SUMMARY AND GENERAL REMARKS

The magnetization of uranium hydride and uranium deuteride has been measured as a function of temperature at constant magnetic fields and as a function of magnetic field at constant low temperatures. We have interpreted magnetization results on the basis of internal molecular fields. A strong remanence is obtained and may be associated with strong anisotropy fields (a possible result of partial quenching of the orbital angular momentum) as well as huge exchange interactions. The high saturation magnetization, 1.18 ± 0.03 Bohr magnetons per atom of uranium in UH_3, may be near the correct value and is in accord with the neutron diffraction value, which is not dependent on purity in first approximation. The lower apparent saturation magnetization for uranium deuteride may conceivably be due to change in interatomic distances from those in UH_3, but the possibility of contamination should be kept in mind, particularly since the magnetization of UO_2 is very small under the conditions for measuring saturation magnetization for the deuteride and hydride. Although the measurement of a magnetization of 0.025 Bohr magneton per atom of uranium in UO_2 at 1.3°K and 35 000 gauss can be explained in terms of antiferromagnetic ordering under a strong molecular field, some consideration should be given to a possible reduction in the intrinsic moment through splitting of the ground state by interactions similar to that in potassium ferrocyanide[32] in which application of the Pauli principle leads to restricted magnetization. In view of the small moment of UO_2, whatever the mechanism, it is possibly worthwhile to keep open the question of saturation magnetization of uranium hydride to see if an even higher value than 1.2 Bohr magnetons per atom can be determined from absolutely pure UH_3. Further work should be done to examine carefully the possibility of a real difference between the magnetization at absolute zero of uranium hydride and of uranium deuteride, as a possible difference is suggested also by unpublished work[33] which shows from specific heat measurements that the sub-Curie regions have different entropies.

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[32] W. E. Henry, Phys. Rev. **106**, 465 (1957).

[33] B. Abraham (private communication).

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Piezoresistance Constants of *P*-Type InSb†*

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The change of resistance in uniaxial tension was measured for several single-crystal specimens of *p*-type InSb over the range 77°K to 350°K. The difference between the donor and acceptor concentrations for these specimens ranges from 3×10^{15} cm^{-3} to 7×10^{17} cm^{-3}. The piezoresistance coefficients were found to depend on impurity concentration. The elasto-Hall constant was measured at 77°K for the purest specimens and the experimental results indicate that, in the extrinsic range, the large piezoresistance is primarily due to stress induced changes in the tensor mobility of the holes. The shear coefficients were found to vary approximately linearly with T^{-1} in the extrinsic range. It is concluded that the valence band extrema occur at, or very near, $\mathbf{K}=0$ in the Brillouin zone with energy surfaces similar to those of Si and Ge.

1. INTRODUCTION

PIEZORESISTANCE measurements have recently been helpful in the study of the band structure of semiconductors.[1–5] Smith[1] points out that the anisotropy of the piezoresistance effect offers information concerning the directions in **K** space along which the energy extrema lie for a simple "many-valley" semiconductor. Theoretical predictions[5–9] have been made about the temperature dependence of the various components of the piezoresistance tensor for both the "many-valley" model and the "degenerate" model. With this in mind, the present work was undertaken with the hope that the results would shed some light on the band structure of *p*-type InSb.

The piezoresistance constants of *p*-type InSb were measured as a function of temperature and impurity concentration from 77°K to 350°K. The Hall constant and resistivity were measured as a function of tempera-

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† Based on work performed at The Institute for the Study of Metals and submitted to the University of Chicago in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

* This work was supported in part by the Office of Naval Research.

[1] C. S. Smith, Phys. Rev. **94**, 42 (1954), and private communication.

[2] R. W. Keyes, Phys. Rev. **100**, 1104 (1955).

[3] R. W. Keyes, Phys. Rev. **103**, 1240 (1956).

[4] R. W. Keyes, Westinghouse Research Laboratory Scientific Paper 8-1038-P18, 1957 (unpublished).

[5] Morin, Geballe, and Herring, Phys. Rev. **105**, 525 (1957).

[6] C. Herring, Bell System Tech. J. **34**, 237 (1955).

[7] E. N. Adams, Chicago Midway Laboratories Technical Report, CML-TN-P8, 1954 (unpublished).

[8] E. N. Adams, Phys. Rev. **96**, 803 (1954).

[9] C. Herring and E. Vogt, Phys. Rev. **101**, 944 (1956).

ture so that a determination of the extrinsic temperature ranges for the samples studied could be made. In addition, the Hall voltage was measured as a function of stress at 77°K in order to separate the effect of a carrier concentration change on the resistivity from a mobility change.

The work reported here represents a refinement and extension of experiments previously published,[10] where preliminary values for the piezoresistance constants were reported which are in essential agreement with the values reported below. During the course of this investigation, it was learned that R. F. Potter was engaged in an independent measurement of the piezoresistance constants of n- and p-type InSb. Dr. Potter was kind enough to send me his results before publication. Insofar as the measurements overlap, agreement is satisfactory.

2. MACROSCOPIC THEORY

The detailed theory has been presented elsewhere[1–9] and only the main features will be outlined below for convenience. The equations presented are valid for cubic crystals with the space groups T_d, O, and O_h only.[1] The equations can be generalized to apply to other crystals.[11,12]

When a stress is applied to a crystal in which the current density is kept constant, the stress induced change in a resistivity component $\delta\rho_{ij}$ is given by[6]

$$\frac{\delta\rho_{ij}}{\rho}=\sum_{kl}\Pi^{ijkl}X^{kl}=\sum_{kl}m^{ijkl}e^{kl}. \tag{1}$$

In Eq. (1) the piezoresistance constants Π^{ijkl} and the elastoresistance constants m^{ijkl} are the components of fourth rank tensors, X^{kl} and e^{kl} are components of the stress tensor and strain tensor, respectively, and ρ is the resistivity of the unstrained crystal. Since $\delta\rho_{ij}/\rho$, X^{kl}, and e^{kl} are the components of symmetric tensors, Eq. (1) may be written as[1]

$$(\delta\rho/\rho)=(\Pi)(X)=(m)(e), \tag{2}$$

where $(\delta\rho/\rho)$, (X), and (e) are six element column matrices of the resistivity components, the stress components, and strain components, respectively, and (Π) and (m) are six by six matrices of piezoresistance constants and elastoresistance constants, respectively. The elastoresistance matrix can be determined from the (Π) matrix and the known elastic constants[13,1] of InSb. There is a one to one correspondence between the piezoresistance matrix elements Π_{ij} and the piezoresistance tensor components Π^{ijkl}. The correspondence is completely analogous to that existing between the elastic constant tensor components c^{ijkl} and the elastic constant matrix elements c_{ij} which is discussed in detail elsewhere.[14]

When the reference axes coincide with the cubic axes, the Π matrix reduces to a simple form involving only three independent quantities, Π_{11}, Π_{12}, and Π_{44}.[1] When the reference axes are not the cubic axes, the elements Π_{ij} will be given as a linear combination of Π_{11}, Π_{12}, and Π_{44}. These three coefficients were determined by combining longitudinal measurements with transverse measurements.[1] The orientations employed for the longitudinal measurements and the pertinent equations are identical with those described by Burns and Fleischer.[15] The orientations used for the transverse measurements are similar to those employed by Smith.[1] The piezoresistance effect is reported as fractional change in resistivity per unit stress $1/X\ \delta\rho/\rho$. This was obtained from the measured fractional change of resistance by correcting for the dimensional change.[1,15] For the case of constant current, $\delta R/R=\delta V/V$ where R is the resistance and V the voltage between the potential leads.[1] For the transverse measurements, an additional correction was applied. This correction arises from the "spreading" of the lines of current in transverse geometry.[1] It is also possible to use longitudinal measurements combined with a hydrostatic pressure measurement as the two independent types of measurements.[1,15] This method was used in a preliminary report of some of the results of this paper.[10]

The magnetic measurements consisted of measuring the change in Hall voltage due to stress. To determine the change in hole concentration in the extrinsic range due to stress, the measurements had to be carried out at 77°K, where the electron concentration is negligible. Assuming only one kind of hole, the Hall constant is[16]

$$R\equiv E_H/jH=r/ep, \tag{3}$$

where e is the electronic charge, p is the hole concentration, E_H is the Hall electric field, j is the current density, and H is the magnetic field. The quantity r is of the order of unity and depends on the scattering mechanisms, the statistics of the carriers, and the detailed structure of the valence band near the band extremum.[16] The quantities E_H, j, and H are mutually

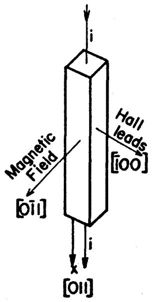

FIG. 1. Electrode arrangement for the elasto-Hall effect measurements.

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[10] A. J. Tuzzolino, Phys. Rev. **105**, 1411 (1957).

[11] R. W. Keyes, Westinghouse Research Laboratory Scientific Paper 8-1038-P12, 1956 (unpublished).

[12] R. W. Keyes, Phys. Rev. **104**, 665 (1956).

[13] R. F. Potter, Phys. Rev. **103**, 47 (1956).

[14] W. P. Mason, *Piezoelectric Crystals and Their Application to Ultrasonics* (D. Van Nostrand Company, Inc., Princeton, 1950).

[15] F. P. Burns and A. A. Fleischer, Phys. Rev. **107**, 1281 (1957).

[16] H. Brooks, Advances in Electronics **7**, 85 (1955).

perpendicular (see Fig. 1). From Eq. (3) we have, neglecting dimensional changes,

$$\frac{1}{X}\frac{\delta E_H}{E_H}=\frac{1}{X}\frac{\delta V_H}{V_H}=\frac{1}{X}\frac{\delta r}{r}-\frac{1}{X}\frac{\delta p}{p}, \tag{4}$$

where X is the stress and V_H is the Hall voltage. In general, $(1/X)(\delta r/r)$ may be a function of the directions of the stress and magnetic field with respect to the cubic axes; therefore, if the elasto-Hall effect $(1/X)(\delta V_H/V_H)$ is of the order of magnitude of the piezoresistance effect $(1/X)(\delta\rho/\rho)$, one does not know whether the former is caused by a change in r or by a change in p. On the other hand, if $(1/X)(\delta V_H/V_H)\ll(1/X)(\delta\rho/\rho)$, it is reasonably certain that both $\delta r/r$ and $\delta p/p$ are small, since it is very unlikely that both are large and cancel one another. The magnetic field and stress were oriented with respect to the cubic axes so that no transverse voltage would appear in the direction of the Hall electric field as a result of the piezoresistance effect alone, i.e., a set of axes which would correspond to the principal axes of the resistivity tensor after stressing the crystal were chosen. The electrode arrangement for the elasto-Hall measurements is shown in Fig. 1.

3. EXPERIMENTAL TECHNIQUE

The single crystal ingots of InSb were obtained from the semiconductor group of the Chicago Midway Laboratories. The InSb was prepared from purified In and Sb. This polycrystalline material was zone-refined and three single crystal ingots were grown from melts doped with Cd, which is known to introduce acceptor levels in InSb.[17] Laue back-reflection x-ray pictures showed that the axis of growth was within 5° of the [111] axis in all cases. Samples were cut from the ingots in the form of rectangular rods of approximate dimensions 2 mm×2 mm×20 mm, with the long axes in the desired directions. The cutting was performed with an ultrasonic cutting tool. Samples taken from each of these ingots were cut from adjacent slices of the ingot to keep the resistivity variation between the samples to within 30%. These three groups of specimens will be referred to as samples A, B, and C from here on (see Table I).

The orientation of each specimen was redetermined by means of an x-ray picture and the orientation error was found to be about 1° in most cases. Some of the samples were etched to study the effect of surface conditions on the experimental results. In all cases, the piezoresistance constants for the etched samples agree to within experimental error (see Sec. 4) with the results for the unetched specimens.

TABLE I. Electrical properties of samples A, B, and C at 77°K.

Samples	Resistivity (ohm-cm)	N_A-N_D (cm^{-3})	Mobility (cm^2 volt^{-1} sec^{-1})
A	0.54	3.0×10^{15}	3.8×10^3
B	0.057	6.3×10^{16}	1.8×10^3
C	0.010	6.6×10^{17}	0.91×10^3

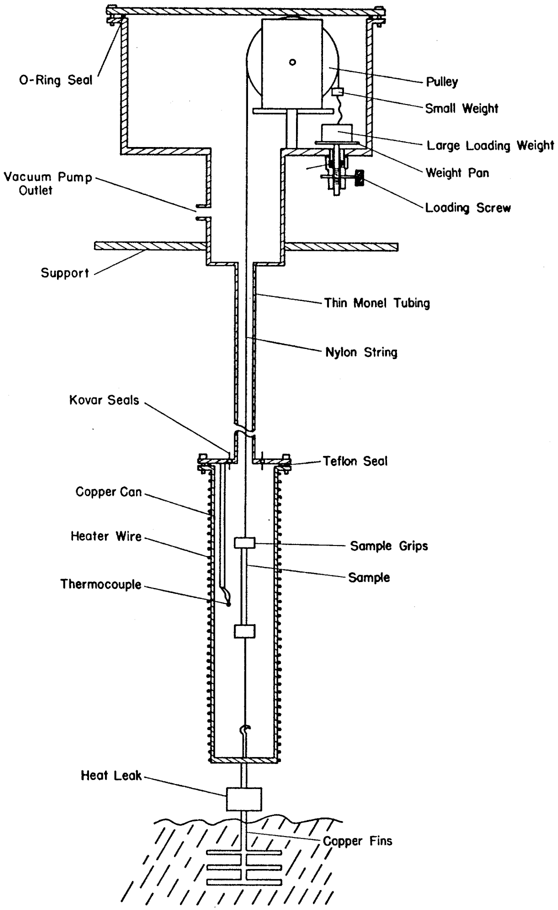

FIG. 2. Cryostat for low-temperature measurements.

The Hall, potential, and current leads were soldered directly to the specimen with indium. The crystals were mounted in small metal grips (see Fig. 2) with sealing wax. The mounting of the samples was done manually and consequently, when the tensile stress was applied, a small amount of shear stress was imposed on the crystal whenever the alignment of grips and specimen was not perfect. All samples were remounted several times and a measurement taken at each mounting. The results agreed in all cases to within experimental error, indicating that the error introduced by misalignment was negligible. The electrical arrangement for the piezoresistance measurements was similar to that employed by Smith.[1] The Hall coefficient and resistivity were measured in the conventional manner.[18] For the low temperature measurements a cryostat shown in Fig. 2 was used. By adjusting the current in the heater

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[17] K. F. Hulme and J. B. Mullin, J. Electronics and Control **3**, 160 (1957).

[18] See, for example, G. L. Pearson and J. Bardeen, Phys. Rev. **75**, 865 (1949).

TABLE II. Piezo- and elastoresistance constants for samples A, B, and C at 77°K. The Π constants are expressed in units of 10^{-12} cm^2 dyne^{-1} while the m constants are dimensionless.

Samples	Π_{11}	Π_{12}	$\Pi_{11}+2\Pi_{12}$	$\Pi_{11}-\Pi_{12}$	Π_{44}	m_{11}	m_{12}	$(m_{11}+2m_{12})/3$	$(m_{11}-m_{12})/2$	m_{44}
A	96	−46	4	142	424	33	−14	1.7	23.5	133
B	93	−44	5	137	365	32	−13	2.0	22.5	115
C	41	−18	5	59	241	15	−4	2.3	9.5	76

windings and the liquid level, a temperature range from 77°K to 350°K was available. The cryostat was mounted between the poles of an electromagnet for the elasto-Hall measurements. Precautions were taken to keep the sample from rotating in the magnetic field upon applying the stress.

4. ERRORS

The temperature was measured with a copper-constantan thermocouple which was standardized in the conventional manner.[19] The error in temperature was less than 1°K throughout the temperature range employed. The error in the assumed magnitude of the stress, introduced by frictional effects in the pulley system, was measured and found to be negligible. The Π constants were found to be independent of stress throughout the stress range employed, which was from 10^6 to 10^7 (dynes/cm^2). For the longitudinal measurements, δV could be measured with a precision of 1–2% at the lower temperatures, where the resistivity was high, and about 5% at the higher temperatures, where the resistivity was quite small. For the transverse measurements, the errors in δV were much larger at all temperatures because of the small transverse voltages compared to the corresponding large longitudinal voltages. The error in δV was about 5% at the low temperatures and about 20% at the higher temperatures.

To investigate the effect of an impurity concentration gradient along the length of the samples, the Hall constant was measured as a function of position along the specimen length for two different samples. For a [111] specimen from samples A, it was found that the impurity concentration varied by approximately 60% in a distance equal to the distance between the potential leads. From Table II, one sees that the Π constants vary by approximately 10% between samples A and B. Furthermore, the impurity concentration of samples B is greater than that of samples A by a factor of approximately 20. Therefore, a concentration gradient of 60% introduces a negligible error in the Π constants for samples A. For a [110] specimen from samples B, the concentration was found to vary by approximately 30% in a distance equal to the distance between the potential leads. A corresponding argument applied to samples B and C reveals that the existence of a concentration gradient of this order of magnitude may lead to an error in the Π constants for these samples of about 3%.

In the magnetic measurements, fluctuations in the magnetic field limited the sensitivity of the elasto-Hall measurements to 3 μv. Because of this uncertainty in δV_H, only an upper limit can be quoted for the elasto-Hall constant. Fortunately, such a limit suffices for our purposes.

5. RESULTS

Figures 3 and 4 present the Hall constant and resistivity for samples A, B, and C. The Hall coefficient shows a reversal of sign at a temperature of about 161°K, 208°K, and 286°K for samples A, B, and C, respectively. All three samples are p-type. Table I lists the excess concentration of acceptors over donors N_A-N_D and the mobility of the holes μ_p for samples A, B, and C at 77°K. These quantities were determined from the Hall coefficient and resistivity by using the formulas[16]

$$N_A-N_D=r/eR,\qquad \mu_p=R\sigma/r, \tag{5}$$

and assuming that $r=3\pi/8$. The uncertainty in r due to the complicated structure of the valence band and

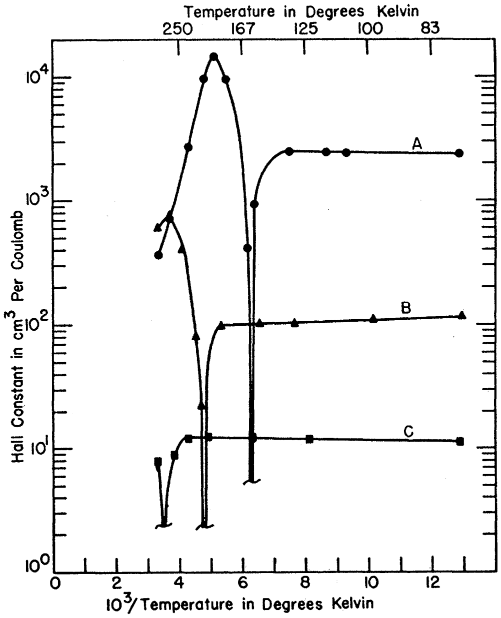

FIG. 3. Hall constant as a function of reciprocal temperature for samples A, B, and C.

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[19] R. B. Scott, *Temperature Its Measure and Control in Science and Industry* (Reinhold Publishing Corporation, New York, 1952).

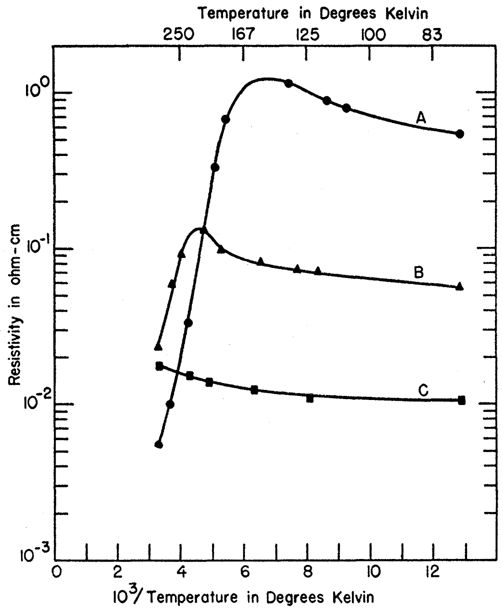

FIG. 4. Resistivity as a function of reciprocal temperature for samples *A*, *B*, and *C*.

the various scattering mechanisms may introduce an error in $N_A - N_D$ and μ_p as large as 50%. In addition, the concentration gradient along the length of the samples, discussed in Sec. (4), increases the uncertainty in the quantities $N_A - N_D$ to about a factor of two. However, these large uncertainties do not affect the conclusions which may be drawn from the experimental results for the Π constants, since much larger concentration difference between the groups of samples will be compared. The resistivity and Hall coefficient curves shown in Figs. 3 and 4 are in satisfactory agreement with published results for *p*-type InSb.[20–24] A comparison of the results of a study of the Hall mobility of holes in *p*-type InSb as a function of impurity concentration[25] and the mobilities given in Table I, shows that samples *A* are compensated, with $N_A + N_D \approx 3 \times 10^{16}$ cm^{-3}. A similar comparison for samples *B* and *C* shows that they are essentially uncompensated.

Figures 5–7 present Π_{11} and Π_{12} as a function of temperature for samples *A*, *B*, and *C*, respectively. Figure 8 shows the pressure coefficients as a function of temperature and Figs. 9 and 10 show the shear coefficients $\Pi_{11} - \Pi_{12}$ and Π_{44}, respectively. Table II presents the piezoresistance and elastoresistance constants at 77°K.

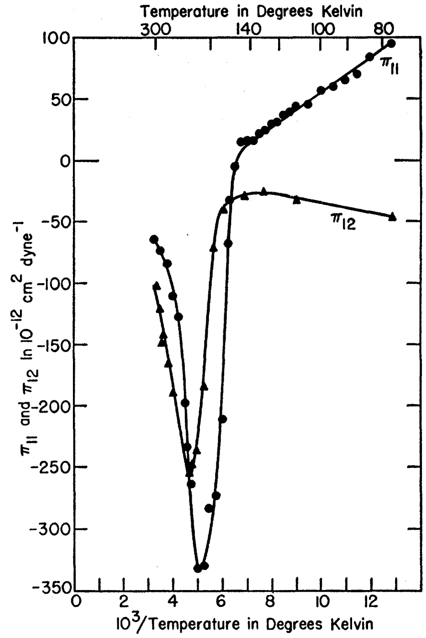

FIG. 5. Π_{11} and Π_{12} as a function of reciprocal temperature for samples *A*.

6. DISCUSSION OF RESULTS

Figures 5, 6, and 7 show that Π_{11} and Π_{12} are impurity dependent. However, the curves of Π_{11} as well as those of Π_{12} show a variation with temperature that is similar for samples *A*, *B*, and *C*. The behavior of Π_{11} and Π_{12} from 77°K to approximately 150°K, 200°K, and 250°K for samples *A*, *B*, and *C* respectively, is characteristic of the extrinsic material (see Figs. 3 and 4). In the extrinsic range, Π_{11} decreases and Π_{12} increases with increasing temperature. In the mixed conduction range, Π_{11} and Π_{12} for samples *A* and *B* vary with temperature in the same manner, both passing through a minimum at approximately the same

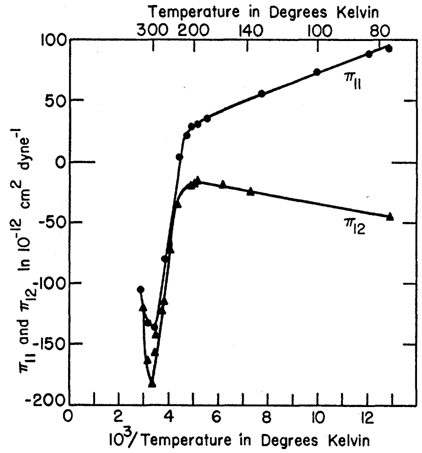

FIG. 6. Π_{11} and Π_{12} as a function of reciprocal temperature for samples *B*.

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[20] M. Tanenbaum and J. P. Maita, Phys. Rev. **91**, 1009 (1953).

[21] Hrostowski, Morin, Geballe, and Wheatley, Phys. Rev. **100**, 1672 (1955).

[22] H. Fritzsche and K. Lark-Horovitz, Phys. Rev. **99**, 400 (1955).

[23] Breckenridge, Blunt, Hosler, Frederikse, Becker, and Oshinsky, Phys. Rev. **96**, 571 (1954).

[24] H. Welker, Physica **20**, 893 (1954).

[25] R. N. Zitter and A. J. Strauss, Chicago Midway Laboratories (private communication).

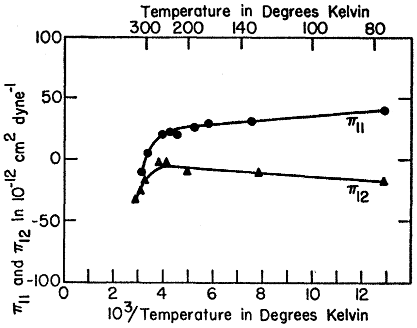

FIG. 7. Π_{11} and Π_{12} as a function of reciprocal temperature for samples C.

temperature. For samples A, the pressure coefficient $\Pi_{11}+2\Pi_{12}$ decreases and becomes negative immediately upon raising the temperature about 77°K. No significance is attached to this initial decrease. $\Pi_{11}+2\Pi_{12}$ for samples B and C remains practically constant in the extrinsic range, as is to be expected.[1]

The variation of $\Pi_{11}+2\Pi_{12}$ for samples A and B in the range of mixed conduction is similar to that for Π_{11} and Π_{12}. The pressure coefficient passes through a minimum at approximately the same temperature at which the minimum for Π_{11} and Π_{12} occurs. If it is assumed that the large pressure effect found in the mixed conduction range can be ascribed to the effect of the lattice dilatation on the energy gap, the following expression may be derived for the change of resistivity due to a pressure induced volume change,[26]

$$\frac{\delta \ln\rho}{\delta \ln V}=\frac{\delta E_g}{\delta \ln V}\frac{np}{kT}\frac{(\mu_n+\mu_p)}{(n\mu_n+p\mu_p)}\frac{1}{(n+p)}, \tag{6}$$

where n and p are the electron and hole concentrations, μ_n and μ_p are the electron and hole mobilities, V is the volume, k is the Boltzmann constant, T is the absolute temperature, and E_g is the energy gap. It is assumed in the above derivation that the energy gap is the only pressure dependent quantity. It is convenient to rewrite this expression in the form employed by Potter,[27] which shows explicitly the dependence of the pressure effect on the quantity N_A-N_D. Equation (6) may be written

$$\Pi_{11}+2\Pi_{12}=\left(\frac{3}{c_{11}+2c_{12}}\right)\frac{\delta E_g}{\delta \ln V}\frac{(a-1)}{2kTa}\times\left\{\frac{1+(\mu_p/\mu_n)}{(a-1)/(a+1)+(\mu_p/\mu_n)}\right\}, \tag{7}$$

where the c_{ij} are the elastic constants of InSb, and

$$a=\left[1+\frac{4np}{(N_A-N_D)^2}\right]^{\frac{1}{2}}.$$

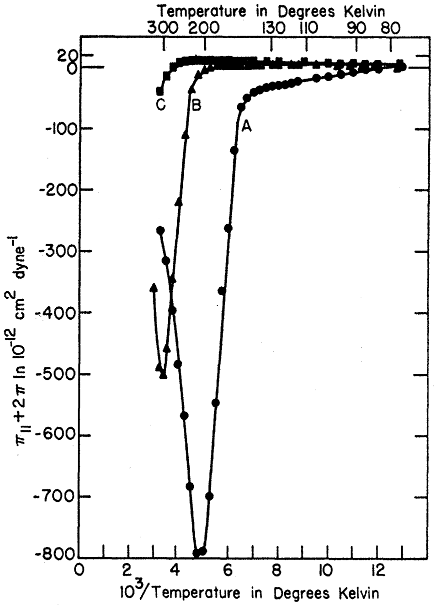

FIG. 8. Pressure coefficient, $\Pi_{11}+2\Pi_{12}$ as a function of reciprocal temperature for samples A, B, and C.

The above equation possesses a minimum at a temperature determined by the impurity concentration N_A-N_D. The minimum shifts to higher temperatures as the quantity N_A-N_D increases. The experimental results for Π_{11}, Π_{12}, and $\Pi_{11}+2\Pi_{12}$ are in qualitative agreement with these facts and therefore, it is reasonable to assume, with Potter,[27] that in the range of mixed conduction, the dominant effect of stress on the resistivity is to change the number of carriers by changing the band gap.

The behavior of the shear constant $\Pi_{11}-\Pi_{12}$ (shown

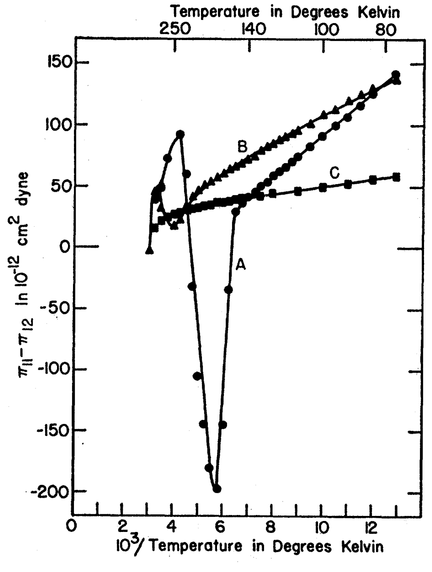

FIG. 9. $\Pi_{11}-\Pi_{12}$ as a function of reciprocal temperature for samples A, B, and C.

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[26] J. H. Taylor, Phys. Rev. **80**, 919 (1950).

[27] R. F. Potter, National Bureau of Standards Report No. 5292, 1957 (unpublished).

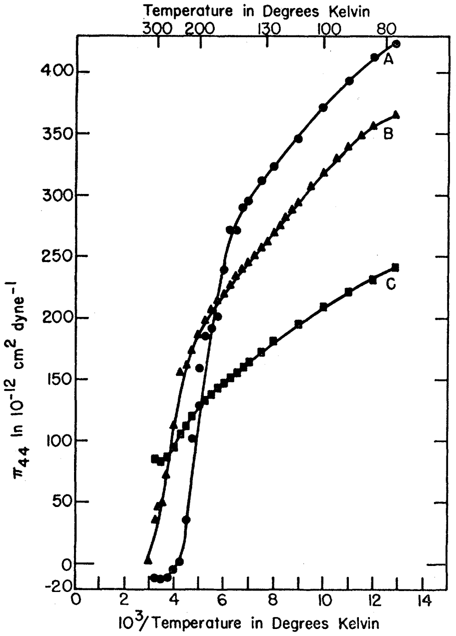

FIG. 10. Π_{44} as a function of reciprocal temperature for samples A, B, and C.

in Fig. 9) is similar for all three samples. In each case, $\Pi_{11}-\Pi_{12}$ varies linearly with T^{-1} throughout the extrinsic range. In the region of mixed conduction, $\Pi_{11}-\Pi_{12}$ for samples A and B passes through a minimum and then rises and passes through a maximum with increasing temperature. As was mentioned in Sec. 4, at these higher temperatures, where the maxima and minima occur for samples A and B, the experimental errors are large because of the small values of δV and consequently, the respective magnitudes of the maxima and minima are uncertain, but are much larger than the experimental error.

The shear constant Π_{44} shown in Fig. 10 for samples A, B, and C, decreases approximately linearly with T^{-1} in the extrinsic range and continues decreasing throughout the remainder of the temperature range investigated. The energy gap effect described above gives a zero contribution to both shear coefficients.

The curves for Π_{11}, Π_{12}, $\Pi_{11}+2\Pi_{12}$, and the shear coefficients for samples A are in qualitative agreement with the concurrent and overlapping measurements of Potter,[27] which are in the process of publication. His p-type samples have electrical properties similar to those of samples A. He also finds a minimum for $\Pi_{11}-\Pi_{12}$, although it is not as large in magnitude as that found for samples A. He finds a linear relationship between both shear coefficients and $1/T$ in the extrinsic range, which is approximately from 77°K to 150°K for his samples, just as for samples A. Although the shear coefficient $\Pi_{11}-\Pi_{12}$ for samples A, B, and C varies linearly with $1/T$ in the extrinsic range, it is evident from Fig. 10 that the measured points for the shear coefficient Π_{44} show deviations from a straight line.

From the results of the elasto-Hall effect measurements, it was found that the upper limit for $(1/X)(\delta p/p)$ is less than 10% of the piezoresistance effect $(1/X)(\delta\rho/\rho)$; therefore, the change in the number of carriers caused by a change in the band gap or in the ionization energy of the acceptor impurities is negligible in the extrinsic range and the large piezoresistance can be ascribed to a stress induced change in the tensor mobility of the holes. To hope to understand the large magnitudes of the Π constants at 77°K and their temperature and impurity dependence in the extrinsic range, we must consider the details of the valence band structure of InSb.

Recent investigations of the valence-band of InSb suggest that it is similar to the valence bands of Si and Ge.[21,28–31] The variation of Hall coefficient with magnetic field which Hrostowski *et al.*[21] finds for p-type InSb is interpreted in terms of light and heavy holes. The magneto-resistance data of Frederikse and Hosler[28] also indicates that two kinds of holes contribute to the electrical conduction.

Kane[29] calculates the band structure of InSb by using the $\mathbf{K}\cdot\mathbf{P}$ perturbation method and assuming that the conduction and valence band extrema are at $\mathbf{K}=0$. He finds that the heavy hole band has maxima located on the $\langle 111\rangle$ axes, very close to $\mathbf{K}=0$. These $\langle 111\rangle$ maxima are probably important only at liquid helium temperatures. At liquid nitrogen temperatures, the extrema may be assumed to occur at $\mathbf{K}=0$ and the band structure will be quite similar to that of germanium. Kane[29] calculates the optical absorption constant and finds good agreement with the experimental data of Fan and Gobeli.[30] This agreement supports his original assumption concerning the band structure.

Dumke[31] analyzed the absorption edge data of Roberts and Quarrington[32] by extending the theory of indirect transitions to the case where both the valence band and the conduction band extrema are at $\mathbf{K}=0$. The results of his analysis are consistent with the experimental data.

On the basis of the symmetry properties of the zincblende structure, Dresselhaus[33] suggests several points in the Brillouin zone as likely points for the occurrence of the energy extrema. By comparing the relative magnitudes of the elastoresistance constants in the extrinsic range with the magnitudes predicted for various band models by theory,[6,7,34] it was concluded earlier[10] that the valence band extrema of InSb most

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likely occur either at $\mathbf{K}=0$, with the energy surfaces similar to those of Si and Ge, or at a general point in the zone.

It is of interest to compare the results shown in Table II, and the temperature dependence found for the shear constants, with the same quantities measured for p-type Si and Ge, where the large piezoresistance effect is assumed to be caused by a large change in mobility of the holes due to stress.[7,8] From Table II, one sees that m_{44} is large and about six times are large as $(m_{11}-m_{12})/2$. Both m_{44} and $(m_{11}-m_{12})/2$ are much larger than $(m_{11}+2m_{12})/3$. It is also seen that the constants are strongly dependent on impurity concentration. Smith[1] found similar relationships between the relative magnitudes of the shear and pressure coefficients for p-type Si and Ge. In addition, he found the constants to be impurity dependent. Morin *et al.*[5] measured the Π constants of high purity n- and p-type Si and Ge as a function of temperature and found the constants for the p-type material to depend strongly on impurity concentration, with some tendency to depend linearly on reciprocal temperature in the extrinsic range. Morin *et al.*[5] suggest that the strong dependence of the shear constants on purity for p-type Si and Ge may be due to the different contributions to the piezoresistance of the light and heavy holes, with the light holes having a greater sensitivity to impurity scattering. On the other hand, for InSb, the very large change in the constants (see Table II) in going from samples B to C, in contrast to the relatively small change in going from samples A to B, suggests that the effects of degeneracy may be more important. Samples A and B are nondegenerate at 77°K, whereas samples C are degenerate.

Adams'[7] theory of the piezoresistance of p-type Si and Ge, which assumes classical statistics for the carriers and a strong strain induced warping of the energy surfaces, predicts that $(m_{11}+2m_{12})/3$ should be small compared to m_{44} and that both m_{44} and $(m_{11}-m_{12})/2$ should be proportional to T^{-1} in the extrinsic range. The magnitudes and temperature dependence found for the shear constants of InSb in the extrinsic range are reasonably consistent with these predictions. It should be pointed out that the theory of the elastoresistance constants for a simple "many-valley" semiconductor[6,7] also predicts a T^{-1} temperature dependence for the shear constants; therefore, the temperature dependence cannot distinguish between the two models. However, when both the magnitudes and temperature dependence of the shear constants are compared with the recent theoretical and experimental investigations on InSb, as discussed in the foregoing, we conclude that the valence band of InSb is quite similar to that of Si and Ge, with modifications as suggested by Kane's work.

7. CONCLUSIONS

The results of this experiment may be summarized as follows: In the extrinsic range, a change in carrier concentration caused by a change in band gap or a change in acceptor ionization energy is negligible and the large piezoresistance is caused by a large change in the tensor mobility of the holes. From the relative magnitudes of the shear and pressure coefficients at 77°K, it is concluded that the energy extrema of the valence band of InSb most likely occur at, or in the immediate vicinity of $\mathbf{K}=0$, with the surfaces of constant energy resembling those for the valence bands of Si and Ge. The temperature dependence found for the shear coefficients in the extrinsic range is not inconsistent with that predicted by Adams' theory, which is based on the valence band structure of Ge.

An explanation for the strong impurity dependence of the Π constants at 77°K is not available at present. An analysis of the large pressure effect found in the transition range of conduction indicates that this large effect can be explained in terms of the effect of lattice dilatation on the forbidden energy gap.

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