(0)}-3^{-\frac{1}{2}}b_{11}^{(0)}+5^{-\frac{1}{2}}b_{22}^{(0)})-[6^{-\frac{1}{2}}b_{01}^{(1)}+\tfrac{1}{6}(2^{-\frac{1}{2}})b_{11}^{(1)}-\tfrac{1}{2}(7/30)^{\frac{1}{2}}b_{12}^{(1)}-\tfrac{1}{2}(10)^{-\frac{1}{2}}b_{22}^{(1)}]p_1P_1(\cos\theta)$$
$$-[\tfrac{1}{2}(6^{-\frac{1}{2}})b_{11}^{(2)}+(14)^{-\frac{1}{2}}b_{02}^{(2)}+\tfrac{1}{2}(14)^{-\frac{1}{2}}b_{12}^{(2)}-3(14)^{-\frac{1}{2}}b_{22}^{(2)}]P_2(\cos\theta)$$
$$+[2(35)^{-\frac{1}{2}}b_{12}^{(3)}+(4/7)(2/5)^{\frac{1}{2}}b_{22}^{(3)}]p_1P_3(\cos\theta). \tag{55}$$

Here $p_1=+1(-1)$ for left (right) circular polarization. From Eq. (55), the directional correlation is obtained by dropping the $P_1(\cos\theta)$ and $P_3(\cos\theta)$ terms. Equation (55) is applicable to Cl^{38},[21] K^{42},[22] As^{76},[23] Rb^{86},[24] Sb^{122},[25] and I^{126},[22] for which the directional correlations are anisotropic, and to Au^{198},[26] for which the directional correlation is isotropic but the polarization correlation is asymmetric.

(2) $3^-(\beta)2^+(\gamma)0^+$.

$$W(\theta,p_1:\beta-\gamma)=(-3^{-\frac{1}{2}}b_{11}^{(0)}+5^{-\frac{1}{2}}b_{22}^{(0)})+[\tfrac{1}{3}(2^{-\frac{1}{2}})b_{11}^{(1)}+(15)^{-\frac{1}{2}}b_{12}^{(1)}]p_1P_1(\cos\theta)$$
$$+[\tfrac{1}{7}(6^{-\frac{1}{2}})b_{11}^{(2)}+\tfrac{1}{7}b_{12}^{(2)}+(2/7)^{\frac{3}{2}}b_{22}^{(2)}]P_2(\cos\theta)-[\tfrac{1}{7}(\tfrac{2}{5})^{\frac{1}{2}}b_{12}^{(3)}+\tfrac{1}{7}(\tfrac{5}{2})^{\frac{1}{2}}b_{22}^{(3)}]p_1P_3(\cos\theta). \tag{56}$$

Here the coefficient of $b_{22}^{(1)}$ is canceled accidentally. Equation (56) is applicable to Sb^{124}, for which the directional correlation is anisotropic[24,27,28] and the polarization correlation is asymmetric.[29] In Eqs. (55) and (56), we use the $b_{LL'}^{(n)}$'s of Eqs. (7)–(34) or Eqs. (A1)–(A28), or Eqs. (A35)–(A48) if those approximations are valid. Furthermore, in the case, where the beta interaction is STP with $C_i=-C_i'$ and real coupling constants, Eqs. (55) and (56) have the form of Eq. (53):

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[28] M. Morita and M. Yamada, Progr. Theoret. Phys. (Japan) **8**, 449 (1952); **10**, 111 and 641 (1953).
[29] H. Appel and H. Schopper, Z. Physik **149**, 103 (1957).

(1) $2^-(\beta)2^+(\gamma)0^+$.

$$A=\frac{-2(6)^{-\frac{1}{2}}XY+\frac{1}{6}Y^2-\frac{1}{6}(\frac{7}{2})^{\frac{1}{2}}WY+[(1/48)K^2+(11/112)p^2]-(1/7)p^2\cos^2\theta}{X^2+Y^2+\frac{1}{2}(21)^{-\frac{1}{2}}(p^2/W)X-\frac{1}{4}(14)^{-\frac{1}{2}}(p^2/W)Y+[(1/12)K^2+(59/672)p^2]-[\frac{1}{2}(3/7)^{\frac{1}{2}}X-\frac{3}{4}(14)^{-\frac{1}{2}}Y+(3/224)W](p^2/W)\cos^2\theta}, \tag{57}$$

where $X\equiv\xi/\zeta$ and $Y\equiv\eta/\zeta$.

(2) $3^-(\beta)2^+(\gamma)0^+$.

$$A=\frac{-\frac{1}{3}Y^2-\frac{1}{3}WY-(3/56)p^2+(5/56)p^2\cos^2\theta}{Y^2+(1/14)(p^2/W)Y+[(1/12)K^2+(2/21)p^2]-[(3/14)Y+(1/28)W](p^2/W)\cos^2\theta}. \tag{58}$$

Equations (57) and (58) are also valid for VA with $C_j=C_j'$, and in the special case of $STPVA$ when $C_i=-C_i'$ for $i=S$, T, P and $C_j=C_j'$ for $j=V$, A together with $\xi=\pm\lambda$, $\eta=\pm\mu$, and $\zeta=\pm\lambda$, and the assumption of time-reversal invariance.

4. CONCLUDING REMARKS

In the beta-ray angular distribution, Eq. (2), the $\bar{f}_n(j)$ with odd n vanish except for polarized nuclei. Consequently, $W(\theta:\beta)$ is a linear combination of even powers of $\cos\theta$ in aligned nuclei. In this case, the $b_{LL'}^{(\mathrm{even})}$ given in references 8–10 are determined up to the second forbidden transitions. These authors took into account all the interferences among S, T, P, V, and A, but assumed invariance under time reversal for beta decay. For polarized nuclei, $W(\theta:\beta)$ is, of course, a linear combination of odd and even powers of $\cos\theta$.

In the case of special decay schemes such as $j(\beta)2L(\gamma_1)L(\gamma_2)0$ with pure electric and/or magnetic 2^L-pole gamma rays [for example, $j(\beta)4(\gamma_1)2(\gamma_2)0$ with pure quadrupole gamma rays], the angular correlation between the beta rays and the circularly polarized γ_1 rays is equal to that between the beta rays and the circularly polarized γ_2 rays. This implies that, for example, we need not discriminate experimentally between the two gamma rays with energies of 0.89 Mev and 1.12 Mev following the beta decay of Sc^{46} to the 2.01-Mev state of Ti^{46} (assuming a low intensity of the beta group of Sc^{46} to the 1.12-Mev state of Ti^{46}). This equality also holds in the case where the circular polarization of the gamma rays is not observed, and in the case of gamma-ray angular distributions from oriented nuclei in similar decay schemes.

The approximation for the $b_{LL'}^{(n)}$ in Appendix 2 is at least valid for the nuclei Rb^{86}, Sb^{122}, I^{126}, and Au^{198}. In these nuclei, the beta-ray spectra have an allowed shape which may support the assumption of $\alpha Z/2\rho\gg W_0$. (Actually, $\alpha Z/2\rho W_0\sim 6$ for these nuclei.) One of the reasons why we do not neglect the $\mathfrak{M}(B_{ij}{}^\beta)$ term in the $b_{LL'}^{(n)}$ is that a small mixture of $\mathfrak{M}(B_{ij}{}^\beta)$ greatly affects the beta-ray angular distributions and beta-gamma angular correlations through its cross terms with the other matrix elements. Furthermore, if we assume that the j–j coupling shell model or A. Bohr's model of the nucleus is strictly valid, the nuclear matrix elements other than $\mathfrak{M}(B_{ij}{}^\beta)$ cancel. The arguments in this paragraph were once justified by us in the theoretical analysis of beta-gamma directional correlations for K^{42}, As^{76}, Rb^{86}, Sb^{122}, and I^{126}.[30] The result was that the approximation for the $b_{LL'}^{(n)}$ in Appendix 2 is good for Rb^{86}, Sb^{122}, I^{126}, and rather poor for K^{42} and As^{76}.

Upon using the $b_{LL'}^{(n)}$, applications to many phenomena involving double or triple cascade transitions in oriented nuclei are easy.

An analysis of the experimental data on Au^{198},[26] and Sb^{124},[29] is given in Appendix 3.

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APPENDIX 1. $b_{LL'}^{(n)}$ IN THE APPROXIMATION $(\alpha Z)^2\ll 1$

Using Eqs. (36)–(52), the $b_{LL'}^{(n)}$ in Eqs. (7)–(34) become the following:

STP

$$b_{00}^{(0)}=|\mathfrak{M}(\beta\boldsymbol{\sigma}\cdot\mathbf{r})|^2(|C_T|^2+|C_T'|^2)\{(1/9)[K^2-2K(p^2/W)+p^2]+\tfrac{2}{3}[-K+(p^2/W)]V+V^2\}$$
$$+|\mathfrak{M}(\beta\gamma_5)|^2(|C_P|^2+|C_P'|^2)+\{i\mathfrak{M}^*(\beta\boldsymbol{\sigma}\cdot\mathbf{r})\mathfrak{M}(\beta\gamma_5)\}2\,\mathrm{Re}(C_T^*C_P+C_T'^*C_P')\{\tfrac{1}{3}[-K+(p^2/W)]+V\}. \tag{A1}$$

[30] Matumoto, Morita, and Yamada, Bull. Kobayasi Inst. Phys. Research (in Japanese) 5, 210 (1955). Their calculation assumed the STP combination with real coupling constants. In this approximation, their result is also the same for VA or the special case of $STPVA$, as is written under Eq. (58).

$$b_{01}^{(1)}=\{\mathfrak{M}^*(\beta\boldsymbol{\sigma}\cdot\mathbf{r})\mathfrak{M}(\beta\mathbf{r})\}\{2\,\mathrm{Re}(C_T^*C_S'+C_T'^*C_S)(p/W)\{(1/9)(K^2-4KW+3p^2)+\tfrac{2}{3}(-K+2W)V+V^2\}$$
$$+2\,\mathrm{Im}(C_T^*C_S'+C_T'^*C_S)(\alpha Z/W)[(1/18)(-3p^2K-4K+3p^2W)+\tfrac{1}{6}(3p^2+4)V]\}$$
$$-\{i\mathfrak{M}^*(\beta\boldsymbol{\sigma}\cdot\mathbf{r})\mathfrak{M}(\beta\boldsymbol{\alpha})\}4\,\mathrm{Re}(C_T^*C_T')(p/W)[\tfrac{1}{3}(K-W)-V]$$
$$+\{i\mathfrak{M}^*(\beta\boldsymbol{\sigma}\cdot\mathbf{r})\mathfrak{M}(\beta\boldsymbol{\sigma}\times\mathbf{r})\}4\,\mathrm{Re}(C_T^*C_T')(p/W)[(1/9)K(K-W)-\tfrac{1}{3}WV-V^2]$$
$$+\{i\mathfrak{M}^*(\beta\gamma_5)\mathfrak{M}(\beta\mathbf{r})\}\{2\,\mathrm{Re}(C_P^*C_S'+C_P'^*C_S)(p/W)[(\tfrac{1}{3}K-W)-V]$$
$$-2\,\mathrm{Im}(C_P^*C_S'+C_P'^*C_S)(\alpha Z/W)(1+\tfrac{1}{2}p^2)\}+\{\mathfrak{M}^*(\beta\gamma_5)\mathfrak{M}(\beta\boldsymbol{\alpha})\}2\,\mathrm{Re}(C_P^*C_T'+C_P'^*C_T)(p/W)$$
$$-\{\mathfrak{M}^*(\beta\gamma_5)\mathfrak{M}(\beta\boldsymbol{\sigma}\times\mathbf{r})\}[2\,\mathrm{Re}(C_P^*C_T'+C_P'^*C_T)(p/W)(\tfrac{1}{3}K+V)$$
$$-2\,\mathrm{Im}(C_P^*C_T'+C_P'^*C_T)(\alpha Z/W)(p^2/4)]. \quad \text{(A2)}$$

$$b_{11}^{(0)}=-\sqrt{3}\{|\mathfrak{M}(\beta\mathbf{r})|^2(|C_S|^2+|C_S'|^2)\{(1/9)[3K^2-2K(p^2/W)+3p^2]+\tfrac{2}{3}[-K+(p^2/W)]V+V^2\}$$
$$+|\mathfrak{M}(\beta\boldsymbol{\alpha})|^2(|C_T|^2+|C_T'|^2)+|\mathfrak{M}(\beta\boldsymbol{\sigma}\times\mathbf{r})|^2(|C_T|^2+|C_T'|^2)\{(1/18)[3K^2+4K(p^2/W)+3p^2]$$
$$+\tfrac{2}{3}[K+(p^2/W)]V+V^2\}-\{\mathfrak{M}^*(\beta\boldsymbol{\alpha})\mathfrak{M}(\beta\boldsymbol{\sigma}\times\mathbf{r})\}2(|C_T|^2+|C_T'|^2)\{\tfrac{1}{3}[K+(p^2/W)]+V\}$$
$$-\{i\mathfrak{M}^*(\beta\mathbf{r})\mathfrak{M}(\beta\boldsymbol{\alpha})\}2\,\mathrm{Re}(C_S^*C_T+C_S'^*C_T')\{\tfrac{1}{3}[K-(p^2/W)]-V\}$$
$$-\{i\mathfrak{M}^*(\beta\mathbf{r})\mathfrak{M}(\beta\boldsymbol{\sigma}\times\mathbf{r})\}2\,\mathrm{Re}(C_S^*C_T+C_S'^*C_T')[(2p^2/3W)V+V^2]\}. \quad \text{(A3)}$$

$$b_{11}^{(1)}=\sqrt{2}\{-|\mathfrak{M}(\beta\mathbf{r})|^2 2\,\mathrm{Re}(C_S^*C_S')(p/W)(-\tfrac{2}{3}KV+V^2)-|\mathfrak{M}(\beta\boldsymbol{\alpha})|^2 2\,\mathrm{Re}(C_T^*C_T')(p/W)$$
$$-|\mathfrak{M}(\beta\boldsymbol{\sigma}\times\mathbf{r})|^2 2\,\mathrm{Re}(C_T^*C_T')(p/W)[(1/12)(K^2+4KW+3p^2)+(\tfrac{2}{3}K+W)V+V^2]$$
$$+\{\mathfrak{M}^*(\beta\boldsymbol{\alpha})\mathfrak{M}(\beta\boldsymbol{\sigma}\times\mathbf{r})\}4\,\mathrm{Re}(C_T^*C_T')(p/W)[\tfrac{1}{6}(2K+3W)+V]$$
$$+\{i\mathfrak{M}^*(\beta\mathbf{r})\mathfrak{M}(\beta\boldsymbol{\alpha})\}[2\,\mathrm{Re}(C_S^*C_T'+C_S'^*C_T)(p/W)(\tfrac{1}{3}K-V)-2\,\mathrm{Im}(C_S^*C_T'+C_S'^*C_T)(\alpha Z/W)(p^2/4)]$$
$$+\{i\mathfrak{M}^*(\beta\mathbf{r})\mathfrak{M}(\beta\boldsymbol{\sigma}\times\mathbf{r})\}\{2\,\mathrm{Re}(C_S^*C_T'+C_S'^*C_T)(p/W)[-\tfrac{1}{6}(K^2+KW)+\tfrac{1}{2}WV+V^2]$$
$$+2\,\mathrm{Im}(C_S^*C_T'+C_S'^*C_T)(\alpha Z/W)[(1/24)(Kp^2-4K+3p^2W)+\tfrac{1}{8}(3p^2+4)V]\}\}. \quad \text{(A4)}$$

$$b_{11}^{(2)}=-\sqrt{6}\{-|\mathfrak{M}(\beta\mathbf{r})|^2(|C_S|^2+|C_S'|^2)(p^2/W)[(1/9)(-2K+3W)+\tfrac{2}{3}V]$$
$$+|\mathfrak{M}(\beta\boldsymbol{\sigma}\times\mathbf{r})|^2(|C_T|^2+|C_T'|^2)(p^2/W)[(1/36)(4K+3W)+\tfrac{1}{3}V]-\{\mathfrak{M}^*(\beta\boldsymbol{\alpha})\mathfrak{M}(\beta\boldsymbol{\sigma}\times\mathbf{r})\}$$
$$\times 2(|C_T|^2+|C_T'|^2)(p^2/6W)+\{i\mathfrak{M}^*(\beta\mathbf{r})\mathfrak{M}(\beta\boldsymbol{\alpha})\}[-2\,\mathrm{Re}(C_S^*C_T+C_S'^*C_T')(p^2/3W)$$
$$+2\,\mathrm{Im}(C_S^*C_T+C_S'^*C_T')(p/4)\alpha Z]+\{i\mathfrak{M}^*(\beta\mathbf{r})\mathfrak{M}(\beta\boldsymbol{\sigma}\times\mathbf{r})\}[2\,\mathrm{Re}(C_S^*C_T+C_S'^*C_T')(p^2/6W)(K+V)$$
$$-2\,\mathrm{Im}(C_S^*C_T+C_S'^*C_T')\alpha Z(p/24)\{[K+3(p^2/W)]+9V\}]\}. \quad \text{(A5)}$$

$$b_{02}^{(2)}=(1/\sqrt{6})\{-\{i\mathfrak{M}^*(\beta\gamma_5)\mathfrak{M}(B_{ij}{}^{\beta})\}[2\,\mathrm{Re}(C_P^*C_T+C_P'^*C_T')(p^2/W)+2\,\mathrm{Im}(C_P^*C_T+C_P'^*C_T')\tfrac{3}{4}p\alpha Z]$$
$$+\{\mathfrak{M}^*(\beta\boldsymbol{\sigma}\cdot\mathbf{r})\mathfrak{M}(B_{ij}{}^{\beta})\}2(|C_T|^2+|C_T'|^2)(p^2/W)[\tfrac{1}{3}(-K+W)+V]\}. \quad \text{(A6)}$$

$$b_{12}^{(1)}=-\tfrac{1}{2}(5/3)^{\frac{1}{2}}\{\{\mathfrak{M}^*(\beta\mathbf{r})\mathfrak{M}(B_{ij}{}^{\beta})\}\{2\,\mathrm{Re}(C_S^*C_T'+C_S'^*C_T)(p/W)[(1/15)(5K^2-5KW+6p^2)+WV]$$
$$+2\,\mathrm{Im}(C_S^*C_T'+C_S'^*C_T)(\alpha Z/W)[(1/12)(-3Kp^2-4K+3p^2W)+(\tfrac{3}{4}p^2+1)V]\}$$
$$-\{i\mathfrak{M}^*(\beta\boldsymbol{\alpha})\mathfrak{M}(B_{ij}{}^{\beta})\}4\,\mathrm{Re}(C_T^*C_T')p$$
$$+\{i\mathfrak{M}^*(\beta\boldsymbol{\sigma}\times\mathbf{r})\mathfrak{M}(B_{ij}{}^{\beta})\}4\,\mathrm{Re}(C_T^*C_T')(p/W)[(1/30)(5K^2+10KW+9p^2)+WV]\}. \quad \text{(A7)}$$

$$b_{12}^{(2)}=-\{\mathfrak{M}^*(\beta\mathbf{r})\mathfrak{M}(B_{ij}{}^{\beta})\}\{2\,\mathrm{Re}(C_S^*C_T+C_S'^*C_T')(p^2/2W)(-\tfrac{1}{3}K+V)+2\,\mathrm{Im}(C_S^*C_T+C_S'^*C_T')\alpha Z(p/8)$$
$$\times[\{-K+(p^2/W)\}+3V]\}+\{i\mathfrak{M}^*(\beta\boldsymbol{\alpha})\mathfrak{M}(B_{ij}{}^{\beta})\}2(|C_T|^2+|C_T'|^2)(p^2/2W)$$
$$-\{i\mathfrak{M}^*(\beta\boldsymbol{\sigma}\times\mathbf{r})\mathfrak{M}(B_{ij}{}^{\beta})\}2(|C_T|^2+|C_T'|^2)(p^2/2W)[\tfrac{1}{6}(2K+3W)+V]. \quad \text{(A8)}$$

$$b_{12}^{(3)}=(1/\sqrt{10})\{\{\mathfrak{M}^*(\beta\mathbf{r})\mathfrak{M}(B_{ij}{}^{\beta})\}2\,\mathrm{Re}(C_S^*C_T'+C_S'^*C_T)(p^3/W)$$
$$-\{i\mathfrak{M}^*(\beta\boldsymbol{\sigma}\times\mathbf{r})\mathfrak{M}(B_{ij}{}^{\beta})\}4\,\mathrm{Re}(C_T^*C_T')(p^3/2W)\}. \quad \text{(A9)}$$

$$b_{22}^{(0)}=|\mathfrak{M}(B_{ij}{}^{\beta})|^2(|C_T|^2+|C_T'|^2)(K^2+p^2)(\sqrt{5})/12. \quad \text{(A10)}$$

$$b_{22}^{(1)}=|\mathfrak{M}(B_{ij}{}^{\beta})|^2 2\,\mathrm{Re}(C_T^*C_T')(p/W)[(1/24)K^2+(1/40)p^2]\sqrt{10}. \quad \text{(A11)}$$

$$b_{22}^{(2)}=-|\mathfrak{M}(B_{ij}{}^{\beta})|^2(|C_T|^2+|C_T'|^2)(1/12)(\tfrac{7}{2})^{\frac{1}{2}}p^2. \quad \text{(A12)}$$

$$b_{22}^{(3)}=-|\mathfrak{M}(B_{ij}{}^{\beta})|^2 2\,\mathrm{Re}(C_T^*C_T')(1/2\sqrt{10})(p^3/W). \quad \text{(A13)}$$

$$b_{22}^{(4)}=0. \quad \text{(A14)}$$

VA

$$b_{00}^{(0)}=|\mathfrak{M}(\boldsymbol{\sigma}\cdot\mathbf{r})|^2(|C_A|^2+|C_A'|^2)\{(1/9)[K^2+2K(p^2/W)+p^2]+\tfrac{2}{3}[K+(p^2/W)]V+V^2\}$$
$$+|\mathfrak{M}(\gamma_5)|^2(|C_A|^2+|C_A'|^2)+\{i\mathfrak{M}^*(\boldsymbol{\sigma}\cdot\mathbf{r})\mathfrak{M}(\gamma_5)\}2(|C_A|^2+|C_A'|^2)\{\tfrac{1}{3}[K+(p^2/W)]+V\}. \tag{A15}$$

$$b_{01}^{(1)}=-\{\mathfrak{M}^*(\boldsymbol{\sigma}\cdot\mathbf{r})\mathfrak{M}(\mathbf{r})\}\{2\,\mathrm{Re}(C_A^*C_V'+C_A'^*C_V)(p/W)[(1/9)(K^2+4KW+3p^2)+\tfrac{2}{3}(K+2W)V+V^2]$$
$$+2\,\mathrm{Im}(C_A^*C_V'+C_A'^*C_V)(\alpha Z/W)\{(1/18)(3p^2K+4K+3p^2W)+\tfrac{1}{6}(3p^2+4)V\}\}-\{i\mathfrak{M}^*(\boldsymbol{\sigma}\cdot\mathbf{r})\mathfrak{M}(\boldsymbol{\alpha})\}$$
$$\times\{2\,\mathrm{Re}(C_A^*C_V'+C_A'^*C_V)(p/W)[\tfrac{1}{3}(K+W)+V]-2\,\mathrm{Im}(C_A^*C_V'+C_A'^*C_V)(\alpha Z/3W)\}$$
$$+\{i\mathfrak{M}^*(\boldsymbol{\sigma}\cdot\mathbf{r})\mathfrak{M}(\boldsymbol{\sigma}\times\mathbf{r})\}4\,\mathrm{Re}(C_A^*C_A')(p/W)[-(1/9)K(K+W)+\tfrac{1}{3}WV+V^2]$$
$$+\{i\mathfrak{M}^*(\gamma_5)\mathfrak{M}(\mathbf{r})\}\{2\,\mathrm{Re}(C_A^*C_V'+C_A'^*C_V)(p/W)[(\tfrac{1}{3}K+W)+V]$$
$$+2\,\mathrm{Im}(C_A^*C_V'+C_A'^*C_V)(\alpha Z/W)(1+\tfrac{1}{2}p^2)\}-\{\mathfrak{M}^*(\gamma_5)\mathfrak{M}(\boldsymbol{\alpha})\}2\,\mathrm{Re}(C_A^*C_V'+C_A'^*C_V)(p/W)$$
$$-\{\mathfrak{M}^*(\gamma_5)\mathfrak{M}(\boldsymbol{\sigma}\times\mathbf{r})\}4\,\mathrm{Re}(C_A^*C_A')(p/W)(\tfrac{1}{3}K-V). \tag{A16}$$

$$b_{11}^{(0)}=-\sqrt{3}\{|\mathfrak{M}(\mathbf{r})|^2(|C_V|^2+|C_V'|^2)\{(1/9)[3K^2+2K(p^2/W)+3p^2]+\tfrac{2}{3}[K+(p^2/W)]V+V^2\}$$
$$+|\mathfrak{M}(\boldsymbol{\alpha})|^2(|C_V|^2+|C_V'|^2)+|\mathfrak{M}(\boldsymbol{\sigma}\times\mathbf{r})|^2(|C_A|^2+|C_A'|^2)\{(1/18)[3K^2-4K(p^2/W)+3p^2]$$
$$+\tfrac{2}{3}[-K+(p^2/W)]V+V^2\}-\{\mathfrak{M}^*(\boldsymbol{\alpha})\mathfrak{M}(\boldsymbol{\sigma}\times\mathbf{r})\}2\,\mathrm{Re}(C_V^*C_A+C_V'^*C_A')\{\tfrac{1}{3}[-K+(p^2/W)]+V\}$$
$$+\{i\mathfrak{M}^*(\mathbf{r})\mathfrak{M}(\boldsymbol{\alpha})\}2(|C_V|^2+|C_V'|^2)\{\tfrac{1}{3}[K+(p^2/W)]+V\}$$
$$-\{i\mathfrak{M}^*(\mathbf{r})\mathfrak{M}(\boldsymbol{\sigma}\times\mathbf{r})\}2\,\mathrm{Re}(C_V^*C_A+C_V'^*C_A')[(2p^2/3W)V+V^2]\}. \tag{A17}$$

$$b_{11}^{(1)}=\sqrt{2}\{|\mathfrak{M}(\mathbf{r})|^2 2\,\mathrm{Re}(C_V^*C_V')(p/W)(\tfrac{2}{3}KV+V^2)+|\mathfrak{M}(\boldsymbol{\alpha})|^2 2\,\mathrm{Re}(C_V^*C_V')(p/W)$$
$$+|\mathfrak{M}(\boldsymbol{\sigma}\times\mathbf{r})|^2 2\,\mathrm{Re}(C_A^*C_A')(p/W)[(1/12)(K^2-4KW+3p^2)+(-\tfrac{2}{3}K+W)V+V^2]$$
$$-\{\mathfrak{M}^*(\boldsymbol{\alpha})\mathfrak{M}(\boldsymbol{\sigma}\times\mathbf{r})\}\{2\,\mathrm{Re}(C_V^*C_A'+C_V'^*C_A)(p/W)[\tfrac{1}{6}(-2K+3W)+V]$$
$$+2\,\mathrm{Im}(C_V^*C_A'+C_V'^*C_A)(\alpha Z/2W)(\tfrac{1}{4}p^2+1)\}+\{i\mathfrak{M}^*(\mathbf{r})\mathfrak{M}(\boldsymbol{\alpha})\}4\,\mathrm{Re}(C_V^*C_V')(p/W)(\tfrac{1}{3}K+V)$$
$$+\{i\mathfrak{M}^*(\mathbf{r})\mathfrak{M}(\boldsymbol{\sigma}\times\mathbf{r})\}\{2\,\mathrm{Re}(C_V^*C_A'+C_V'^*C_A)(p/W)[\tfrac{1}{6}(K^2-KW)-\tfrac{1}{2}WV-V^2]$$
$$+2\,\mathrm{Im}(C_V^*C_A'+C_V'^*C_A)(\alpha Z/W)[(1/24)(Kp^2-4K-3p^2W)-\tfrac{1}{8}(3p^2+4)V]\}\}. \tag{A18}$$

$$b_{11}^{(2)}=-6^{\frac{1}{2}}\{-|\mathfrak{M}(\mathbf{r})|^2(|C_V|^2+|C_V'|^2)(p^2/W)[(1/9)(2K+3W)+\tfrac{2}{3}V]+|\mathfrak{M}(\boldsymbol{\sigma}\times\mathbf{r})|^2(|C_A|^2+|C_A'|^2)(p^2/W)$$
$$\times[(1/36)(-4K+3W)+\tfrac{1}{3}V]-\{\mathfrak{M}^*(\boldsymbol{\alpha})\mathfrak{M}(\boldsymbol{\sigma}\times\mathbf{r})\}[2\,\mathrm{Re}(C_V^*C_A+C_V'^*C_A')(p^2/6W)$$
$$+2\,\mathrm{Im}(C_V^*C_A+C_V'^*C_A')(p/8)\alpha Z]-\{i\mathfrak{M}^*(\mathbf{r})\mathfrak{M}(\boldsymbol{\alpha})\}2(|C_V|^2+|C_V'|^2)(p^2/3W)$$
$$+\{i\mathfrak{M}^*(\mathbf{r})\mathfrak{M}(\boldsymbol{\sigma}\times\mathbf{r})\}\{2\,\mathrm{Re}(C_V^*C_A+C_V'^*C_A')(p^2/6W)(-K+V)$$
$$-2\,\mathrm{Im}(C_V^*C_A+C_V'^*C_A')\alpha Z(p/24)[\{-K+3(p^2/W)\}+9V]\}\}. \tag{A19}$$

$$b_{02}^{(2)}=6^{-\frac{1}{2}}\{-\{i\mathfrak{M}^*(\gamma_5)\mathfrak{M}(B_{ij})\}2(|C_A|^2+|C_A'|^2)(p^2/W)$$
$$+\{\mathfrak{M}^*(\boldsymbol{\sigma}\cdot\mathbf{r})\mathfrak{M}(B_{ij})\}2(|C_A|^2+|C_A'|^2)(p^2/W)[\tfrac{1}{3}(K+W)+V]\}. \tag{A20}$$

$$b_{12}^{(1)}=-\tfrac{1}{2}(5/3)^{\frac{1}{2}}\{-\{\mathfrak{M}^*(\mathbf{r})\mathfrak{M}(B_{ij})\}\{2\,\mathrm{Re}(C_V^*C_A'+C_V'^*C_A)(p/W)[(1/15)(5K^2+5KW+6p^2)+WV]$$
$$+2\,\mathrm{Im}(C_V^*C_A'+C_V'^*C_A)(\alpha Z/W)[(1/12)(3Kp^2+4K+3p^2W)+(\tfrac{3}{4}p^2+1)V]\}$$
$$+\{i\mathfrak{M}^*(\boldsymbol{\alpha})\mathfrak{M}(B_{ij})\}[2\,\mathrm{Re}(C_V^*C_A'+C_V'^*C_A)p+2\,\mathrm{Im}(C_V^*C_A'+C_V'^*C_A)(\alpha Z/W)(\tfrac{3}{4}p^2+1)]$$
$$-\{i\mathfrak{M}^*(\boldsymbol{\sigma}\times\mathbf{r})\mathfrak{M}(B_{ij})\}4\,\mathrm{Re}(C_A^*C_A')(p/W)[(1/30)(5K^2-10KW+9p^2)+WV]\}. \tag{A21}$$

$$b_{12}^{(2)}=-\{\mathfrak{M}^*(\mathbf{r})\mathfrak{M}(B_{ij})\}\{2\,\mathrm{Re}(C_V^*C_A+C_V'^*C_A')(p^2/2W)(\tfrac{1}{3}K+V)$$
$$+2\,\mathrm{Im}(C_V^*C_A+C_V'^*C_A')\alpha Z(p/8)[\{K+(p^2/W)\}+3V]\}$$
$$+\{\mathfrak{M}^*(\boldsymbol{\alpha})\mathfrak{M}(B_{ij})\}[2\,\mathrm{Re}(C_V^*C_A+C_V'^*C_A')(p^2/2W)+2\,\mathrm{Im}(C_V^*C_A+C_V'^*C_A')\tfrac{3}{8}p\alpha Z]$$
$$-\{i\mathfrak{M}^*(\boldsymbol{\sigma}\times\mathbf{r})\mathfrak{M}(B_{ij})\}2(|C_A|^2+|C_A'|^2)(p^2/2W)[\tfrac{1}{6}(-2K+3W)+V]. \tag{A22}$$

$$b_{12}^{(3)}=10^{-\frac{1}{2}}\{-\{\mathfrak{M}^*(\mathbf{r})\mathfrak{M}(B_{ij})\}2\,\mathrm{Re}(C_V^*C_A'+C_V'^*C_A)(p^3/W)+\{i\mathfrak{M}^*(\boldsymbol{\sigma}\times\mathbf{r})\mathfrak{M}(B_{ij})\}4\,\mathrm{Re}(C_A^*C_A')(p^3/2W)\}. \tag{A23}$$

$$b_{22}^{(0)}=|\mathfrak{M}(B_{ij})|^2(|C_A|^2+|C_A'|^2)(K^2+p^2)(\sqrt{5})/12. \tag{A24}$$

$$b_{22}^{(1)}=-|\mathfrak{M}(B_{ij})|^2 2\,\mathrm{Re}(C_A^*C_A')(p/W)[(1/24)K^2+(1/40)p^2]\sqrt{10}. \tag{A25}$$

$$b_{22}^{(2)}=-|\mathfrak{M}(B_{ij})|^2(|C_A|^2+|C_A'|^2)(1/12)(\tfrac{7}{2})^{\frac{1}{2}}p^2. \tag{A26}$$

$$b_{22}^{(3)}=|\mathfrak{M}(B_{ij})|^2 2\,\mathrm{Re}(C_A^*C_A')(1/2\sqrt{10})p^3/W. \tag{A27}$$

$$b_{22}^{(4)}=0. \tag{A28}$$

STPVA with Assumption of No Interference between *STP* and *VA*

See Eq. (35). Here the $b_{LL'}^{(n)}$ ($L=L'$, for STP) are equal to the s_K given by Alder, Stech, and Winther[31] except for constant factors which come from a difference in definition. Again, the real part of the $b_{LL'}^{(\text{even})}$ are equal to those given for directional correlations by Morita.[11] The $b_{LL'}^{(n)}$ above are given for electron decay. For positron decay, see the last paragraph in Sec. 2.

APPENDIX 2. FURTHER APPROXIMATION FOR $b_{LL'}^{(n)}$

In this Appendix we assume that $V\equiv\alpha Z/2\rho\gg W_0$ and that $\mathfrak{M}(B_{ij}{}^{\beta})$ is large compared with the other first forbidden nuclear matrix elements. Furthermore, we make the following abbreviations:

$$iVC_T{}^{(\prime)}\mathfrak{M}(\beta\boldsymbol{\sigma}\cdot\mathbf{r})-C_P{}^{(\prime)}\mathfrak{M}(\beta\gamma_5)\equiv\xi^{(\prime)}. \tag{A29}$$

$$iVC_S{}^{(\prime)}\mathfrak{M}(\beta\mathbf{r})-C_T{}^{(\prime)}\mathfrak{M}(\beta\boldsymbol{\alpha})+VC_T{}^{(\prime)}\mathfrak{M}(\beta\boldsymbol{\sigma}\times\mathbf{r})\equiv\eta^{(\prime)}. \tag{A30}$$

$$iC_T{}^{(\prime)}\mathfrak{M}(B_{ij}{}^{\beta})\equiv\zeta^{(\prime)}. \tag{A31}$$

$$iVC_A{}^{(\prime)}\mathfrak{M}(\boldsymbol{\sigma}\cdot\mathbf{r})-C_A{}^{(\prime)}\mathfrak{M}(\gamma_5)\equiv\lambda^{(\prime)}. \tag{A32}$$

$$iVC_V{}^{(\prime)}\mathfrak{M}(\mathbf{r})-C_V{}^{(\prime)}\mathfrak{M}(\boldsymbol{\alpha})+VC_A{}^{(\prime)}\mathfrak{M}(\boldsymbol{\sigma}\times\mathbf{r})\equiv\mu^{(\prime)}. \tag{A33}$$

$$iC_A\mathfrak{M}(B_{ij})\equiv\nu^{(\prime)}. \tag{A34}$$

Here the prime in parentheses means the quantity either with or without a prime. Upon using these abbreviations and Eqs. (A1)–(A28), the $b_{LL'}^{(n)}$ for $STPVA$ are greatly simplified as follows:

$$b_{00}^{(0)}=|\xi|^2+|\xi'|^2+|\lambda|^2+|\lambda'|^2. \tag{A35}$$

$$b_{01}^{(1)}=(p/W)[\xi^*\eta'+\xi'^*\eta-\lambda^*\mu'-\lambda'^*\mu]+\text{c.c.} \tag{A36}$$

$$b_{11}^{(0)}=-\sqrt{3}[|\eta|^2+|\eta'|^2+|\mu|^2+|\mu'|^2]. \tag{A37}$$

$$b_{11}^{(1)}=\sqrt{2}(p/W)[-\eta^*\eta'+\mu^*\mu']+\text{c.c.} \tag{A38}$$

$$b_{11}^{(2)}=0. \tag{A39}$$

$$b_{02}^{(2)}=6^{-\frac{1}{2}}(p^2/W)[\xi^*\zeta+\xi'^*\zeta'+\lambda^*\nu+\lambda'^*\nu']+\text{c.c.} \tag{A40}$$

$$b_{12}^{(1)}=\tfrac{1}{2}(5/3)^{\frac{1}{2}}p[-\eta^*\zeta'-\eta'^*\zeta+\mu^*\nu'+\mu'^*\nu]+\text{c.c.} \tag{A41}$$

$$b_{12}^{(2)}=-(p^2/2W)[\eta^*\zeta+\eta'^*\zeta'+\mu^*\nu+\mu'^*\nu']+\text{c.c.} \tag{A42}$$

$$b_{12}^{(3)}=0. \tag{A43}$$

$$b_{22}^{(0)}=(5^{\frac{1}{2}}/12)(K^2+p^2)[|\zeta|^2+|\zeta'|^2+|\nu|^2+|\nu'|^2]. \tag{A44}$$

$$b_{22}^{(1)}=10^{\frac{1}{2}}[(1/24)K^2+(1/40)p^2](p/W)[\zeta^*\zeta'-\nu^*\nu']+\text{c.c.} \tag{A45}$$

$$b_{22}^{(2)}=-\tfrac{1}{12}(7/2)^{\frac{1}{2}}p^2[|\zeta|^2+|\zeta'|^2+|\nu|^2+|\nu'|^2]. \tag{A46}$$

$$b_{22}^{(3)}=\tfrac{1}{2}(10)^{-\frac{1}{2}}(p^3/W)[-\zeta^*\zeta'+\nu^*\nu']+\text{c.c.} \tag{A47}$$

$$b_{22}^{(4)}=0. \tag{A48}$$

If we assume $C_i=-C_i'$ for $i=S$, T, P and $C_j=C_j'$ for $j=V$, A, then $\xi=-\xi'$, $\eta=-\eta'$, $\zeta=-\zeta'$, $\lambda=\lambda'$, $\mu=\mu'$, and $\nu=\nu'$. In the case of STP, the terms containing λ, μ, and ν should be dropped. The above $b_{LL'}^{(n)}$ are given for electron decay. For the case of positron decay, see the last paragraph in Sec. 2.

APPENDIX 3. FIRST FORBIDDEN BETA DECAYS OF Sb^{124} AND Au^{198}

A recent experiment on the beta-gamma polarization correlation from Sc^{46} has shown that the interference between Fermi and Gamow-Teller interactions, $\text{Re}(C_S^*C_T'+C_S'^*C_T-C_V^*C_A'-C_V'^*C_A)$, has the maximum possible value.[32] This would imply that the beta interaction is invariant under time reversal. In contrast to the

[31] s_2 for $|\int\beta\mathbf{r}|\cdot|\int\beta\boldsymbol{\sigma}\times\mathbf{r}|$ in reference 7 should be read as

$$s_2=-\tfrac{1}{6}\,\text{Re}[C_SC_T^*+C_S'C_T'^*](p^2/E)[q+\xi]-\tfrac{3}{8}\,\text{Im}[C_SC_T^*+C_S'C_T'^*]Z\alpha p[(p^2/3E)+(1/9)q+\xi].$$

These authors may have dropped the L_1 term in the real part and a factor $\frac{1}{2}$ in the imaginary part.

[32] F. Boehm and A. H. Wapstra, Phys. Rev. **107**, 1202 (1957).

TABLE II. Theoretical values of the anisotropy[a] $a(W)$ and the asymmetry[b] A of beta-gamma directional and polarization correlations as functions of Y or[c] z for Sb^{124} at the electron energy $W=5mc^2$ and $\theta=150°$.

z	Y	$a(W)$	A
-343	0.04	-0.39	0.16
-86	0.16	-0.43	0.03
-55	0.25	-0.45	-0.07
-45	0.30	-0.47	-0.14
-8.8	1.55	-0.47	-0.91
-7.2	1.90	-0.43	-0.91
-6.1	2.26	-0.39	-0.89
Experimental value		-0.43 ± 0.04[d]	-0.13 ± 0.06[e]

[a] $a(W)=\{W(\pi)-W(\pi/2)\}/W(\pi/2)$.
[b] The definition of A is given Eq. (53).
[c] The quantity z is the magnitude of $\mathfrak{M}(B_{ij}{}^{\beta})$ or $\mathfrak{M}(B_{ij})$; see text.
[d] See reference 27.
[e] See reference 29.

situation with Sc^{46}, the same interference appears to be very small from the experimental data on the beta-ray angular distribution from polarized neutrons.[33,34] Similar interferences are also expected in the first-forbidden transitions of beta decay. Fortunately, we have data on the beta-gamma polarization correlations from Sb^{124} measured by Appel and Schopper,[35] and from Au^{198} by Boehm and Wapstra.[36] We shall analyze these data together with data concerning other phenomena of the same nuclei.

To reduce the numerical task, we make several assumptions: namely, $(\alpha Z)^2 \ll 1$, $\alpha Z/2\rho \gg W_0$, and real coupling constants. The beta interaction is assumed to be STP with $C_i=-C_i'$, or VA with $C_j=C_j'$, or a linear combination of these two interactions[16] with $\xi=\pm\lambda$, etc. Under the above assumptions, the asymmetry, A, of the beta-gamma polarization correlation is expressed by Eq. (57) for Au^{198} and by Eq. (58) for Sb^{124}. Furthermore, the beta-gamma directional correlation functions are equal to the denominators of Eqs. (57) and (58).

In the case of Sb^{124}, the results are shown in Table II. If we choose $Y=0.25-0.30$, the theoretical values of the asymmetry A, and the anisotropy $a(W)$, of the beta-gamma polarization and directional correlations fit the experimental data well. The parameter $z=-i\mathfrak{M}(B_{ij}{}^{\beta})/\mathfrak{M}(\beta\boldsymbol{\sigma}\times\mathbf{r})$ [or $-i\mathfrak{M}(B_{ij})/\mathfrak{M}(\boldsymbol{\sigma}\times\mathbf{r})$ for VA], is a measure of the magnitude of $\mathfrak{M}(B_{ij}{}^{\beta})$ [or $\mathfrak{M}(B_{ij})$] and is related to Y by $zY=-(\alpha Z/2\rho)$ with the assumption $x-y+1=1$. (The definition of x, y, and z was given in reference 28, where an extensive analysis was performed on the beta decay of Sb^{124}.) It is interesting to notice that the values of z listed in the first column of Table II coincide with those listed in Figs. 1–4 of reference 28; however, those listed in Figs. 1 and 3 disagree with the data given by Appel and Schopper.[29] With our choice of $Y=0.25-0.30$ we can fit the beta spectrum, ft value, and decay scheme $3^-(\beta)2^+(\gamma)0^+$, for the same reason as given in reference 28. It should be noticed that the interaction STP without $\mathfrak{M}(B_{ij}{}^{\beta})$ [or VA without $\mathfrak{M}(B_{ij})$, or a linear combination of these two] disagrees with the large anisotropy of the beta-gamma directional correlation of Sb^{124}.

In the case of Au^{198}, the decay scheme is $2^-(\beta)2^+(\gamma)0^+$. A small anisotropy of the beta-gamma directional correlation requires a relation $Y=2(\frac{2}{3})^{\frac{1}{2}}X$. The maximum asymmetry of the beta-gamma polarization correlation is $A=0.60$ at $Y=-0.33$, while $A_{\text{exp}}=0.52\pm0.09$.[26] As the allowed shape of beta spectrum of Au^{198} requires $Y\gtrsim 3$, we cannot obtain a consistent explanation with large values of $\mathfrak{M}(B_{ij}{}^{\beta})$ and $\mathfrak{M}(B_{ij})$. Therefore, we omit these matrix elements. The analysis without these terms is equivalent to that given by Boehm and Wapstra.[26] In this case, it is necessary that the interferences among STP or VA or both be maximum.[37] These interactions are also consistent with the data on the longitudinal polarization of beta particles from Au^{198}.[38]

For both Sb^{124} and Au^{198}, the asymmetry and the anisotropy of the beta-gamma polarization and directional correlations may be reduced by assuming complex coupling constants. If the coupling constants have very small phase differences, we have no hope of detecting them.

Concluding the above analysis, the beta interaction is STP with $C_i=-C_i'$, or VA with $C_j=C_j'$, or a linear combination of these two. All of the data on Au^{198} are favorable to the assumption of real coupling constants. From the data on Sb^{124}, the reality of the coupling constants is indeterminate, because of large $\mathfrak{M}(B_{ij}{}^{\beta})$ and $\mathfrak{M}(B_{ij})$ terms.

[33] Burgy, Epstein, Krohn, Novey, Raboy, Ringo, and Telegdi, Phys. Rev. **107**, 1731 (1957).
[34] The data from polarized Co^{58} do not result in unique conclusions, because the beta-ray angular distribution shows that $\mathrm{Re}(C_S{}^*C_T'+C_S'{}^*C_T-C_V{}^*C_A'-C_A'{}^*C_A)M_F/M_{GT}\approx 0$ [Ambler, Hayward, Hoppes, Hudson, and Wu, reference 20], and the beta-gamma directional correlation shows $\mathrm{Im}(C_S{}^*C_T'+C_S'{}^*C_T-C_V{}^*C_A'-C_V'{}^*C_A)M_F/M_{GT}\approx 0$ [Ambler, Hayward, Hoppes, and Hudson, Phys. Rev. **108**, 503 (1957)]. This implies that $(C_S{}^*C_T'+C_S'{}^*C_T-C_V{}^*C_A'-C_V'{}^*C_A)\approx 0$ and/or $M_F/M_{GT}\approx 0$.
[35] See reference 29. We have assumed that $W=5mc^2$ and $\theta=150°$ in their experiment.
[36] See reference 26. We have assumed that $W=2.8mc^2$ and $\cos^2\theta=\frac{1}{2}$ in their experiment.
[37] Fujita, Yamada, Matumoto, and Nakamura, Phys. Rev. **108**, 1104 (1957). They obtained a conclusion similar to ours by a theoretical analysis of the beta-ray spectrum of RaE.
[38] Benczer-Koller, Schwarzschild, Vise, and Wu, Phys. Rev. **109**, 85 (1958).