Theory of the Fermi Interaction

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The representation of Fermi particles by two-component Pauli spinors satisfying a second order differential equation and the suggestion that in β decay these spinors act without gradient couplings leads to an essentially unique weak four-fermion coupling. It is equivalent to equal amounts of vector and axial vector coupling with two-component neutrinos and conservation of leptons. (The relative sign is not determined theoretically.) It is taken to be "universal"; the lifetime of the μ agrees to within the experimental errors of 2%. The vector part of the coupling is, by analogy with electric charge, assumed to be not renormalized by virtual mesons. This requires, for example, that pions are also "charged" in the sense that there is a direct interaction in which, say, a π^0 goes to π^- and an electron goes to a neutrino. The weak decays of strange particles will result qualitatively if the universality is extended to include a coupling involving a Λ or Σ fermion. Parity is then not conserved even for those decays like $K\rightarrow 2\pi$ or 3π which involve no neutrinos. The theory is at variance with the measured angular correlation of electron and neutrino in He^6, and with the fact that fewer than 10^{-4} pion decay into electron and neutrino.

THE failure of the law of reflection symmetry for weak decays has prompted Salam, Landau, and Lee and Yang[1] to propose that the neutrino be described by a two-component wave function. As a consequence neutrinos emitted in β decay are fully polarized along their direction of motion. The simplicity of this idea makes it very appealing, and considerable experimental evidence is in its favor. There still remains the question of the determination of the coefficients of the scalar, vector, etc., couplings.

There is another way to introduce a violation of parity into weak decays which also has a certain amount of theoretical *raison d'être*. It has to do with the number of components used to describe the electron in the Dirac equation,

$$(i\boldsymbol{\nabla}-\mathbf{A})\psi=m\psi. \tag{1}$$

Why must the wave function have four components? It is usually explained by pointing out that to describe the electron spin we must have two, and we must also represent the negative-energy states or positrons, requiring two more. Yet this argument is unsatisfactory. For a particle of spin zero we use a wave function of only one component. The sign of the energy is determined by how the wave function varies in space and time. The Klein-Gordon equation is second order and we need both the function and its time derivative to predict the future. So instead of two components for spin zero we use one, but it satisfies a second order equation. Initial states require specification of that one and its time derivative. Thus for the case of spin $\frac{1}{2}$ we would expect to be able to use a simple two-component spinor for the wave function, but have it satisfy a second order differential equation. For example, the wave function for a free particle would look like $U\exp[-i(Et-\mathbf{P}\cdot\mathbf{x})]$, where U has just the two components of a Pauli spinor and whether the particle refers to electron or positron depends on the sign of E in the four-vector $p_\mu=(E,\mathbf{P})$.

In fact it is easy to do this. If we substitute

$$\psi=\frac{1}{m}(i\nabla-\mathbf{A}+m)\chi \tag{2}$$

in the Dirac equation, we find that χ satisfies

$$(i\boldsymbol{\nabla}-\mathbf{A})^2\chi=[(i\nabla_\mu-A_\mu)\cdot(i\nabla_\mu-A_\mu) -\tfrac{1}{2}\sigma_{\mu\nu}F_{\mu\nu}]\chi=m^2\chi, \tag{3}$$

where $F_{\mu\nu}=\partial A_\nu/\partial x_\mu-\partial A_\mu/\partial x_\nu$ and $\sigma_{\mu\nu}=\frac{1}{2}i(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu)$. Now we have a second order equation, but χ still has four components and we have twice as many solutions as we want. But the operator $\gamma_5=\gamma_x\gamma_y\gamma_z\gamma_t$ commutes with $\sigma_{\mu\nu}$; therefore there are solutions of (3) for which $i\gamma_5\chi=\chi$ and solutions for $i\gamma_5\chi=-\chi$. We may select, say, the first set. We always take

$$i\gamma_5\chi=\chi. \tag{4}$$

Then we can put the solutions of (3) into one-to-one correspondence with the Dirac equation (1). For each ψ there is a unique χ; in fact we find

$$\chi=\tfrac{1}{2}(1+i\gamma_5)\psi \tag{5}$$

by multiplying (2) by $1+i\gamma_5$ and using (4). The function χ has really only two independent components. The conventional ψ requires knowledge of both χ and its time derivative [see Eq. (2)]. Further, the six $\sigma_{\mu\nu}$ in (3) can be reduced to just the three σ_{xy}, σ_{yz}, σ_{zx}. Since $\sigma_{zt}=i\gamma_z\gamma_t=i\sigma_{xy}\cdot i\gamma_5$, Eq. (4) shows that σ_{zt} may be replaced by $i\sigma_{xy}$ when operating on χ as it does in (3).

Let us use the representation

$$\gamma_t=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix},\quad \boldsymbol{\gamma}=\begin{pmatrix}0 & \boldsymbol{\sigma}\\ -\boldsymbol{\sigma} & 0\end{pmatrix},\quad i\gamma_5=-\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix},$$

where $\sigma_{x,y,z}$ are the Pauli matrices. If

$$\psi=\begin{pmatrix}a\\ b\end{pmatrix},$$

[1] A. Salam, Nuovo cimento **5**, 299 (1957); L. Landau, Nuclear Phys. **3**, 127 (1957); T. D. Lee and C. N. Yang, Phys. Rev. **105**, 1671 (1957).

where a, b are two-component spinors, we find from (5) that

$$\chi=\begin{pmatrix}\varphi\\ -\varphi\end{pmatrix},$$

where $\varphi=\frac{1}{2}(a-b)$. Our Eq. (3) for the two-component spinor φ is

$$[(i\nabla_\mu-A_\mu)^2+\boldsymbol{\sigma}\cdot(\mathbf{B}+i\mathbf{E})]\varphi=m^2\varphi, \tag{6}$$

where $B_x=F_{yz}$, $E_x=F_{tx}$, etc., which is the equation we are looking for.

Rules of calculation for electrodynamics which involve only the algebra of the Pauli matrices can be worked out on the basis of (6). They, of course, give results exactly the same as those calculated with Dirac matrices. The details will perhaps be published later.

One of the authors has always had a predilection for this equation.[2] If one tries to represent relativistic quantum mechanics by the method of path integrals, the Klein-Gordon equation is easily handled, but the Dirac equation is very hard to represent directly. Instead, one is led first to (3), or (6), and from there one must work back to (1).

For this reason let us imagine that (6) had been discovered first, and (1) only deduced from it later. It would make no difference for any problem in electrodynamics, where electrons are neither created nor destroyed (except with positrons). But what would we do if we were trying to describe β decay, in which an electron is created? Would we use a field operator ψ directly in the Hamiltonian to represent the annihilation of an electron, or would we use φ? Now everything we can do one way, we can represent the other way. Thus if ψ were used it could be replaced by

$$\frac{1}{m}(p-A+m)\begin{pmatrix}\varphi\\ -\varphi\end{pmatrix}, \tag{a}$$

while an expression in which φ was used could be rewritten by substituting

$$\tfrac{1}{2}(1+i\gamma_5)\psi. \tag{b}$$

If φ were really fundamental, however, we might be prejudiced against (a) on the grounds that gradients are involved. That is, an expression for β coupling which does not involve gradients from the point of view of ψ, does from the point of view of φ. So we are led to suggest *φ as the field annihilation operator to be used in β decay without gradients*. If φ is written as in (b), we see this does not conserve parity, but now we know that that is consistent with experiment.

For this reason one of us suggested the rule[3] that the electron in β decay is coupled directly through φ, or, what amounts to the same thing, in the usual four-particle coupling

$$\textstyle\sum_i C_i(\bar\psi_n O_i\psi_p)(\bar\psi_\nu O_i\psi_e), \tag{7}$$

we always replace ψ_e by $\frac{1}{2}(1+i\gamma_5)\psi_e$.

One direct consequence is that the electron emitted in β decay will always be left-hand polarized (and the positron right) with polarization approaching 100% as $v\to c$, irrespective of the kind of coupling. That is a direct consequence of the projection operator

$$a=\tfrac{1}{2}(1+i\gamma_5).$$

A priori we could equally well have made the other choice and used

$$\bar a=\tfrac{1}{2}(1-i\gamma_5);$$

electrons emitted would then be polarized to the right. We appeal to experiment[4] to determine the sign. Notice that $a^2=a$, $\bar a a=0$.

But now we go further, and suppose that the same rule applies to the wave functions of all the particles entering the interaction. We take for the β-decay interaction the form

$$\textstyle\sum C_i(\overline{a\psi_n}O_i a\psi_p)(\overline{a\psi_\nu}O_i a\psi_e),$$

and we should like to discuss the consequences of this hypothesis.

The coupling is now essentially completely determined. Since $\overline{a\psi}=\bar\psi\bar a$, we have in each term expressions like $\bar a O_i a$. Now for S, T, and P we have O_i commuting with γ_5 so that $\bar a O_i a=O_i\bar a a=0$. For A and V we have $aO_ia=O_ia^2=O_ia$ and the coupling survives. Furthermore, for axial vector $O_i=i\gamma_\mu\gamma_5$, and since $i\gamma_5 a=a$, we find $O_ia=\gamma_\mu a$; thus A leads to the same coupling as V:

$$(8)^{\frac{1}{2}}G(\bar\psi_n\gamma_\mu a\psi_p)(\bar\psi_\nu\gamma_\mu a\psi_e), \tag{8}$$

the most general β-decay interaction possible with our hypothesis.[5]

This coupling is not yet completely unique, because our hypothesis could be varied in one respect. Instead of dealing with the neutron and proton, we could have made use of the antineutron and antiproton, considering them as the "true particles." Then it would be the wave function $\psi_{\bar n}$ of the antineutron that enters with the factor a. We would be led to

$$(8)^{\frac{1}{2}}G(\bar\psi_{\bar p}\gamma_\mu a\psi_{\bar n})(\bar\psi_\nu\gamma_\mu a\psi_e). \tag{9}$$

This amounts to the same thing as

$$(8)^{\frac{1}{2}}G(\bar\psi_n\gamma_\mu\bar a\psi_p)(\bar\psi_\nu\gamma_\mu a\psi_e), \tag{9'}$$

and from the *a priori* theoretical standpoint is just as good a choice as (8).

We have assumed that the neutron and proton are

[2] R. P. Feynman, Revs. Modern Phys. **20**, 367 (1948); Phys. Rev. **84**, 108 (1951).

[3] R. P. Feynman, *Proceedings of the Seventh Annual Rochester Conference on High-Energy Nuclear Physics, 1957* (Interscience Publishers, Inc., New York, 1957).

[4] See, for example, Boehm, Novey, Barnes, and Stech, Phys. Rev. **108**, 1497 (1957).

[5] A universal V, A interaction has also been proposed by E. C. G. Sudarshan and R. E. Marshak (to be published).

either both "particles" or both "antiparticles." We have defined the electron to be a "particle" and the neutrino must then be a particle too.

We shall further assume the interaction "universal," so for example it is

$$(8)^{\frac{1}{2}}G(\bar{\psi}_\mu\gamma_\mu a\psi_\nu)(\bar{\psi}_\nu\gamma_\mu a\psi_e) \tag{10}$$

for μ decay, as currently supposed; the μ^- is then a particle. Here the other choice, that the μ^- is an antiparticle, leads to $(8)^{\frac{1}{2}}G(\bar{\psi}_\nu\gamma_\mu a\psi_{\bar{\mu}})(\bar{\psi}_\nu\gamma_\mu a\psi_e)$, which is excluded by experiment since it leads to a spectrum falling off at high energy (Michel's $\rho=0$).

Since the neutrino function always appears in the form $a\psi_\nu$ only neutrinos with left-hand spin can exist. That is, the two-component neutrino theory with conservation of leptons is valid. Our neutrinos spin oppositely to those of Lee and Yang.[6] For example, a β particle is a lepton and spins to the left; emitted with it is an antineutrino which is an antilepton and spins to the right. In a transition with $\Delta J=0$ they tend to go parallel to cancel angular momentum. This is the angular correlation typical of vector coupling.

We have conservation of leptons and double β decay is excluded.

There is a symmetry in that the incoming particles can be exchanged without affecting the coupling. Thus if we define the symbol

$$(\bar{A}B)(\bar{C}D)=(\bar{\psi}_A\gamma_\mu a\psi_B)(\bar{\psi}_C\gamma_\mu a\psi_D),$$

we have $(\bar{A}B)(\bar{C}D)=(\bar{C}B)(\bar{A}D)$. (We have used anticommuting ψ's; for C-number ψ's the interchange gives a minus sign.[7])

The capture of muons by nucleons results from a coupling $(\bar{n}p)(\bar{\nu}\mu)$. It is already known that this capture is fitted very well if the coupling constant and coupling are the same as in β decay.[8]

If we postulate that the universality extends also to the strange particles, we may have couplings such as $(\bar{\Lambda}^0p)(\bar{\nu}\mu)$, $(\bar{\Lambda}^0p)(\bar{\nu}e)$, and $(\bar{\Lambda}^0p)(\bar{p}n)$. The $(\bar{\Lambda}^0p)$ might be replaced by $(\overline{\Sigma^-}n)$, etc. At any rate the existence of such couplings would account qualitatively for the existence of all the weak decays. Consider, for example, the decay of the K^+. It can go virtually into an anti-Λ^0 and a proton by the fairly strong coupling of strange particle production. This by the weak decay $(\bar{\Lambda}^0p)(\bar{p}n)$ becomes a virtual antineutron and proton. These become, on annihilating, two or three pions. The parity is not conserved because of the a in front of the nucleons in the virtual transition. The theory in which only the neutrino carries the a cannot explain the parity failure for decays not involving neutrinos (the τ-θ puzzle). Here we turn the argument around; both the lack of parity conservation for the K and the fact that neutrinos are always fully polarized are consequences of the same universal weak coupling.

For β decay the expression (8) will be recognized as that for the two-component neutrino theory with couplings V and A with equal coefficients and opposite signs [expression (9) or (9′) makes the coupling $V+A$]. The coupling constant of the Fermi (V) part is equal to G. This constant has been determined[9] from the decay of O^{14} to be $(1.41\pm0.01)\times10^{-49}$ erg/cm^3. In units where $\hbar=c=1$, and M is the mass of the proton, this is

$$G=(1.01\pm0.01)\times10^{-5}/M^2. \tag{11}$$

At the present time several β-decay experiments seem to be in disagreement with one another. Limiting ourselves to those that are well established, we find that the most serious disagreement with our theory is the recoil experiment in He^6 of Rustad and Ruby[10] indicating that the T interaction is more likely than the A. Further check on this is obviously very desirable. Any experiment indicating that the electron is not 100% left polarized as $v\rightarrow c$ for any transition allowed or forbidden would mean that (8) and (9) are incorrect. An interesting experiment is the angular distribution of electrons from polarized neutrons for here there is an interference between the V and A contributions such that if the coupling is $V-A$ there is no asymmetry, while if it is $V+A$ there is a maximal asymmetry. This would permit us to choose between the alternatives (8) and (9). The present experimental results[11] agree with neither alternative.

We now look at the muon decay. The fact that the two neutrinos spin oppositely and the ρ parameter is $\frac{3}{4}$ permitted us to decide that the μ^- is a lepton if the electron is, and determines the order of $(\bar{\mu},\nu)$ which we write in (10). But now we can predict the direction of the electron in the $\pi^-\rightarrow\mu^-+\bar{\nu}\rightarrow e^-+\nu+\bar{\nu}$ sequence. Since the muon comes out with an antineutrino which spins to the right, the muon must also be spinning to the right (all senses of spin are taken looking down the direction of motion of the particle in question). When the muon disintegrates with a high-energy electron the two neutrinos are emitted in the opposite direction. They have spins opposed. The electron emitted must spin to the left, but must carry off the angular momentum of the muon, so it must proceed in the direction opposite to that of the muon. This direction agrees with experiment. The proposal of Lee and Yang predicted

[6] This is only because they used S and T couplings in β decay; had they used V and A, their theory would be similar to ours, with left-handed neutrinos.

[7] We can express $(\bar{A}B)(\bar{C}D)$ directly in terms of the two-component spinors φ: $(\bar{A}B)(\bar{C}D)=4(\varphi_A{}^*\varphi_B)(\varphi_C{}^*\varphi_D)-4(\varphi_A{}^*\boldsymbol{\sigma}\varphi_B)\cdot(\varphi_C{}^*\boldsymbol{\sigma}\varphi_D)$. If we put $\varphi_A=\begin{pmatrix}A_1\\A_2\end{pmatrix}$, etc., where A_1 and A_2 are complex numbers, we obtain $8(A_1{}^*C_2{}^*-A_2{}^*C_1{}^*)(B_1D_2-B_2D_1)$ and the symmetry is evident.

[8] See, for example, J. L. Lopes, Phys. Rev. (to be published); L. Michel, *Progress in Cosmic-Ray Physics*, edited by J. G. Wilson (Interscience Publishers, Inc., New York, 1952), Vol. 1, p. 125.

[9] Bromley, Almquist, Gove, Litherland, Paul, and Ferguson, Phys. Rev. **105**, 957 (1957).

[10] B. M. Rustad and S. L. Ruby, Phys. Rev. **97**, 991 (1955).

[11] Burgy, Epstein, Krohn, Novey, Raboy, Ringo, and Telegdi, Phys. Rev. **107**, 1731 (1957).

the electron spin here to be opposite to that in the case of β decay. Our β-decay coupling is V, A instead of S, T and this reverses the sign. That the electron have the same spin polarization in all decays (β, muon, or strange particles) is a consequence of putting $a\psi_e$ in the coupling for this particle. It would be interesting to test this for the muon decay.

Finally we can calculate the lifetime of the muon, which comes out

$$\tau = 192\pi^3/G^2\mu^5 = (2.26 \pm 0.04)\times 10^{-6} \text{ sec}$$

using the value (11) of G. This agrees with the experimental lifetime[12] $(2.22\pm 0.02)\times 10^{-6}$ sec.

It might be asked why this agreement should be so good. Because nucleons can emit virtual pions there might be expected to be a renormalization of the effective coupling constant. On the other hand, if there is some truth in the idea of an interaction with a universal constant strength it may be that *the other interactions are so arranged so as not to destroy this constant.* We have an example in electrodynamics. Here the coupling constant e to the electromagnetic field is the same for all particles coupled. Yet the virtual mesons do not disturb the value of this coupling constant. Of course the distribution of charge is altered, so the coupling for high-energy fields is apparently reduced (as evidenced by the scattering of fast electrons by protons), but the coupling in the low-energy limit, which we call the total charge, is not changed.

Using this analogy to electrodynamics, we can see immediately how the Fermi part, at least, can be made to have no renormalization. For the sake of this discussion imagine that the interaction is due to some intermediate (electrically charged) vector meson of very high mass M_0. If this meson is coupled to the "current" $(\bar{\psi}_p\gamma_\mu a\psi_n)$ and $(\bar{\psi}_\mu\gamma_\mu a\psi_\nu)$ by a coupling $(4\pi f^2)^{\frac{1}{2}}$, then the interaction of the two "currents" would result from the exchange of this "meson" if $4\pi f^2 M_0^{-2} = (8)^{\frac{1}{2}}G$. Now we must arrange that the total current

$$J_\mu = (\bar{\psi}_p\gamma_\mu a\psi_n) + (\bar{\psi}_\nu\gamma_\mu a\psi_e) + (\bar{\psi}_\nu\gamma_\mu a\psi_\mu) + \cdots \quad (12)$$

be not renormalized. There are no known large interaction terms to renormalize the $(\bar{\nu}e)$ or $(\bar{\nu}\mu)$, so let us concentrate on the nucleon term. This current can be split into two: $J_\mu = \frac{1}{2}(J_\mu{}^V + J_\mu{}^A)$, where $J_\mu{}^V = \bar{\psi}_p\gamma_\mu\psi_n$ and $J_\mu{}^A = \bar{\psi}_p i\gamma_\mu\gamma_5\psi_n$. The term $J_\mu{}^V = \bar{\psi}\gamma_\mu\tau_+\psi$, in isotopic spin notation, is just like the electric current. The electric current is

$$J_\mu{}^{\text{el}} = \bar{\psi}\gamma_\mu(\tfrac{1}{2}+\tau_z)\psi.$$

The term $\frac{1}{2}\bar{\psi}\gamma_\mu\psi$ is conserved, but the term $\bar{\psi}\gamma_\mu\tau_z\psi$ is not, unless we add the current of pions, $i[\varphi^* T_z\nabla_\mu\varphi - (\nabla_\mu\varphi^*)T_z\varphi]$, because the pions are charged. Likewise $\bar{\psi}\gamma_\mu\tau_+\psi$ is not conserved but the sum

$$J_\mu{}^V = \bar{\psi}\gamma_\mu\tau_+\psi + i[\varphi^* T_+\nabla_\mu\varphi - (\nabla_\mu\varphi)^* T_+\varphi] \quad (13)$$

is conserved, and, like electricity, leads to a quantity whose value (for low-energy transitions) is unchanged by the interaction of pions and nucleons. If we include interactions with hyperons and K particles, further terms must be added to obtain the conserved quantity.

We therefore suppose that this conserved quantity be substituted for the vector part of the first term in (12). Then the Fermi coupling constant will be strictly universal, except for small electromagnetic corrections. That is, the constant G from the μ decay, which is accurately $V-A$, should be also the exact coupling constant for at least the vector part of the β decay. (Since the energies involved are so low, the spread in space of $J_\mu{}^V$ due to the meson couplings is not important, only the total "charge.") It is just this part which is determined by the experiment with O^{14}, and that is why the agreement should be so close.

The existence of the extra term in (13) means that other weak processes must be predicted. In this case there is, for example, a coupling

$$(8)^{\frac{1}{2}}Gi(\varphi^*\nabla_\mu T_+\varphi - (\nabla_\mu\varphi)^* T_+\varphi)(\bar{\psi}_\nu\gamma_\mu a\psi_e),$$

by which a π^- can go to a π^0 with emission of $\bar{\nu}$ and e. The amplitude is

$$4G(p_\mu{}^- + p_\mu{}^0)(\bar{\psi}_\nu\gamma_\mu a\psi_e),$$

where p^-, p^0 are the four-momenta of π^- and π^0. Because of the low energies involved, the probability of the disintegration is too low to be observable. To be sure, the process $\pi^-\rightarrow\pi^0+e+\bar{\nu}$ could be understood to be qualitatively necessary just from the existence of β decay. For the π^- may become virtually an antiproton and neutron, the neutron decay virtually to a proton, e, and $\bar{\nu}$ by β decay and the protons annihilate forming the π^0. But the point is that by our principle of a universal coupling whose vector part requires no renormalization we can calculate the rate directly without being involved in closed loops, strong couplings, and divergent intervals.

For any transition in which strangeness doesn't change, the current $J_\mu{}^V$ is the total current density of isotopic spin T_+. Thus the vector part gives transitions $\Delta T=0$ with square matrix element $T(T+1)-T_zT_z'$ if we can neglect the energy release relative to the rest mass of the particle decaying. For the nucleon and $K^-\rightarrow K^0+e+\bar{\nu}$ the square of the matrix element is 1, for the pion and $\Sigma^-\rightarrow\Sigma^0+e+\bar{\nu}$ it is 2. The axial coupling in the low-energy limit is zero between states of zero angular momentum like the π meson or O^{14}, so for both of these we can compute the lifetime knowing only the vector part. Thus the $\pi^-\rightarrow\pi^0+e+\bar{\nu}$ decay should have the same ft value as O^{14}. Unfortunately because of the very small energies involved (because isotopic spin is such a good quantum number) none of these decays of mesons or hyperons are fast enough to observe in competition to other decay processes in which T or strangeness changes.

[12] W. E. Bell and E. P. Hincks, Phys. Rev. 84, 1243 (1951).

This principle, that the vector part is not renormalized, may be useful in deducing some relations among the decays of the strange particles.

Now with present knowledge it is not so easy to say whether or not a pseudovector current like $\bar{\psi}i\gamma_5\gamma_\mu\tau_+\psi$ can be arranged to be not renormalized. The present experiments[13] in β decay indicate that the ratio of the coupling constant squared for Gamow-Teller and Fermi is about 1.3 ± 0.1. This departure from 1 might be a renormalization effect.[14] On the other hand, an interesting theoretical possibility is that it is exactly unity and that the various interactions in nature are so arranged that it need not be renormalized (just as for V). It might be profitable to try to work out a way of doing this. Experimentally it is not excluded. One would have to say that the $ft_{\frac{1}{2}}$ value of 1220 ± 150 measured[15] for the neutron was really 1520, and that some uncertain matrix elements in the β decay of the mirror nuclei were incorrectly estimated.

The decay of the π^- into a μ^- and $\bar{\nu}$ might be understood as a result of a virtual process in which the π becomes a nucleon loop which decays into the $\mu+\bar{\nu}$. In any event one would expect a decay into $e+\bar{\nu}$ also. The ratio of the rates of the two processes can be calculated without knowledge of the character of the closed loops. It is $(m_e/m_\mu)^2(1-m_\mu^2/m_\pi^2)^{-2}=13.6\times10^{-5}$. Experimentally[16] no $\pi\rightarrow e+\nu$ have been found, indicating that the ratio is less than 10^{-5}. This is a very serious discrepancy. The authors have no idea on how it can be resolved.

We have adopted the point of view that the weak interactions all arise from the interaction of a current J_μ with itself, possibly via an intermediate charged vector meson of high mass. This has the consequence that any term in the current must interact with all the rest of the terms and with itself. To account for β decay and μ decay we have to introduce the terms in (12) into the current; the phenomenon of μ capture must then also occur. In addition, however, the pairs $e\nu$, $\mu\nu$, and pn must interact with themselves. In the case of the $(\bar{e}\nu)(\bar{\nu}e)$ coupling, experimental detection of electron-neutrino scattering might some day be possible if electron recoils are looked for in materials exposed to pile neutrinos; the cross section[17] with our universal coupling is of the order of 10^{-45} cm^2.

[13] A. Winther and O. Kofoed-Hansen, Kgl. Danske Vidensakb. Selskab, Mat.-fys. Medd. (to be published).

[14] This slight inequality of Fermi and Gamow-Teller coupling constants is not enough to account for the experimental results of reference 11 on the electron asymmetry in polarized neutron decay.

[15] Spivac, Sosnovsky, Prokofiev, and Sokolov, *Proceedings of the International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1955* (United Nations, New York, 1956), A/Conf. 8/p/650.

[16] C. Lattes and H. L. Anderson, Nuovo cimento (to be published).

[17] For neutrinos of energy ω (in units of the electron mass m) the total cross section is $\sigma_0\omega^2/(1+2\omega)$, and the spectrum of recoil energies ϵ of the electron is uniform $d\epsilon$. For antineutrinos it is $\sigma_0(\omega/6)[1-(1+2\omega)^{-3}]$ with a recoil spectrum varying as $(1+\omega-\epsilon)^2$. Here $\sigma_0=2G^2m^2/\pi=8.3\times10^{-45}$ cm^2.

To account for all observed strange particle decays it is sufficient to add to the current a term like $(\bar{p}\Lambda^0)$, $(\bar{p}\Sigma^0)$, or $(\overline{\Sigma^-}n)$, in which strangeness is increased by one as charge is increased by one. For instance, $(\bar{p}\Lambda^0)$ gives us the couplings $(\bar{p}\Lambda^0)(\bar{e}\nu)$, $(\bar{p}\Lambda^0)(\bar{\mu}\nu)$, and $(\bar{p}\Lambda^0)(\bar{n}p)$. A direct consequence of the coupling $(\bar{p}\Lambda^0)(\bar{e}\nu)$ would be the reaction

$$\Lambda^0\rightarrow p+e+\bar{\nu} \tag{14}$$

at a rate 5.3×10^7 sec^{-1}, assuming no renormalization of the constants.[18] Since the observed lifetime of the Λ^0 (for disintegration into other products, like $p+\pi^-$, $n+\pi^0$) is about 3×10^{-10} sec, we should observe process (14) in about 1.6% of the disintegrations. This is not excluded by experiments. If a term like $(\overline{\Sigma^-}n)$ appears, the decay $\Sigma^-\rightarrow n+e^-+\nu$ is possible at a predicted rate 3.5×10^8 sec^{-1} and should occur (for $\tau_{\Sigma^-}=1.6\times10^{-10}$ sec) in about 5.6% of the disintegrations of the Σ^-. Decays with μ replacing the electron are still less frequent. That such disintegrations actually occur at the above rates is not excluded by present experiments. It would be very interesting to look for them and to measure their rates.

These rates were calculated from the formula Rate$=(2G^2W^5c/30\pi^3)$ derived with neglect of the electron mass. Here $W=(M_\Lambda^2-M_p^2)/2M_\Lambda$ is the maximum electron energy possible and c is a correction factor for recoil. If $x=W/M_\Lambda$ it is

$$c=-\tfrac{15}{16}x^{-5}(1-2x)^2\ln(1-2x)-\tfrac{5}{8}x^{-4}(1-x)(3-6x-2x^2),$$

and equals 1 for small x, about 1.25 for the Σ decay, and 2.5 for $M_p=0$.

It should be noted that decays like $\Sigma^+\rightarrow n+e^++\nu$ are forbidden if we add to the current only terms for which $\Delta S=+1$ when $\Delta Q=+1$. In order to cause such a decay, the current would have to contain a term with $\Delta S=-1$ when $\Delta Q=+1$, for example $(\overline{\Sigma^+}n)$. Such a term would then be coupled not only to $(\bar{\nu}e)$, but also to all the others, including one like $(\bar{p}\Lambda^0)$. But a coupling of the form $(\overline{\Sigma^+}n)(\bar{\Lambda}^0p)$ leads to strange particle decays with $\Delta S=\pm2$, violating the proposed rule $\Delta S=\pm1$. It is important to know whether this rule really holds; there is evidence for it in the apparent absence of the decay $\Xi^-\rightarrow\pi^-+n$, but so few Ξ particles have been seen that this is not really conclusive. We are not sure, therefore, whether terms like $(\overline{\Sigma^+}n)$ are excluded from the current.

We deliberately ignore the possibility of a neutral current, containing terms like $(\bar{e}e)$, $(\bar{\mu}e)$, $(\bar{n}n)$, etc., and possibly coupled to a neutral intermediate field. No weak coupling is known that requires the existence of such an interaction. Moreover, some of these couplings, like $(\bar{e}e)(\bar{\mu}e)$, leading to the decay of a muon into three electrons, are excluded by experiment.

It is amusing that this interaction satisfies simultaneously almost all the principles that have been

[18] R. E. Behrends and C. Fronsdal, Phys. Rev. **106**, 345 (1957).

proposed on simple theoretical grounds to limit the possible β couplings. It is universal, it is symmetric, it produces two-component neutrinos, it conserves leptons, it preserves invariance under CP and T, and it is the simplest possibility from a certain point of view (that of two-component wave functions emphasized in this paper).

These theoretical arguments seem to the authors to be strong enough to suggest that the disagreement with the He^6 recoil experiment and with some other less accurate experiments indicates that these experiments are wrong. The $\pi \rightarrow e+\bar{\nu}$ problem may have a more subtle solution.

After all, the theory also has a number of successes. It yields the rate of μ decay to 2% and the asymmetry in direction in the $\pi \rightarrow \mu \rightarrow e$ chain. For β decay, it agrees with the recoil experiments[19] in A^{35} indicating a vector coupling, the absence of Fierz terms distorting the allowed spectra, and the more recent electron spin polarization[4] measurements in β decay.

[19] Herrmansfeldt, Maxson, Stähelin, and Allen, Phys. Rev. **107**, 641 (1957).

Besides the various experiments which this theory suggests be done or rechecked, there are a number of directions indicated for theoretical study. First it is suggested that all the various theories, such as meson theory, be recast in the form with the two-component wave functions to see if new possibilities of coupling, etc., are suggested. Second, it may be fruitful to analyze further the idea that the vector part of the weak coupling is not renormalized; to see if a set of couplings could be arranged so that the axial part is also not renormalized; and to study the meaning of the transformation groups which are involved. Finally, attempts to understand the strange particle decays should be made assuming that they are related to this universal interaction of definite form.

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Dispersion Relations for Dirac Potential Scattering

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Dispersion relations for scattering of a Dirac particle by a potential are shown to hold for a broad class of potentials. In contrast to the field theoretic case, the derivation here makes no use of the concept of causality but is instead based directly on the analytic properties of the Fredholm solution of the scattering integral equation. It is shown that the scattering amplitude, considered as a function of energy and momentum transfer, can be extended to a function analytic in the complex energy plane, for real momentum transfer. The dispersion relations then follow in the standard way from Cauchy's theorem. The final results involve one "subtraction." It is also shown that the analytic continuation into the unphysical region for nonforward scattering can be carried out by means of a partial wave expansion.

I. INTRODUCTION

It has recently been shown[1] that, under certain broad conditions, dispersion relations of the type so much discussed for relativistic field theories[2] also hold in ordinary nonrelativistic quantum mechanics for scattering of a particle by a potential. The treatment of this problem is quite straightforward and explicit; in contrast to the field theoretic case, one can show explicitly that the dispersion relations involve no "subtractions" and that the scattering amplitude can be analytically continued into the unphysical region for nonforward scattering by means of a partial wave expansion. In this sense, nonrelativistic quantum mechanics provides a complete and simple model of a system for which dispersion relations are valid. It has already been used as a basis for investigating to what extent the dispersion relations, taken together with the unitarity of the S-matrix, constitute a self-contained formulation of scattering theory.[3]

In the present paper, the discussion of dispersion relations in ordinary quantum mechanics is extended to the case of scattering of a Dirac particle by a potential. Using arguments similar to those employed for the Schrödinger case,[1] one again finds that dispersion relations hold for a broad class of potentials. The restrictions on the potentials are now somewhat more severe; and in the present case one finds that the dispersion

* Lockheed Fellow, 1956–1957.
[1] N. N. Khuri, Phys. Rev. **107**, 1148 (1957).
[2] Chew, Goldberger, Low, and Nambu, Phys. Rev. **106**, 1337 (1957). For a complete list of references see R. H. Capps and G. Takeda, Phys. Rev. **103**, 1877 (1956).
[3] S. Gasiorowicz and M. Ruderman (to be published).