Decay of the Neutral π Meson*

JEREMY BERNSTEIN† AND KENNETH A. JOHNSON‡
Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico
(Received September 9, 1957)

The possibilities of detecting the parity of the π^0 meson by electromagnetic experiments involving the decay γ rays are discussed. It is shown that no experiment can be performed which distinguishes scalar and pseudoscalar decays unless azimuthal angular correlations are measured, assuming that the π^0 decays at rest. As an example, the total coincidence Compton scattering cross section from polarized electron targets is calculated explicitly and shown to be identical for the scalar, pseudoscalar, and parity-nonconserving decay.

I. INTRODUCTION

ALTHOUGH it is generally assumed that the parity of the π^0 is pseudoscalar there has not been, as yet, any direct experimental confirmation of this supposition. As is well known, if the π^0 is a pseudoscalar, a wave function describing the two-photon state into which it decays will be of the general form $\mathbf{k}\cdot\boldsymbol{\varepsilon}_1\times\boldsymbol{\varepsilon}_2$, where $\mathbf{k}$ is the relative photon wave number and $\boldsymbol{\varepsilon}_1$ and $\boldsymbol{\varepsilon}_2$ are the photon polarization vectors, assuming both that the π^0 decays at rest and that parity is conserved in the decay. In Sec. II we shall exhibit the wave function for the two-photon state produced by a π^0 decaying in flight. If the π^0 were a scalar, then the wave function would be of the form $\boldsymbol{\varepsilon}_1\cdot\boldsymbol{\varepsilon}_2$. Conventionally one says that for a pseudoscalar π^0 the decay photons have orthogonal polarization vectors and for a scalar π^0 they are parallel; always assuming that the π^0 decays at rest and that the decay mechanism is parity-conserving. Yang[1] suggested an experiment to measure the parity of the π^0 based on this observation. One would attempt to measure the angular correlation in the distribution of electron-positron pairs produced in coincidence by the π^0 decay γ's. Another experiment along similar lines was suggested by Wightman[2] who proposed a measurement of the azimuthal angular distributions for photons which undergo Compton scattering from a suitable electron target. He was actually interested in an experiment to determine the parity of the ${}^1S$ ground state of positronium, but the same calculation serves for the two-photon decay of the π^0. These experiments have in common that one must measure azimuthal angular distributions. Hence they are very difficult in practice and to our knowledge have never been tried for the π^0 decay.

One might hope to construct an experiment which would involve the determination of total cross sections rather than angular distributions. For example, Marshall[3] recently made the suggestion that a coincidence Compton scattering experiment, in which the electron targets were polarized and in which one would measure just the total number of scattered electrons, might produce spin correlations in the cross section which would serve to distinguish between a scalar and a pseudoscalar two-photon state. In this note we shall show that no such experiment is possible, providing one neglects effects produced by the pion motion and integrates over azimuthal angles. The azimuthal integration, in effect, washes out any correlations which depend on the parity of the two-photon state. The proof which is given in the next section covers any electromagnetic process involving the two photons including the type of experiment in which one would observe, say the pairs produced by one photon and the total Compton scattering of the other. As an example we have made an explicit calculation of the experiment suggested by Marshall, coincidence Compton scattering from polarized electron targets. In this case one finds for the number of coincidences per π^0 decaying at rest with respect to the electron targets

$$\sigma(\theta_1,\theta_2)=4\pi^2\left(\frac{\alpha^2}{2m^2}\right)^2\left(\frac{k_f^1}{k_0}\right)^2\left(\frac{k_f^2}{k_0}\right)^2 \times[f_0(\theta_1)f_0(\theta_2)+\mathbf{n}_1\cdot\boldsymbol{\sigma}_1\mathbf{n}_2\cdot\boldsymbol{\sigma}_2 f_s(\theta_1)f_s(\theta_2)]N(\Omega_1)N(\Omega_2),$$

where

$$f_0(\theta)=\left(\frac{k_f}{k_0}\right)^2\left(\frac{k_0}{k_f}+\frac{k_f}{k_0}-\sin^2\theta\right),$$

and

$$f_s(\theta)=\left(\frac{k_f}{k_0}\right)^2\cos\theta\left(\frac{k_f}{k_0}-\frac{k_0}{k_f}\right).$$

Here k_f is the final photon energy, k_0 the initial photon energy ($\sim$70 Mev), $\mathbf{n}_1$ and $\mathbf{n}_2$ are the photon directions, and $N(\Omega)$ is a geometrical factor given by the experimental setup. One can see that the spin correlation effects are as large as the Compton cross section itself, but, unfortunately, they are identical in the scalar, pseudoscalar, and parity nonconserving cases.

As Wightman noted,[2] such long-range correlations arise because of purely quantum mechanical coherence effects. Namely, one must treat the annihilation photons

* Work performed under the auspices of the U. S. Atomic Energy Commission.
† Present address: Institute for Advanced Study, Princeton, New Jersey.
‡ Present address: Institute for Theoretical Physics, Copenhagen, Denmark.
1 C. N. Yang, Phys. Rev. **77**, 722 (1950).
2 A. S. Wightman, Phys. Rev. **74**, 1813 (1948).
3 L. Marshall (private communication).

as particles in intermediate states whose possible polarizations are summed over, in the usual quantum mechanical sense, before one squares the matrix element to find the transition probability. In the experiments under discussion no observation of the photon polarization is made between the decay and the Compton scattering so that no measurement intervenes to spoil the coherence of the possible intermediate photon polarization states. The formalism in the next section will reflect and make precise these remarks.

II. CALCULATION

We wish to compute the transition amplitude from an initial state consisting of two targets and two γ's in the state prepared by the decay of the π^0, to some state produced by the interaction of the decay γ's with the targets. For example, the final state might be two Compton electrons and two scattered γ's. The state of the γ's produced by the π^0 decay may be described by a wave function in the representation afforded by the polarization vectors and photon wave numbers of the two photons. On the basis of invariance principles, the most general two-photon state of angular momentum zero which is Lorentz- and gauge-invariant is of the form

$$(\epsilon_1\epsilon_2|\pi^0)=\alpha\epsilon_1{}^\mu\left(\delta_{\mu\nu}-\frac{k_2{}^\mu k_1{}^\nu}{k_1k_2}\right)\epsilon_2{}^\nu-\beta\frac{k_1{}^\mu k_2{}^\nu}{k_1k_2}\epsilon_{\mu\nu\alpha\beta}\epsilon_1{}^\alpha\epsilon_2{}^\beta. \quad (1)$$

Here k_μ and ϵ_μ are the four-dimensional photon wave and polarization vectors, respectively. In writing Eq. (1) we are allowing for the possibility that the decay interaction might not be parity-conserving and hence the scalar and pseudoscalar wave functions are added with amplitudes α and β. We shall now show that in this case α and β must be taken as real numbers. To see this we recall that the decay interaction, H', will effectively take the form (Note that H' is not invariant under CP as well as P.)

$$H'=\alpha\int(d\mathbf{x})\varphi_{\pi^0}(\mathbf{x})(E^2-H^2)+\beta\int(d\mathbf{x})\varphi_{\pi^0}(\mathbf{x})(\mathbf{E}\cdot\mathbf{H}). \quad (2)$$

Since we deal with a spin-zero, chargeless particle, φ_{π^0}, which is the π^0 annihilation operator, will be real. Hence, from the Hermitian character of H' it follows that α and β are real, as stated above. We shall normalize Eq. (1) in such a way that $\alpha^2+\beta^2=1$.

Let us denote by $t_\mu\epsilon^\mu$ the transition amplitude leading from a photon of a given initial polarization ϵ^μ and some target state to any specified final state. In terms of this notation any coincidence photon process involving the π^0 decay γ's will be described by a transition amplitude of the form

$$\sum_{\epsilon_1\epsilon_2}\epsilon_1{}^\mu t_{1\mu}{}^a\epsilon_2{}^\mu t_{2\mu}{}^b(\epsilon_1\epsilon_2|\pi^0). \quad (3)$$

$(\epsilon_1\epsilon_2|\pi^0)$ is given by Eq. (1) and the superscripts a and b on the t_μ indicate the possibility of measuring distinct processes for each of the two photons; for example, one might wish to measure Compton scattering in one target and pair production in the other. To find the transition probability, we multiply Eq. (3) by its complex conjugate and learn that

$$w=(D^{1a})_{\mu\alpha}S_{\alpha\beta}(D^{2b})_{\nu\beta}S_{\mu\nu}{}^*=\mathrm{Tr}[D^{1a}S(D^{2b})^TS^{T*}], \quad (4)$$

where Tr denotes the trace over the indicated dyadic indices and A^T is the transpose of A while A^{T*} is its Hermitian conjugate. In writing Eq. (4) we have introduced the notation

$$S_{\alpha\beta}=\sum\epsilon_1{}^\alpha\epsilon_2{}^\beta(\epsilon_1\epsilon_2|\pi^0). \quad (5)$$

Physically, S represents the density matrix of the initial two-photon state and in virtue of the reality of α and β it is real. By way of additional notation,

$$\begin{aligned}(D^a)_{\mu\alpha}&=\sum t_\mu{}^{a*}t_\alpha{}^a,\\ (D^b)_{\nu\beta}&=\sum t_\nu{}^{b*}t_\beta{}^b\end{aligned} \quad (6)$$

are the dyadics which describe the processes by which the photons are transformed.

$\sum$ means a sum over those final quantum numbers which are not measured in the final state. If we do not sum over an invariant subset of final states then D will no longer be a true tensor. As a first case let us imagine that we sum over all final states so that the D's can be taken as Lorentz tensors. Hence it is important to ask what the most general form of D is, consistent with the invariance principles which are at hand.

To get an insight into this question, let us consider the problem in the frame in which the π^0 decays at rest, and let us use the radiation gauge appropriate to this frame. Now the dyadic D must be transverse to the incident photon directions since it is multiplied on both sides by the initial photon polarization vectors. We shall denote the direction of the relative photon wave number by the unit vector $\mathbf{n}$. Further let ϵ' and ϵ'' specify the two independent directions transverse to $\mathbf{n}$. Then D can always be expressed as a linear combination of the dyadics $\epsilon_a{}'\epsilon_b{}'$, $\epsilon_a{}'\epsilon_b{}''$, $\epsilon_a{}''\epsilon_b{}'$, and $\epsilon_a{}''\epsilon_b{}''$. Since we have assumed that all final states are summed over, including final directions, D must be invariant against rotations about $\mathbf{n}$ or, in other words, under the transformation $\epsilon_a{}'\rightarrow\epsilon_a{}''\epsilon_a{}''\rightarrow-\epsilon_a{}'$. From this it follows that D must be composed of just the dyadics $\epsilon_a{}'\epsilon_b{}'+\epsilon_a{}''\epsilon_b{}''=\delta_{ab}-n_an_b=P_{ab}$ and $\epsilon_a{}'\epsilon_b{}''-\epsilon_a{}''\epsilon_b{}'=\epsilon_{abc}n_c$, where ϵ_{abc} is the Levi-Civita symbol. Which is to say

$$D_{ab}=(\delta_{ab}-n_an_b)A+\epsilon_{abc}n_cB, \quad (7)$$

where A and B are, respectively, a scalar and a pseudoscalar function.

We shall now write down the generalization of Eq. (7) to a four-dimensional form and an arbitrary Lorentz-radiation gauge which will be specified by a time-like vector η_α.

$$D_{\alpha\beta}=FP_{\alpha\beta}+G\frac{\eta_\lambda k_\sigma}{(k\eta)}\epsilon_{\lambda\sigma\alpha\beta}. \tag{8}$$

Here $P_{\alpha\beta}=\sum \epsilon^\alpha\epsilon^\beta$, where the ϵ's are photon polarization four-vectors which satisfy the Lorentz condition $\epsilon^\mu k_\mu=0$ where k_μ is the four-vector photon momentum. The sum is over independent polarizations. F and G are invariant functions and $\epsilon_{\lambda\sigma\alpha\beta}$ is the four-dimensional Levi-Civita symbol. Using Eq. (1) and the definition of $P_{\alpha\beta}$, the tensor $S_{\alpha\beta}$ is given by

$$S_{\alpha\beta}=\sum \epsilon_1{}^\alpha\epsilon_2{}^\beta(\epsilon_1\epsilon_2|\pi^0)=\alpha\left[P_1{}^{\alpha\mu}\left(\delta^{\mu\nu}-\frac{k_2{}^\mu k_1{}^\nu}{k_1k_2}\right)P_2{}^{\nu\beta}\right]-\beta\frac{k_1{}^\mu k_2{}^\nu}{k_1k_2}\epsilon_{\mu\nu\lambda\sigma}P_1{}^{\lambda\alpha}P_2{}^{\sigma\beta}. \tag{9}$$

With these definitions we find [see Eq. (4)]

$$\mathrm{Tr}[D^{1a}S(D^{2b})^TS^{T*}]=2(F_1{}^aF_2{}^b+G_1{}^aG_2{}^b), \tag{10}$$

where, as above, F and G are invariant functions which are composed out of the initial photon vectors and quantities characteristic of the targets. The most important thing to notice about Eq. (10) is that the result is completely independent of α and β and hence, in an experiment in which all final states are summed over, no detection of the parity of the state of two photons prepared by the π^0 decay is possible.

We may now consider a somewhat less special situation in which we integrate over azimuthal angles but not zenith angles. For example, in the coincidence Compton scattering experiment we might measure the distribution in the angles between the incident photons and outgoing electrons, summing over final electron spins and integrating over azimuthal angles. This specification of final states is not Lorentz-invariant and for the sake of simplicity we shall do the calculations assuming that the π^0 decays at rest with respect to the electron targets.

If we specialize Eq. (9) to this frame and work in the radiation gauge with $\boldsymbol{\varepsilon}\cdot\mathbf{n}=0$, where $\mathbf{n}$ is the unit relative photon wave vector and $\boldsymbol{\varepsilon}$ is either of the two photon polarization vectors, then we find

$$S_{kl}=\alpha(\delta_{kl}-n_kn_l)+\beta\epsilon_{klm}n_m. \tag{11}$$

If we use Eq. (4) we learn that the transition amplitude can be written

$$\begin{aligned}w=\ &\alpha^2\,\mathrm{Tr}[(D^{1a})^S(1-nn)(D^{2b})^S(1-nn)]\\&+\beta^2\,\mathrm{Tr}[(D^{1a})^S(n\times 1)(D^{2b})^S(n\times 1)]\\&+\alpha^2\,\mathrm{Tr}[(D^{1a})^A(1-nn)(D^{2b})^A(1-nn)]\\&+\beta^2\,\mathrm{Tr}[(D^{1a})^A(n\times 1)(D^{2b})^A(n\times 1)].\end{aligned} \tag{12}$$

In writing Eq. (12) we have introduced some notation. $(1-nn)$ stands for the dyadic $\delta_{ab}-n_an_b$. $n\times 1$ stands for $\epsilon_{abc}n_c$. The dyadics D have been split into their symmetric and antisymmetric parts $D=D^S+D^A$ and the trace, Tr, is always taken over the dyadic indices. It is easy to see that all terms which involve D^S and D^A together, and which might have appeared in Eq. (12), vanish because of the symmetry properties of the trace.

At this point it becomes clear that the coefficients of α^2 and β^2 in Eq. (12) are identical. This follows from the fact that, as above, the dyadics D are transverse to n. Now the operator $(1-nn)$ applied to D picks out its transverse part, and since D is already transverse this has no effect. On the other hand, the operator $n\times 1$ applied to D on both sides will rotate it through 90° about the transverse direction $\mathbf{n}$. Since all of the azimuthal angular integrations have been done in D, this rotation will again have no effect on the dyadic. Thus Eq. (12) can be written, using the fact that $\alpha^2+\beta^2=1$,

$$w=2\{\mathrm{Tr}[(D^{1a})^S(D^{2b})^S]+\mathrm{Tr}[(D^{1a})^A(D^{2b})^A]\}. \tag{13}$$

In this expression the real numbers β and α which weight the components of the incident photon state with different parities do not appear. Hence, in an experiment in which the azimuthal angles have been integrated over, it is not possible to detect the parity of the state of the pion-decay γ's.

In this argument we have used the fact that the π^0 decays at rest with respect to the targets. In reality, if the π^0 is produced in a charge-exchange reaction, it will have at least the velocity characteristic of the π^-, π^0 mass difference $(v/c)^2\sim 1/15$. Hence in an experiment like the one above, in which one integrates over azimuthal but not zenith angles, we might expect parity-dependent effects of this relative order which are then difficult to observe. However, in virtue of the first proof there will be no such effects in the total cross section in which zenith angles are also integrated over.

After these very general statements it may be interesting to consider explicitly the theory of the coincidence Compton scattering of the decay γ's from polarized electron targets.

We may rewrite the well-known matrix element for Compton scattering of a photon from a polarized electron as follows:

$$\bar{u}_{sf}(p_f)\left[\gamma\cdot\boldsymbol{\varepsilon}_f\frac{1}{\gamma(p+k_0)+m}\gamma\cdot\boldsymbol{\varepsilon}+\gamma\cdot\boldsymbol{\varepsilon}\frac{1}{\gamma(p_f-k_f)+m}\gamma\cdot\boldsymbol{\varepsilon}_f\right]u_{s0}(p_0)=(\boldsymbol{\varepsilon}_fA+\boldsymbol{\varepsilon}_f\times\mathbf{B}), \tag{14}$$

where

$$A=u_f{}^\dagger(1-i\gamma_5\boldsymbol{\sigma}\cdot\mathbf{n}^+)u_i, \tag{15}$$

and

$$B=u_f{}^\dagger[\gamma_5(\mathbf{n}^-+i\mathbf{n}^-\times\boldsymbol{\sigma})]u_i. \tag{16}$$

Here the u's are appropriately normalized Dirac spinors; $\boldsymbol{\sigma}$ is the vector spin matrix which is related to the γ's by the equation $i\gamma_0\gamma_5\boldsymbol{\sigma}=\boldsymbol{\gamma}$ where we have chosen γ's with property that $\boldsymbol{\gamma}^{T*}=-\boldsymbol{\gamma}$, $\gamma_0^{T*}=\gamma_0$ and $\gamma_5^2=-1$; $\mathbf{n}^+=(\mathbf{n}+\mathbf{n}')/2$ and $\mathbf{n}^-=(\mathbf{n}-\mathbf{n}')/2$, where $\mathbf{n}$ and $\mathbf{n}'$ are the initial and final photon directions. Equation (14) is a correct expression for the Compton scattering, with the definitions Eq. (15) and Eq. (16), only in the frame of reference in which the electron is initially at rest so that the kinematics are simply $\mathbf{p}_f+\mathbf{k}_f=\mathbf{k}_0$ and $E(p_f)+k_f=m+k_0$. In the present example the dyadic D of Eq. (6) becomes

$$D_{ab}=\sum_{\epsilon_f}\int_0^{2\pi}d\varphi\sum_{s_f}(\boldsymbol{\varepsilon}_fA+\boldsymbol{\varepsilon}_f\times\mathbf{B})_a(\boldsymbol{\varepsilon}_fA+\boldsymbol{\varepsilon}_f\times\mathbf{B})_b, \quad (17)$$

where $\sum_{\epsilon_f}$ stands for the sum over final polarizations, $\int_0^{2\pi}d\varphi$ is the integral over outgoing azimuthal angles and $\sum_{s_f}$ is the sum over final spins. After the indicated operations are performed, D takes the form indicated in Eq. (7):

$$D_{ab}=f_0(\theta)(1-nn)_{ab}+i(\mathbf{n}\cdot\boldsymbol{\sigma})(n\times 1)_{ab}f_s(\theta), \quad (18)$$

where $\boldsymbol{\sigma}$ stands for the matrix element of the spin in the initial state and θ is the angle between the incident and outgoing photon directions. The tedious part of the calculation is the determination of $f_0(\theta)$ and $f_s(\theta)$.

Since this calculation must reduce to the usual Compton cross section when initial spins are summed over, $f_0(\theta)$ is essentially the Klein-Nishina function

$$f_0(\theta)=\pi\left(\frac{k_f}{k_0}+\frac{k_0}{k_f}-\sin^2\theta\right), \quad (19)$$

while for $f_s(\theta)$ we find

$$f_s(\theta)=\pi\left[\cos\theta\left(\frac{k_f}{k_0}-\frac{k_0}{k_f}\right)\right]. \quad (20)$$

From this calculation of the dyadic D and from the form of S_{kl} given by Eq. (11), it is entirely straightforward to calculate the number of coincident Compton scatterings per π^0 decaying at rest with respect to the electron targets. We find

$$\sigma(\theta_1,\theta_2)=4\pi^2\left(\frac{\alpha^2}{2m^2}\right)^2\left(\frac{k_f^1}{k_0}\right)^2\left(\frac{k_f^2}{k_0}\right)^2 \times[f_0(\theta_1)f_0(\theta_2)+\mathbf{n}_1\cdot\boldsymbol{\sigma}_1\mathbf{n}_2 \cdot\boldsymbol{\sigma}_2f_s(\theta_1)f_s(\theta_2)]N(\Omega_1)N(\Omega_2). \quad (21)$$

The notation has been explained; however, we remind reader that $N(\Omega)$ is a geometrical factor which depends upon the experiment.

As we have seen, this result is completely independent of the parity nature of the two-photon state produced by the decaying π^0 and hence one cannot use the zenith angular distributions of the Compton electrons to measure the π^0 parity. In any event it may be of interest that purely quantum mechanical coherence requirements can produce observable correlations in the spins of electron targets which are widely separated in space.

III. ACKNOWLEDGMENTS

We thank Professor L. Marshall for having raised the original question which initiated these calculations. Valuable discussions were had with M. L. Goldberger, H. W. Lewis, and K. A. Watson. We are grateful to F. E. Low for continual advice, encouragement, and criticism.